INFORMATION-THEORETIC BOUNDED RATIONALITY IN PERCEPTION-ACTION SYSTEMS

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Sensorimotor Learning

- Bayesian models
- Structure Learning
- Hierarchies of abstraction

Theory of Decision-Making

- Neuroeconomic principles
- Bounded rationality

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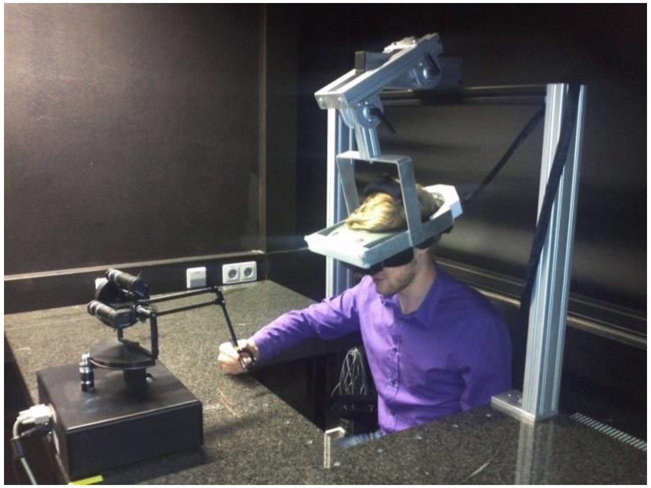
Experiments in virtual-reality (psychophysics)



PhD: Structure Learning with Hierarchical Models for Computational Motor Control

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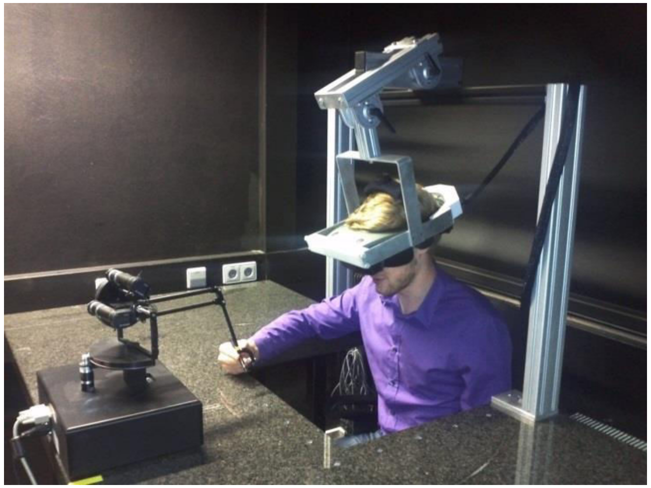

PhD: Structure Learning with Hierarchical Models for Computational Motor Control

From July 2016:

Cognitive Systems Group

Corporate Research - Robert Bosch GmbH

BOSCH

Outline

- Information-theoretic bounded rationality
  - Free energy minimization
  - Lossy compression and emergence of levels of abstraction

- Perception for Action
  - Coupling perception and action
  - Likelihood function synthetization
  - Illustrative example

Utility maximization

Goal:

- Given some world state w, pick best action a
- Desirability of action is specified by utility function $U(w, a)$

Easy…

$$a_w^* = \arg\max_a U(w, a)$$

The problem with utility maximization

Goal:

- Given some world state w, pick best action a
- Desirability of action is specified by utility function $U(w, a)$

Easy?

$$a_w^* = \arg\max_a U(w, a)$$

Problem:

- Searching through a vast set with limited computational capacity
- Finding the best action can easily become intractable

Bounded rational decision-making

Goal:

- Given some world state w, pick best action a
- Desirability of action is specified by utility function $U(w, a)$

Take the process of computation into account

- Modified optimality principle
- Information-theoretic bounded rationality
- *Rather than finding the single best action, find "good" actions that are actually computable*

Information-theoretic bounded rationality

Find a stochastic policy $p(a|w)$ that maximizes

- expected utility $\sum_a p(a|w) U(w,a)$

subject to the constraint:

- "computational effort" $\leq K$

Information-theoretic bounded rationality

Find a stochastic policy $p(a|w)$ that maximizes

- expected utility $\sum_a p(a|w)U(w,a)$

subject to the constraint:

- "computational effort" $\leq K$

Computational effort?

- Transformation of behavior in response to observation w
- Any change of behavior requires **computation**
- Limit the "amount of change" in behavior

$$\boxed{\text{computational effort} = D_{\mathrm{KL}}(p(a|w)||p_0(a))}$$

Information-theoretic bounded rationality

Trade-off:

- Large expected utility
- Low computational effort

$$\boxed{\mathrm{p}^*(\mathrm{a}|\mathrm{w}) = \underset{p(a|w)}{\arg\max}\, \mathbf{E}_{p(a|w)}[U(w,a)] - \frac{1}{\beta} D_{\mathrm{KL}}(p(a|w)||p_0(a))}$$

- Mathematically equivalent to minimization of **free energy** difference
- Also, deep conceptual ties – “the physics of computation”
  - Ortega, Braun 2013, *Thermodynamics as a theory of decision-making with information-processing cos*ts, Royal Society A

Solving the variational problem

Trade-off:

- Large expected utility
- Low computational effort

$$\mathrm{p}^*(\mathrm{a}|\mathrm{w}) = \underset{p(a|w)}{\arg\max}\, \mathbf{E}_{p(a|w)}[U(w,a)] - \frac{1}{\beta} D_{\mathrm{KL}}(p(a|w)||p_0(a))$$

$$\boxed{p^*(a|w) = \frac{1}{Z} p_0(a) e^{\beta U(w,a)}}$$

Z … partition sum, acts as normalization constant $\sum_a p_0(a) e^{\beta U(w,a)}$

β … inverse temperature, governs trade-off

Solving the variational problem

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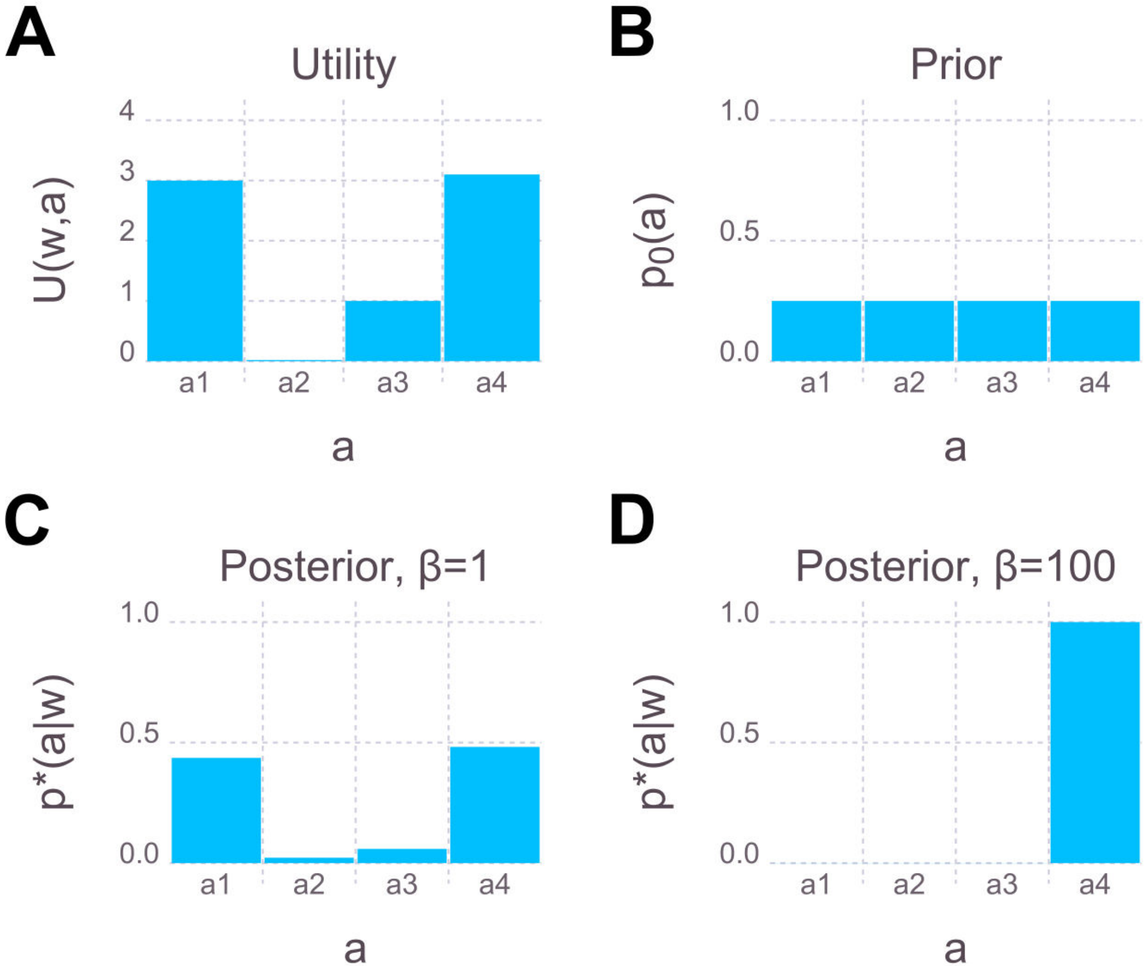
$$p^*(a|w) = \frac{1}{Z} p_0(a) e^{\beta U(w,a)}$$

<u>Special case – Bayes' rule:</u>

$$U(w,a) = \log q(w|a), \qquad \beta = 1$$

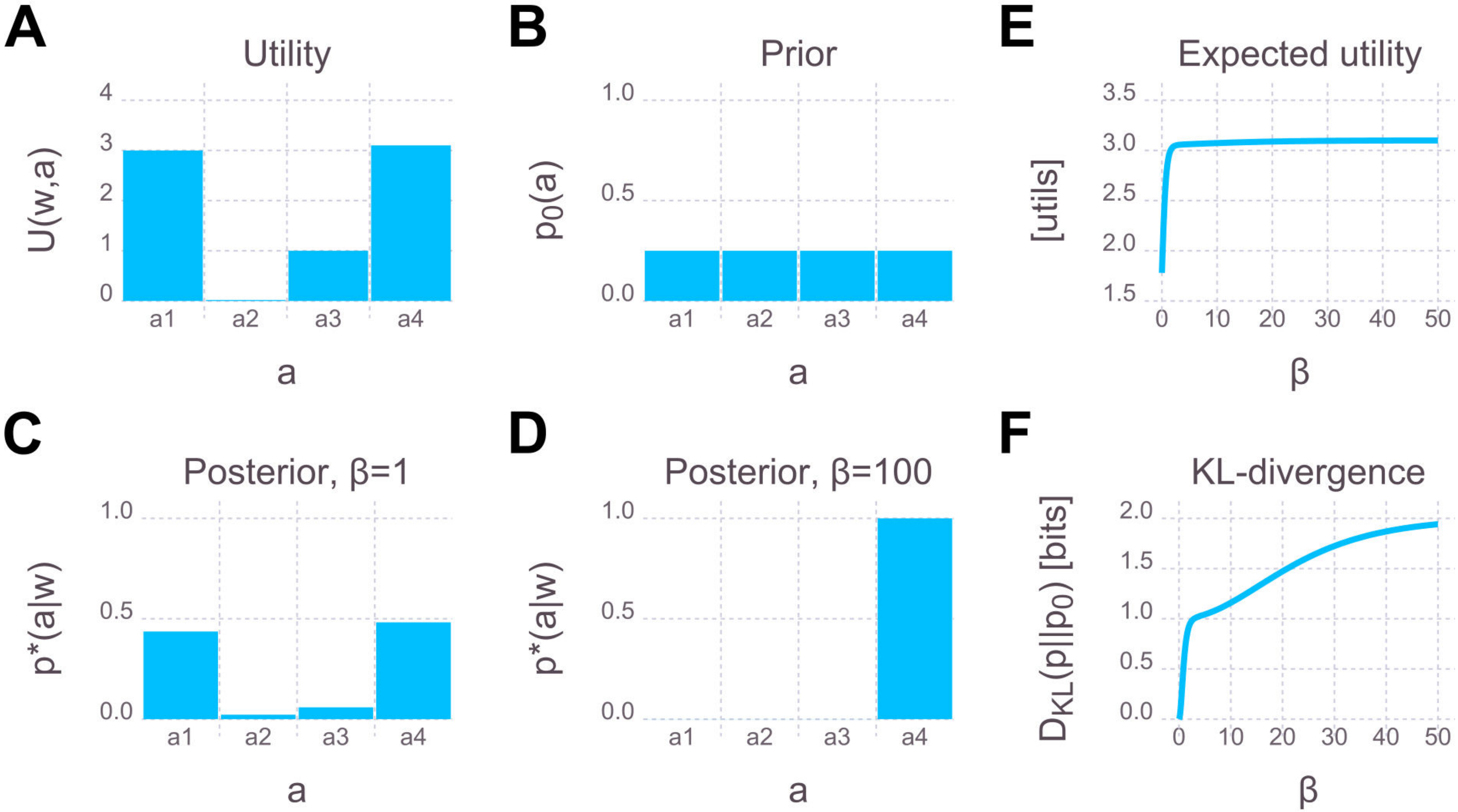
$$p^*(a|w) = \frac{q(w|a)p_0(a)}{Z}$$

Example: grasping movement



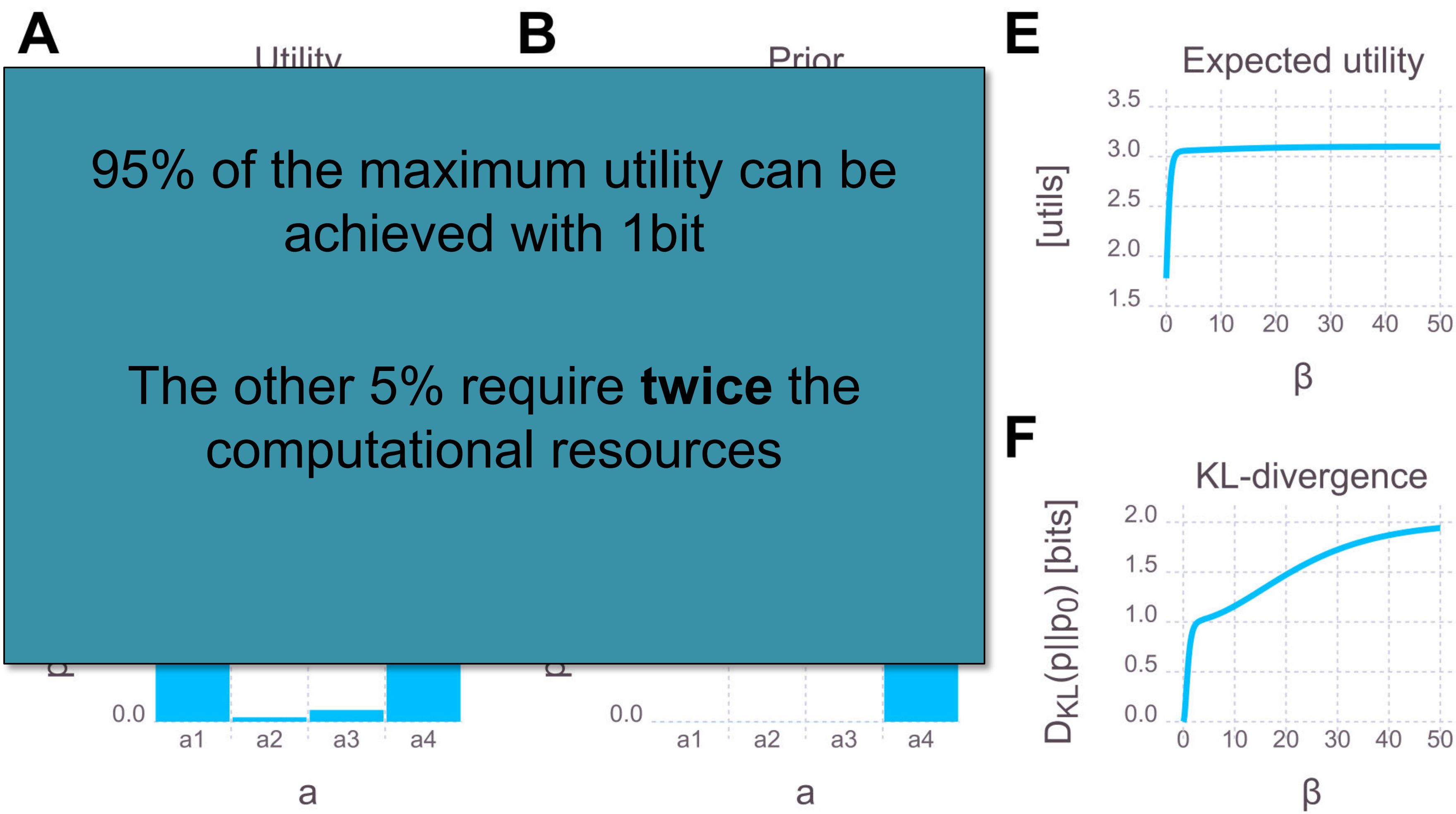
Example: grasping movement

- β governs computational resources



Example: grasping movement

- β governs computational resources



95% of the maximum utility can be achieved with 1bit

The other 5% require **twice** the computational resources

Analytical solution

$$p^*(a|w) = \frac{1}{Z(w)} p_0(a) e^{\beta U(w,a)}$$

- Still intractable (partition sum): $$\mathrm{Z(w)} = \sum_a p_0(a) e^{\beta U(w,a)}$$
- Descriptive framework (external point-of-view)

Analytical solution

$$p^*(a|w) = \frac{1}{Z(w)} p_0(a) e^{\beta U(w,a)}$$

- Still intractable (partition sum):

$$\mathrm{Z(w)} = \sum_a p_0(a) e^{\beta U(w,a)}$$

- Descriptive framework (external point-of-view)

Rejection sampling scheme

- Constructive framework (internal point-of-view)

1. Draw sample $\tilde{a} \sim p_{0(a)}, \quad \tilde{u} \sim U(0,1)$
2. Accept if: $\tilde{u} \leq \frac{e^{\beta U(w,a)}}{e^{\beta U_{\max}(w)}}$
3. Otherwise reject $\tilde{a}$ and go back to 1.

- Guaranteed to produce samples from $p^*(a|w)$
- Expected number of rejections: $\#\mathrm{samples} = e^{\beta U_{\max}(w)} / Z(w)$
  - β controls how many rejections “are allowed” (on average)

Outline

- Information-theoretic bounded rationality
  - Free energy minimization
  - Lossy compression and emergence of levels of abstraction

- Perception for Action
  - Coupling perception and action
  - Likelihood function synthetization
  - Illustrative example

Multiple world-states

Trade off large utility against low computational effort

- **Now:** consider multiple w, more precisely $p(w)$
- What is the optimal prior $p_0(a)$? → the marginal
- What is the (bounded) optimal $p^*(a|w)$?

$$p^*(a|w) = \underset{p(a|w)}{\arg\max}\, \mathbf{E}_{p(w,a)}[U(w,a)] - \frac{1}{\beta} I(W;A)$$

- Mathematically equivalent to **rate-distortion** problem
  - → Lossy compression
  - Channel from observations to actions with limited capacity

Multiple world-states

Trade off large utility against low computational effort

- **Now:** consider multiple w, more precisely $p(w)$
- What is the optimal prior $p_0(a)$? → the marginal
- What is the (bounded) optimal $p^*(a|w)$?

$$p^*(a|w) = \underset{p(a|w)}{\arg\max}\ \mathbf{E}_{p(w,a)}[U(w,a)] - \frac{1}{\beta} I(W;A)$$

Solution:

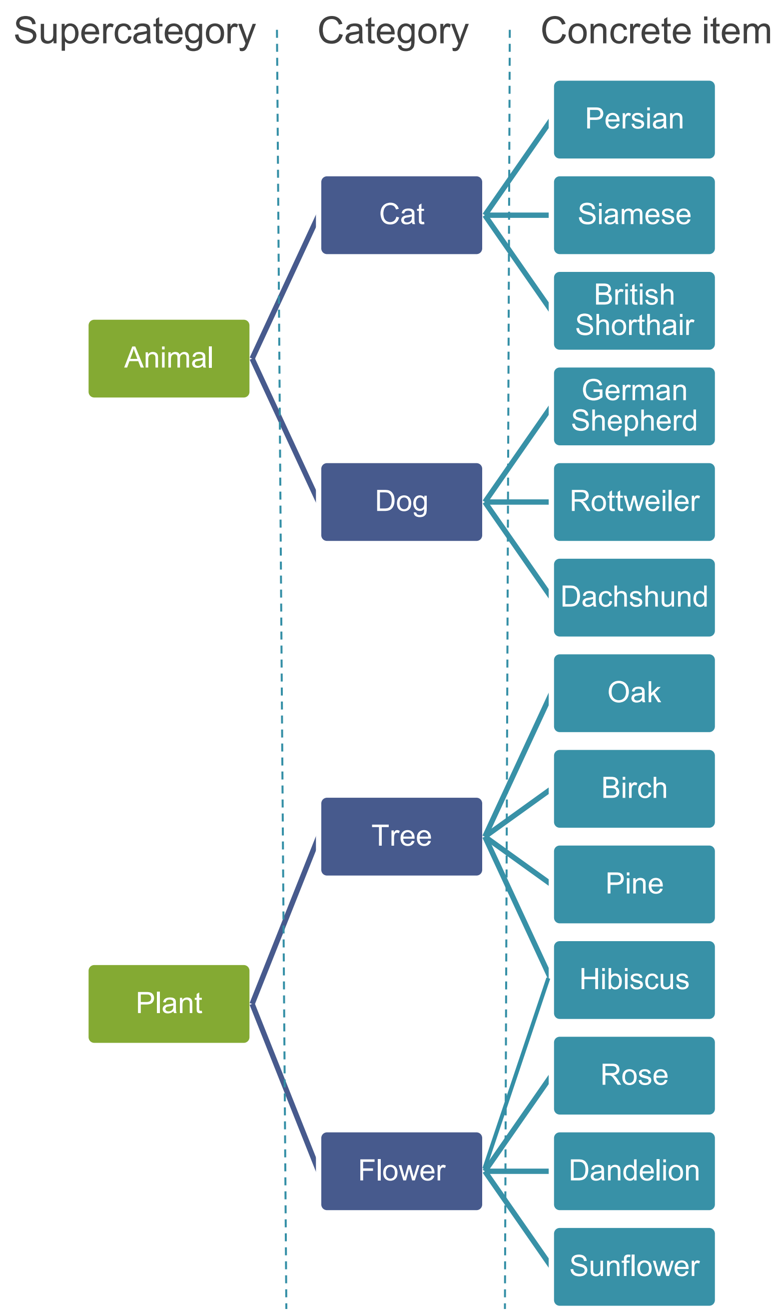
$$\boxed{\begin{aligned} p^*(a|w) &= \frac{1}{Z} p(a) e^{\beta U(w,a)} \\ p(a) &= \sum_w p(w) p^*(a|w) \end{aligned}}$$

$$Z = \sum_a p(a) e^{\beta U(w,a)}$$

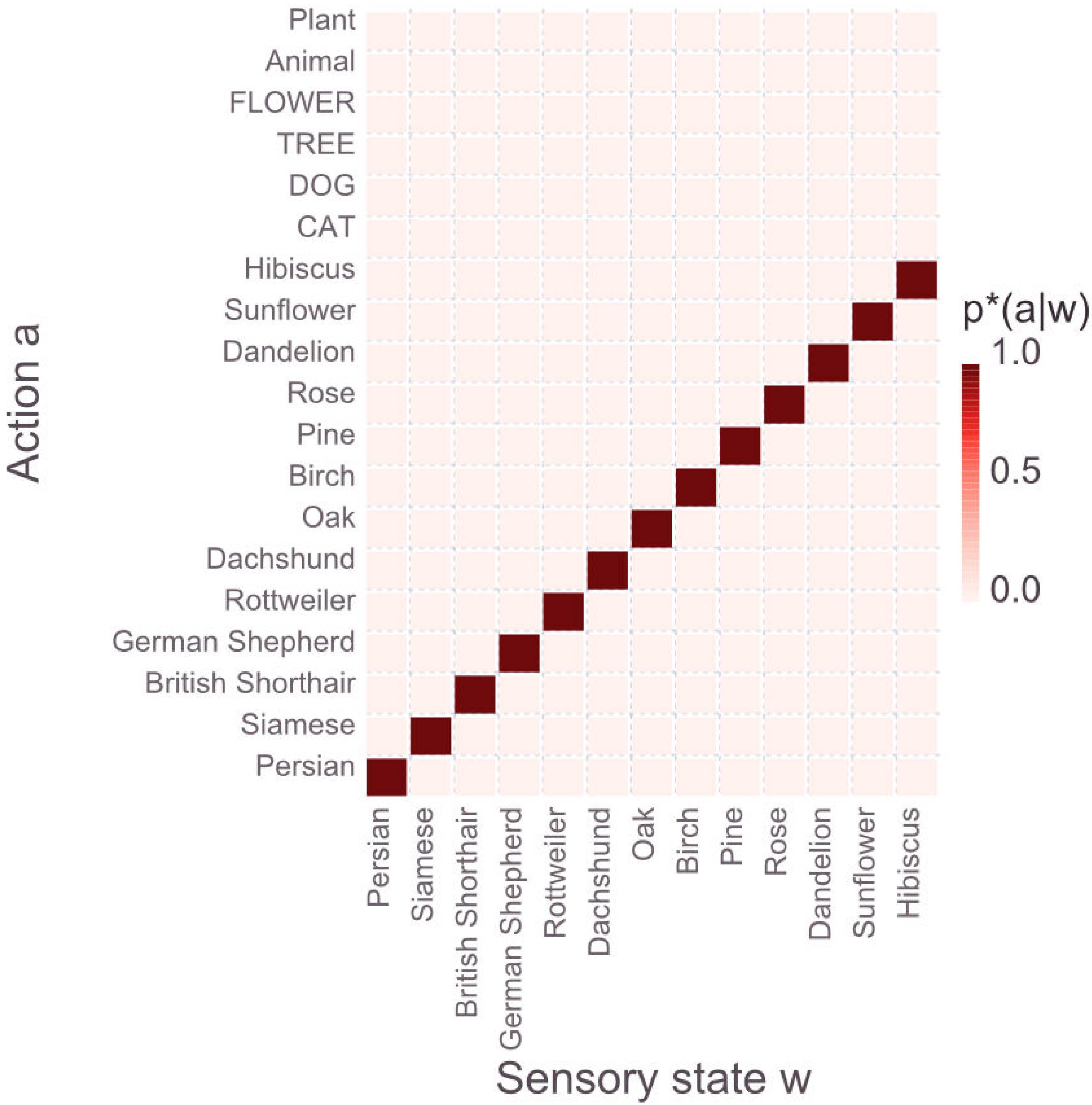
Toy Example

- Sensory state
  $w \in \{\text{concrete items}\}$
- Action $a \in \{\text{concrete items},$
  $\text{categories}, \text{supercategories}\}$

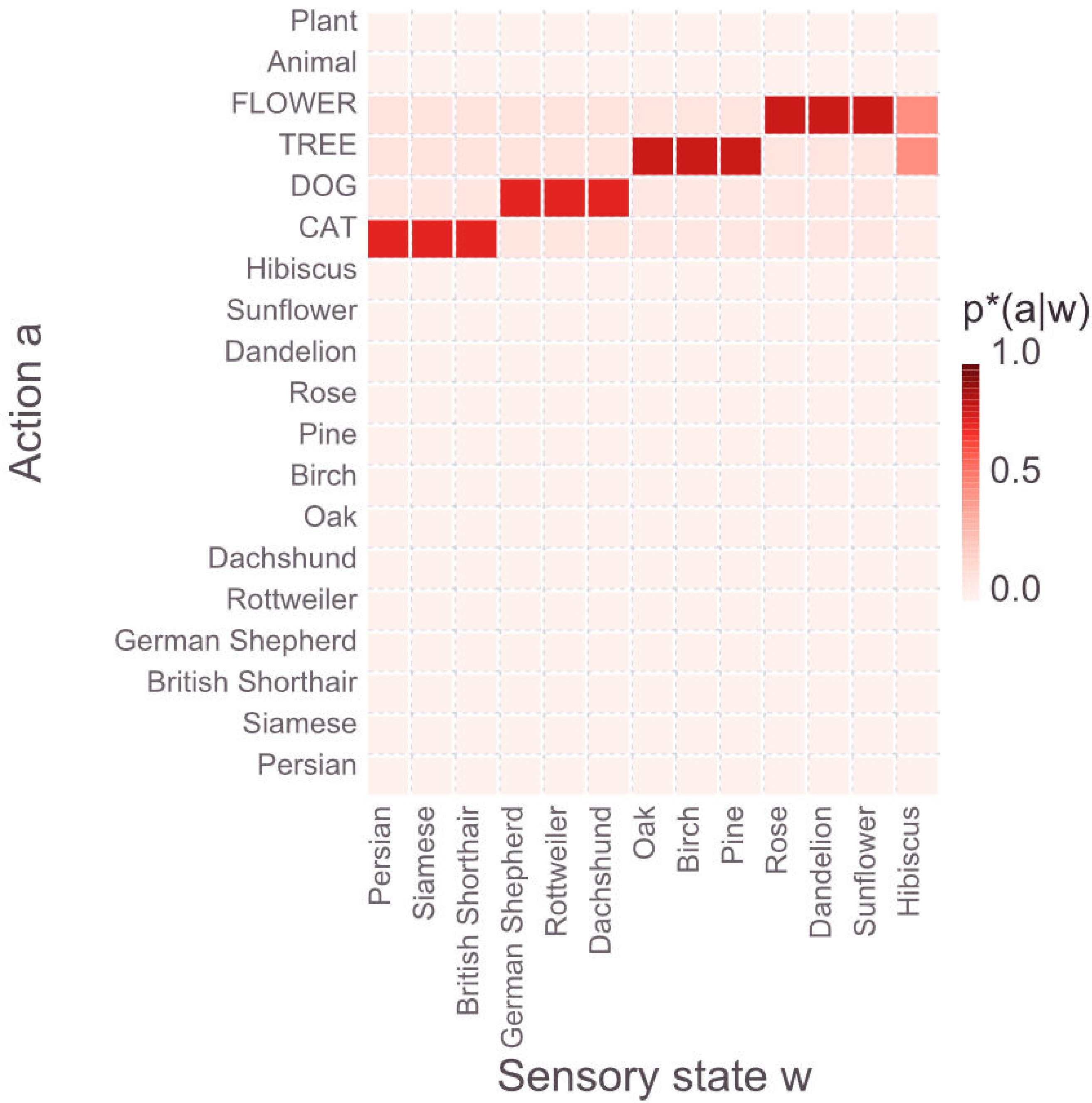
- Rewards/Utilities:
  - 3€ if concrete item correct
  - 2.2€ if category correct
  - 1.6€ if supercategory correct



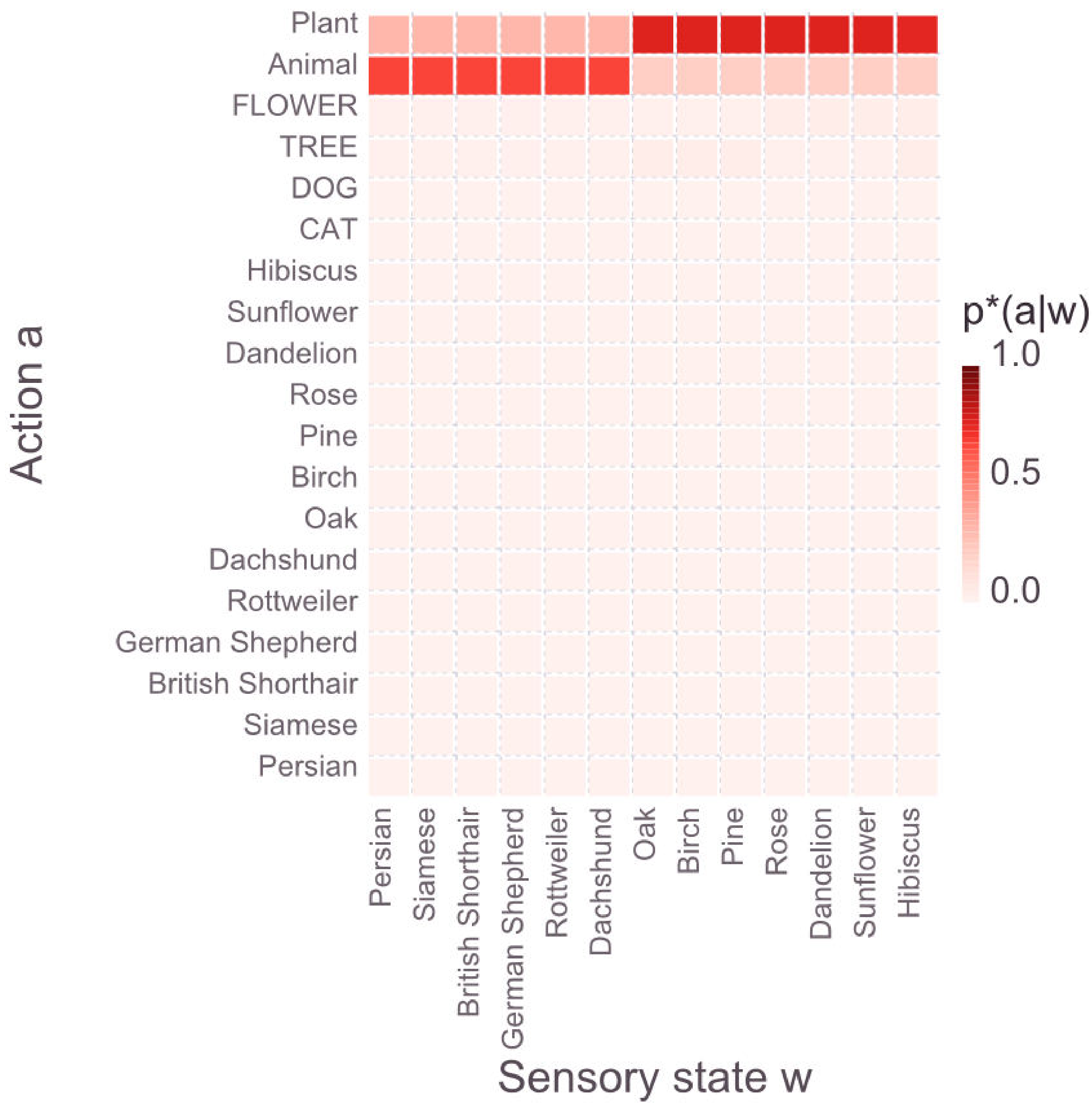
β	10	$[\text{bits}/€]$
I	3.7	[bits]
$\mathbf{E}[U]$	3	[€]



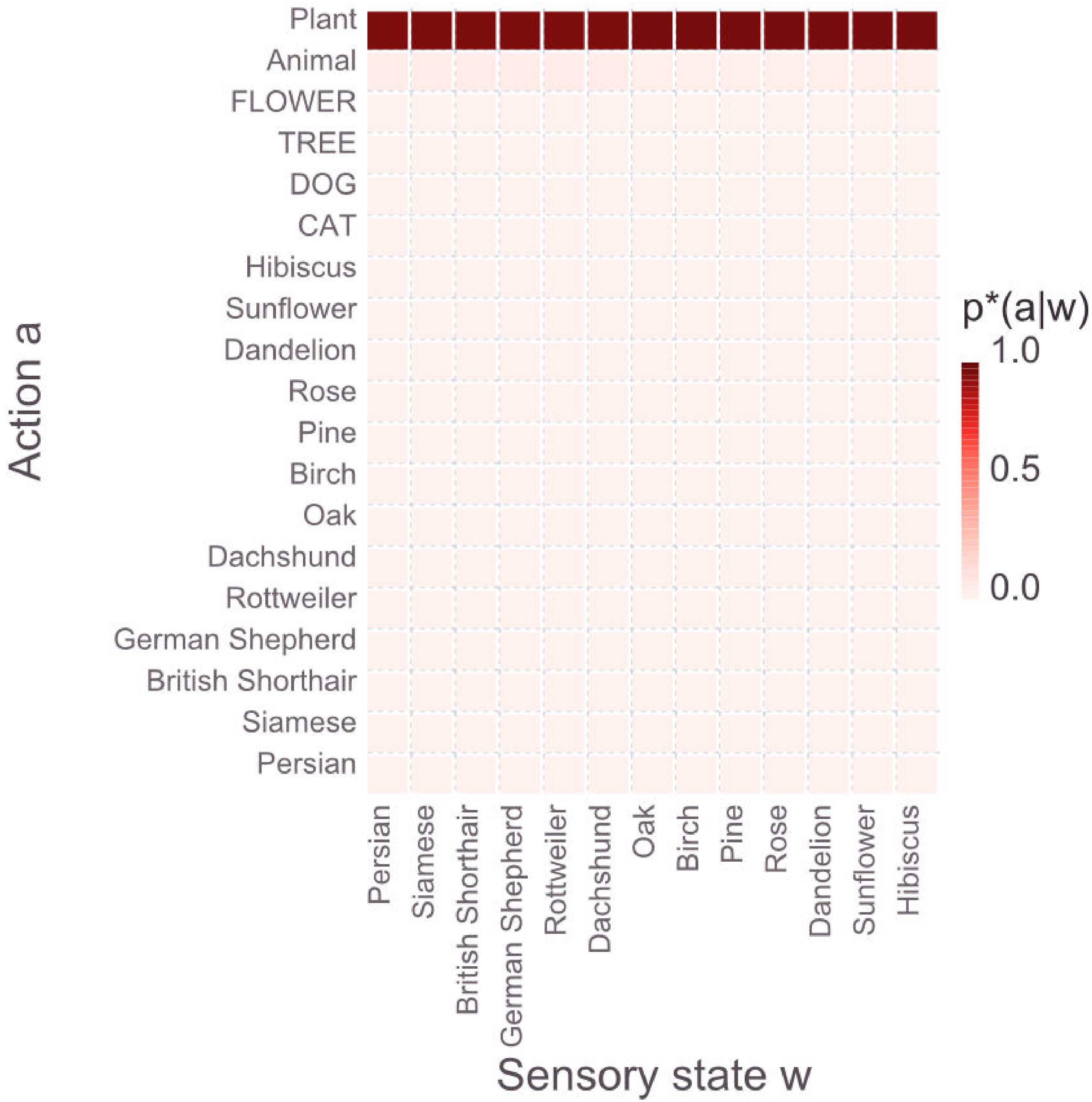
β	1.11	$\left[{}^{\text{bits}}/_{€}\right]$
I	0.9	[bits]
$\mathbf{E}[U]$	1.8	[€]



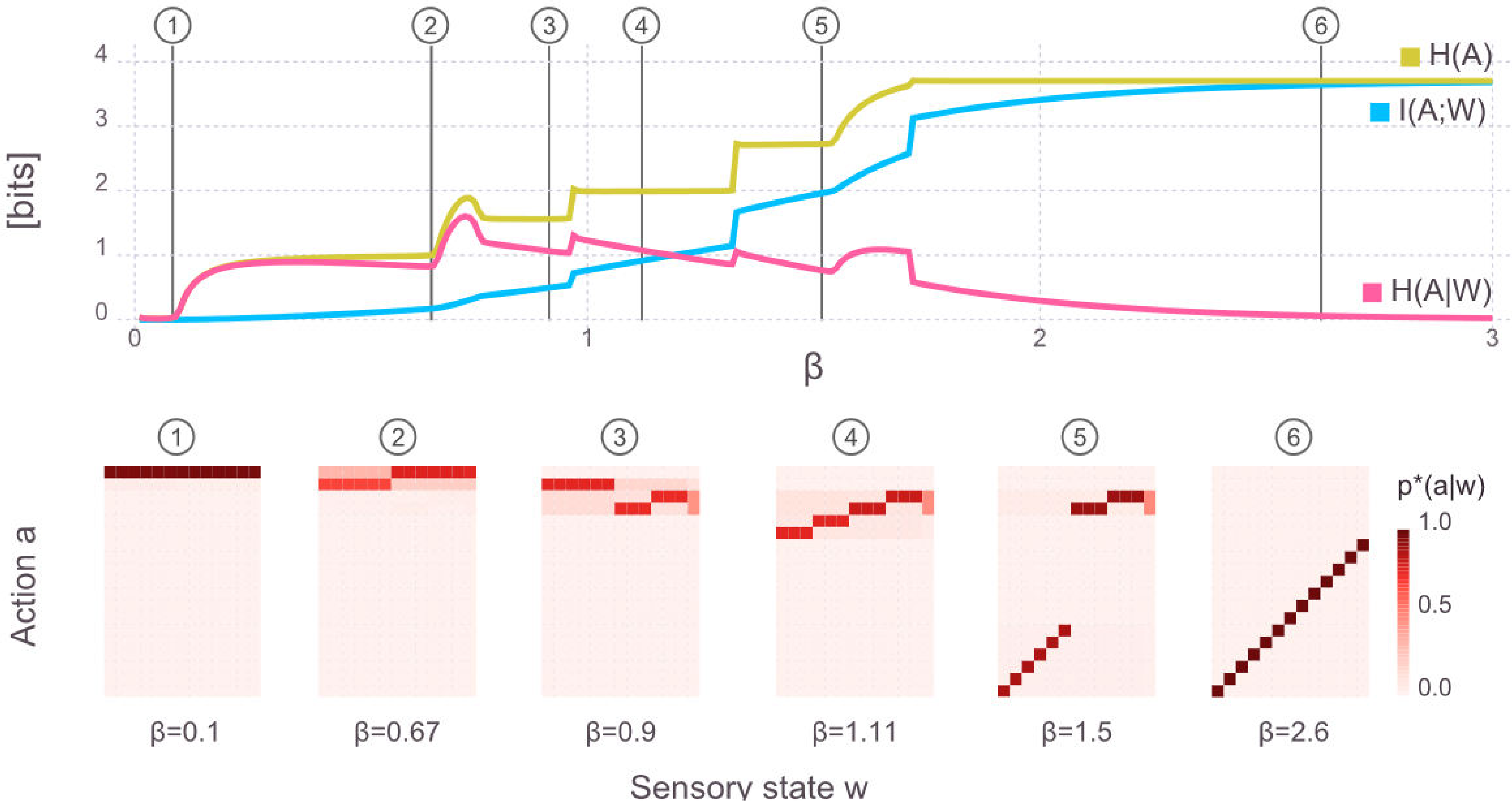
β	0.67	$\left[{}^{\text{bits}}/_{€}\right]$
I	0.18	[bits]
$\mathbf{E}[U]$	1.2	[€]



β	0.1	$[\text{bits}/\text{€}]$
I	0	[bits]
$\mathbf{E}[U]$	0.86	[€]



Emergence of natural levels of abstraction



Summary – bounded rationality

Trade off large utility against low computational cost

Abstractions are induced through limitations in information processing capabilities

- Levels of abstraction are formed through the structure of the utility-function

Extensions:

- Modelling perception-action systems with the principle
- Modelling several layers of abstraction in parallel (not in this talk)

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Perception and action

World state w (not directly accessible), percept x, action a

$$w \rightarrow x \rightarrow a$$

Classical: perception is inference

- Percept x should represent world-state w as faithfully as possible

$$p(w|x) = \frac{p(x|w)p(w)}{Z}$$

Action is decision-making

- Maximize utility under posterior belief over w

$$U(x,a) = \sum_{w} p(w|x)U(w,a)$$

Perception and action

World state w (not directly accessible), percept x, action a

$$w \rightarrow x \rightarrow a$$

Classical: perception is inference

- Percept x should represent world-state w as faithfully as possible

$$p(w|x) = \frac{p(x|w)p(w)}{Z}$$

How to define the likelihood model $p(x|w)$?
Classical: problem independent of action-channel

Perception for action

World state w (not directly accessible), percept x, action a

$$w \to x \to a$$

Information-theoretic bounded rationality:

- Trade off (gains in) expected utility against computational effort

Two coupled stages of computation (“channels”):

- Perception: stochastic mapping from w to x
- Action: stochastic mapping from x to a

Trading off utility against information processing cost

$$\underset{p(x|w),p(a|x)}{\arg\max}\ \mathbf{E}_{p(w,x,a)}[U(w,a)] - \frac{1}{\beta_1} I(W;X) - \frac{1}{\beta_2} I(X;A)$$

Set of self-consistent solutions:

$$p^*(x|w) = \frac{1}{Z(w)} p(x) \exp\left(\beta_1 \Delta F_{\text{ser}}(w,x)\right)$$

$$p(x) = \sum_w p(w) p^*(x|w)$$

$$p^*(a|x) = \frac{1}{Z(x)} p(a) \exp\left(\beta_2 \sum_w p(w|x) U(w,a)\right)$$

$$p(a) = \sum_{w,x} p(w) p^*(x|w) p^*(a|x),$$

Trading off utility against information processing cost

$$\underset{p(x|w),p(a|x)}{\arg\max}\ \mathbf{E}_{p(w,x,a)}[U(w,a)] - \frac{1}{\beta_1} I(W;X) - \frac{1}{\beta_2} I(X;A)$$

Set of self-consistent solutions:

$$\boxed{p^*(x|w) = \frac{1}{Z(w)} p(x) \exp\left(\beta_1 \Delta F_{\text{ser}}(w,x)\right)}$$

$$\Delta F_{\text{ser}}(w,x) := \mathbf{E}_{p^*(a|x)}[U(w,a)] - \frac{1}{\beta_2} D_{\text{KL}}(p^*(a|x)||p(a))$$

Well-defined likelihood model $p(x|w)$

- Maximizes downstream utility-computation trade-off (free energy)
- Tight **coupling** between perception and action!

Trading off utility against information processing cost

$$\underset{p(x|w),p(a|x)}{\arg\max}\ \mathbf{E}_{p(w,x,a)}[U(w,a)] - \frac{1}{\beta_1} I(W;X) - \frac{1}{\beta_2} I(X;A)$$

Set of self-consistent solutions:

$$p^*(x|w) = \frac{1}{Z(w)} p(x) \exp\left(\beta_1 \Delta F_{\text{ser}}(w,x)\right)$$

Bounded-optimal perception should extract the **most relevant** information (for efficient acting) rather than allowing to predict w as well as possible!

Trading off utility against information processing cost

$$\arg\max_{p(x|w),p(a|x)} \mathbf{E}_{p(w,x,a)}[U(w,a)] - \frac{1}{\beta_1} I(W;X) - \frac{1}{\beta_2} I(X;A)$$

Set of self-consistent solutions:

$$\boxed{p^*(a|x) = \frac{1}{Z(x)} p(a) \exp\left(\beta_2 \sum_w p(w|x) U(w,a)\right)}$$

The action-part of the system $p(a|x)$

- Maximizes posterior expected utility in a bounded rational fashion

$$U(x,a) = \sum_w p(w|x) U(w,a)$$

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Predator-Prey example

Three groups of animals

- Small: prey, can’t hear well
- Medium: prey, can hear well
- Large: predators

Three basic actions

- Ambush: works equally well on small and medium-sized animals
- Sneak-up: works well on small animals
- Flee: only sensible actions for large animals

Predator-Prey example

Three groups of animals

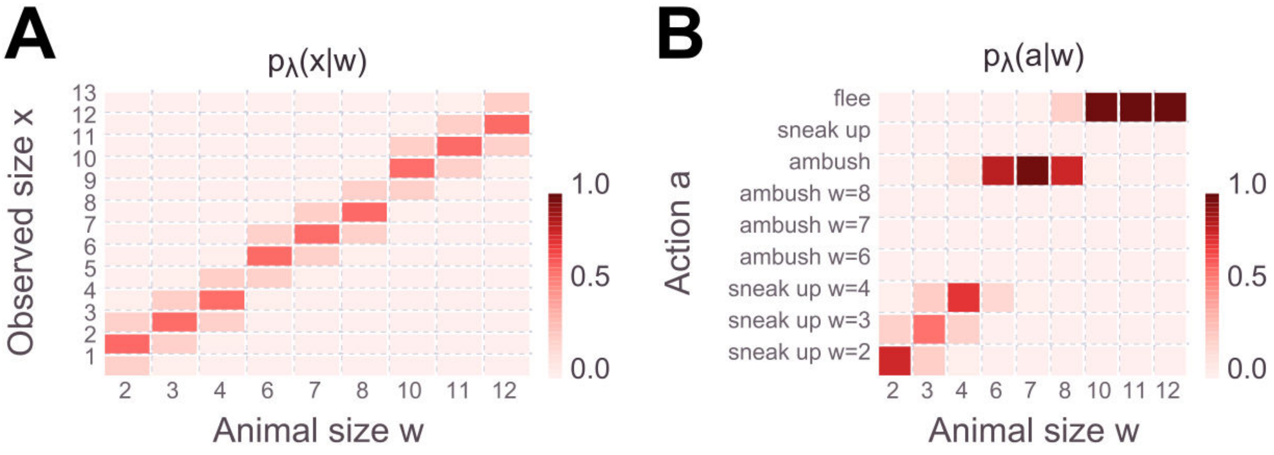
- Small: prey, can’t hear well
- Medium: prey, can hear well
- Large: predators

Three basic actions

- Ambush: works equally well on small and medium-sized animals
- Sneak-up: works well on small animals
- Flee: only sensible actions for large animals

- Design agent with limited perceptual capacity
  - Compare bounded-optimal perception against hand-crafted likelihood model
  - Hand-crafted model is designed to predict actual animal size w from observed animal size x well

Large action capacity (good motor skills)



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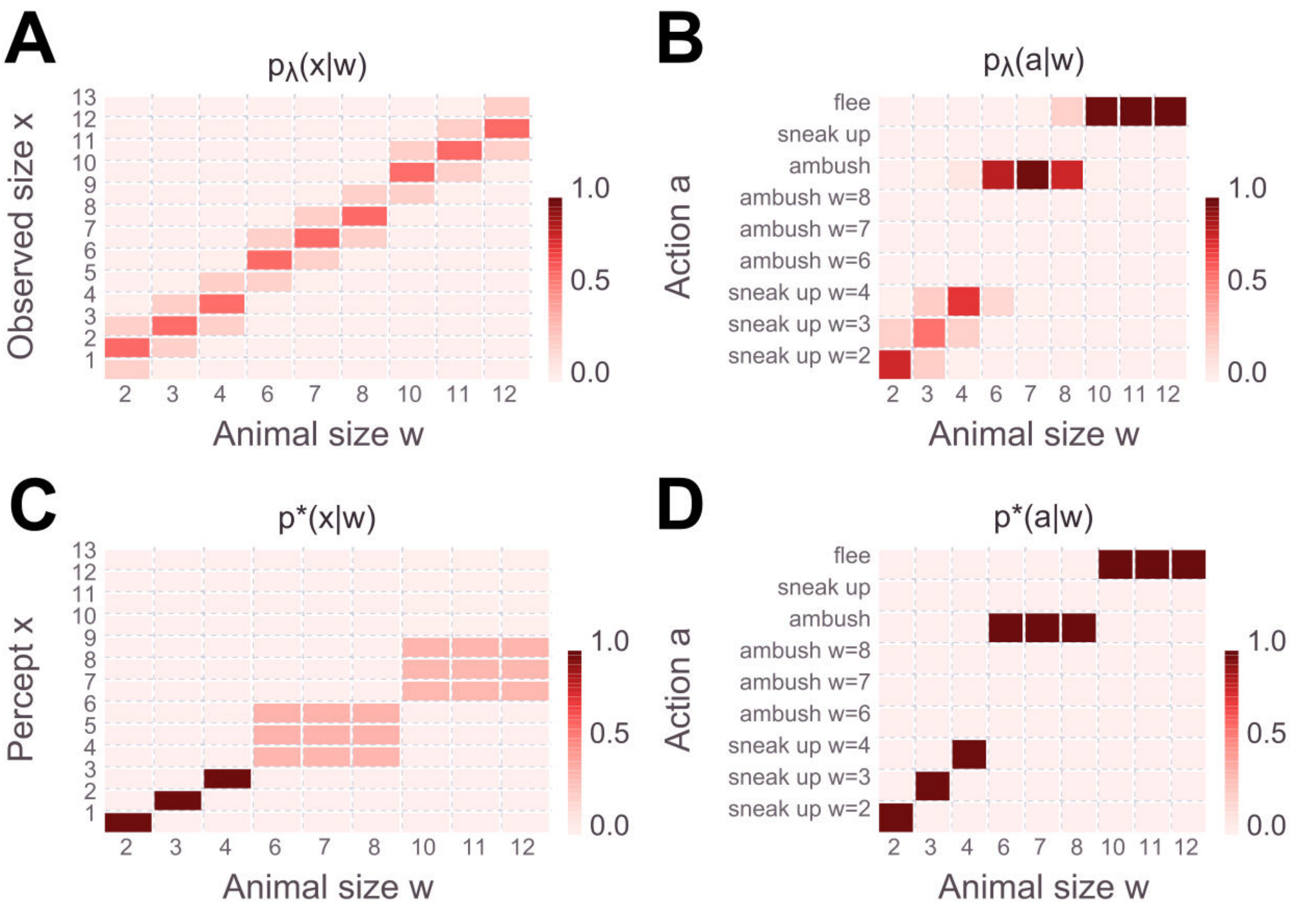

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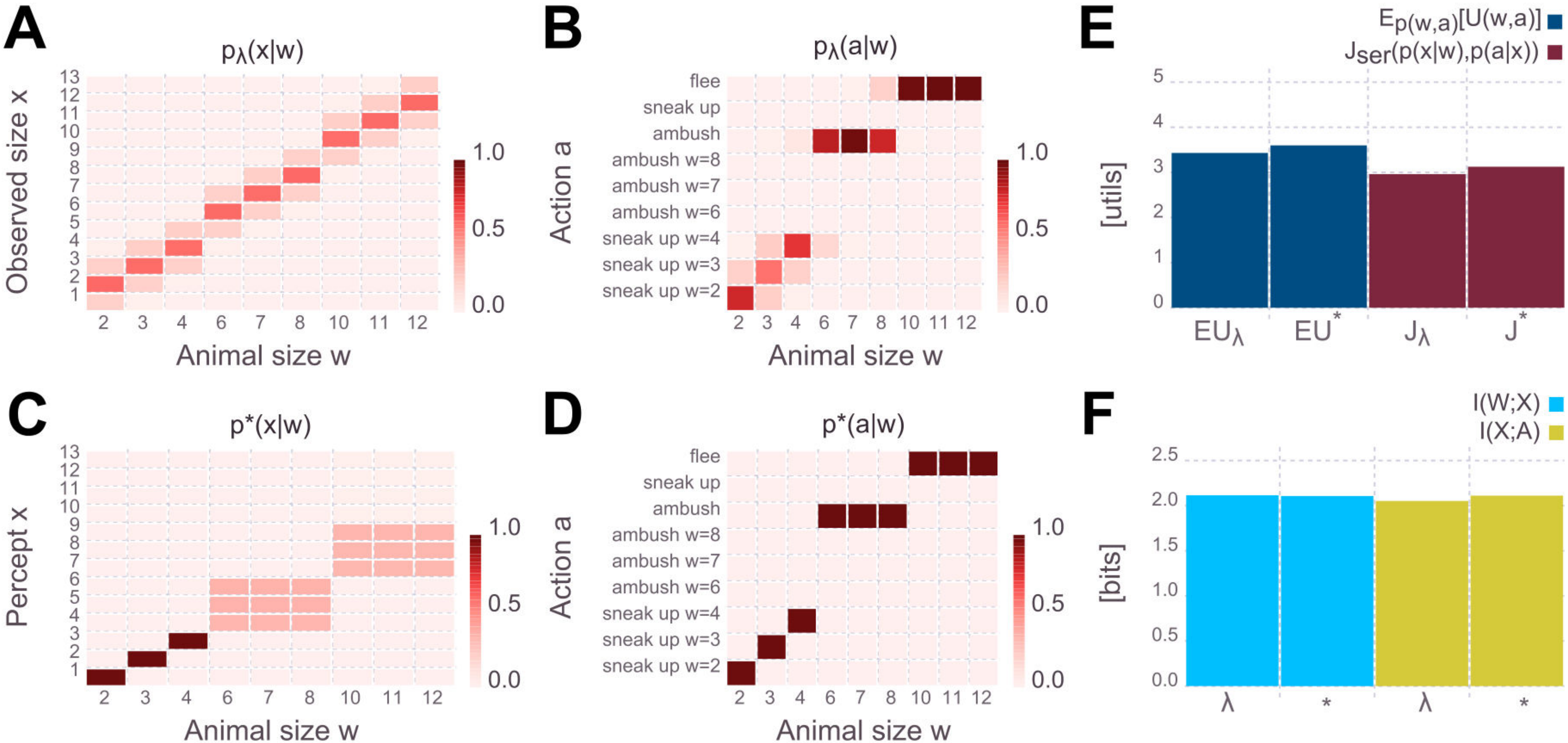
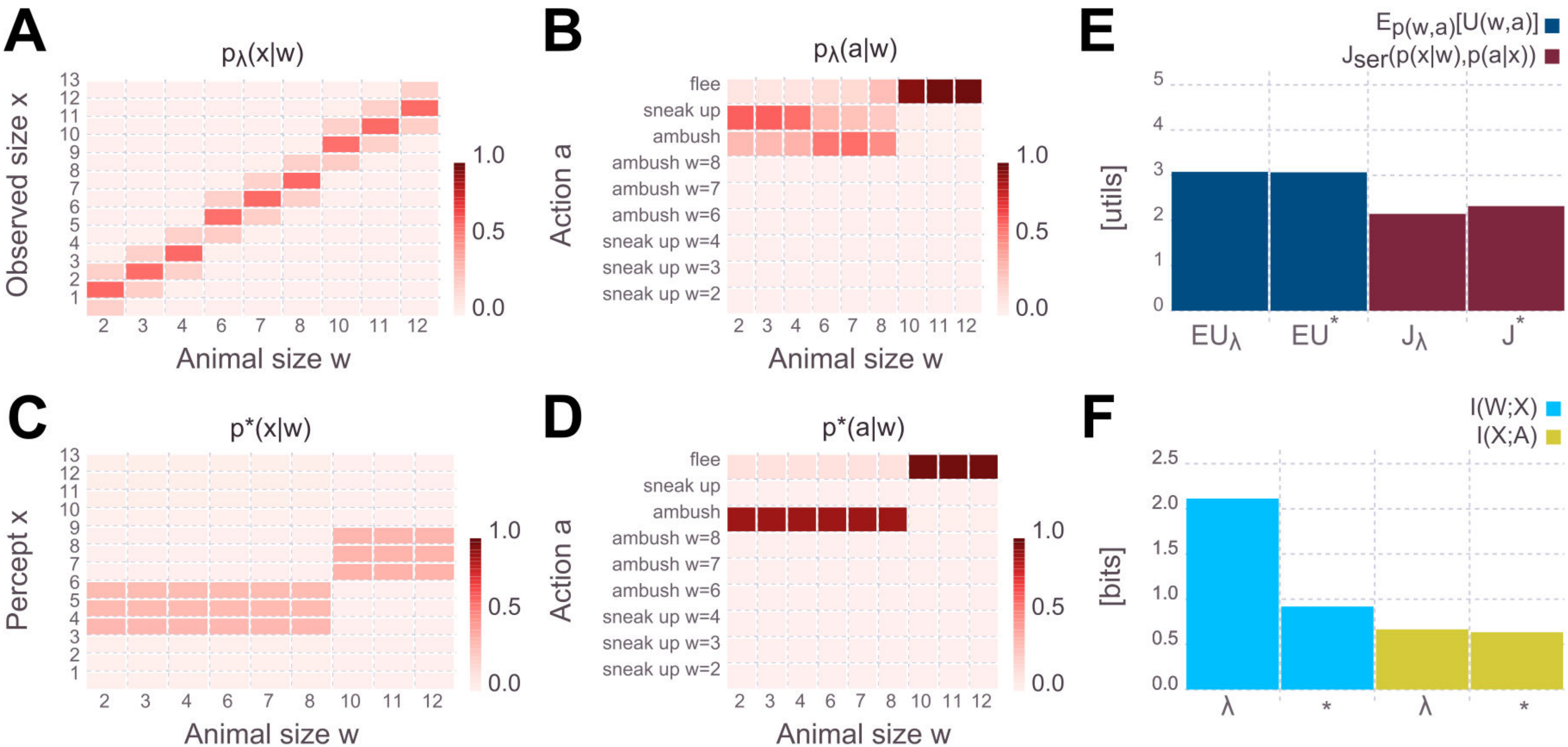
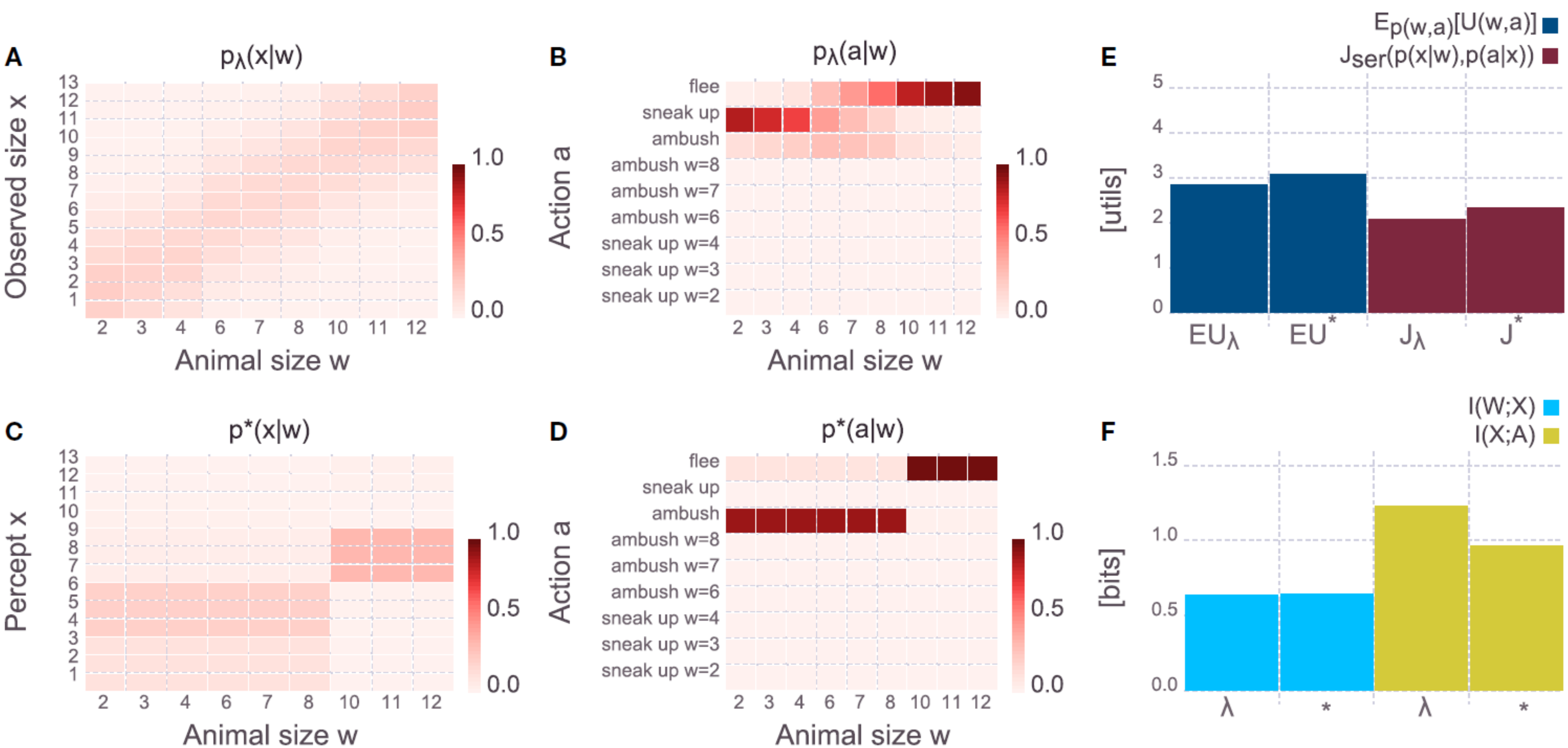


Low action capacity (bad motor skills)



Low perceptual capacity (bad vision)



Conclusions

Information-theoretic optimality principle

- Limited computational resources are part of the optimization problem
  - Formalization of limited resources as KL-divergence
  - Trade off gains in utility against cost of computation

- Mathematical and conceptual relations to:
  - Thermodynamics, Statistical mechanics
  - Information theory, lossy compression
  - Path integral control, variational Bayes, relative entropy policy search, KL-control, G-learning, …

- Modelling perception-action system
  - Bounded-optimal perception is tightly coupled to action
  - Likelihood function synthetization

Future directions:

Continuous problems / parametric distributions, large-scale problems

- Sampling schemes

  Ortega, P.A., Braun, D.A., Dyer, J.S., Kim, K.-E., and Tishby, N.
  Information-Theoretic Bounded Rationality
  ArXiv:1512.06789, 2015

- Regularizer for (deep) neural networks

  Leibfried, F., Braun, D.A
  Bounded Rational Decision-Making in Feedforward Neural Networks
  UAI 2016, ArXiv:1602.08332

Sequential decision-making problems (reinforcement learning)

- Tishby, Polani, and others.

  Fox, R., Pakman, A., Tishby, N.
  G-Learning: Taming the Noise in Reinforcement Learning via Soft Updates
  ArXiv:1512.08562, 2015

- Modeling computational limitations and model uncertainty

  Grau-Moya, J., Leibfried, F., Genewein, T., Braun, D.A
  Planning with Information-Processing Constraints and Model Uncertainty in Markov Decision Processes
  ArXiv:1604.02080, 2016

More information

Today's talk:

- http://tim.inversetemperature.net/research/
- Explore all examples (Jupyter notebooks):
  https://github.com/tgenewein/BoundedRationalityAbstractionAndHierarchicalDecisionMaking
- Paper:
  Genewein T., Leibfried F., Grau-Moya J., Braun D.A. (2015):
  Bounded rationality, abstraction and hierarchical decision-making: an information-theoretic optimality principle
  Frontiers in Robotics and AI, DOI:10.3389/frobt.2015.00027

Information-theoretic bounded rationality

Ortega, P.A., Braun, D.A., Dyer, J.S., Kim, K.-E., and Tishby, N.
Information-Theoretic Bounded Rationality
ArXiv:1512.06789, 2015

- Pedro Ortega's website: http://www.adaptiveagents.org/freeenergy

Thanks!