BAYESIAN MODEL SELECTION IN SENSORIMOTOR TASKS

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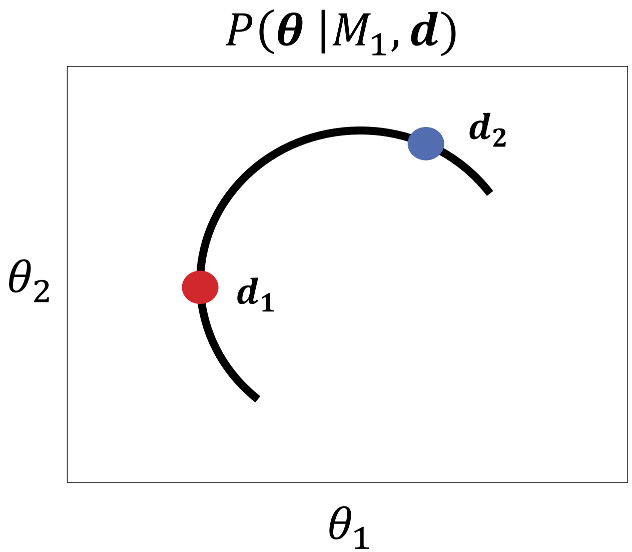
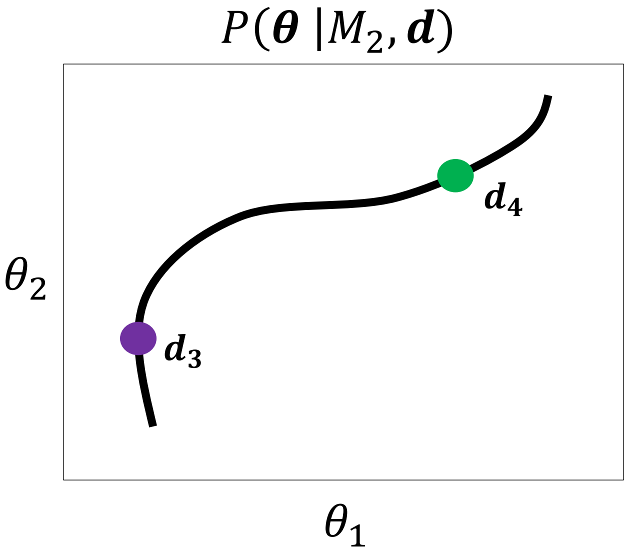
BIOLOGISCHE KYBERNETIK

Outline

- How to select among learned structures?
  - Bayesian model selection
  - Study I
    - Test for consistency with Bayesian model selection
  - Study II
    - Test for preference of simpler models - Occam‘s razor

Model Selection





- How does one select between several learned structures/models with tunable parameters?
  - Probabilistic formulation of this question

$$P(M_i|d) = ?$$

Bayesian model selection

Compare the probability of two models M_1 and M_2, given the observations d:

$$\frac{P(M_1|d)}{P(M_2|d)} = \frac{P(d|M_1)\ P(M_1)}{P(d|M_2)\ P(M_2)}$$

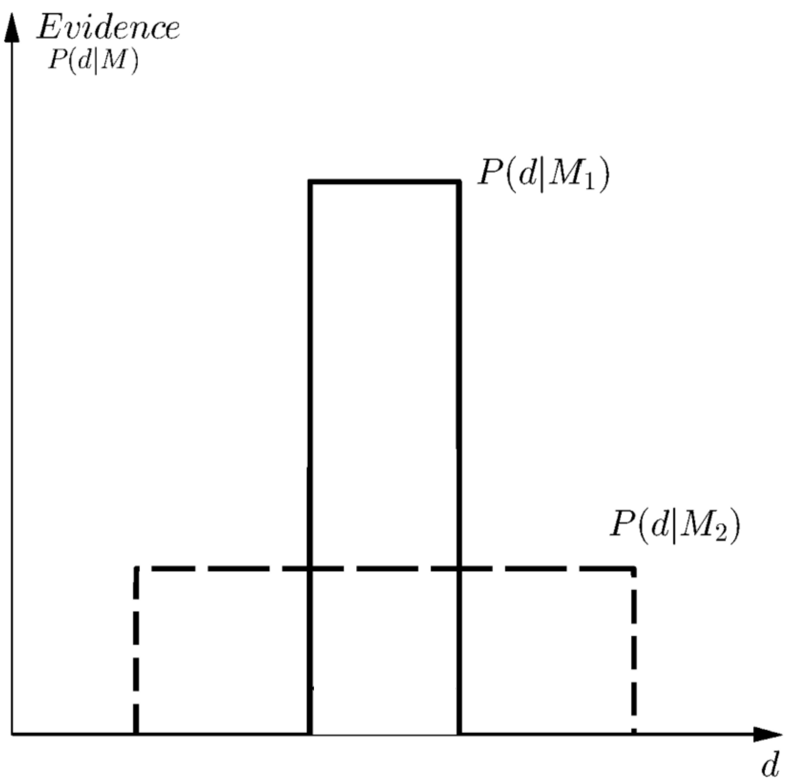
Posterior odds = Bayes factor × Prior odds

if $P(M_1) = P(M_2)$

$$\dots = \frac{P(d|M_1)}{P(d|M_2)}$$

Marginal likelihood or evidence for model M_i
by integration over parameters θ:

$$P(d|M_i) = \int P(d|M_i,\theta)P(\theta|M_i)d\theta$$



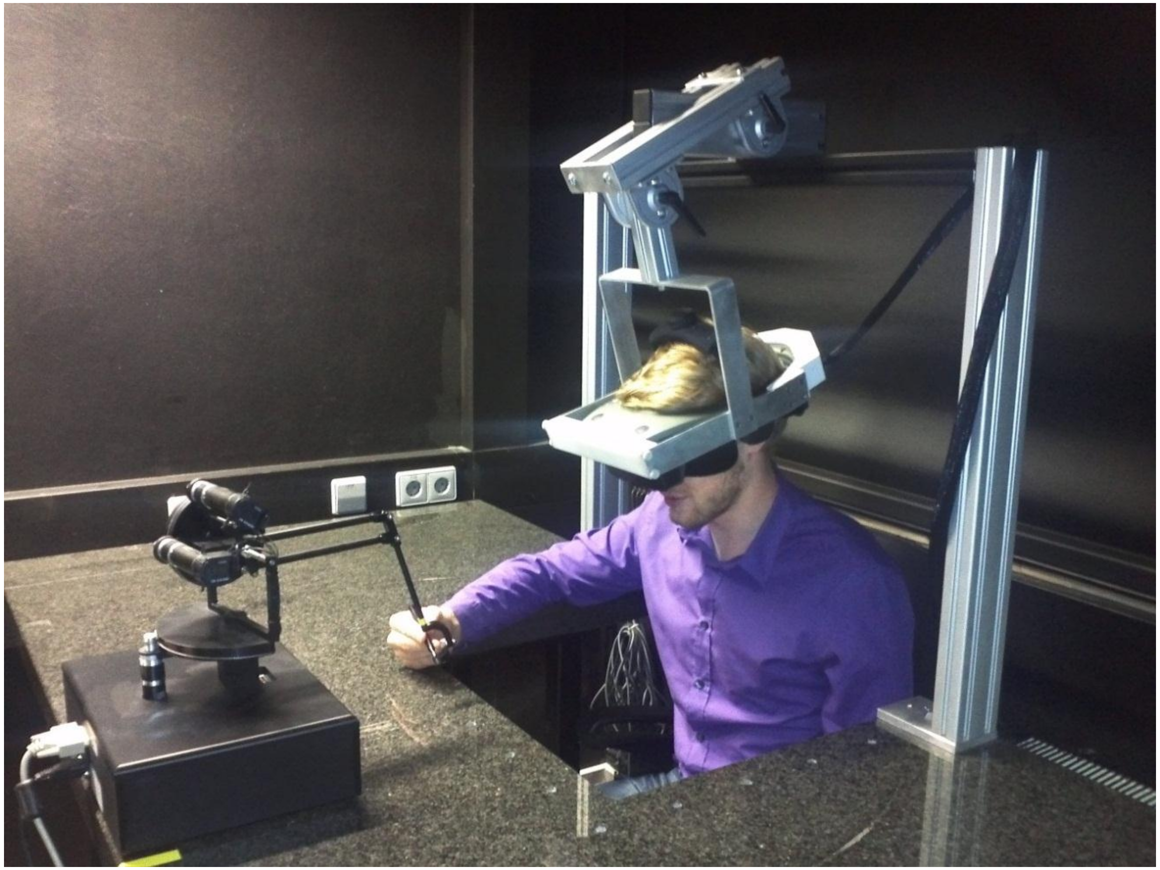
Study I

A sensorimotor paradigm for Bayesian model selection

Genewein, Braun (2012): *A sensorimotor paradigm for Bayesian model selection,* Frontiers in Human Neuroscience

Experimental Apparatus

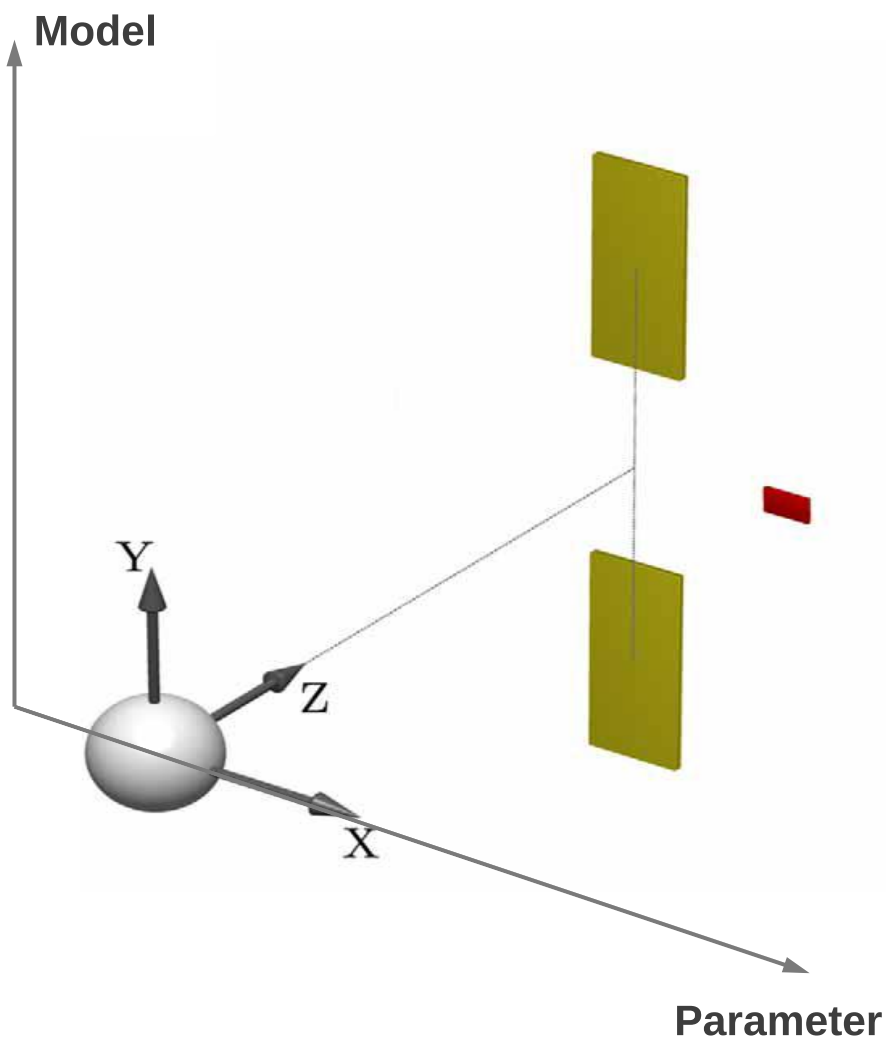
- 3D virtual reality environment for sensorimotor tasks

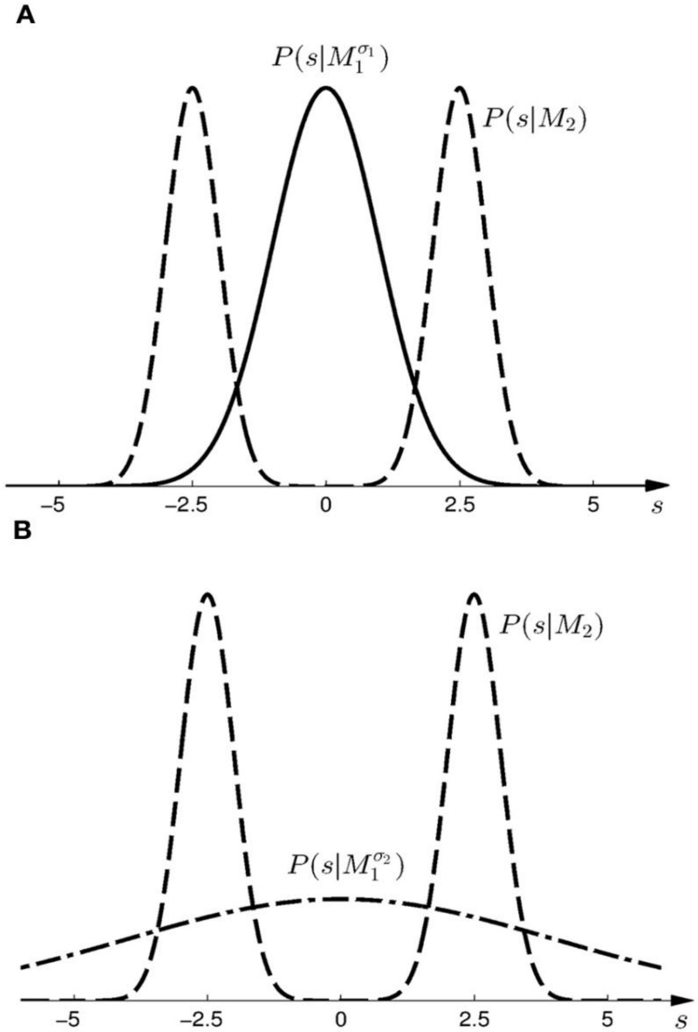


Experimental Demands

- Separate model- and parameter-dimension
- “Integrating out” of the parameters

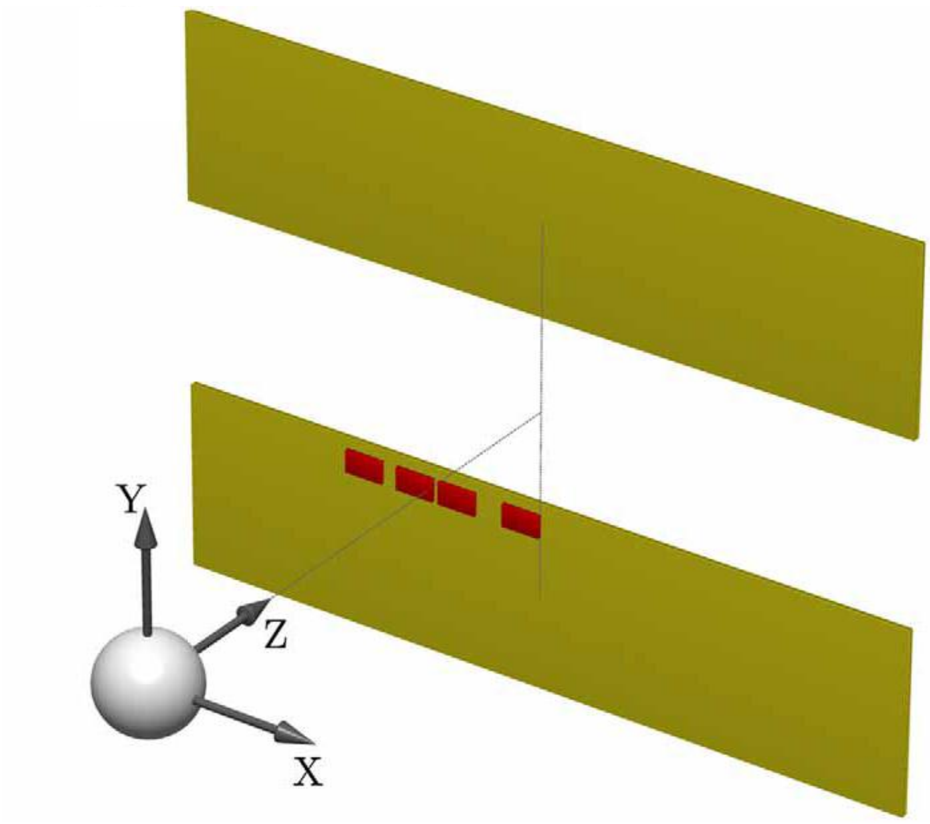
Model Selection – Experimental Setup



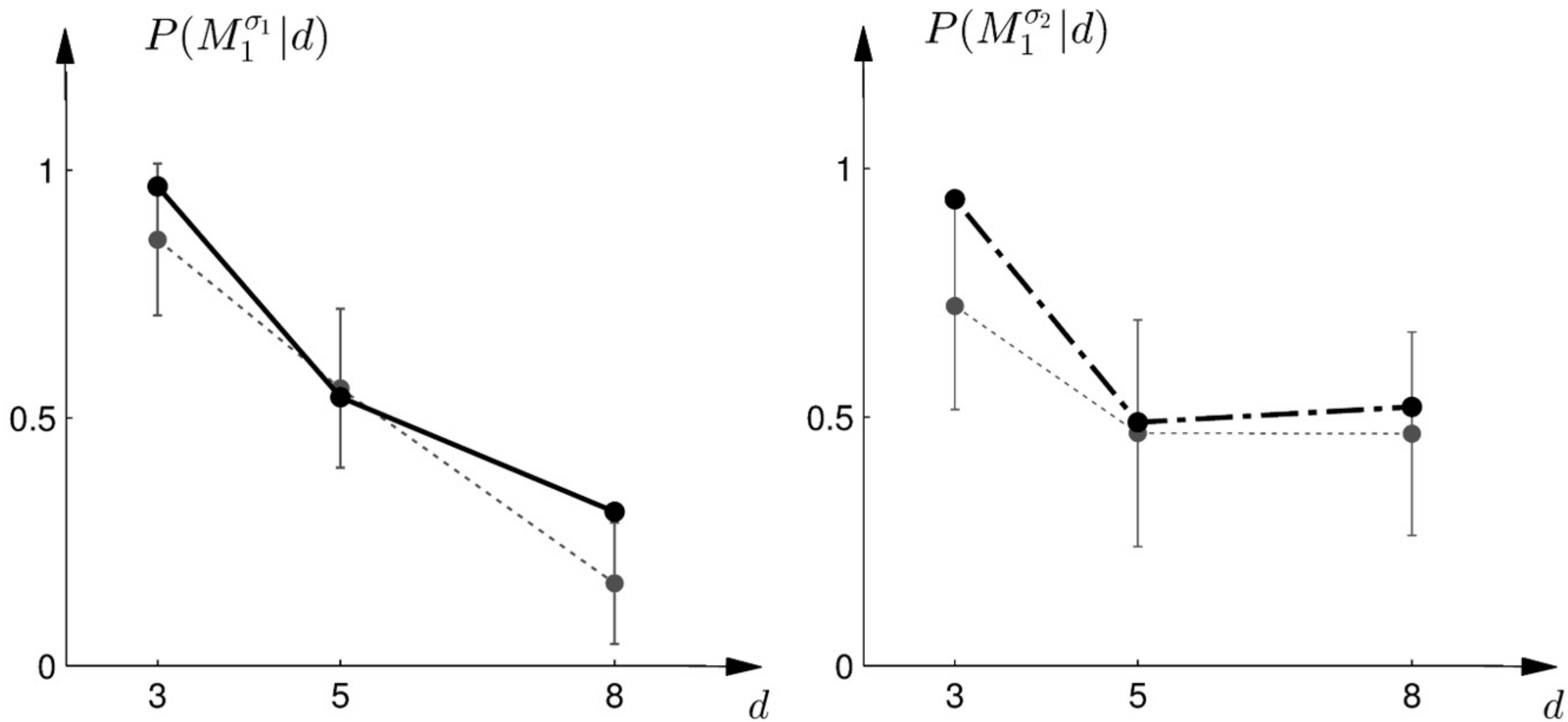


Probe Trials

- Probe trials with **ambiguous** visual feedback
  - Requires “integrating out” all possible shifts
  - Indicate model choice without indicating belief about parameter



Results – Bayes Factors



Conclusions

- Subjects were consistent with predictions given by Bayesian model selection
- Subjects‘ behavior was inconsistent with the predictions of nonprobabilistic heuristics:
  - Average shift – shift is in the middle of the cursor-array
  - Biggest shift – shift lies at the end of the cursor-array
  - Halfway shift – shift is halfway between the end and the middle

Study II

Occam‘s razor in a sensorimotor model selection task

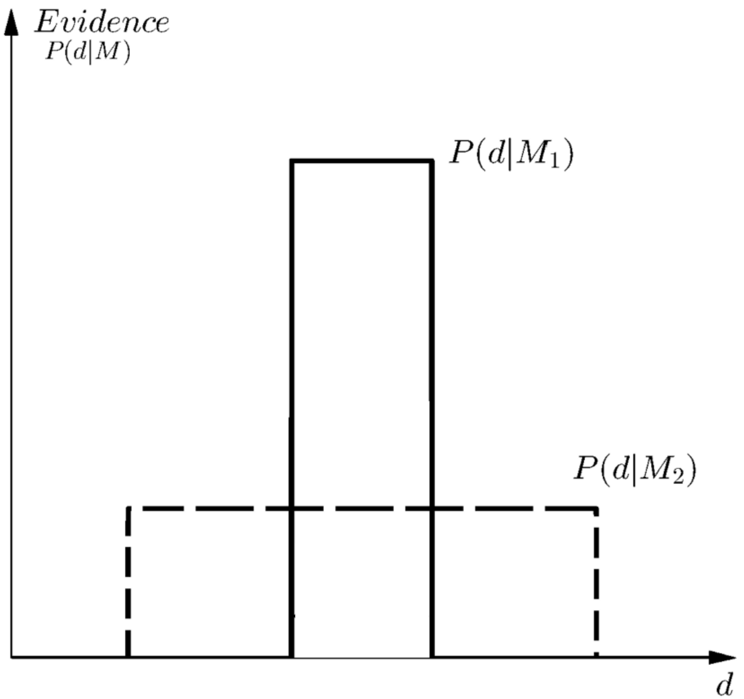
Bayesian Occam‘s Razor

- Occam‘s razor
  - “If two models explain the observations equally well, prefer the simpler model.”

- Bayesian model selection embodies Occam‘s razor
  - Previous study did not allow to directly test for Occams‘s razor because:

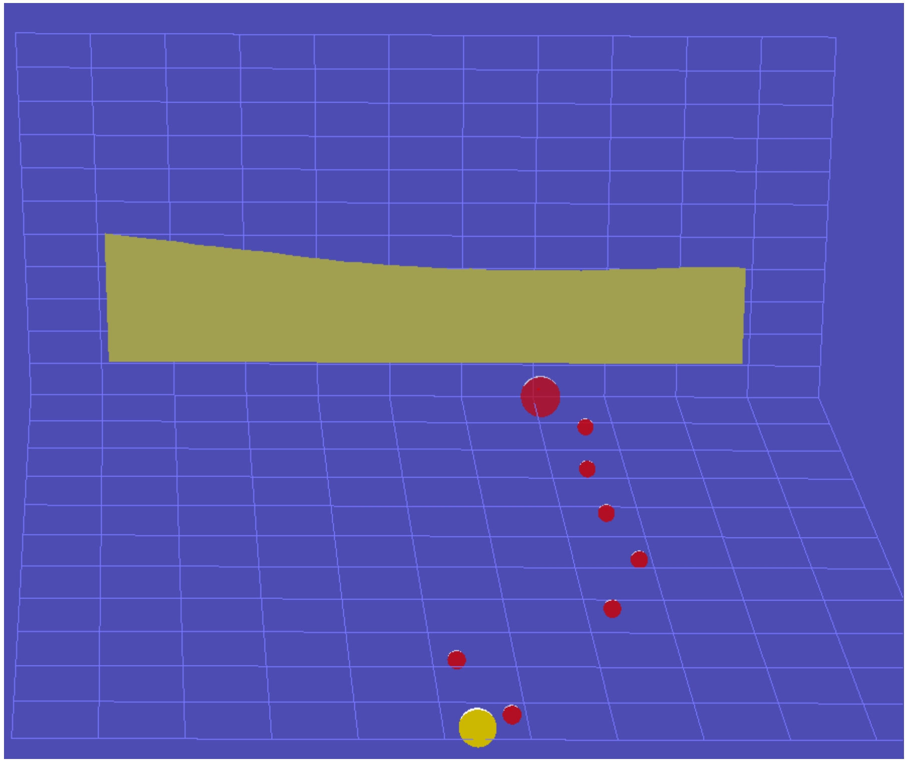
$$P(d|\theta, M_i)=P(d|\theta)$$

$$P(d|M_i) = \int P(d|\theta, M_i)P(\theta|M_i)d\theta$$



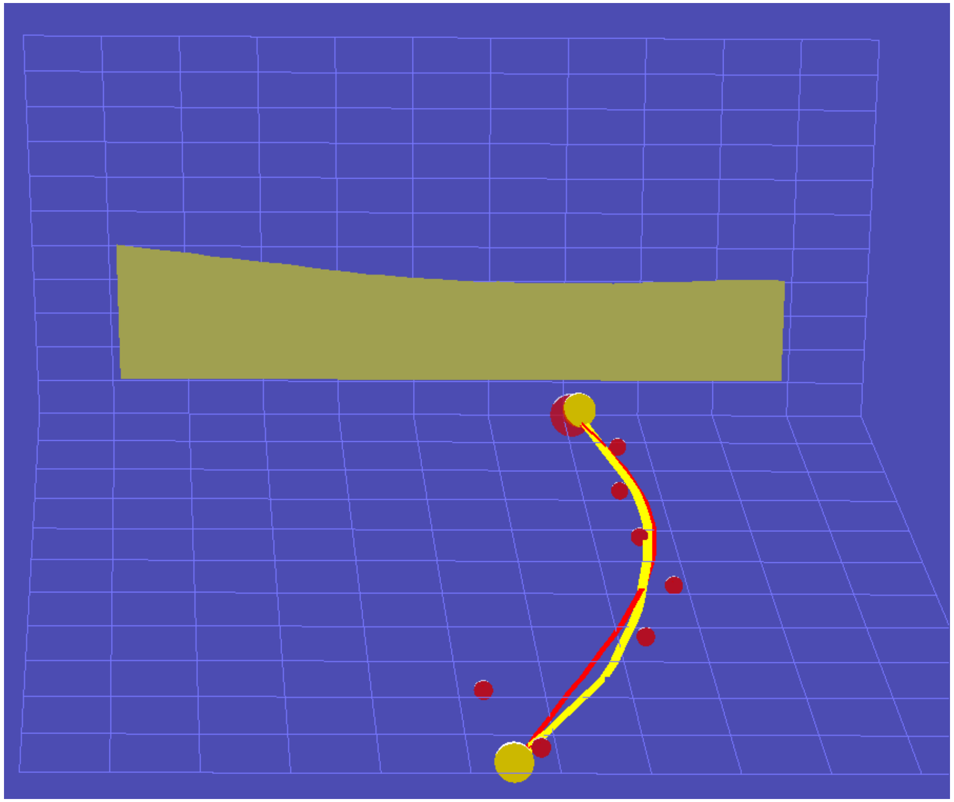
Model Selection - Regression

- Show Occam‘s razor in a more natural task
- Standard textbook task to illustrate complexity: regression
- Models: Gaussian processes with different length-scales



Regression Task

- Feedback at the end of standard trials
- Color- and landscape-cue



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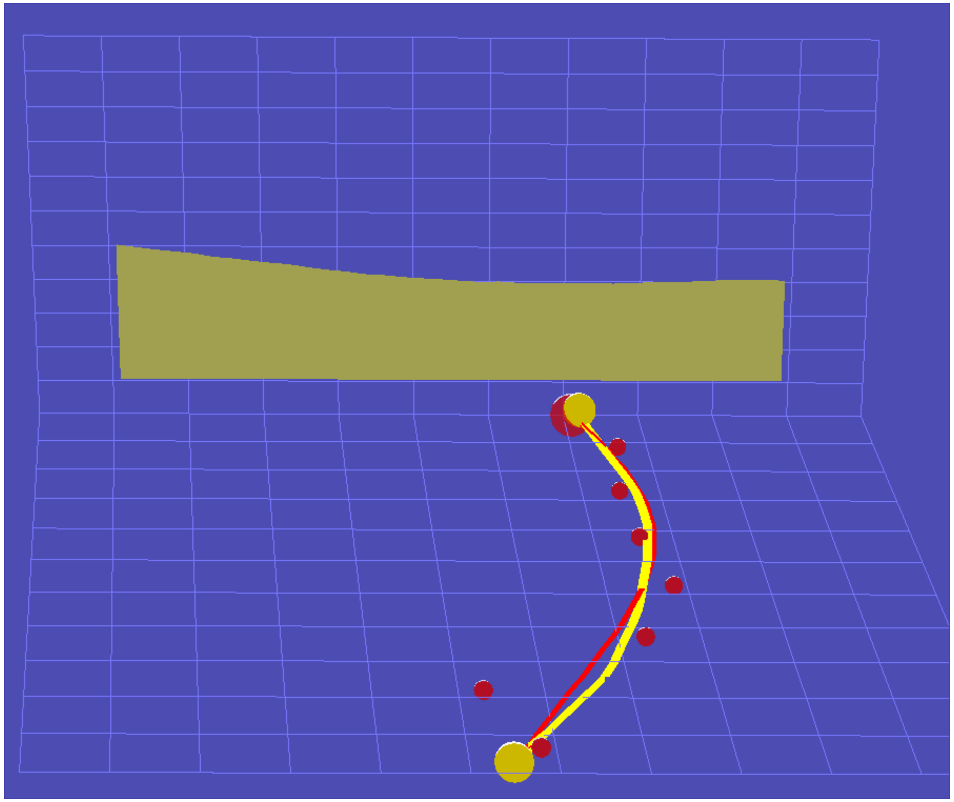
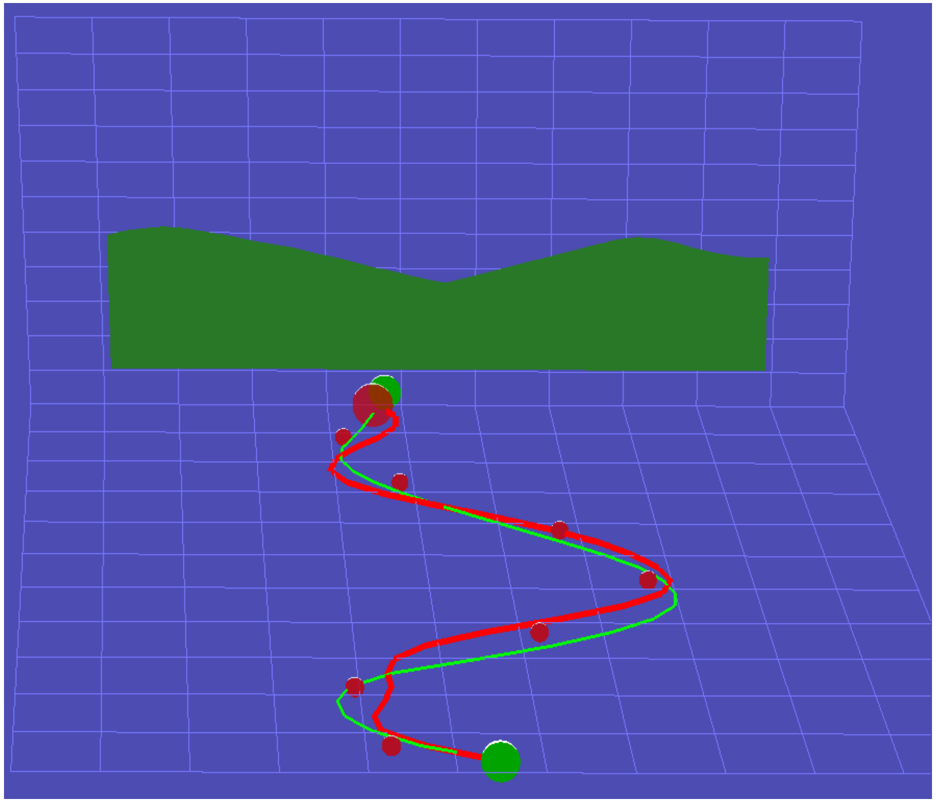
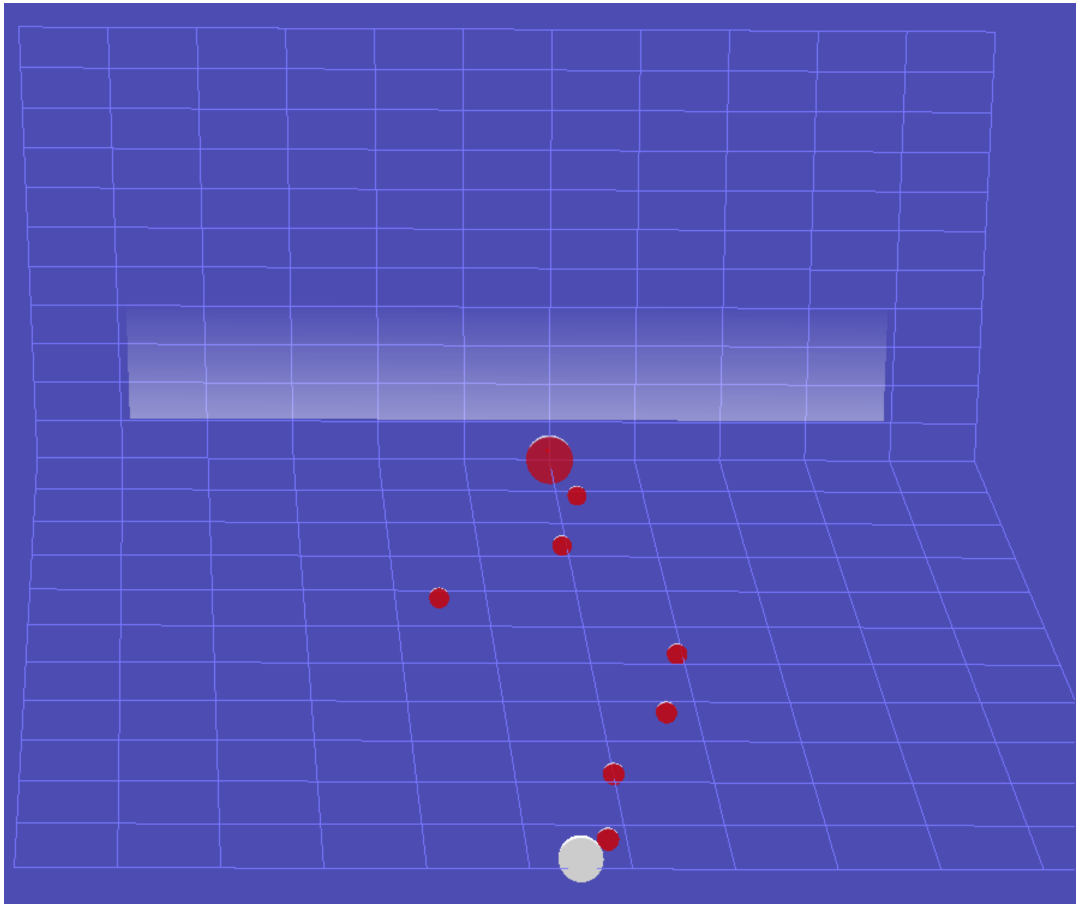




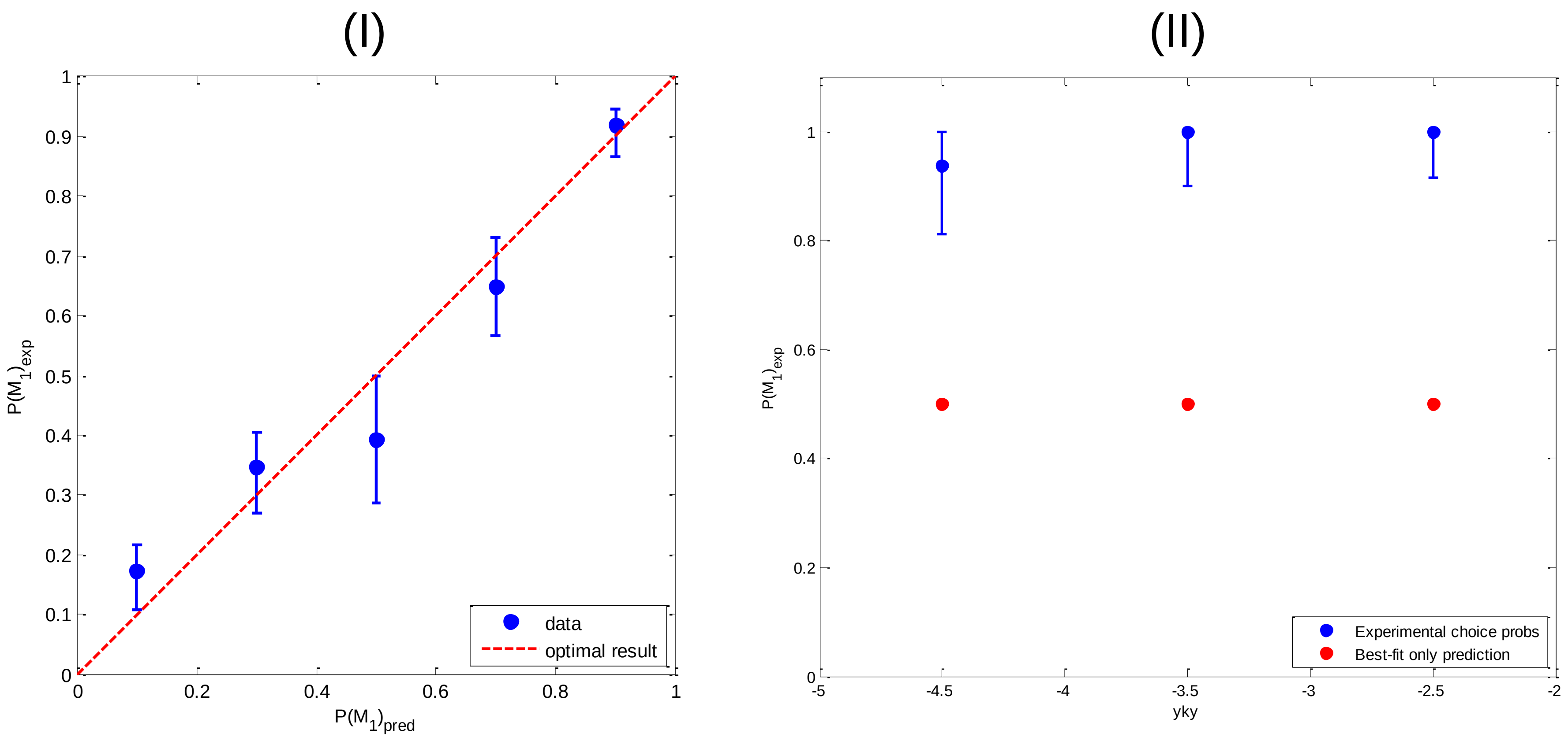
Regression – Probe Trials

- Ambiguous stimulus (neutral color and landscape)
- No feedback on model choice

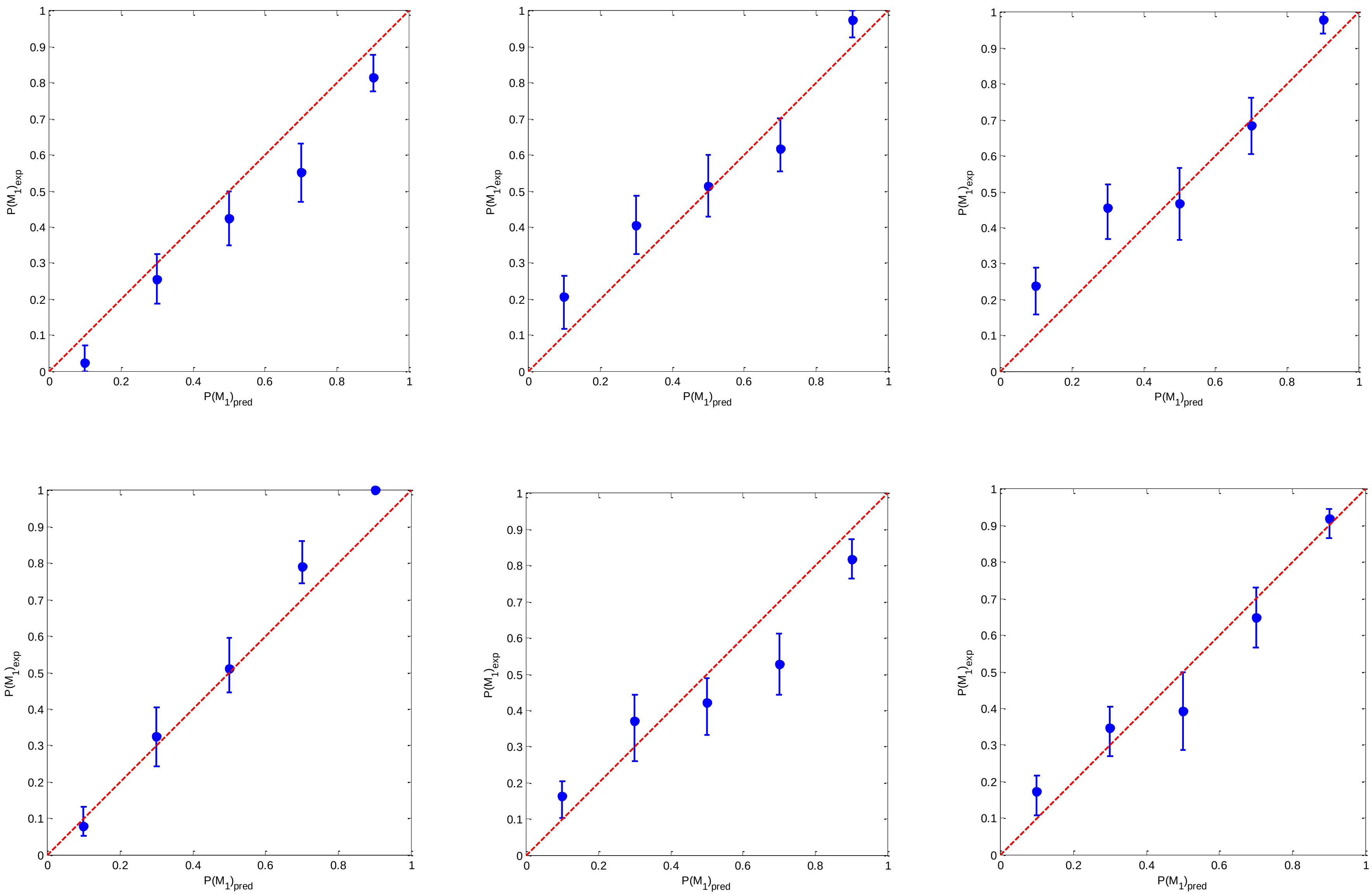


Preliminary Results

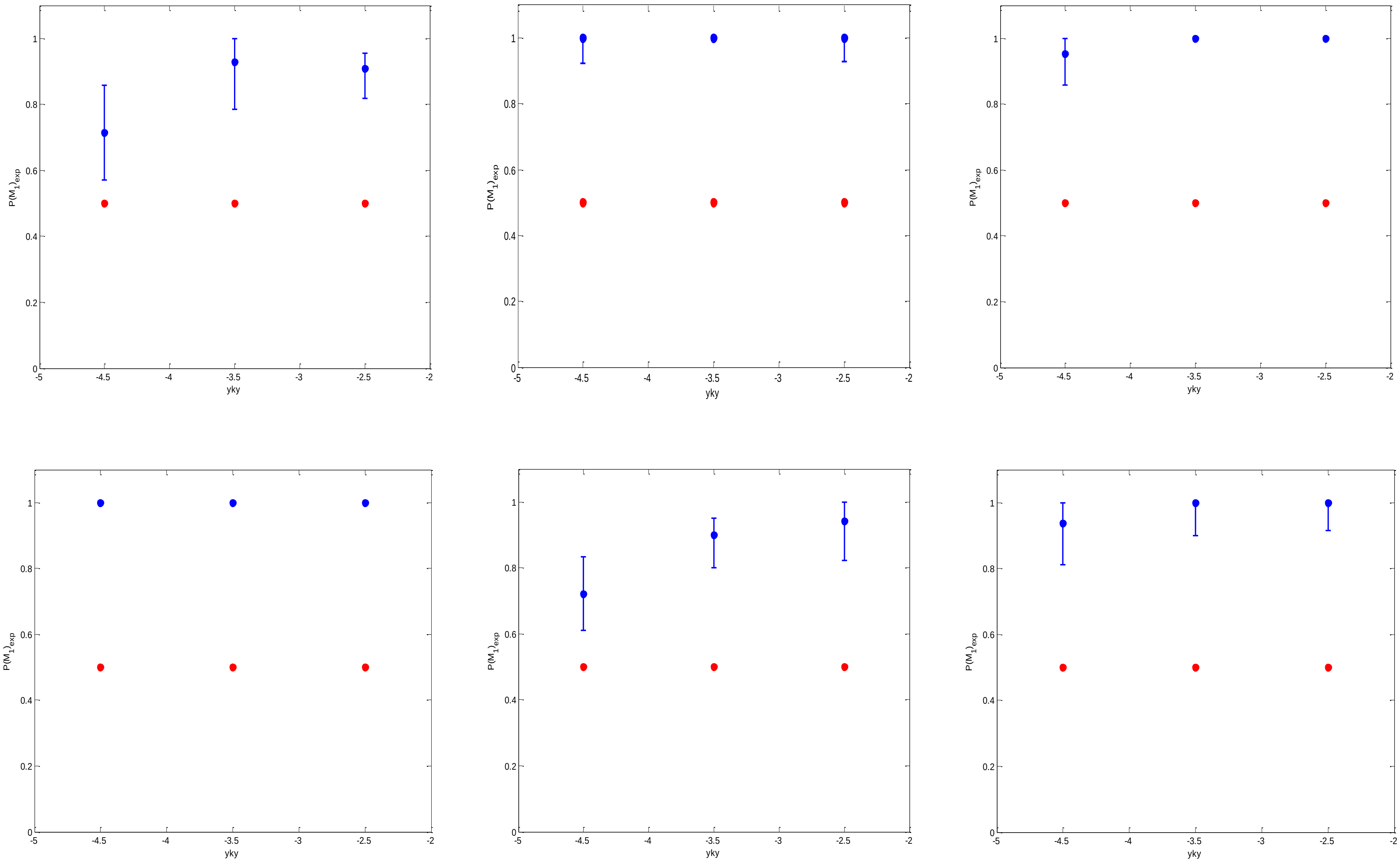
- (I) - Predicted vs. observed choice probabilities
- (II) - Choice probabilities in equal-fit condition



6 subjects – (I)

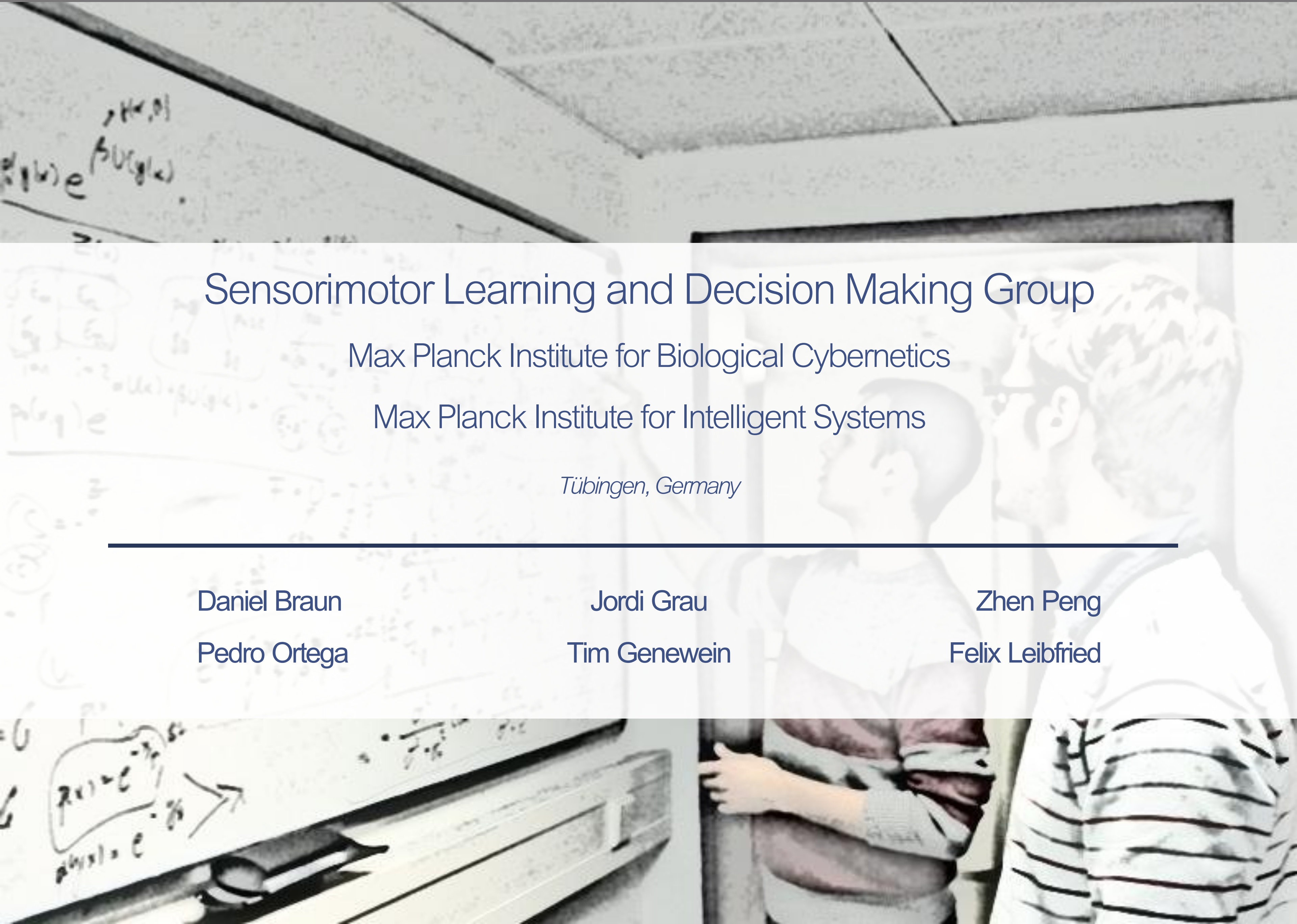


6 subjects – (II)



Summary

- We have developed a paradigm to test the predictive power of Bayesian Model Selection in a sensorimotor task
  - Observed choice behavior could be predicted by a Bayesian model
- We have also tested against alternative hypotheses (see publication)
- We have designed another paradigm to test for the preference of simpler models (Occam‘s razor)
  - Recorded data supports the application of the Bayesian framework for modelling human behavior in sensorimotor choice tasks



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Gaussian Processes

Noisy observations are drawn from a Gaussian posterior:

$$\mathbf{y} \sim \mathcal{N}(\mathbf{0}, K + \sigma_n^2 I)$$

Log marginal likelihood (evidence) as required for computing the Bayes factor:

$$\log p(\mathbf{y}|X) = \underbrace{-\tfrac{1}{2}\mathbf{y}^\top (K + \sigma_n^2 I)^{-1}\mathbf{y}}_{\text{observation dependent term}} - \underbrace{\tfrac{1}{2}\log|K + \sigma_n^2 I|}_{\text{complexity (volume)}} - \tfrac{n}{2}\log 2\pi$$

with:

$\boldsymbol{K} = kernel(\boldsymbol{X}, \boldsymbol{X})$

$\boldsymbol{y}$ ... $observed\ (noisy) values$

$\boldsymbol{X}$ ... $input\ (or\ training) locations$

n ... $number\ of\ input\ locations$

σ_n^2 ... $variance\ of\ observation\ noise$

Rasmussen, Williams: *Gaussian Processes for Machine Learning*, 2006, MIT Press

Modeling

- Bayesian model selection – Bayes factors

Marginal likelihood

$$P(d|M_i) = \int P(d|s, M_i)P(s|M_i)ds$$

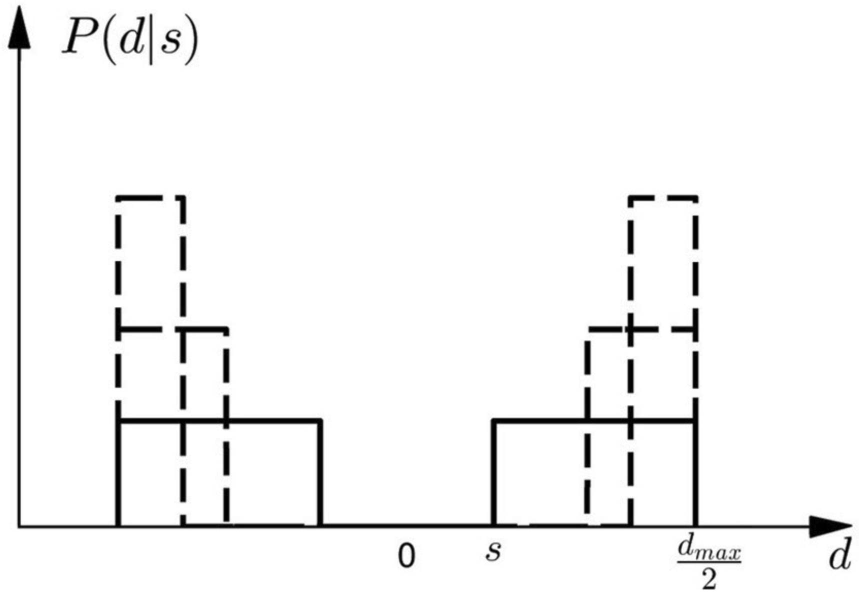
Softmax with Bayes factor

$$P(a = M_1|d) = \frac{1}{1 + e^{-\alpha \log\frac{P(d|M_1)}{P(d|M_2)}}}$$

Modeling – Likelihood Model

- $P(d|s, M_i) = P(d|s)$
- For given shift s, all array-widths d are equally probable; but the array must contain the shift (i.e. $d \geq s$)!

A



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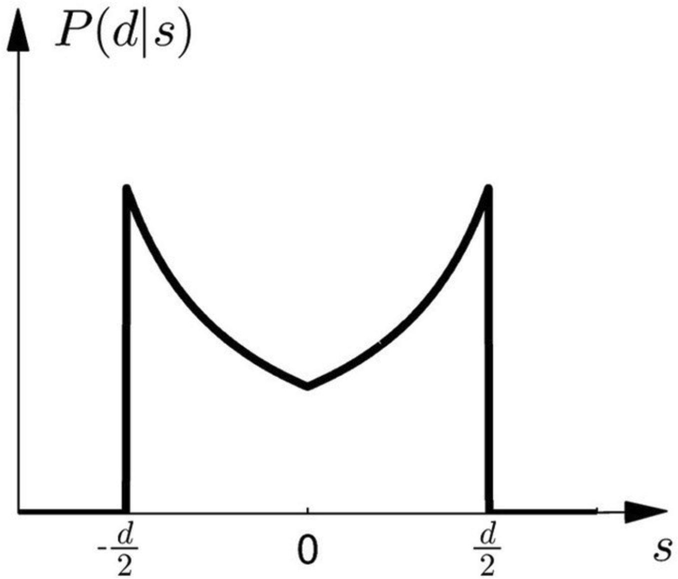


Modeling – Alternative Models

- Bayesian policy inference

$$P(a = M_1|d) = \int P(a = M_1|s)P(s|d)ds$$

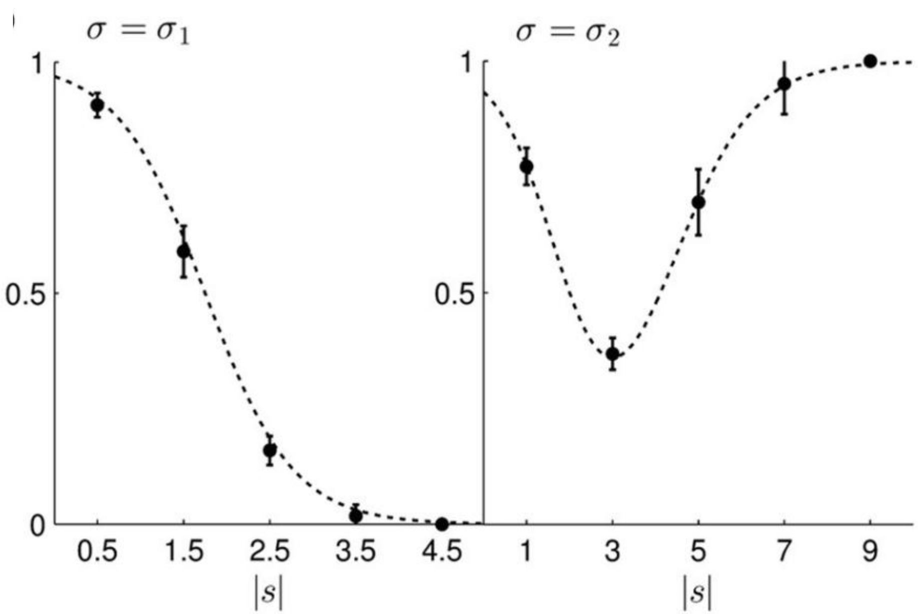
  - Stochastic superposition of given policies
  - i.e. weighted average of all shifts inside the cursor-array

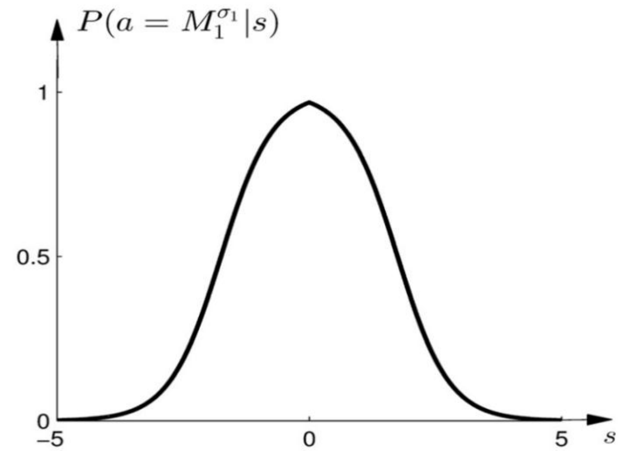
- Heuristics
  - Average shift $P(a = M_1|d) = P(a = M_1|s = 0)$
  - Biggest shift $P(a = M_1|d) = P\left(a = M_1\middle|s = \frac{d}{2}\right)$
  - Halfway shift $P(a = M_1|d) = P\left(a = M_1\middle|s = \frac{d}{4}\right)$

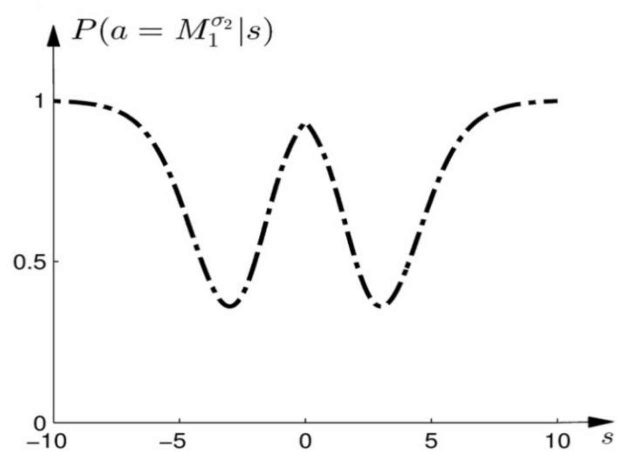
Ortega, Braun (2010): *A minimum relative entropy principle for learning and acting,* Journal of Artificial Intelligence Research

Modeling – Indiviual Fits

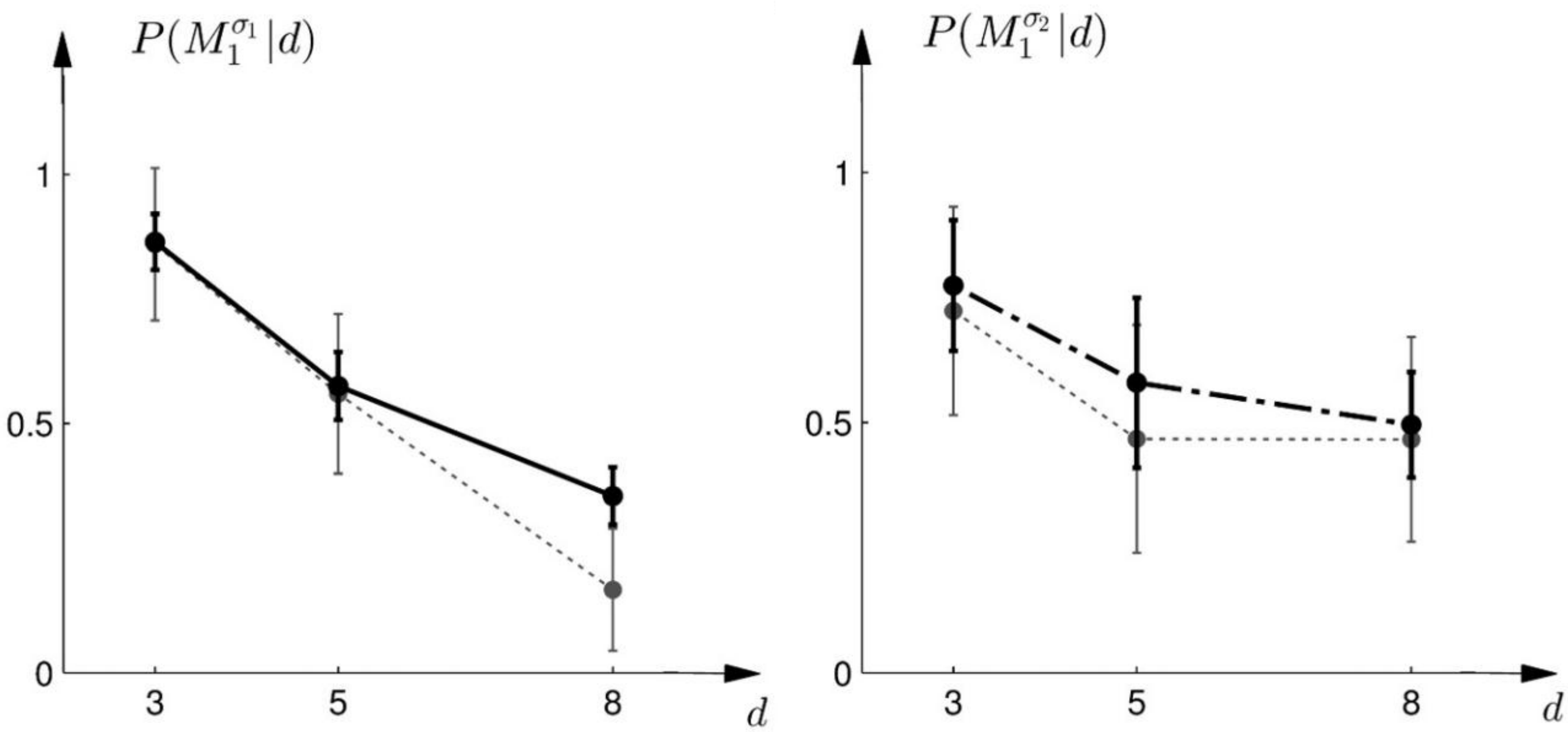
- Psychometric functions fitted to the standard trials
- $P(a = M_1|s)$





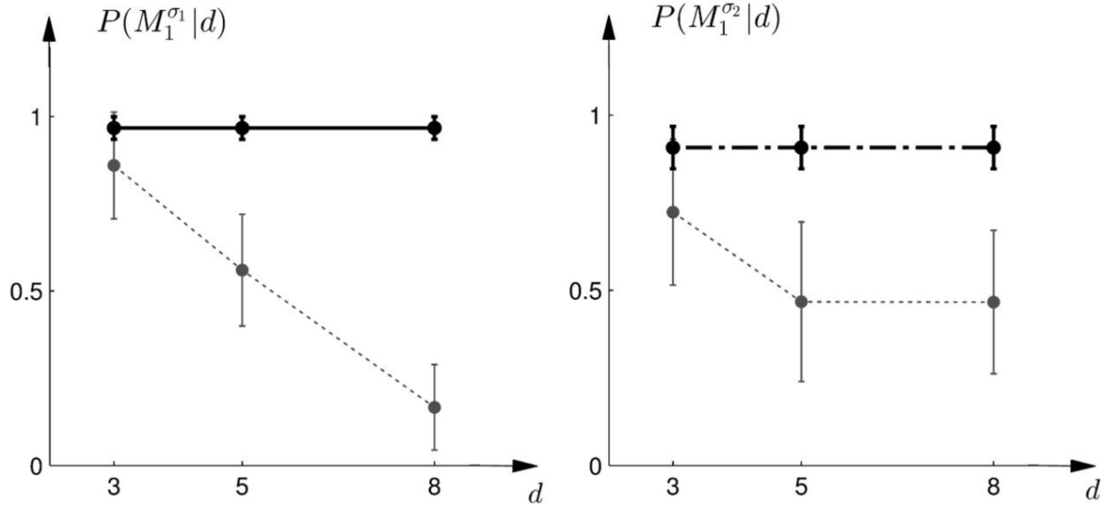


Results – Bayesian Policy Inference

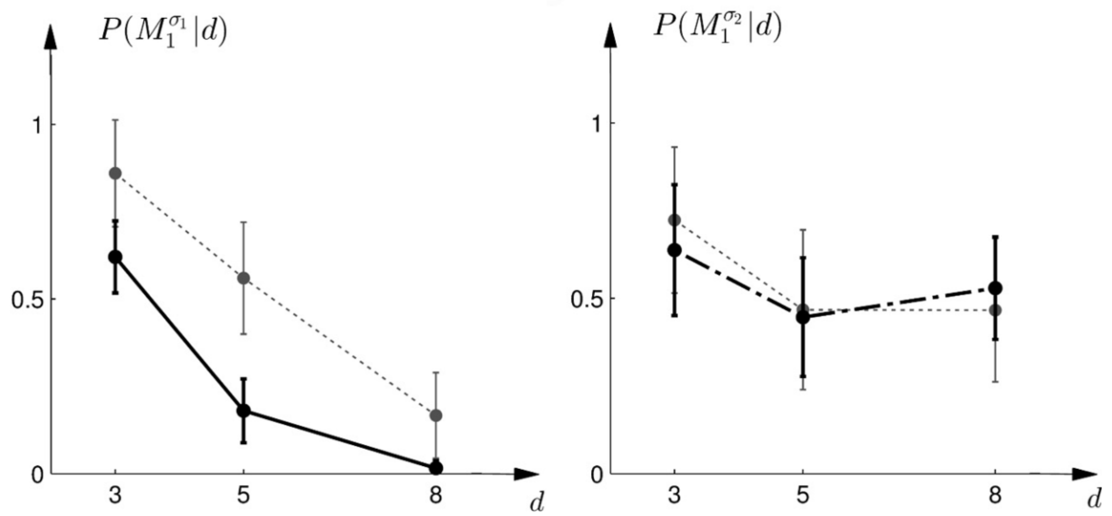


Results - Heuristics

Average shift



Biggest shift



Results – Halfway-Shift Heuristic