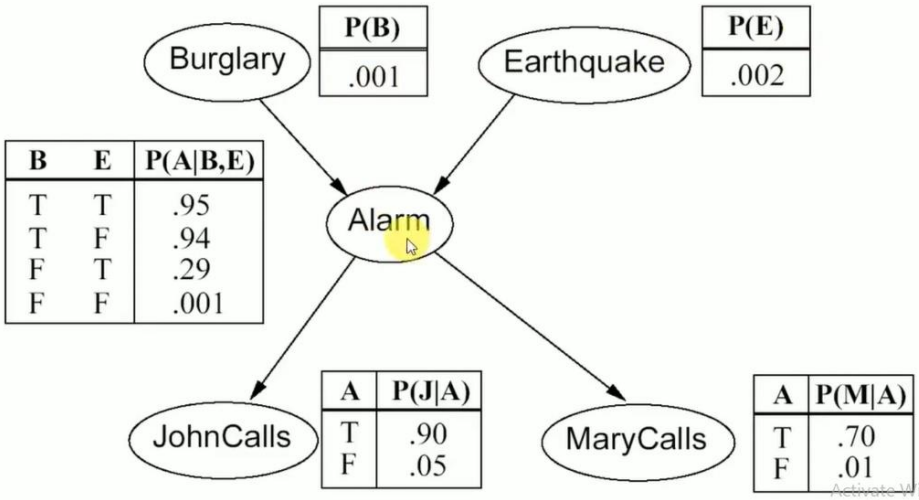
BAYESIAN BELIEF NETWORKS – EXAMPLE – 1

- You have a new burglar alarm installed at home.
- It is fairly reliable at detecting burglary, but also sometimes responds to minor earthquakes.
- You have two neighbors, John and Merry , who promised to call you at work when they hear the alarm.
- John always calls when he hears the alarm, but sometimes confuses telephone ringing with the alarm and calls too.
- Merry likes loud music and sometimes misses the alarm.
- Given the evidence of who has or has not called, we would like to estimate the probability of a burglary.

BAYESIAN BELIEF NETWORKS – EXAMPLE – 1



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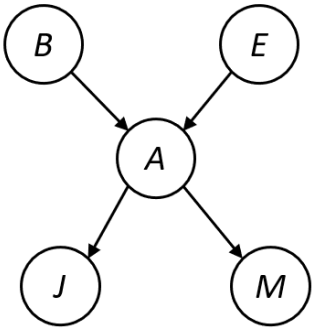
Example: Alarm Network

B	P(B)
+b	0.001
-b	0.999

E	P(E)
+e	0.002
-e	0.998



A	J	P(J|A)
+a	+j	0.9
+a	-j	0.1
-a	+j	0.05
-a	-j	0.95

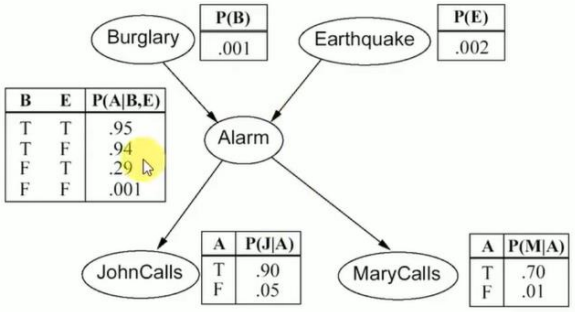
A	M	P(M|A)
+a	+m	0.7
+a	-m	0.3
-a	+m	0.01
-a	-m	0.99

B	E	A	P(A|B,E)
+b	+e	+a	0.95
+b	+e	-a	0.05
+b	-e	+a	0.94
+b	-e	-a	0.06
-b	+e	+a	0.29
-b	+e	-a	0.71
-b	-e	+a	0.001
-b	-e	-a	0.999

$$P(+b,-e,+a,-j,+m) =$$
$$P(+b)P(-e)P(+a|+b,-e)P(-j|+a)P(+m|+a) =$$
$$0.001 \times 0.998 \times 0.94 \times 0.1 \times 0.7$$

BAYESIAN BELIEF NETWORKS – EXAMPLE – 1

1. What is the probability that the alarm has sounded but neither a burglary nor an earthquake has occurred, and both John and Merry call?



Solution:

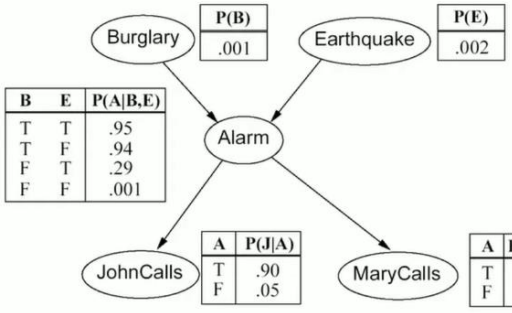
$P(j \wedge m \wedge a \wedge \neg b \wedge \neg e) = P(j \mid a)\, P(m \mid a)\, P(a \mid \neg b, \neg e)\, P(\neg b)\, P(\neg e)$

$= 0.90 \times 0.70 \times 0.001 \times 0.999 \times 0.998$

$= 0.00062$

BAYESIAN BELIEF NETWORKS – EXAMPLE – 1

2. What is the probability that John call?



Solution:

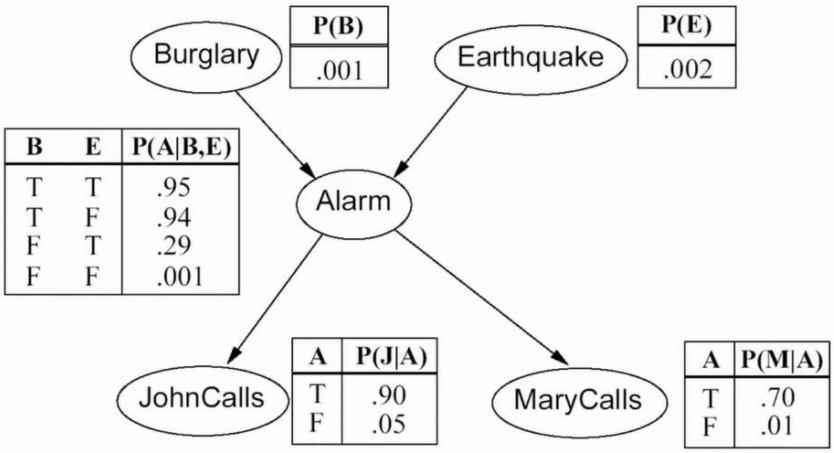
$P(j) = P(j \mid a)\,P(a) + P(j \mid \neg a)\,P(\neg a)$

$= P(j|a)\{P(a|b,e)*P(b,e)+P(a|\neg b,e)*P(\neg b,e)+P(a|b,\neg e)*P(b,\neg e)+P(a|\neg b,\neg e)*P(\neg b,\neg e)\}$

$+ P(j|\neg a)\{P(\neg a|b,e)*P(b,e)+P(\neg a|\neg b,e)*P(\neg b,e)+P(\neg a|b,\neg e)*P(b,\neg e)+P(\neg a|\neg b,\neg e)*P(\neg b,\neg e)\}$

$= 0.90 * 0.00252 + 0.05 * 0.9974 = 0.0521$

BAYESIAN BELIEF NETWORKS – EXAMPLE – 2



3. What is the probability that there is a burglary given that John and Merry calls?

BAYESIAN BELIEF NETWORKS – EXAMPLE – 2

- Suppose, we are given for the evidence variables $E_1,\ldots,E_m$, their values $e_1,\ldots,e_m$, and we want to predict whether the query variable X has the value x or not.
- For this we compute and compare the following:

$$P(x \mid e_1,\ldots,e_m) = \frac{P(x,e_1,\ldots,e_m)}{P(e_1,\ldots,e_m)} = \alpha P(x,e_1,\ldots,e_m)$$

$$P(\neg x \mid e_1,\ldots,e_m) = \frac{P(\neg x,e_1,\ldots,e_m)}{P(e_1,\ldots,e_m)} = \alpha P(\neg x,e_1,\ldots,e_m)$$

$$\propto = \frac{1}{(P(x,e_1,\ldots em) + P(\neg x,e_1,\ldots em))}$$

BAYESIAN BELIEF NETWORKS – EXAMPLE – 2

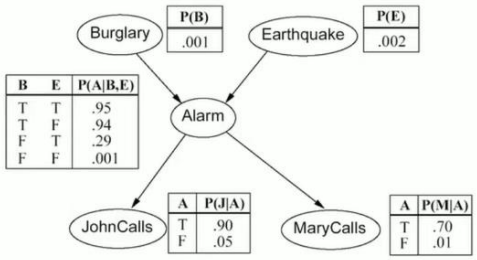
3. What is the probability that there is a burglary given that John and Merry calls?

$$P(b \mid j,m) = \alpha P(b) \sum_a P(j|a)\, P(m|a) \sum_e P(a|b,e)P(e)$$

$$= \alpha P(b) \sum_a P(j|a)\, P(m|a)\, \{P(a|b,e)P(e) + P(a|b,\neg e)P(\neg e)\}$$

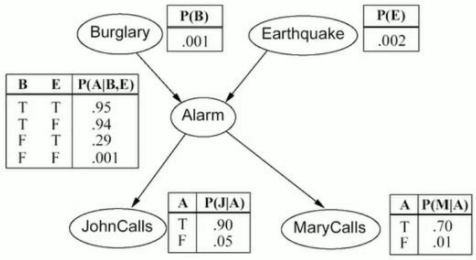
$$= \alpha P(b)\, [\, P(j|a)P(m|a)\, \{P(a|b,e)P(e) + P(a|b,\neg e)P(\neg e)\, \}$$

$$+ P(j|\neg a)P(m|\neg a)\, \{P(\neg a|b,e)P(e) + P(\neg a|b,\neg e)P(\neg e)\}\,]$$



BAYESIAN BELIEF NETWORKS – EXAMPLE – 2

3. What is the probability that there is a burglary given that John and Merry calls?



$$P(\neg b \mid j,m) = \alpha\, P(\neg b) \sum_a P(j|a)P(m|a) \sum_e P(a|\neg b,e)P(e)$$

$$= \alpha\, P(\neg b) \sum_a P(j|a)P(m|a) \{ P(a|\neg b,e)P(e) + P(a|\neg b,\neg e)P(\neg e)\}$$

$$= \alpha\, P(\neg b) [\, P(j|a)P(m|a) \{ P(a|\neg b,e)P(e) + P(a|\neg b,\neg e)P(\neg e) \}$$

$$+ P(j|\neg a)P(m|\neg a) \{ P(\neg a|\neg b,e)P(e) + P(\neg a|\neg b,\neg e)P(\neg e) \}]$$

$$= \alpha * .999*(.9*.7*(.29*.002 + .001*.998) + .05*.01*(.71*.002 + .999*.998))$$

$$= \alpha * .0015$$

BAYESIAN BELIEF NETWORKS – EXAMPLE – 2

3. What is the probability that there is a burglary given that John and Merry calls?

$$\alpha = \frac{1}{(P(b,j,m) + P(\neg b,j,m))}$$

$$\alpha = \frac{1}{(.00059 + .0015)}$$

$$= 478.5$$

$$P(b \mid j, m) = \propto * P(b, j, m)$$

$$= 478.5 * .00059$$

$$= 0.28$$

$$P(\neg b \mid j,m) = \propto * P(\neg\, b, j, m)$$

$$= 478.5 * .0015$$

$$= 0.72$$