Bayesian Occam's Razor in sensorimotor learning

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Introduction

- Learning structure is a key-element for achieving flexible and adaptive control in real-world environments (Braun et al., 2009). If several structural models explain observed data equally well, the question of how to select one particular structure arises (Körding, 2007).
- *Occam's Razor* is a general principle that suggests the preference of simpler explanations that make fewer assumptions.
- Here we investigate in a quantitative manner how humans select between several learned structures of different complexity when faced with novel adaptation problems.

Bayesian Occam's Razor

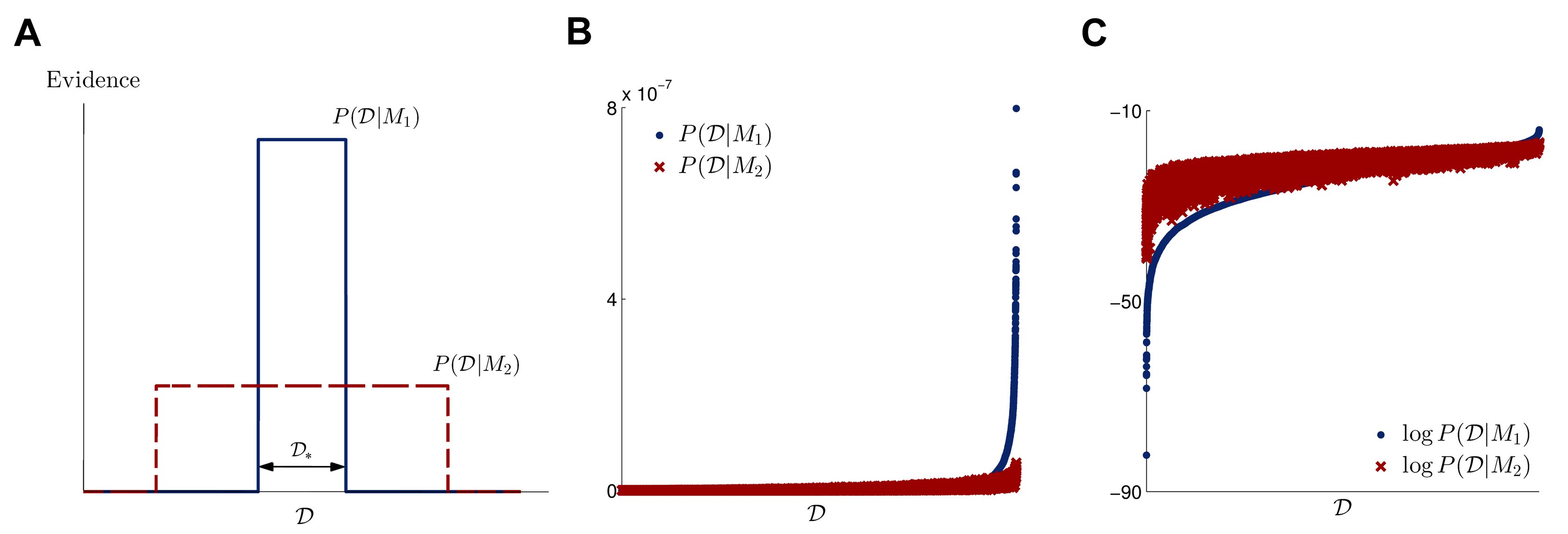
Bayesian model selection answers the question which of two probabilistic models M_1 and M_2 is more likely given the observation data $\mathbf{y}$ by computing the Bayes factor (Kass and Raftery, 1993):

$$\frac{P(M_1|\mathbf{y})}{P(M_2|\mathbf{y})} = \frac{P(\mathbf{y}|M_1)P(M_1)}{P(\mathbf{y}|M_2)P(M_2)} = \frac{P(\mathbf{y}|\mathbf{x}, M_1)}{P(\mathbf{y}|\mathbf{x}, M_2)} = \mathcal{BF},$$

with $P(M_1) = P(M_2)$.
The marginalization over model parameters θ, $P(\mathbf{y}|M) = \int P(\mathbf{y}|\theta, M)P(\theta|M)$, leads to an implicit penalization of complex models that allow for a large range of different parameter-settings.
The marginal likelihood of a Gaussian process (GP) model M_λ for N noisy observations $\mathbf{y}$, given input locations X can be derived analytically:

$$\log p(\mathbf{y}|X, M_\lambda) = \underbrace{-\frac{1}{2}\mathbf{y}^T(K_\lambda + \sigma_n^2\mathbb{I})^{-1}\mathbf{y}}_{\text{data fit error term}} \underbrace{-\frac{1}{2}\log|K_\lambda + \sigma_n^2\mathbb{I}| - \frac{N}{2}\log 2\pi}_{\text{complexity term}},$$

where K_λ is the kernel matrix with characteristic length scale parameter λ and σ_n^2 is the observation noise variance.



A Evidence $P(\mathcal{D}|M)$ for a simple model M_1 and a complex model M_2. Because both models have to spread unit probability mass over all compatible observations, the simpler model M_1 has a higher evidence in the overlapping region $\mathcal{D}_*$. **B** Evidence for two GP models with different length scales for simulated sets of random observations **C** Same as in B on log scale.

Complexity and free energy

For exponential family distributions $p(x) = \frac{1}{Z_\beta}e^{-\beta U(x)}$ the number of bits to record a data point x is simply

$$-\log p(x) = \beta U(x) + \log Z_\beta,$$

where $\beta U(x)$ can be thought of as the bits incurred by x in particular, and $\log Z_\beta$ as the number of bits induced by the model. Z_β is a partition sum and counts the effective amount of possibilities and is thus a measure for model complexity.
The average complexity of data and model is given by the entropy

$$H[p] = \beta < U > + \log Z_\beta.$$

When considering a model M with parameters θ:

$$P(\mathcal{D}|M) = \sum_\theta P(\mathcal{D}, \theta|M) = \sum_\theta \frac{e^{-\beta U_M(\mathcal{D},\theta)}}{Z_\beta^M},$$

the description length has the following form that yields a free energy term:

$$-\log P(\mathcal{D}|M) = \underbrace{-\log\sum_\theta e^{-\beta U_M(\mathcal{D},\theta)} + \log Z_\beta^M}_{\text{free energy}}$$

References

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Kass, R. E. and Raftery, A. E. (1993). Bayes factors and model uncertainty. *J. Am. Stat. Assoc.*, 90(254):466.

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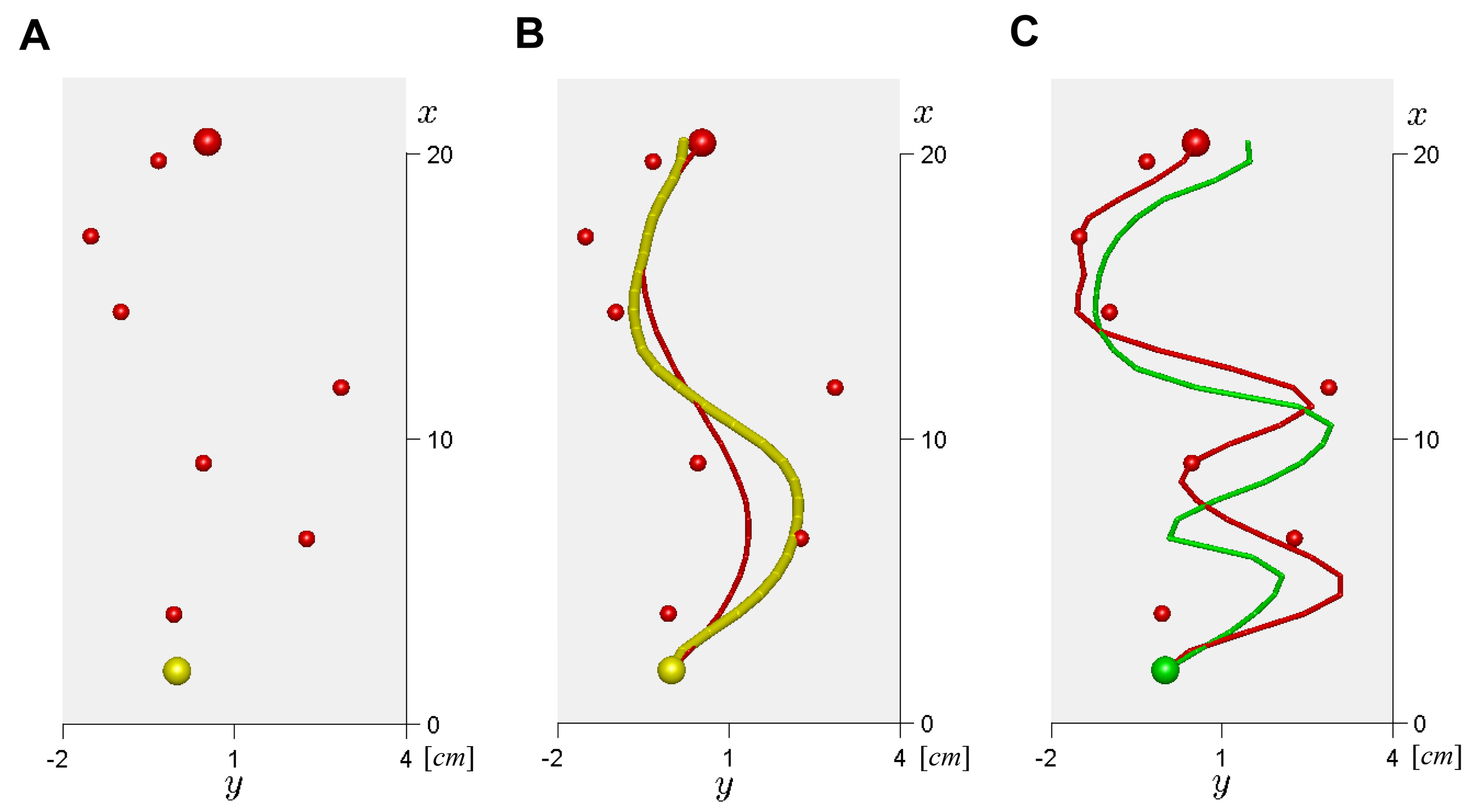
This study was supported by the DFG, Emmy Noether grant BR4164/1-1.

Regression task

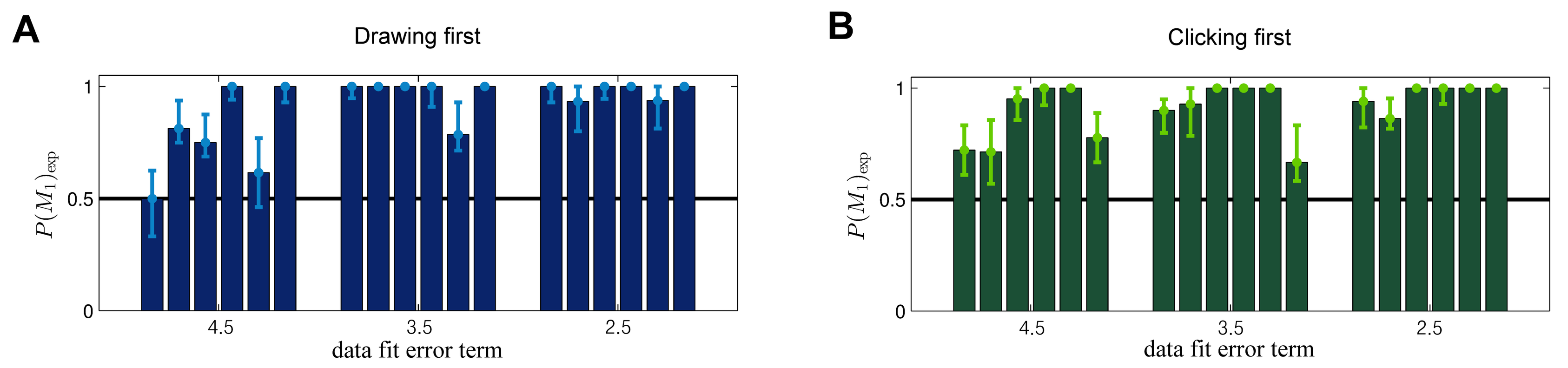
We designed a sensorimotor regression task, where participants had to draw a regression curve after seeing a number of noisy observations from an underlying trajectory. The underlying trajectories were generated by one of two possible GP models, which allowed us to test for the preference of the simpler model.



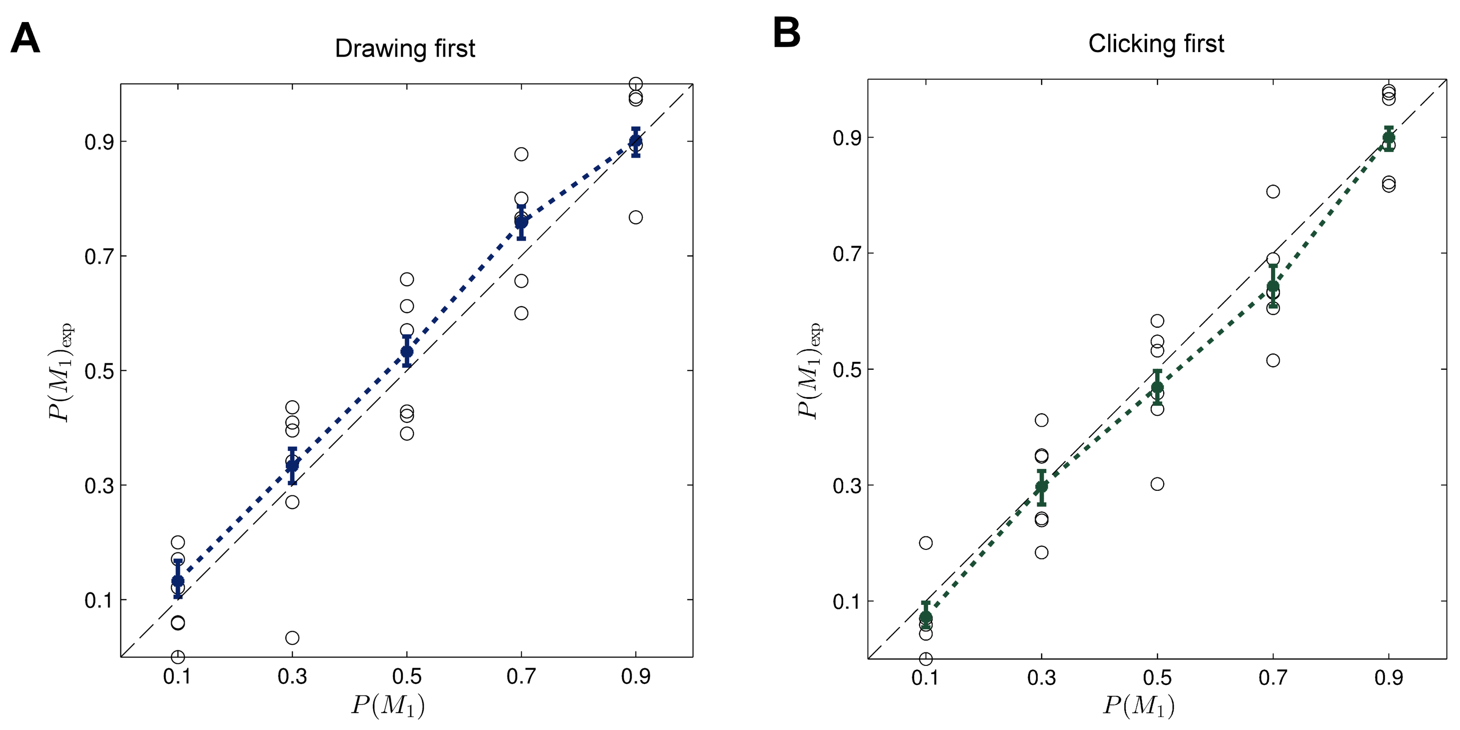
A Noisy observations are small red spheres with the start position as a yellow sphere and the end position as a larger red sphere. The color indicates the underlying generative model (simple model: yellow, complex model: green). **B** After completion of a *standard trial*, the underlying trajectory (red) is revealed and shown along with the participants trajectory (yellow). **C** Example of a standard trial where the short length scale model M_2 was the generating model.
In *probe trials*, participants were shown an ambiguous stimulus (no indication of the true generative model), which allowed us to test for the model choice behavior.

Experimental results

Occam's Razor observed in the experiment—participants were shown stimuli that had equal data fit error for both models. We found that participants were not indifferent about the underlying model complexity and showed a strong preference for the simpler model M_1.



A Choice probability for the simple model M_1 for the drawing first group. Each bar corresponds one participant in a particular error condition. **B** Clicking first group.



Quantitative consistency with Bayesian Occam's Razor in all trials (without special conditions on the data fit error). Circles represent individual participants median choice probability in different stimulus conditions. The dotted line shows the median using pooled data of all participants. **A** Drawing first group. **B** Clicking first group.

Conclusions

- We designed a sensorimotor task where we could analytically express the trade-off between goodness-of-fit and model complexity and test for this trade-off in the human sensorimotor system.
- We found that participants strongly favored the simpler model, in case both models were supported by the observed data equally well. In general, we found that participants' choice behavior was quantitatively consistent with Bayesian statistics. These results suggest that Occam's Razor is a general principle already present during sensorimotor processing.
- The approach presented in this work lends itself for general application especially if one of several structural models has to be selected or when sensory data has to be disambiguated.