Current and future constraints on f(Q) cosmology with ΛCDM background

Spanish and Portuguese Relativity Meeting 2023, Bilbao

Nelson J. Nunes, Tiago Barreiro, Bruno Barros, José Ferreira, Tomi Koivisto and José Mimoso

arxiv: 2004.07867, 2203.13788, 2306.10176,

This work was supported by FCT through the grants UIDP/04434/2020 & UIDB/04434/2020, PTDC/FIS-AST/0054/2021, EXPL/FIS-AST/1368/2021

instituto de astrofísica e ciências do espaço

Ciências ULisboa

Fundação para a Ciência e a Tecnologia

COMPETE 2020

União Europeia Fundo Europeu de Desenvolvimento Regional

PORTUGAL 2020

Introduction

Gravity as Geometry

- Spacetime is described with two (in principle independent) objects: the metric $g_{\mu\nu}$, and the, affine connection $\Gamma^{\lambda}_{\mu\nu}$

- Metric defines distances, lengths and angles

$$ds^2 = g_{\mu\nu} dx^\mu dx^\nu \,, \qquad \cos(V,U) = \frac{g_{\mu\nu} V^\mu U^\nu}{\sqrt{g_{\mu\nu} V^\mu V^\nu g_{\rho\sigma} U^\rho U^\sigma}}$$

- The affine connection relates two nearby tangent spaces

$$V^\mu(x) - \tilde{V}^\mu(x + \delta x) = \Gamma^\mu{}_{\alpha\beta} V^\alpha(x) \delta x^\beta$$

defines the covariant derivative

$$\nabla_\mu V^\nu \equiv \partial_\mu V^\nu + \Gamma^\nu{}_{\alpha\mu} V^\alpha$$

and the Riemann tensor

$$R^\sigma{}_{\rho\mu\nu} \equiv \partial_\mu \Gamma^\sigma{}_{\nu\rho} - \partial_\nu \Gamma^\sigma{}_{\mu\rho} + \Gamma^\alpha{}_{\nu\rho} \Gamma^\sigma{}_{\mu\alpha} - \Gamma^\alpha{}_{\mu\rho} \Gamma^\sigma{}_{\nu\alpha}$$

Decomposition of the affine connection

$$\Gamma^\lambda{}_{\mu\nu} = \{^\lambda{}_{\mu\nu}\} + K^\lambda{}_{\mu\nu} + L^\lambda{}_{\mu\nu}$$

- Levi Civita connection

$$\{^\lambda{}_{\mu\nu}\} \equiv \frac{1}{2} g^{\lambda\beta} \left(\partial_\mu g_{\beta\nu} + \partial_\nu g_{\beta\mu} - \partial_\beta g_{\mu\nu}\right)$$

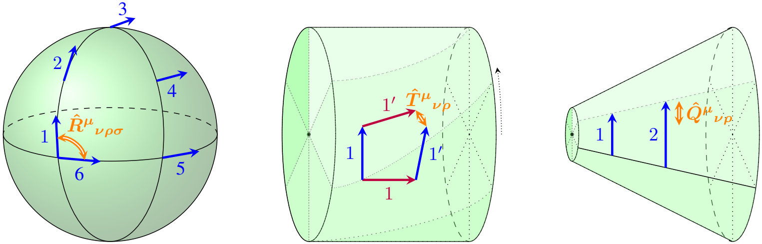
- Contortion and tortion

$$K^\lambda{}_{\mu\nu} \equiv \frac{1}{2} g^{\lambda\beta} \left(T_{\mu\beta\nu} + T_{\nu\beta\mu} + T_{\beta\mu\nu}\right) , \qquad T^\lambda{}_{\mu\nu} \equiv \Gamma^\lambda{}_{\mu\nu} - \Gamma^\lambda{}_{\nu\mu}$$

- Disformation and non-metricity

$$L^\lambda{}_{\mu\nu} \equiv \frac{1}{2} g^{\lambda\beta} \left(-Q_{\mu\beta\nu} - Q_{\nu\beta\mu} + Q_{\beta\mu\nu}\right) , \qquad Q_{\alpha\mu\nu} \equiv \nabla_\alpha g_{\mu\nu}$$

Interpretation of curvature, torsion and non-metricity



By Laur Järv, from Bahamonde 2021.

Construction of GR equivalent theories

$$S_{GR} = \int dx^4 \sqrt{-g}\, R$$

$$S_{TEGR} = \int dx^4 \sqrt{-g}\, T \qquad \longleftrightarrow \qquad S_{STEGR} = \int dx^4 \sqrt{-g}\, Q$$

Trinity of Gravity as in Beltrán Jiménez, Heisenberg, Koivisto 2019

Extending GR equivalent theories

$$
\begin{tikzcd}
& S_{F(R)} = \int dx^4 \sqrt{-g}\, F(R) & \\
& S_{GR} = \int dx^4 \sqrt{-g}\, R \arrow[u] \arrow[dl, leftrightarrow] \arrow[dr, leftrightarrow] & \\
S_{TEGR} = \int dx^4 \sqrt{-g}\, T \arrow[rr, leftrightarrow] \arrow[d] & & S_{STEGR} = \int dx^4 \sqrt{-g}\, Q \arrow[d] \\
S_{TG} = \int dx^4 \sqrt{-g}\, F(T) & & S_{STG} = \int dx^4 \sqrt{-g}\, F(Q)
\end{tikzcd}
$$

Symmetric Teleparallel Gravity (STG)

$$S = \int \sqrt{-g}\left[-\frac{1}{16\pi G}F(Q) + \mathcal{L}_m\right] d^4x$$

$$Q \equiv -\frac{1}{4}Q_{\alpha\beta\gamma}Q^{\alpha\beta\gamma} + \frac{1}{2}Q_{\alpha\beta\gamma}Q^{\gamma\beta\alpha} + \frac{1}{4}Q_\alpha Q^\alpha - \frac{1}{2}Q_\alpha \tilde{Q}^\alpha\,,$$

$$Q_\mu \equiv Q_\mu{}^\alpha{}_\alpha\,, \qquad \tilde{Q}^\mu \equiv Q_\alpha{}^{\alpha\mu}\,.$$

For a flat FLRW, the non-metricity scalar is

$$Q = 6H^2$$

and the Friedmann equations are

$$6F_Q H^2 - \frac{1}{2}F = 8\pi G\rho$$

$$(12H^2 F_{QQ} + F_Q)\dot{H} = -4\pi G\,(\rho + p)$$

Evolution of the matter contrast

The evolution of the matter contrast in the small scales

$$\ddot{\delta} + 2H\dot{\delta} - \frac{4\pi G}{F_Q}\,\rho_m\,\delta = 0$$

Beltrán Jiménez, Heisenberg, Koivisto, Pekar 2020

Propagation of Gravitational Waves

The propagation of the tensorial perturbations $g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$

$$\bar{h}''_A + 2\mathcal{H}(1 + 2\delta(z))\bar{h}'_A + k^2\bar{h}_A = 0$$

where for an $f(Q)$ model

$$\delta(z) = \frac{d\ln F_Q}{2\mathcal{H}d\eta}$$

which leads to modification in the luminosity distance for GW

$$d_L^{(\mathsf{GW})}(z) = \exp\left(\int_0^z \frac{\delta(z)}{1+z}dz\right)d_L(z) = \sqrt{\frac{F_Q^{(0)}}{F_Q}}\, d_L(z)$$

Belgacem 2018.

STG with a ΛCDM background

Imposing a ΛCDM background in the Friedmann equation, the general solution for $F(Q)$ is

$$F = Q + M\sqrt{Q} + C$$

then the evolution of matter contrast is

$$\delta'' + \delta'\left(2 + \frac{H'}{H}\right) - \frac{3\sqrt{6}H}{2\sqrt{6}H + M}\Omega_m\delta = 0$$

and the relation between d_L and $d_L^{(GW)}$

$$d_L^{(GW)}(z) = \sqrt{\frac{2\sqrt{6} + M}{2\sqrt{6} + M/E(z)}}\, d_L(z)$$

where we take

$$E^2(z) \equiv \frac{H^2}{H_0^2} = \Omega_m(1+z)^3 + 1 - \Omega_m$$

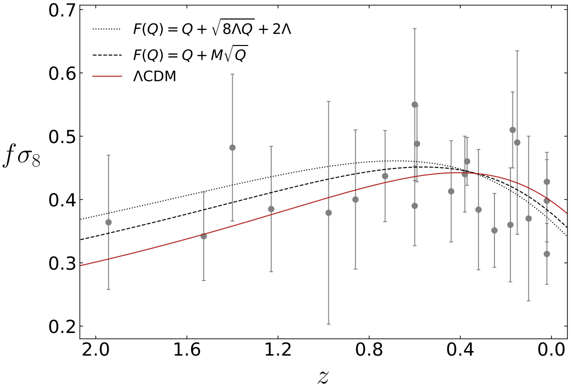
Constraints with Redshift Space Distortions data

Dataset

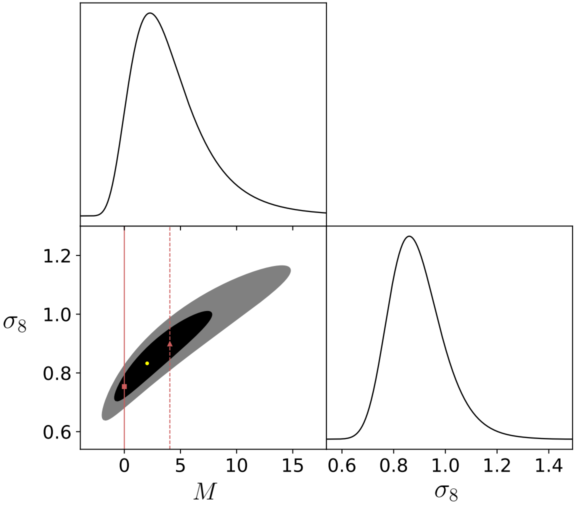
RSD data constrain the combination

$$f\sigma_8(N) = \sigma_8 \frac{\delta'(N)}{\delta(0)}$$

Use RSD data from Sagredo 2018.

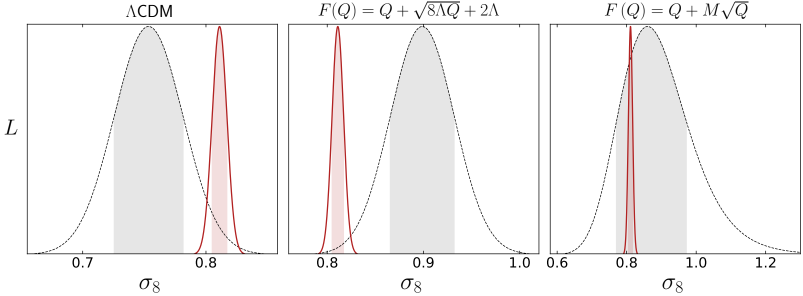


Likelihood analysis results



Barros, Barreiro, Koivisto, NJN 2020

Likelihood analysis results



Possible alleviation of the σ_8 tension with the 3rd model?

Likelihood analysis results

Model	M	σ_8	$\chi^2/$dof	ΔAIC_c
ΛCDM	0	0.75	0.62	0.60
$F(Q) = Q + M\sqrt{Q}$	2.03	0.83	0.60	1.90
$F(Q) = Q + \sqrt{8\Lambda Q} + 2\Lambda$	4.05	0.90	0.60	0

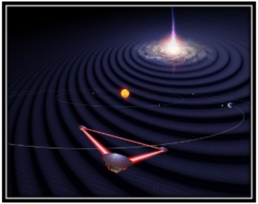
First conclusions

- Model with fixed M has a best fit value of σ_8 beyond the Planck value and also in tension;
- Best fit for M is non-zero;
- For the model with free M, the best fit σ_8 includes the Planck value;
- Both the χ^2 and AIC_c tests favour the model with fixed M.
- As the values for the AIC_c are all relatively close, there is no strong evidence for a preferred model.

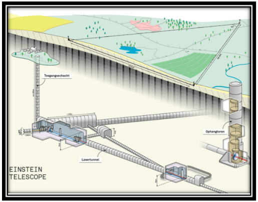
Forecasts for GW interferometers

GW interferometers

Since 2002



2030 - 2040?



From 2035

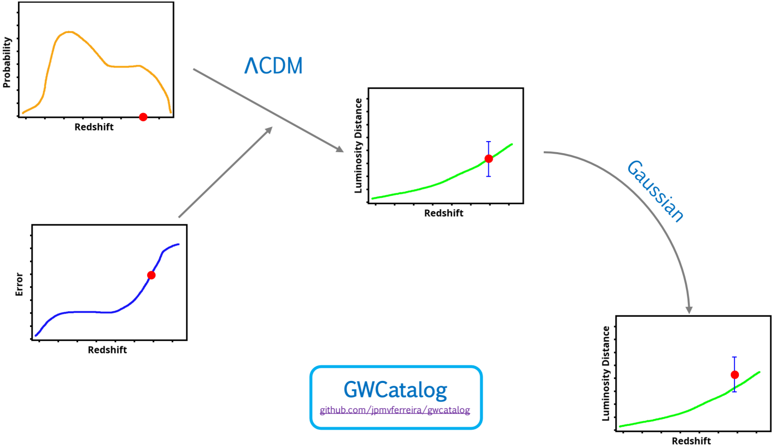
Catalog selection criteria

Standard sirens: Astrophysics events measured in both the electromagnetic spectrum and in gravitational waves.

Examples such as binary neutron star mergers (BNS), black hole binaries with accretion disks (BHB); extreme mass ratio inspirals (EMRI).

- **LIGO**: we generated **15** catalogs, each with **50** events (Macarena 2019, Baker 2020);
- **LISA**: we generated **15** different catalogs, each with **15** events, and picked the best, median and worst catalogs (Speri 2021);
- **ET**: we generated **5** different catalogs, each with **1000** events (Belgacem 2018);
- ... and we used **SnIa** data from the Pantheon compilation.

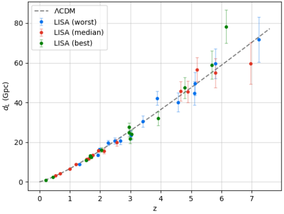
Making a mock catalog

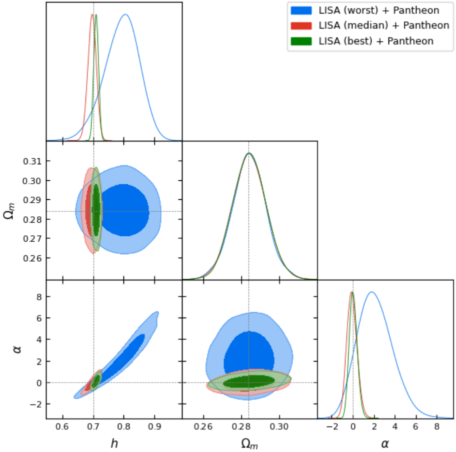


GWCatalog

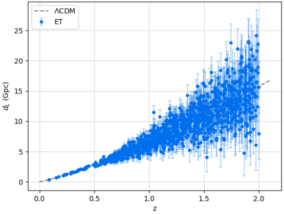
github.com/jpmvferreira/gwcatalog

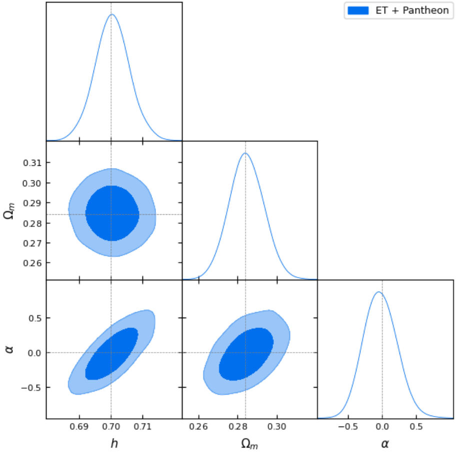
LISA





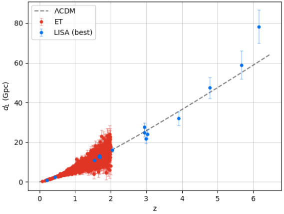
Ferreira, Barreiro, Mimoso, NJN, 2022

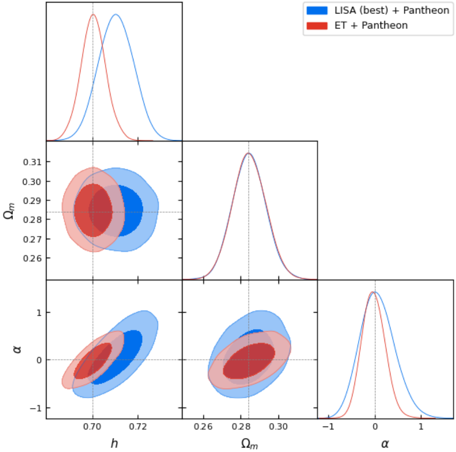




Ferreira, Barreiro, Mimoso, NJN, 2022

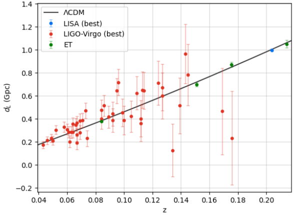
LISA vs ET

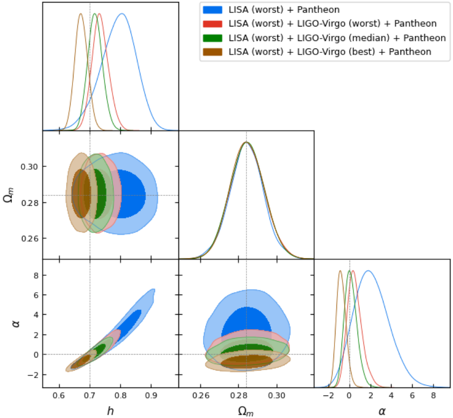




Ferreira, Barreiro, Mimoso, NJN, 2022

Can LIGO help?





Yes. A good catalog can improve LISA's constraints.

Summary of constraints

Catalog	σ_α	Relative Size
ET	0.25	1
LISA (best)	0.37	1.5
LISA (worst) + LIGO-Virgo (best)	0.44	1.8
LISA (median)	0.49	2
LISA (worst)	1.70	6.8

More conclusions

- The Einstein Telescope will perform better than LISA;
- LISA will perform better than LIGO;
- LIGO alone will not be able to constrain this model;
- We obtain different catalogs for LISA and LIGO, but the Einstein Telescope catalogs are all very similar;
- If we obtain a bad LISA catalog, we can use LIGO as a complement to obtain better constrains;
- The optimal region is $z \sim 0.6$ for LISA and $z \sim 1$ for the Einstein Telescope.

Beyond the ΛCDM background assumption

ABS $f(Q)$ model

A model with only one free parameter λ,

$$F(Q) = Q\, e^{\lambda Q_0/Q}$$

$\ll$ one of the first alternatives to the concordance model that apart from the fact that it might be preferred by the data [...] it does not face the cosmological constant problem since it does not include a "hidden" cosmological constant inside the $F(Q)$ form $\gg$

Anagnostopoulos, **B**asilakos, **S**aridakis 2021

$$(E^2 - 2\lambda)e^{\lambda/E^2} = \Omega_m(1+z)^3 + \Omega_r(1+z)^4$$

with

$$\lambda = \frac{1}{2} + W_0\left(-\frac{\Omega_m + \Omega_r}{2e^{1/2}}\right)$$

ABS $f(Q)$ model

Given that

$$(E^2 - 2\lambda)e^{\lambda/E^2} = \Omega_m(1+z)^3 + \Omega_r(1+z)^4$$

In the limit of low redshifts

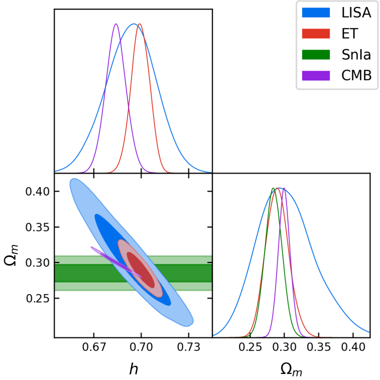
$$E^2 \approx e^{-\lambda}\,\Omega_m(1+z)^3 + e^{-\lambda}\,\Omega_r(1+z)^4 + 2\lambda$$

For high redshifts one recovers ΛCDM

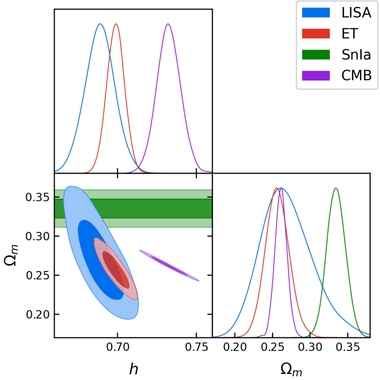
$$E^2 \approx \Omega_m(1+z)^3 + \Omega_r(1+z)^4 + \lambda$$

Supernovae + CMB + standard sirens constraints

ΛCDM



ABS



Actually... CMB already rules out this model as is!