Relationships

Week 10

AEM 2850 / 5850 : R for Business Analytics

Cornell Dyson

Spring 2023

Acknowledgements: Andrew Heiss

Announcements

Reminders:

- Group project due April 14 (link)
  - We set up group-specific workspaces on Posit Cloud for the project to allow simultaneous collaborative editing
  - Instructions are posted there and on canvas
  - Make a plan and start early!
- Local R install instructions posted (link)
  - You are welcome to work locally if that's what your group prefers

Questions before we get started?

Plan for today

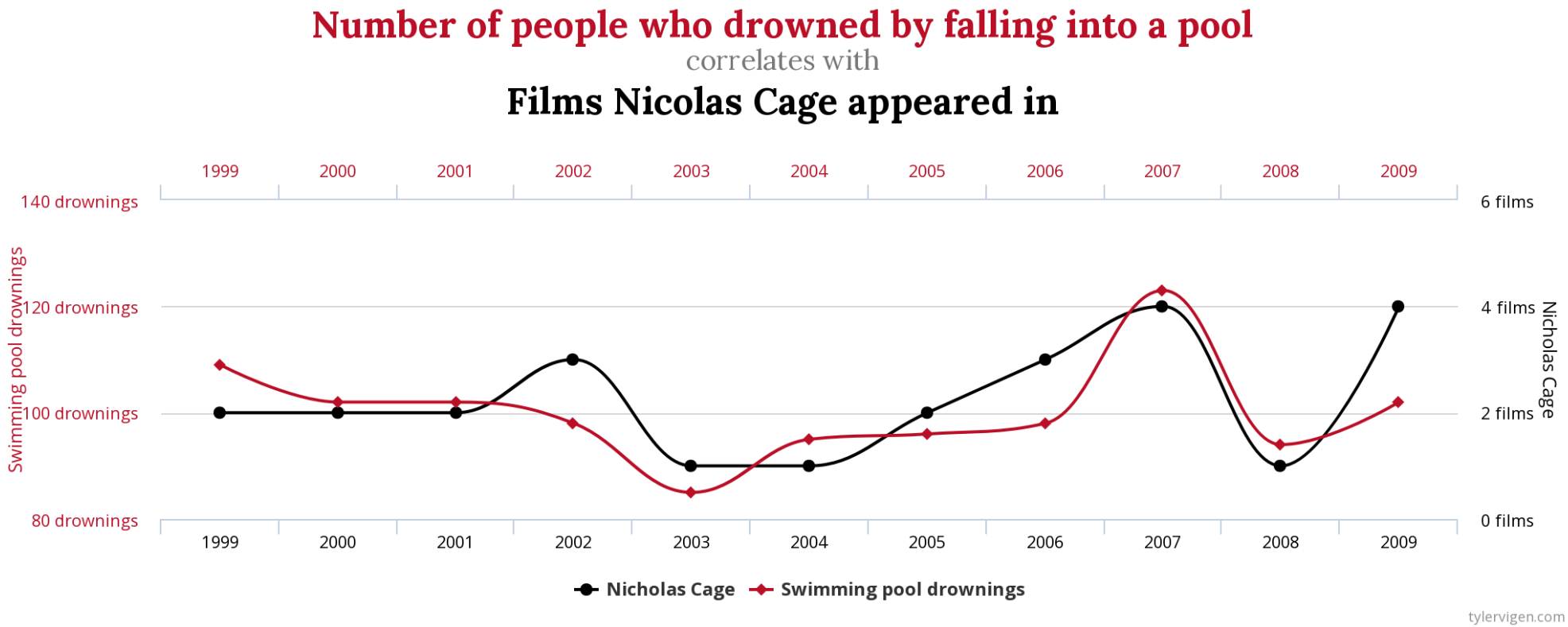
Prologue: The dangers of dual y-axes

Visualizing correlations

Visualizing regressions

Prologue: The dangers of dual y-axes

Oh no!



Source: Tyler Vigen's spurious correlations

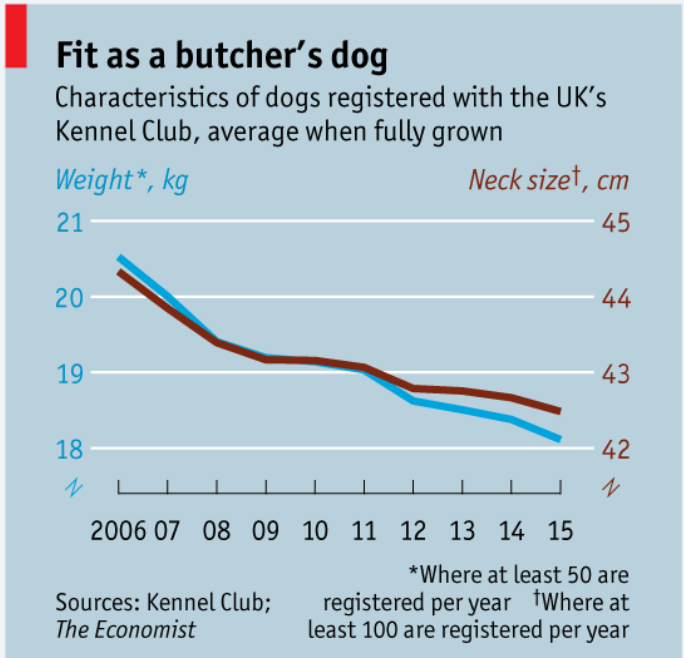
Why not use two y-axes?

You have to choose where the y-axes start and stop, which means...

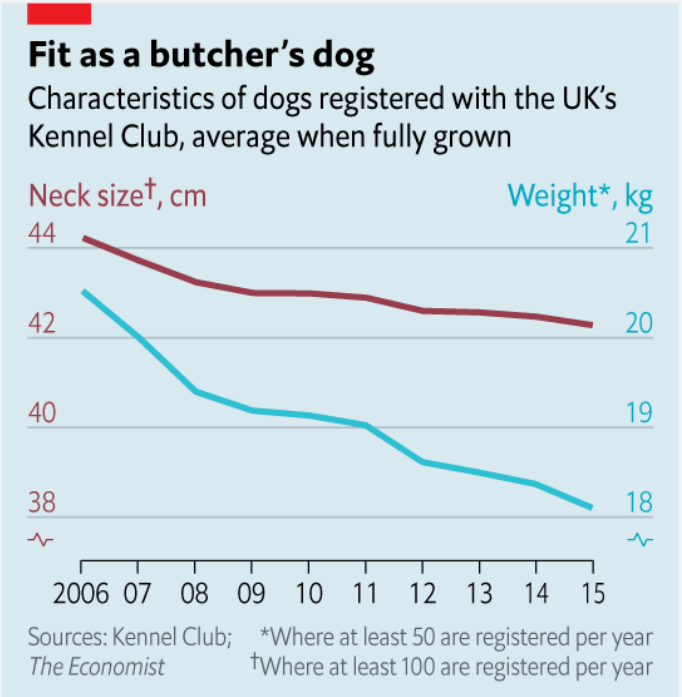
...you can force the two trends to line up however you want!

It even happens in *The Economist*!

Original

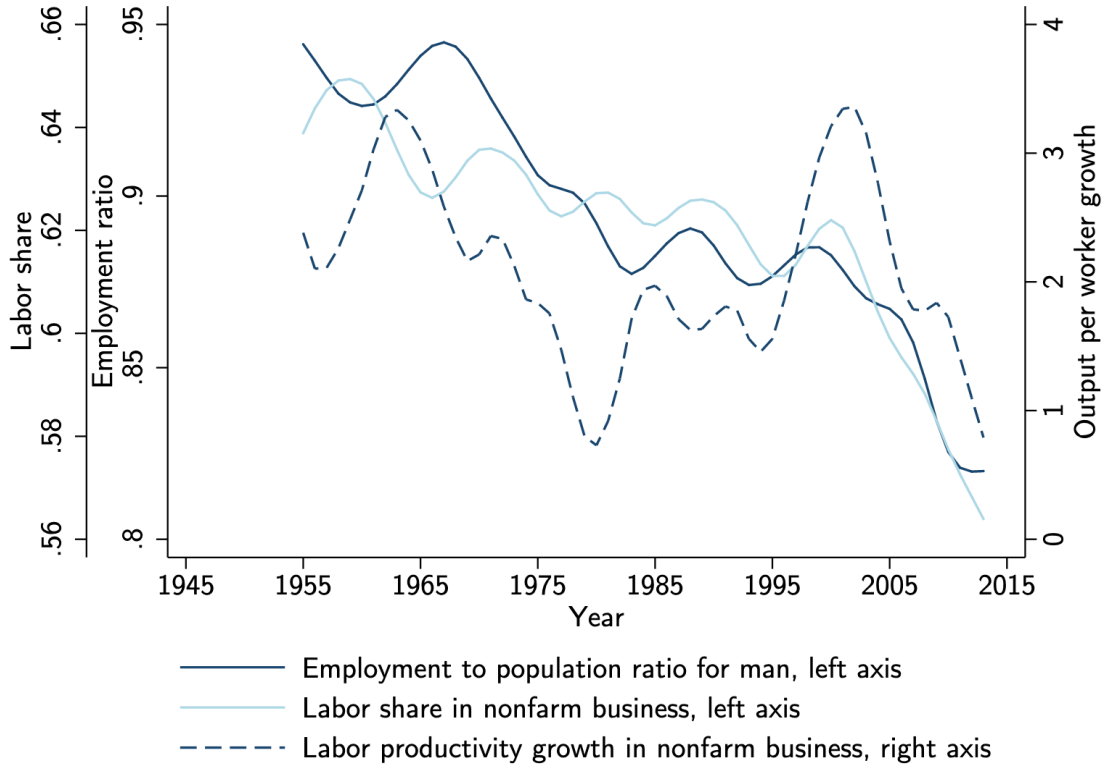


Better



The revised axes ranges reflect a comparable proportional change

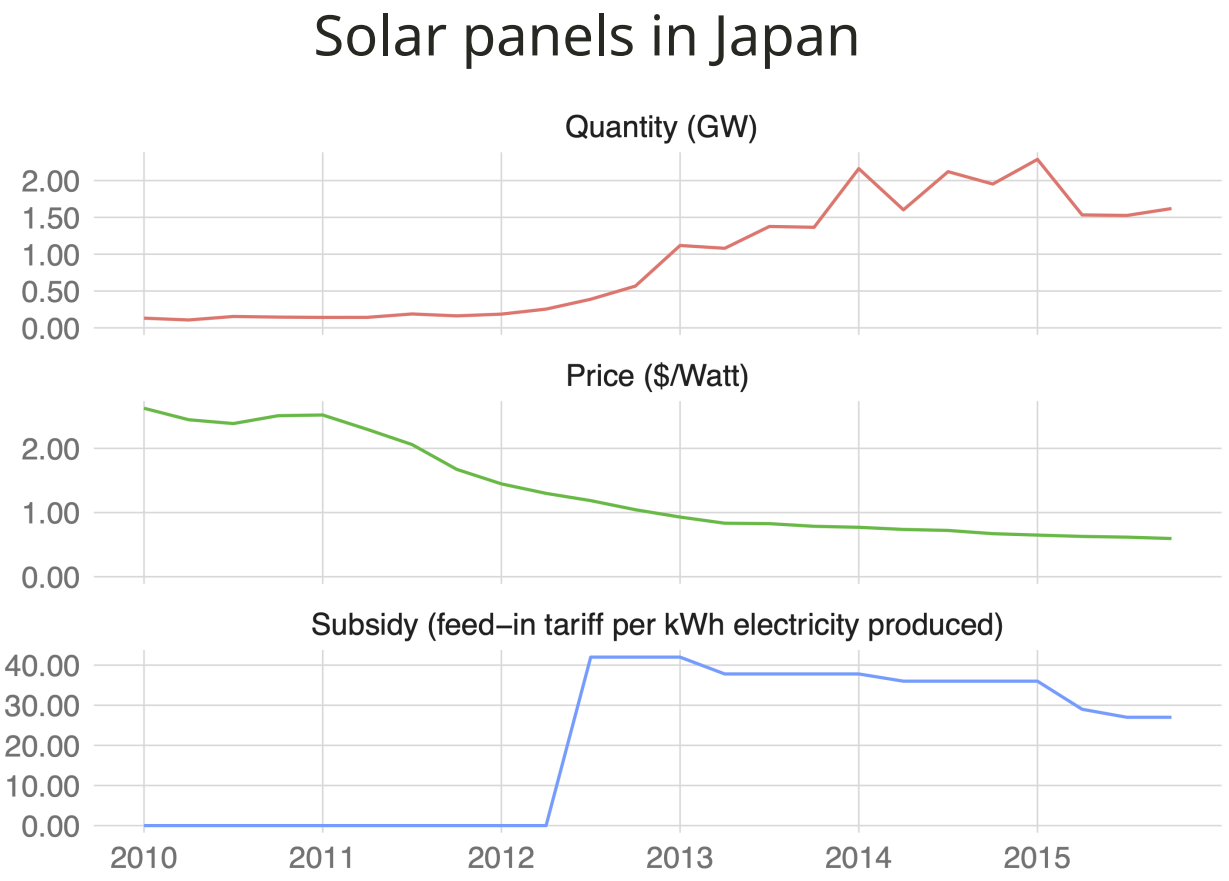
The rare triple y-axis



Source: Daron Acemoglu and Pascual Restrepo, "The Race Between Man and Machine: Implications of Technology for Growth, Factor Shares and Employment"

What could we do instead?

Use multiple plots!



How could we make multiple plots in R?

1. Facets are great when using a common geometry (we've already seen that)

2. Combining multiple plot objects can be more flexible

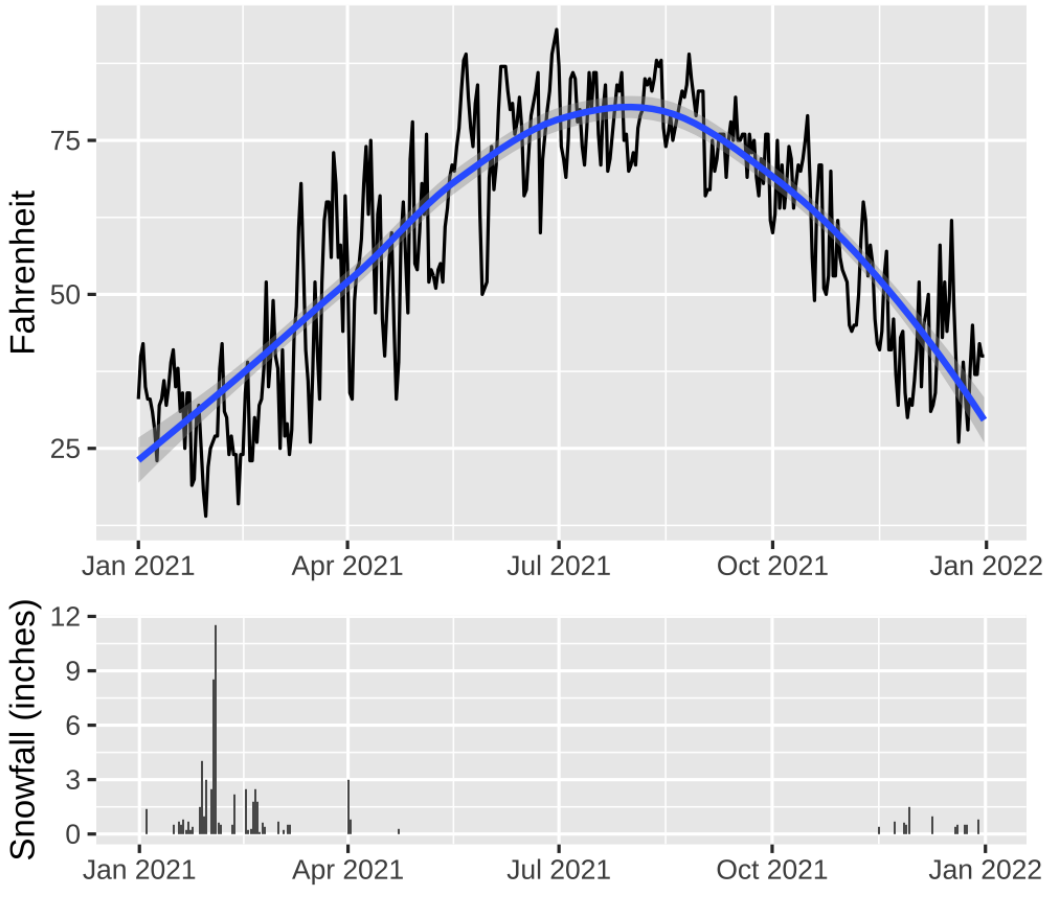
Let's use Ithaca weather data to see an example of combining plots:

```
ithaca_weather <- read_csv("data/ithaca-weather-2021.csv")

ithaca_weather |>
  select(STATION, NAME, DATE, TMAX, SNOW) |>
  head(3)
```

```
## # A tibble: 3 × 5
##   STATION     NAME                                DATE        TMAX  SNOW
##   <chr>       <chr>                               <date>     <dbl> <dbl>
## 1 USC00304174 ITHACA CORNELL UNIVERSITY, NY US 2021-01-01    33     0
## 2 USC00304174 ITHACA CORNELL UNIVERSITY, NY US 2021-01-02    40     0
## 3 USC00304174 ITHACA CORNELL UNIVERSITY, NY US 2021-01-03    42     0
```

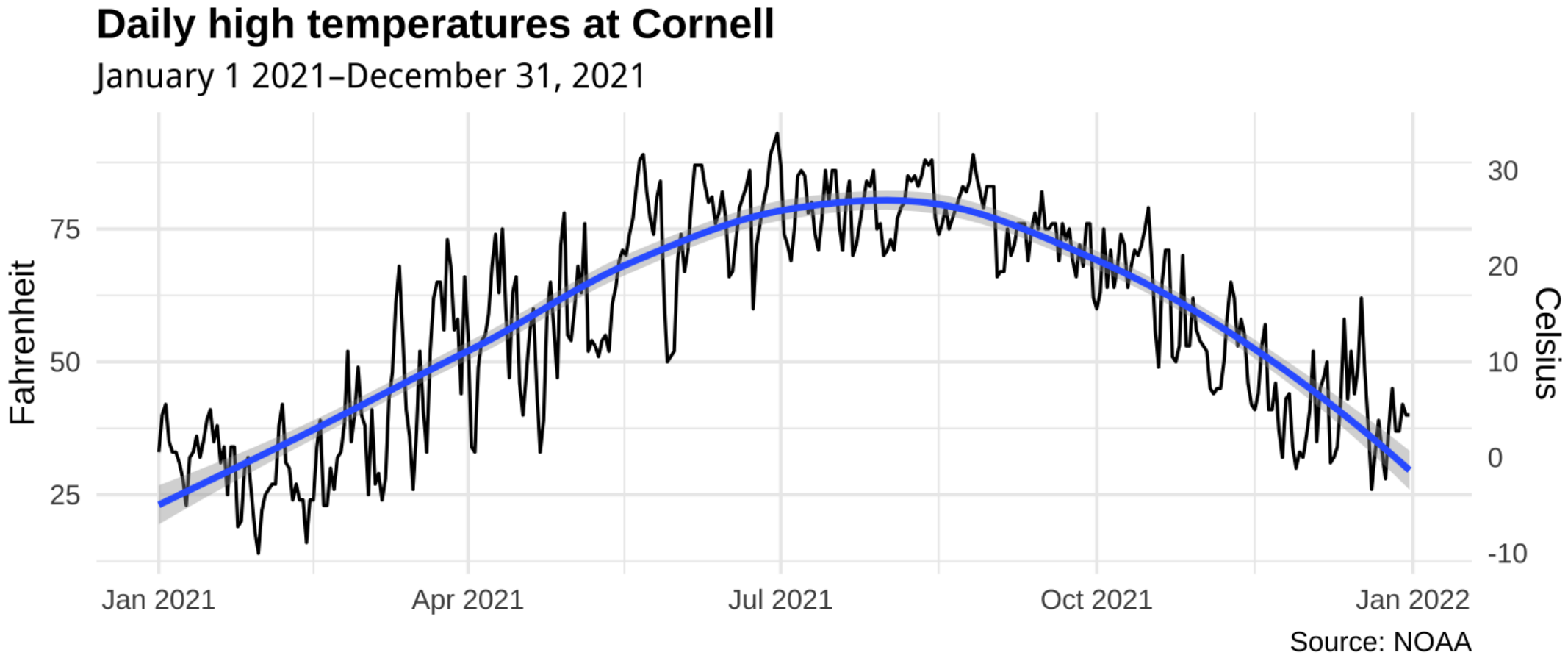
Combining multiple plots in R

```
library(patchwork)

# make a plot of temperatures
temp_plot <- ggplot(ithaca_weather,
                    aes(x = DATE, y = TMAX)) +
  geom_line() + geom_smooth() +
  labs(x = NULL, y = "Fahrenheit")

# make a plot of snowfall
snow_plot <- ggplot(ithaca_weather,
                    aes(x = DATE, y = SNOW)) +
  geom_col() +
  labs(x = NULL, y = "Snowfall (inches)")

# use patchwork to combine the two plots
temp_plot +     # simply use + to combine plots
  snow_plot +   # then add on custom options
  plot_layout(ncol = 1,
              heights = c(0.7, 0.3))
```



When are dual y-axes defensible?

When the two axes measure the same thing

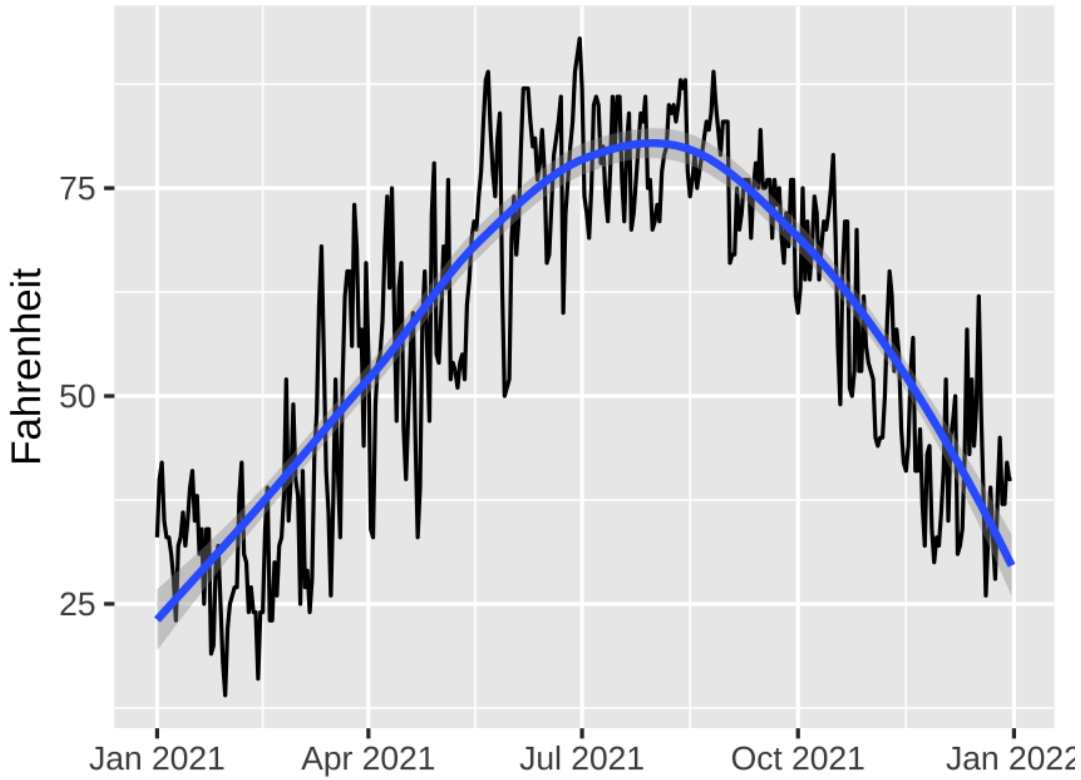


Making the base plot in R

```
ggplot(ithaca_weather,
       aes(x = DATE, y = TMAX)) +
  geom_line() +
  geom_smooth() +
  labs(x = NULL, y = "Fahrenheit")
```

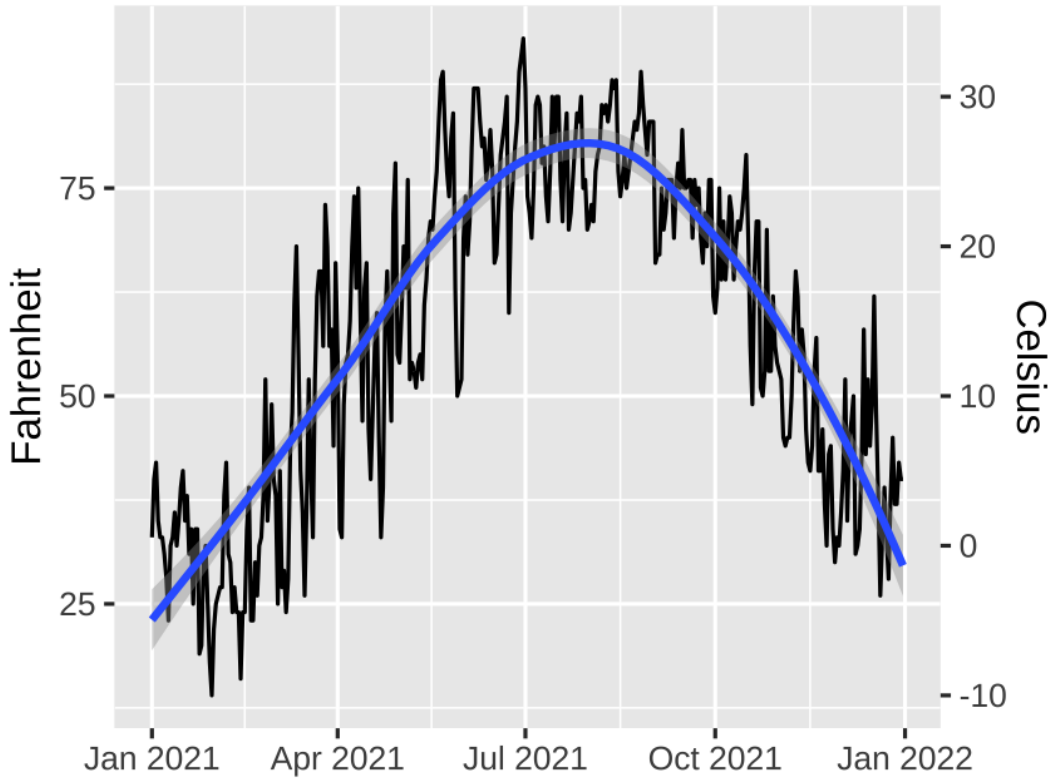
How could we add a second axis?

Do any functions come to mind?



Adding a second scale in R

```
ggplot(ithaca_weather,
       aes(x = DATE, y = TMAX)) +
  geom_line() +
  geom_smooth() +
  scale_y_continuous(
    sec.axis =
      sec_axis(trans = ~ (. - 32) * 5/9,
               name = "Celsius")
  ) +
  labs(x = NULL, y = "Fahrenheit")
```

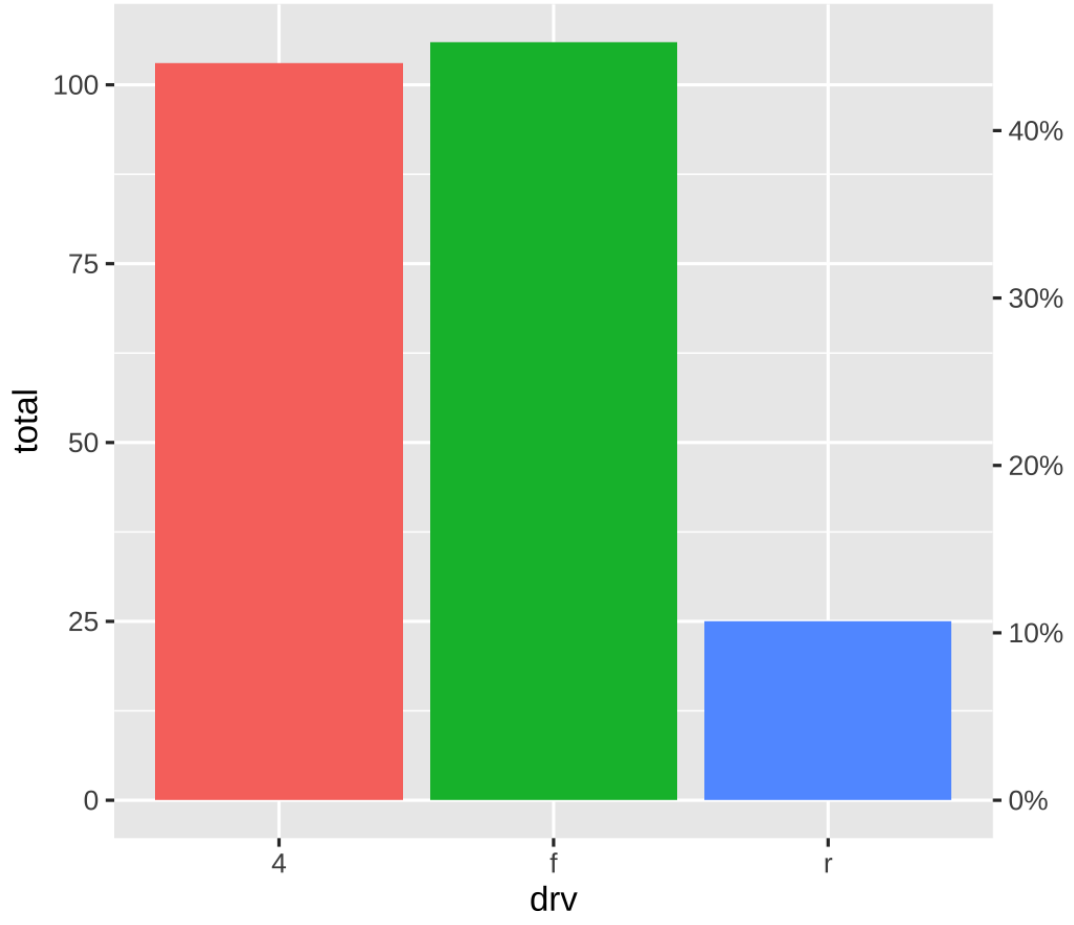


We provided this formula for the **trans**formation argument:

```
Celsius = (Fahrenheit - 32) * 5/9
```

Adding a second scale in R

```
car_counts <- mpg |>
  group_by(drv) |> summarize(total = n())

total_cars <- sum(car_counts$total)

ggplot(car_counts,
       aes(x = drv, y = total, fill = drv)) +
  geom_col() +
  scale_y_continuous(
    sec.axis = sec_axis(
      trans = ~ . / total_cars,
      labels = scales::percent)) +
  guides(fill = "none")
```



This makes it a lot easier to see proportions with side-by-side bars!

Note: **total_cars** is not in **car_counts**

Visualizing correlations

What does "correlation" mean to you?

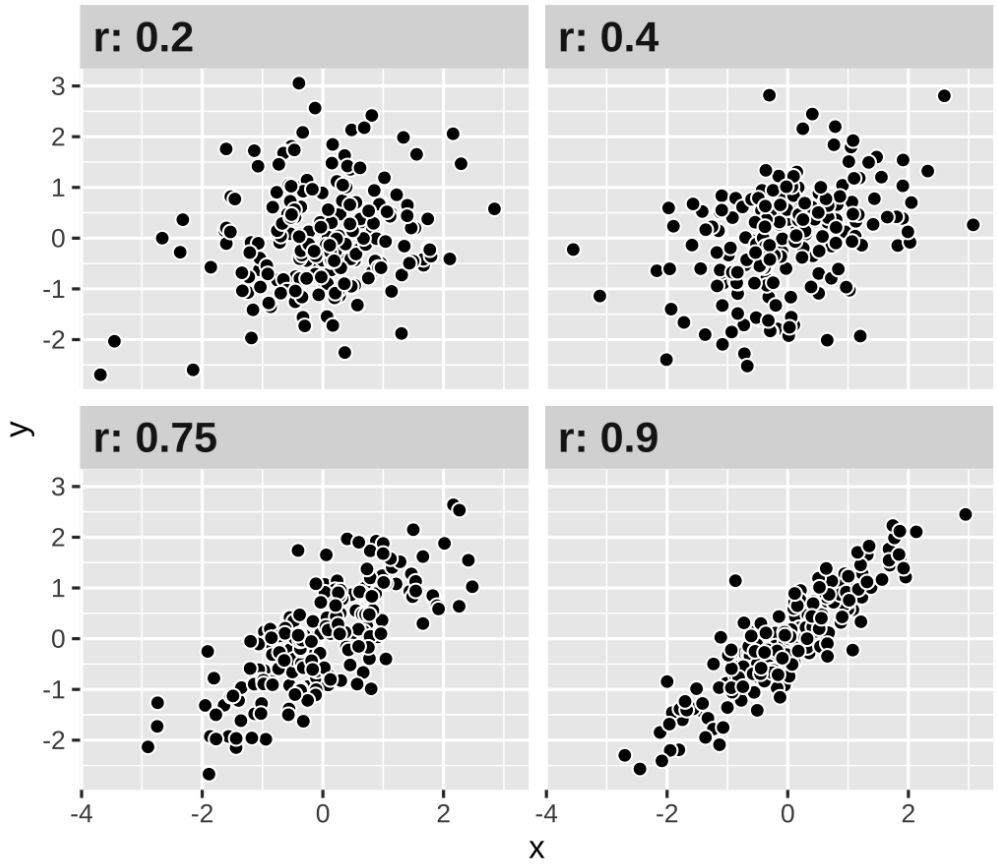
As the value of X goes up, Y tends to go up (down) a lot / a little / not at all

$$\rho_{X,Y} = \frac{\operatorname{cov}(X,Y)}{\sigma_X \sigma_Y}$$

Says nothing about *how much* Y changes when X changes

Correlation values

ρ	**Rough meaning**
±0.1–0.3	Weak
±0.3–0.5	Moderate
±0.5–0.8	Strong
±0.8–0.9	Very strong



Scatter plots

The humble scatter plot is often the best place to start when studying the association between two variables

Example: max and min temperature in Ithaca each day of the year

- Do you think they are highly correlated, somewhat correlated, or not at all correlated?
- What sign do you think this correlation has?
- How would you make a scatter plot of these data in R?

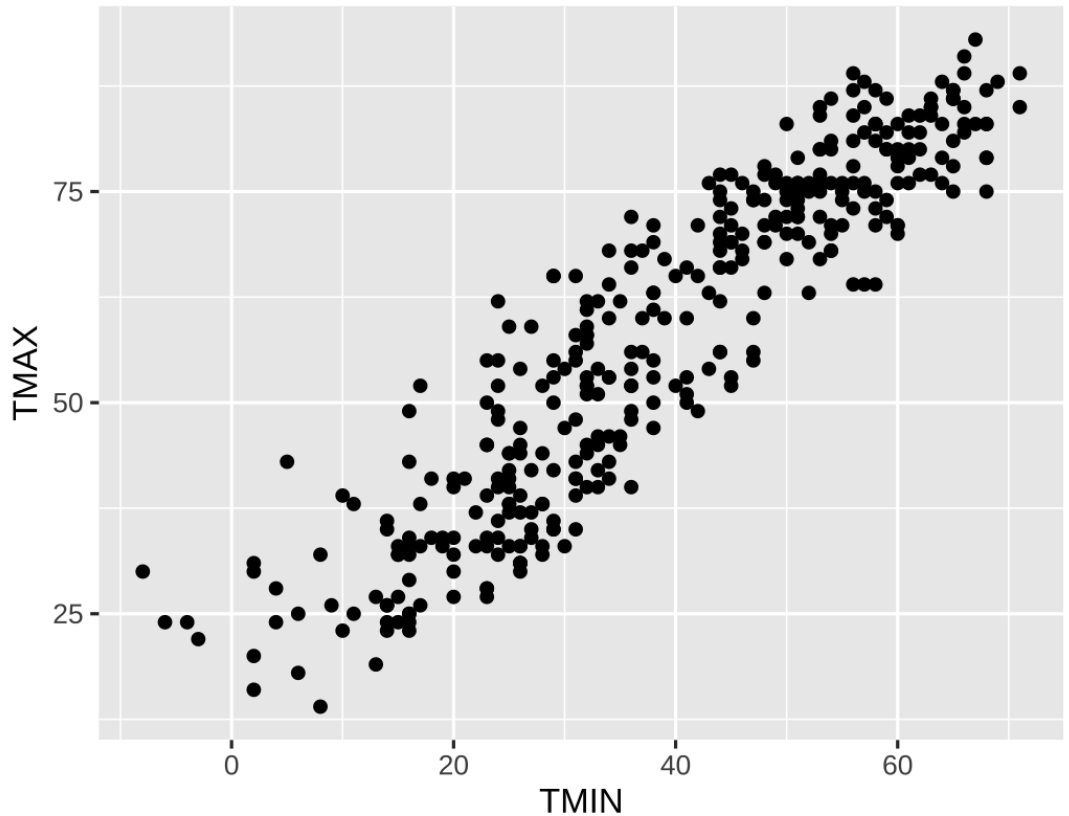
Scatter plots

```
ithaca_weather |>
  ggplot(aes(x = TMIN, y = TMAX)) +
  geom_point()
```

```
cor(ithaca_weather$TMIN,
    ithaca_weather$TMAX) |>
  round(3)
```

```
## [1] 0.919
```

Strong positive correlation



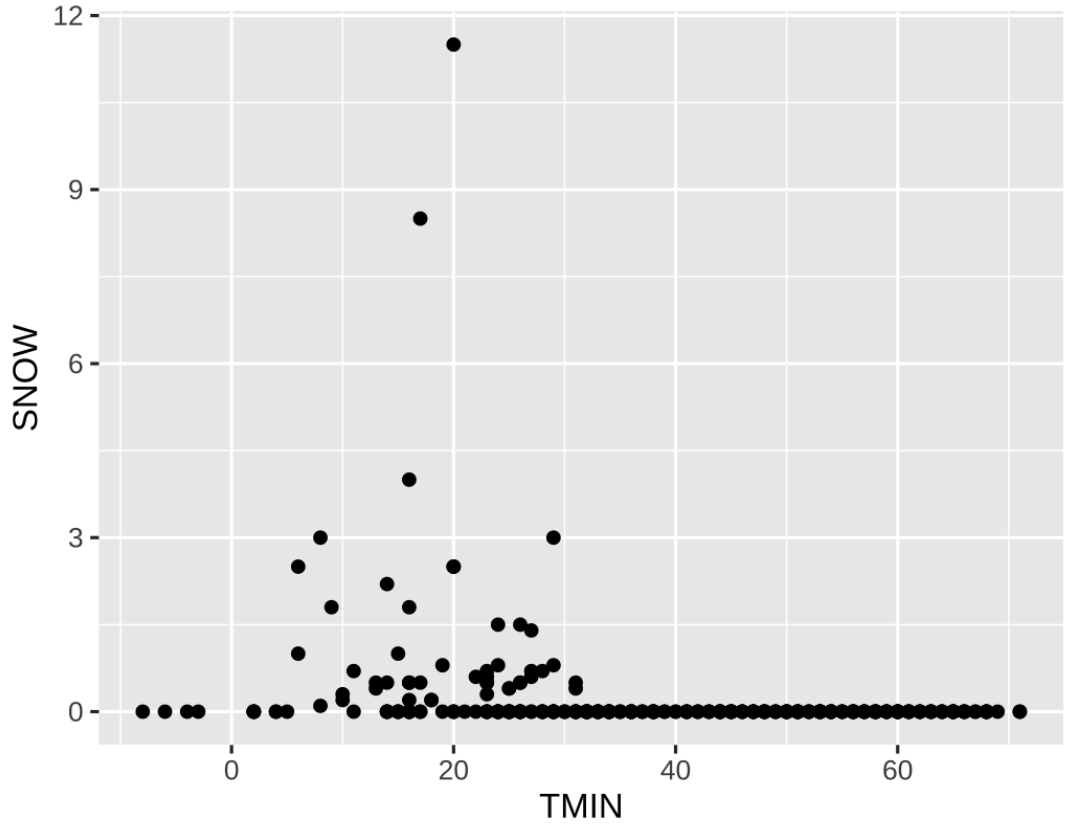
What about min temp and snowfall?

```
ithaca_weather |>
  ggplot(aes(x = TMIN, y = SNOW)) +
  geom_point()
```

```
cor(ithaca_weather$TMIN,
    ithaca_weather$SNOW) |>
  round(3)
```

```
## [1] -0.239
```

Weak negative correlation

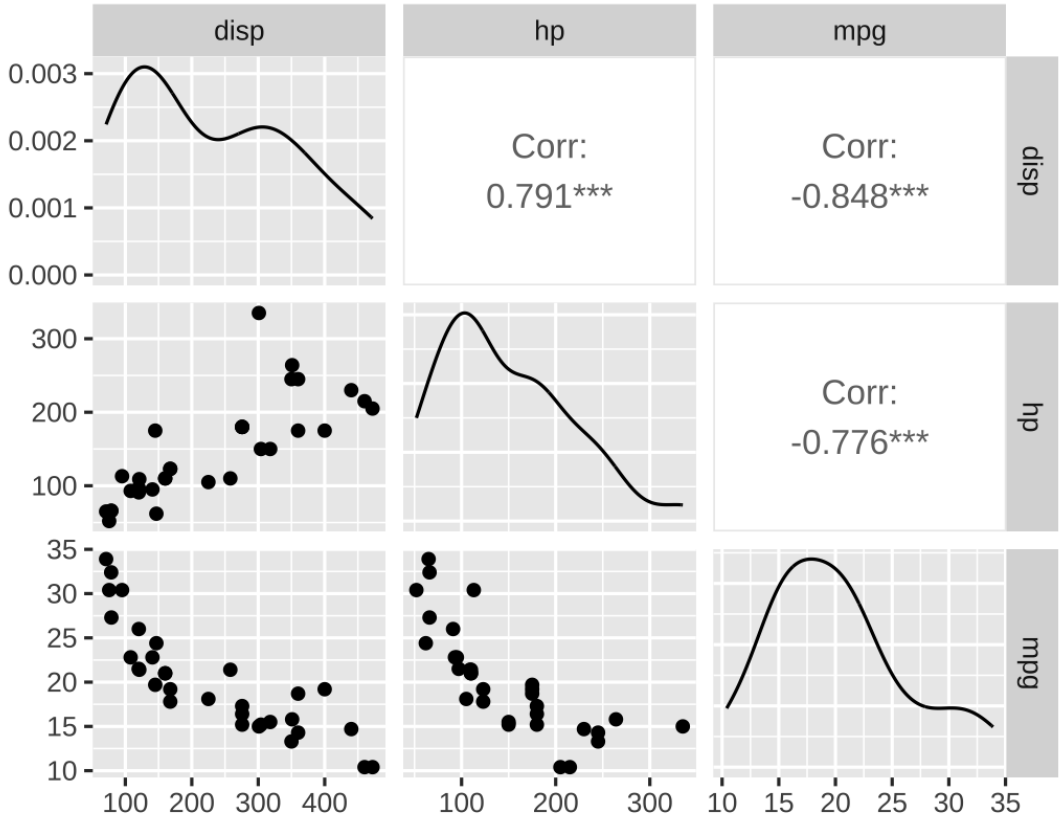


Scatter plot matrices

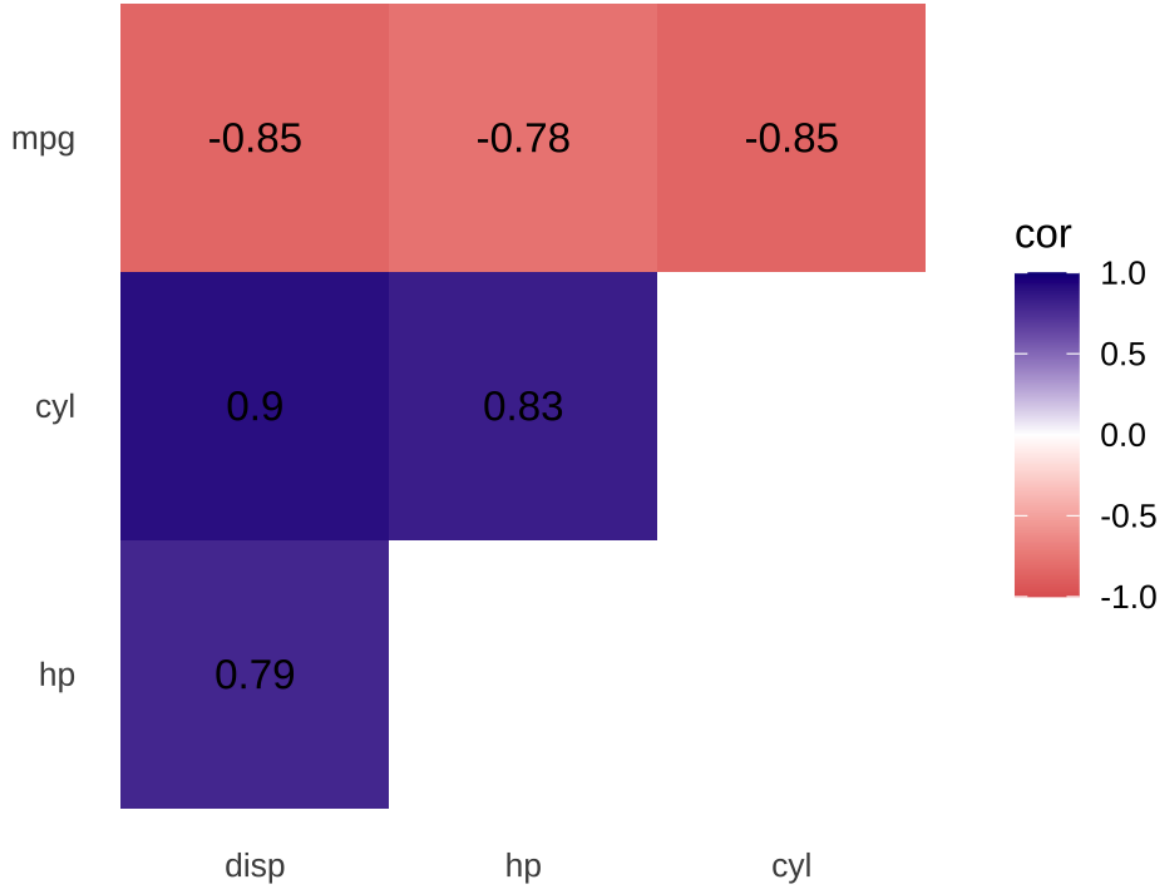
```
library(GGally)

cars_smaller <- mtcars |>
  select(disp, hp, mpg)

ggpairs(cars_smaller)
```

Scatter plots can be scaled up to matrices for several variables

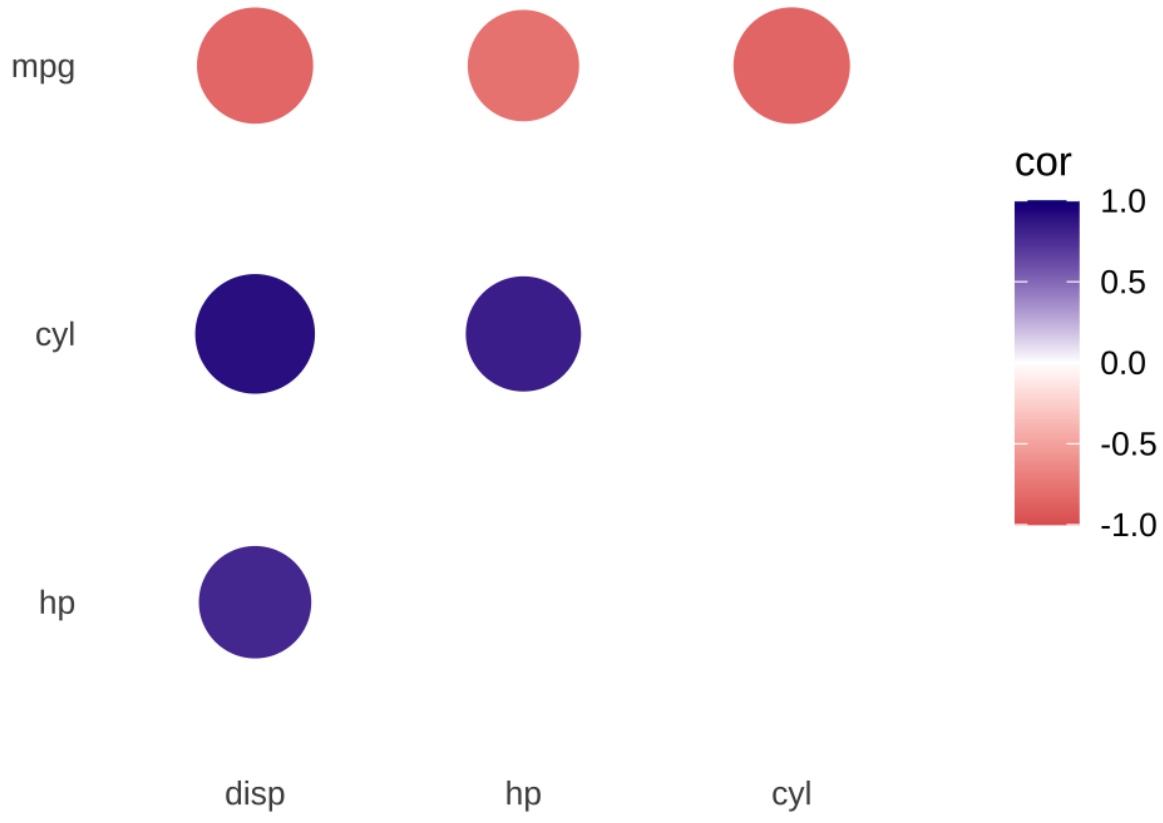
For many variables, correlograms or correlation coefficients often better



Correlograms: Heatmaps



Correlograms: Points



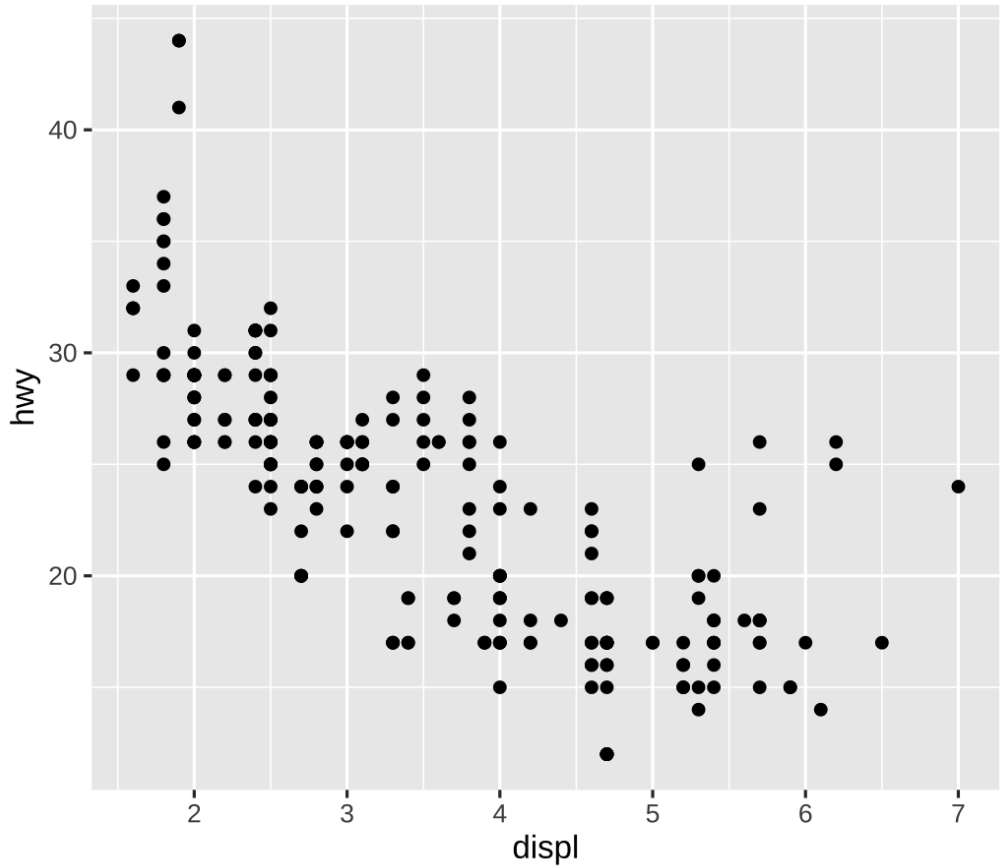
Visualizing regressions

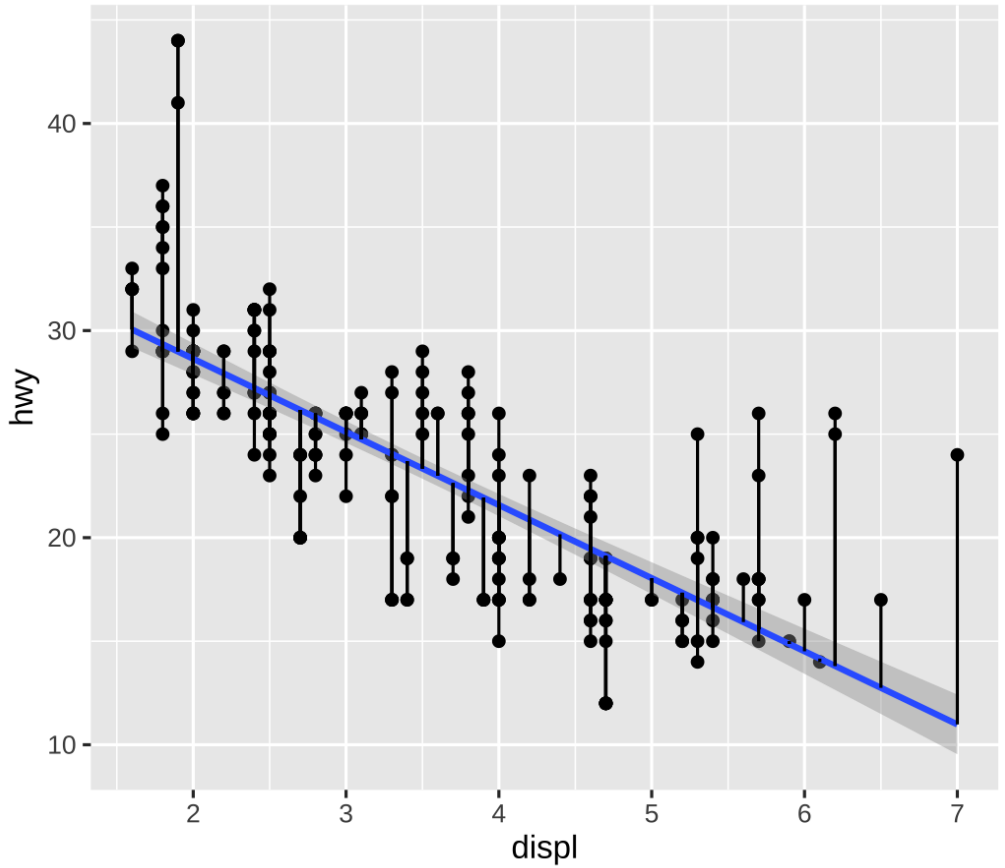
Linear regression reminder

$$y = \beta_0 + \beta_1 x_1 + \varepsilon$$

y	Outcome variable (DV)
x_1	Explanatory variable (IV)
β_1	Slope
β_0	y-intercept
ε	Error (residuals)

Linear regression is just drawing lines





Building models in R

Base R has some basic modeling tools:

```
name_of_model <- lm(<Y> ~ <X>, data = <DATA>) # use lm to fit simple linear models

summary(name_of_model) # see model details
```

The `broom` package provides helpful tools for tidying model output:

```
library(broom)

# convert model estimates to a data frame for plotting
tidy(name_of_model)

# return a data frame that includes predictions, residuals, etc.
augment(name_of_model)
```

Modeling displacement and highway MPG

$$\text{hwy} = \beta_0 + \beta_1 \text{displ} + \varepsilon$$

```
car_model <- lm(hwy ~ displ,
                data = mpg)
```

Note how we didn't write anything for the β_0 or ε terms

What do you think the sign on β_1 is?

```
car_model
```

```
##
## Call:
## lm(formula = hwy ~ displ, data = mpg)
##
## Coefficients:
## (Intercept)        displ
##      35.698       -3.531
```

Modeling displacement and highway MPG

```
summary(car_model)
```

```
##
## Call:
## lm(formula = hwy ~ displ, data = mpg)
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -7.1039 -2.1646 -0.2242  2.0589 15.0105
##
## Coefficients:
##             Estimate Std. Error t value Pr(>|t|)
## (Intercept)  35.6977     0.7204   49.55   <2e-16 ***
## displ        -3.5306     0.1945  -18.15   <2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 3.836 on 232 degrees of freedom
## Multiple R-squared:  0.5868,    Adjusted R-squared:  0.585
## F-statistic: 329.5 on 1 and 232 DF,  p-value: < 2.2e-16
```

Modeling displacement and highway MPG

```
tidy(car_model, conf.int = TRUE)
```

```
## # A tibble: 2 × 7
##   term        estimate std.error statistic   p.value conf.low conf.high
##   <chr>          <dbl>     <dbl>     <dbl>     <dbl>    <dbl>     <dbl>
## 1 (Intercept)    35.7      0.720      49.6 2.12e-125    34.3      37.1
## 2 displ          -3.53     0.195     -18.2 2.04e- 46    -3.91     -3.15
```

Interpretation for a continuous variable

$$y = \beta_0 + \beta_1 x_1 + \varepsilon$$

On average, a one unit increase in x_1 is *associated* with a β_1 change in y

$$\text{hwy} = \beta_0 + \beta_1 \text{displ} + \varepsilon$$

$$\widehat{\text{hwy}} = 35.7 + (-3.53) \times \text{displ}$$

On average, a one unit increase in displacement is associated with 3.53 lower highway fuel economy

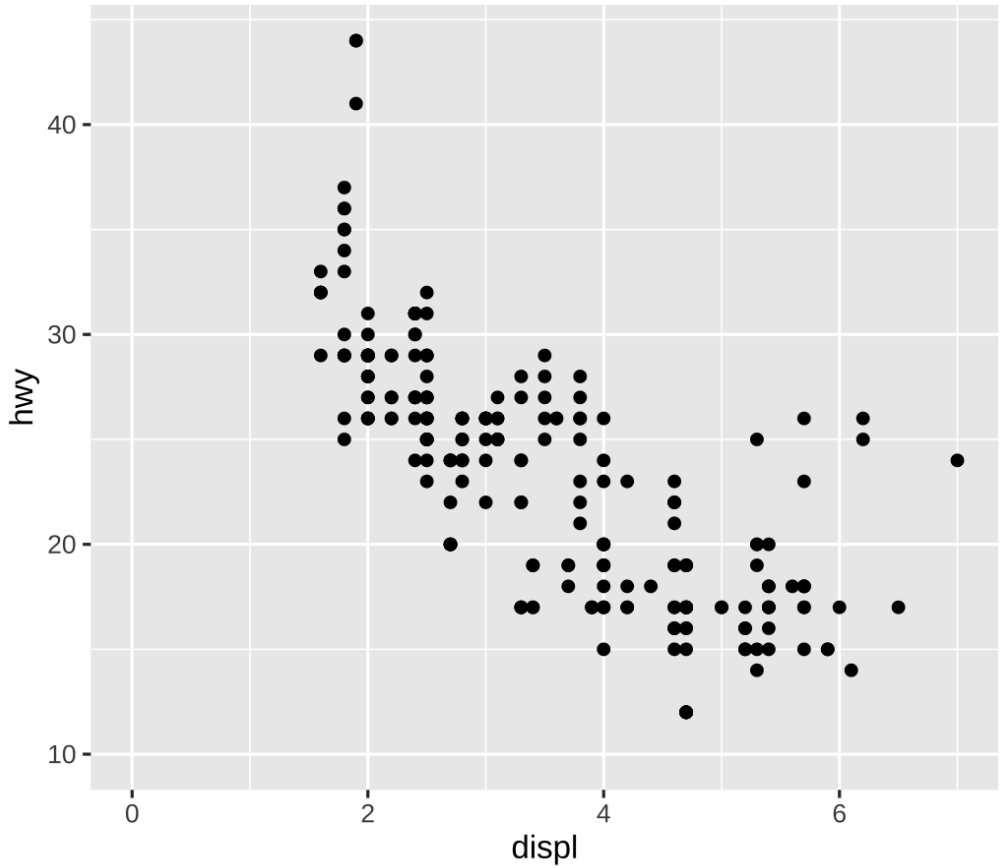
This is easy to visualize: it's a line!

Visualization of a continuous variable

```
tidy(car_model) |>
  select(term, estimate)
```

```
## # A tibble: 2 × 2
##   term         estimate
##   <chr>           <dbl>
## 1 (Intercept)    35.7
## 2 displ          -3.53
```

$$\widehat{\text{hwy}} = 35.7 + (-3.53) \times \text{displ}$$

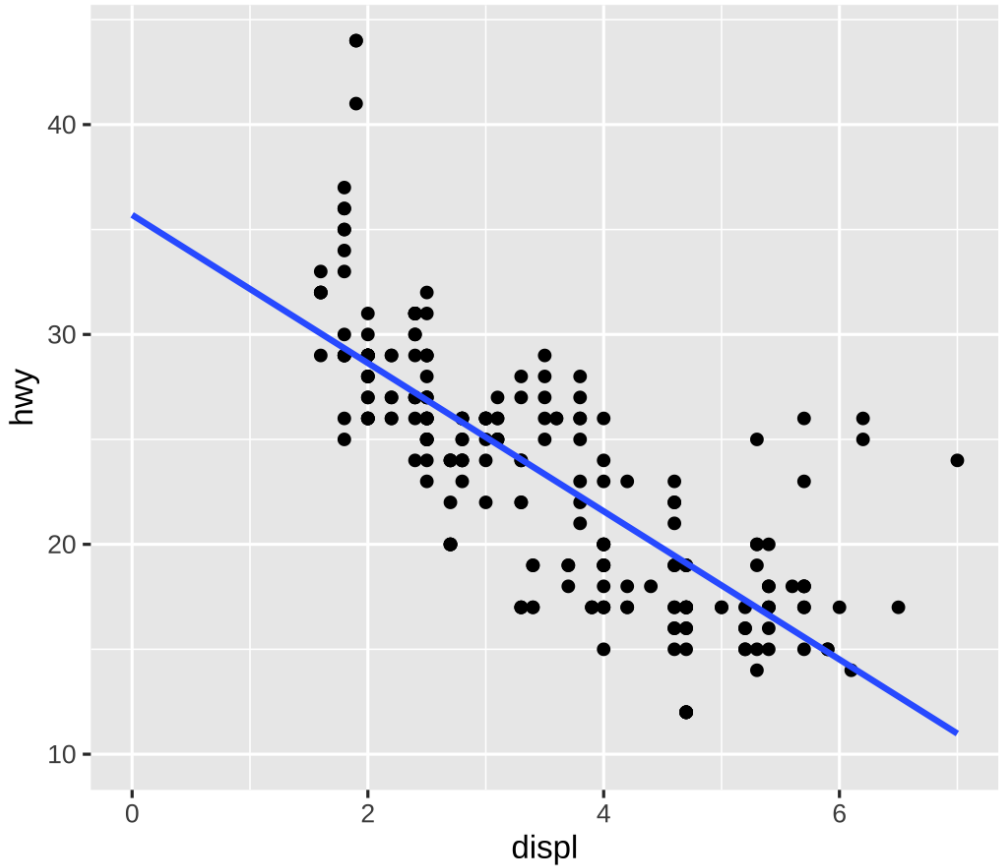


Visualization of a continuous variable

```
tidy(car_model) |>
  select(term, estimate)
```

```
## # A tibble: 2 × 2
##   term         estimate
##   <chr>           <dbl>
## 1 (Intercept)    35.7
## 2 displ          -3.53
```

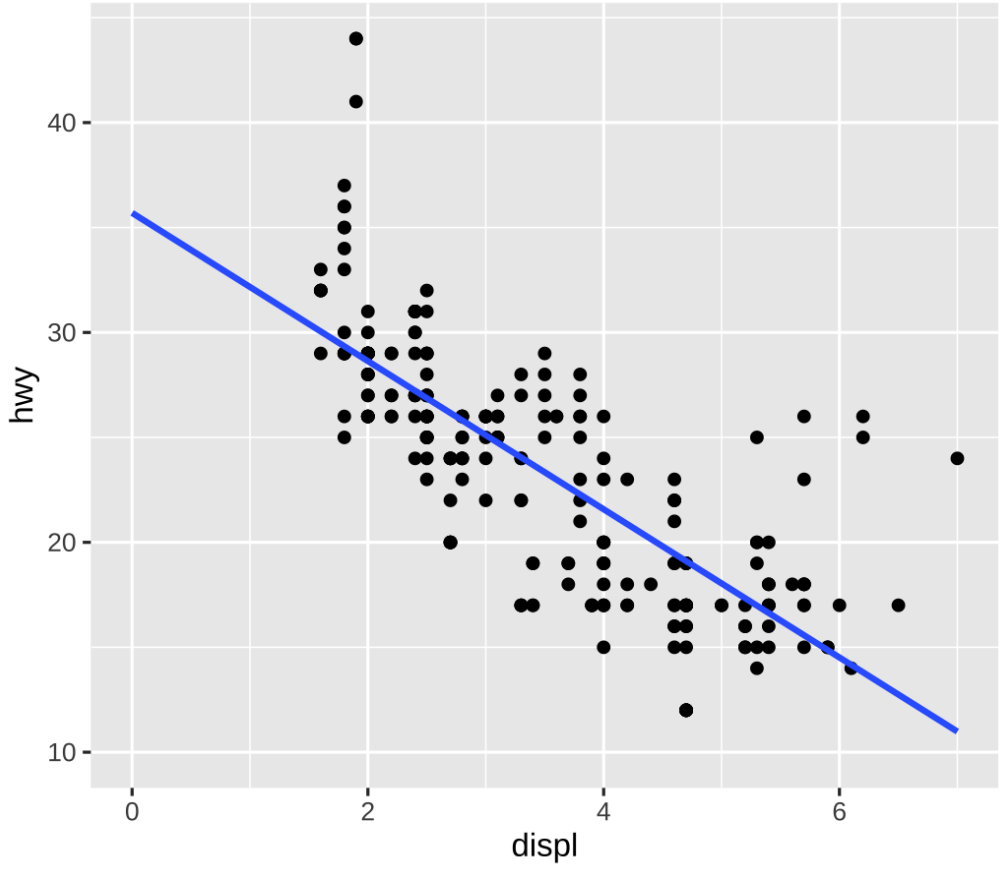
$$\widehat{\text{hwy}} = 35.7 + (-3.53) \times \text{displ}$$



Visualization of a continuous variable

Reminder: `geom_smooth(method = "lm")` allows us to skip the estimation step!

```
mpg |>
  ggplot(aes(x = displ, y = hwy)) +
  geom_point() +
  geom_smooth(
    method = "lm",      # smoothing function
    formula = "y ~ x", # optional in this case
    fullrange = TRUE,  # start line at 0
    se = FALSE         # omit confidence bands
  ) +
  scale_x_continuous(limits = c(0, 7)) +
  scale_y_continuous(limits = c(10, NA))
```



Multiple regression

We're not limited to just one explanatory variable!

$$y = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \cdots + \beta_n x_n + \varepsilon$$

```
car_model_big <- lm(hwy ~ displ + cyl + drv,
                    data = mpg)
```

$$\widehat{\text{hwy}} = \widehat{\beta}_0 + \widehat{\beta}_1 \text{displ} + \widehat{\beta}_2 \text{cyl} + \widehat{\beta}_3 \text{drv:f} + \widehat{\beta}_4 \text{drv:r}$$

Modeling lots of things and MPG

```
tidy(car_model_big, conf.int = TRUE)
```

```
## # A tibble: 5 × 7
##   term        estimate std.error statistic  p.value conf.low conf.high
##   <chr>          <dbl>     <dbl>     <dbl>    <dbl>    <dbl>     <dbl>
## 1 (Intercept)    33.1      1.03      32.1  9.49e-87    31.1     35.1
## 2 displ          -1.12     0.461     -2.44 1.56e- 2    -2.03    -0.215
## 3 cyl            -1.45     0.333     -4.36 1.99e- 5    -2.11    -0.796
## 4 drvf            5.04     0.513      9.83 3.07e-19     4.03     6.06
## 5 drvr            4.89     0.712      6.86 6.20e-11     3.48     6.29
```

$$\widehat{\text{hwy}} = 33.1 + (-1.12) \times \text{displ} + (-1.45) \times \text{cyl} + (5.04) \times \text{drv:f} + (4.89) \times \text{drv:r}$$

Sliders and switches

Sliders and switches

Interpretation for continuous variables

$$y = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \cdots + \beta_n x_n + \varepsilon$$

Holding everything else constant, a one unit increase in x_n is *associated* with a β_n change in y, on average

$$\widehat{\text{hwy}} = 33.1 + (-1.12) \times \text{displ} + (-1.45) \times \text{cyl} + (5.04) \times \text{drv:f} + (4.89) \times \text{drv:r}$$

On average, a one unit increase in displacement is associated with 1.12 lower highway fuel economy, holding everything else constant

Aside: for the earlier model we had said

> On average, a one unit increase in displacement is associated with 3.53 lower highway fuel economy

Interpretation for categorical variables

$$y = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \cdots + \beta_n x_n + \varepsilon$$

Holding everything else constant, y is β_n units larger (or smaller) for category x_n than for the omitted category, on average

$$\widehat{\text{hwy}} = 33.1 + (-1.12) \times \text{displ} + (-1.45) \times \text{cyl} + (5.04) \times \text{drv:f} + (4.89) \times \text{drv:r}$$

On average, front-wheel drive cars have 5.04 higher highway fuel economy than 4-wheel-drive cars, holding everything else constant

Good luck visualizing all this!

You can't just draw a single line! There are too many moving parts!

Main challenges

Each coefficient has its own estimate and standard errors

Solution: Plot the coefficients and their errors with a *coefficient plot*

The results change as you move each slider up and down and flip each switch on and off

Solution: Plot the *marginal effects* for the coefficients you're interested in

Coefficient plots

Convert the model results to a data frame with `tidy()`

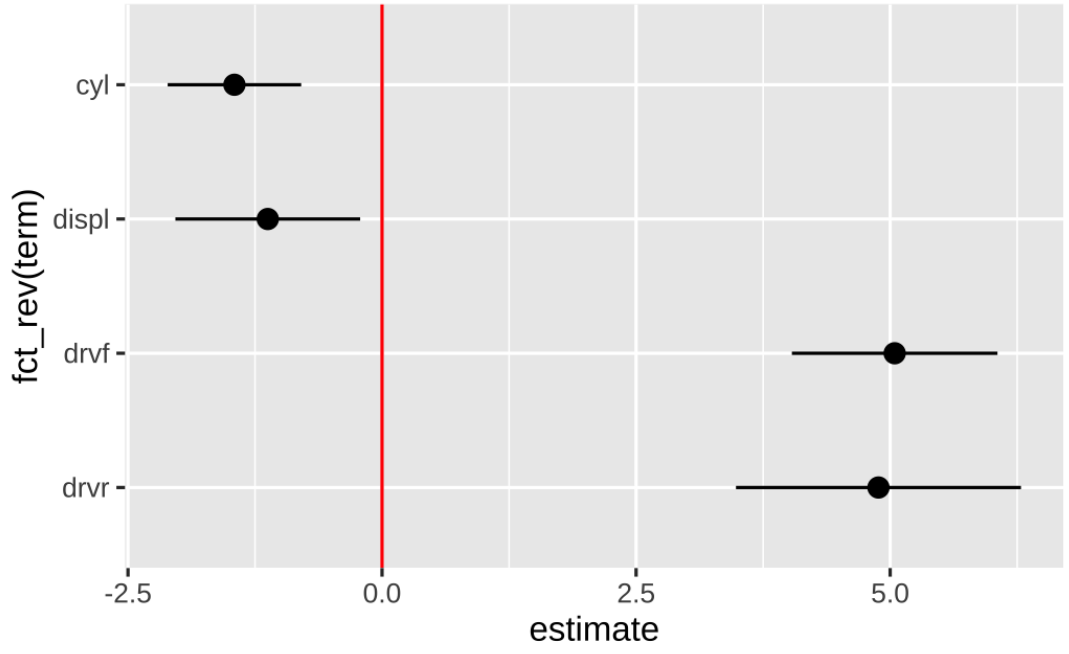
```
car_model_big <- lm(hwy ~ displ + cyl + drv, data = mpg)

car_coefs <- tidy(car_model_big, conf.int = TRUE) |>
  filter(term != "(Intercept)")  # we typically skip plotting the intercept, so remove it
car_coefs
```

```
## # A tibble: 4 × 7
##   term  estimate std.error statistic  p.value conf.low conf.high
##   <chr>    <dbl>     <dbl>     <dbl>    <dbl>    <dbl>     <dbl>
## 1 displ    -1.12     0.461     -2.44 1.56e- 2    -2.03    -0.215
## 2 cyl      -1.45     0.333     -4.36 1.99e- 5    -2.11    -0.796
## 3 drvf      5.04     0.513      9.83 3.07e-19     4.03     6.06
## 4 drvr      4.89     0.712      6.86 6.20e-11     3.48     6.29
```

Coefficient plots

Plot the point estimate and confidence intervals with `geom_pointrange()`

```
car_coefs |>
  ggplot(aes(x = estimate,
             y = fct_rev(term))) +
  geom_pointrange(aes(xmin = conf.low,
                      xmax = conf.high)) +
  geom_vline(xintercept = 0, color = "red")
```



Marginal effects plots

Remember that we interpret individual coefficients while holding the others constant

We move one slider while leaving all the other sliders and switches alone

Same principle applies to visualizing the effect

Plug a bunch of values into the model and find the predicted outcome

Plot the values and predicted outcome

Marginal effects plots

Create a data frame of values you want to manipulate and values you want to hold constant

Must include all the explanatory variables in the model

Marginal effects plots

```
cars_new_data <- tibble(displ = seq(2, 7, by = 0.1),  # create grid for displ
                        cyl = mean(mpg$cyl),          # hold cylinders at its mean
                        drv = "f")                    # drive: front-wheel

cars_new_data
```

```
## # A tibble: 51 × 3
##    displ   cyl drv
##    <dbl> <dbl> <chr>
##  1   2    5.89 f
##  2   2.1  5.89 f
##  3   2.2  5.89 f
##  4   2.3  5.89 f
##  5   2.4  5.89 f
##  6   2.5  5.89 f
##  7   2.6  5.89 f
##  8   2.7  5.89 f
##  9   2.8  5.89 f
## 10   2.9  5.89 f
## # … with 41 more rows
```

Marginal effects plots

Plug each of those rows of data into the model with `augment()`

```
predicted_mpg <- augment(car_model_big,            # our estimated model
                         newdata = cars_new_data, # our new data for plotting
                         se_fit = TRUE)           # add standard errors for fitted values

head(predicted_mpg)
```

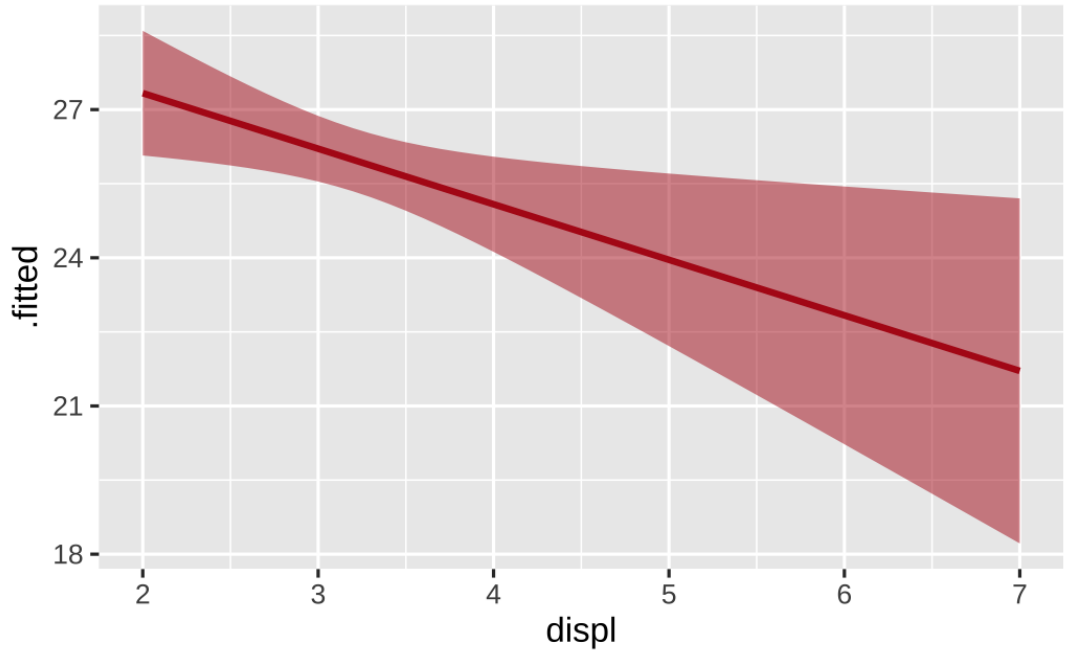
```
## # A tibble: 6 × 5
##   displ   cyl drv   .fitted .se.fit
##   <dbl> <dbl> <chr>   <dbl>   <dbl>
## 1   2    5.89 f        27.3   0.644
## 2   2.1  5.89 f        27.2   0.604
## 3   2.2  5.89 f        27.1   0.566
## 4   2.3  5.89 f        27.0   0.529
## 5   2.4  5.89 f        26.9   0.494
## 6   2.5  5.89 f        26.8   0.460
```

Marginal effects plots

Plot the fitted values for each row

We made predictions for front-wheel drive cars with cylinders held at their mean

```
predicted_mpg |>
  ggplot(aes(x = displ, y = .fitted)) +
  geom_line(color = "#B31B1B",
            linewidth = 1) +
  geom_ribbon(aes(ymin = .fitted +
                    (-1.96 * .se.fit),
                  ymax = .fitted +
                    (1.96 * .se.fit)),
              fill = "#B31B1B",
              alpha = 0.5)
```



Multiple effects at once

We can also move multiple sliders and switches at the same time!

What's the marginal effect of increasing displacement across the front-, rear-, and four-wheel drive cars?

How would you approach this?

Multiple effects at once

Create a new dataset with varying displacement *and* varying drive, holding cylinders at its mean

The `expand_grid()` function comes in handy for this

`expand_grid()` creates a data frame with every possible combination of the variables you supply

Multiple effects at once

```
cars_new_data_fancy <- expand_grid(displ = seq(2, 7, by = 0.1),  # create grid for displ
                                   cyl = mean(mpg$cyl),          # hold cylinders at its mean
                                   drv = c("f", "r", "4"))       # drive: front-wheel

cars_new_data_fancy
```

```
## # A tibble: 153 × 3
##    displ   cyl drv
##    <dbl> <dbl> <chr>
##  1   2    5.89 f
##  2   2    5.89 r
##  3   2    5.89 4
##  4   2.1  5.89 f
##  5   2.1  5.89 r
##  6   2.1  5.89 4
##  7   2.2  5.89 f
##  8   2.2  5.89 r
##  9   2.2  5.89 4
## 10   2.3  5.89 f
## # … with 143 more rows
```

Multiple effects at once

Plug each of those rows of data into the model with `augment()`

```
predicted_mpg_fancy <- augment(car_model_big,                  # our estimated model
                               newdata = cars_new_data_fancy, # our new data for plotting
                               se_fit = TRUE)

head(predicted_mpg_fancy)
```

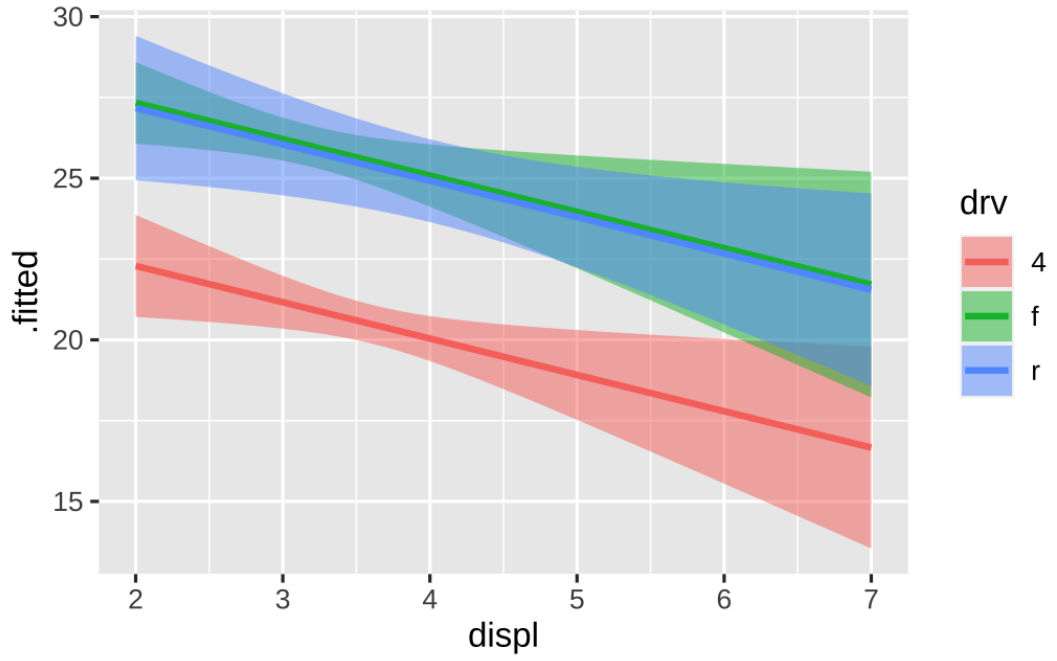
```
## # A tibble: 6 × 5
##   displ   cyl drv   .fitted .se.fit
##   <dbl> <dbl> <chr>   <dbl>   <dbl>
## 1   2    5.89 f        27.3   0.644
## 2   2    5.89 r        27.2   1.14
## 3   2    5.89 4        22.3   0.805
## 4   2.1  5.89 f        27.2   0.604
## 5   2.1  5.89 r        27.1   1.10
## 6   2.1  5.89 4        22.2   0.763
```

Multiple effects at once

Plot the fitted values for each row

Cylinders held at their mean; colored/filled by drive

```
predicted_mpg_fancy |>
  ggplot(aes(x = displ, y = .fitted)) +
  geom_ribbon(aes(ymin = .fitted +
                    (-1.96 * .se.fit),
                  ymax = .fitted +
                    (1.96 * .se.fit),
                  fill = drv),
              alpha = 0.5) +
  geom_line(aes(color = drv), linewidth = 1)
```

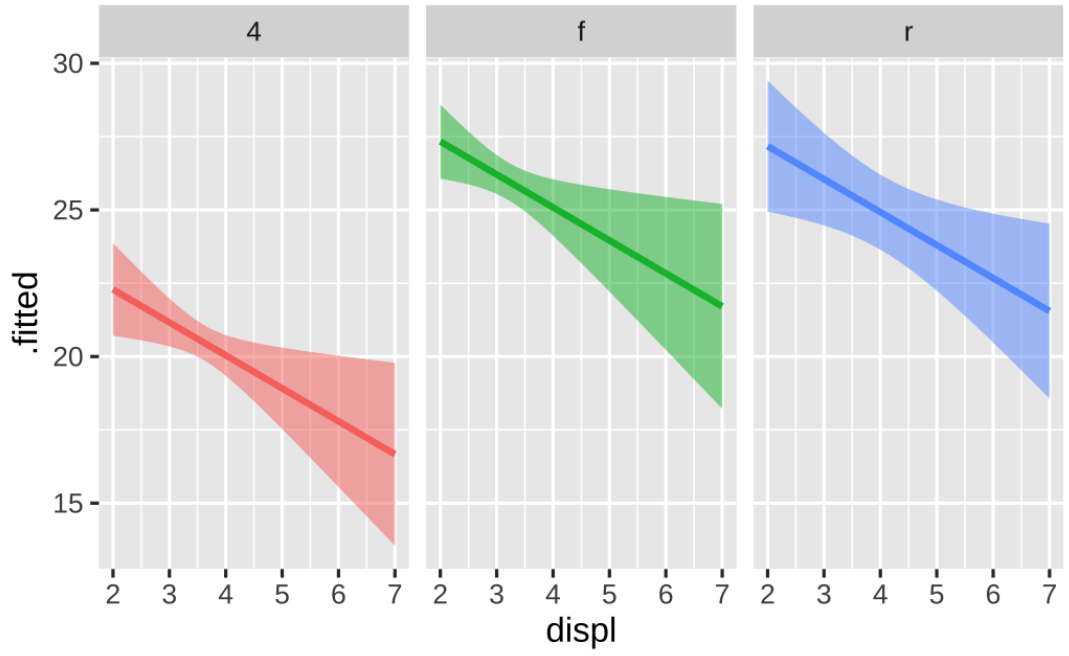


Multiple effects at once

Plot the fitted values for each row

Cylinders held at their mean; colored/filled/facetted by drive

```
predicted_mpg_fancy |>
  ggplot(aes(x = displ, y = .fitted)) +
  geom_ribbon(aes(ymin = .fitted +
                    (-1.96 * .se.fit),
                  ymax = .fitted +
                    (1.96 * .se.fit),
                  fill = drv),
              alpha = 0.5) +
  geom_line(aes(color = drv), linewidth = 1) +
  guides(fill = "none", color = "none") +
  facet_wrap(vars(drv))
```



Estimating regressions with interactions

We can use **interactions** to estimate different slopes for each drive type

Syntax: `drv:displ` adds an interaction between `drv` and `displ`

```
car_model_interactions <- lm(
  hwy ~ drv + drv:displ + cyl,
  data = mpg
)
```

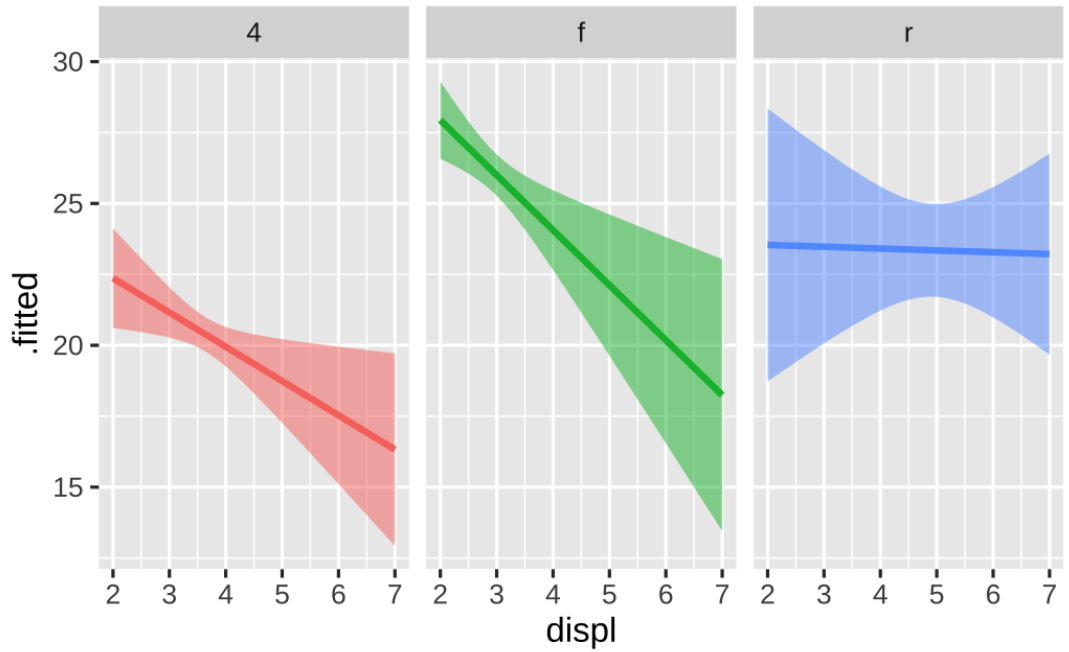
Alternatively: `drv*displ` adds `drv`, `displ`, and their interaction

```
tidy(car_model_interactions) |>
  select(term, estimate)
```

```
## # A tibble: 7 × 2
##   term         estimate
##   <chr>           <dbl>
## 1 (Intercept)   32.5
## 2 drvf           7.03
## 3 drvr          -1.12
## 4 cyl           -1.30
## 5 drv4:displ    -1.21
## 6 drvf:displ    -1.94
## 7 drvr:displ    -0.0645
```

Even more effects at once!

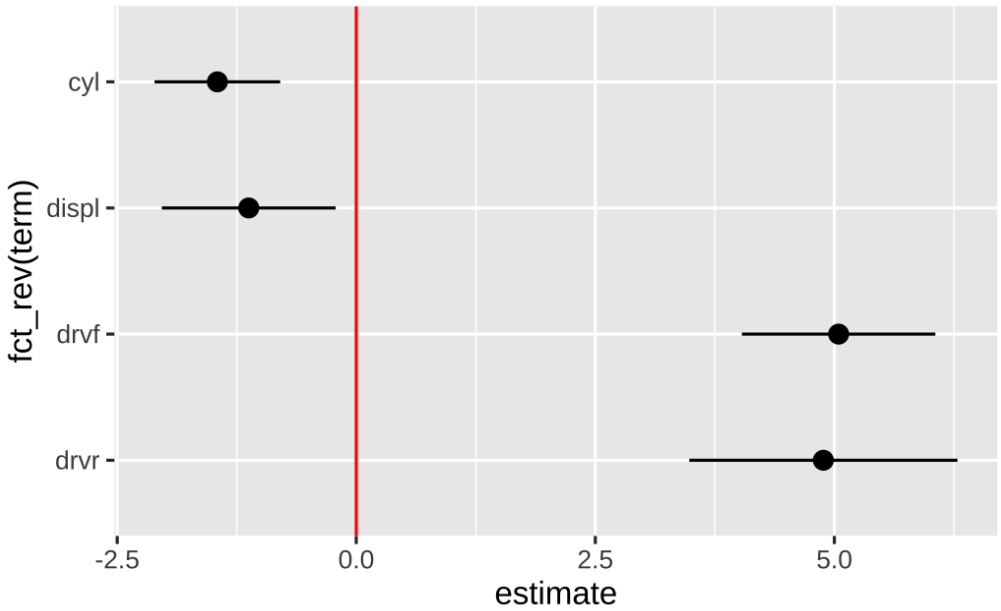
```
predicted_mpg_interactions <- augment(
  car_model_interactions, # interacted model
  newdata = cars_new_data_fancy,
  se_fit = TRUE
)

predicted_mpg_interactions |>
  ggplot(aes(x = displ, y = .fitted)) +
  geom_ribbon(aes(ymin = .fitted +
                    (-1.96 * .se.fit),
                  ymax = .fitted +
                    (1.96 * .se.fit),
                  fill = drv),
              alpha = 0.5) +
  geom_line(aes(color = drv), linewidth = 1) +
  guides(fill = "none", color = "none") +
  facet_wrap(vars(drv))
```

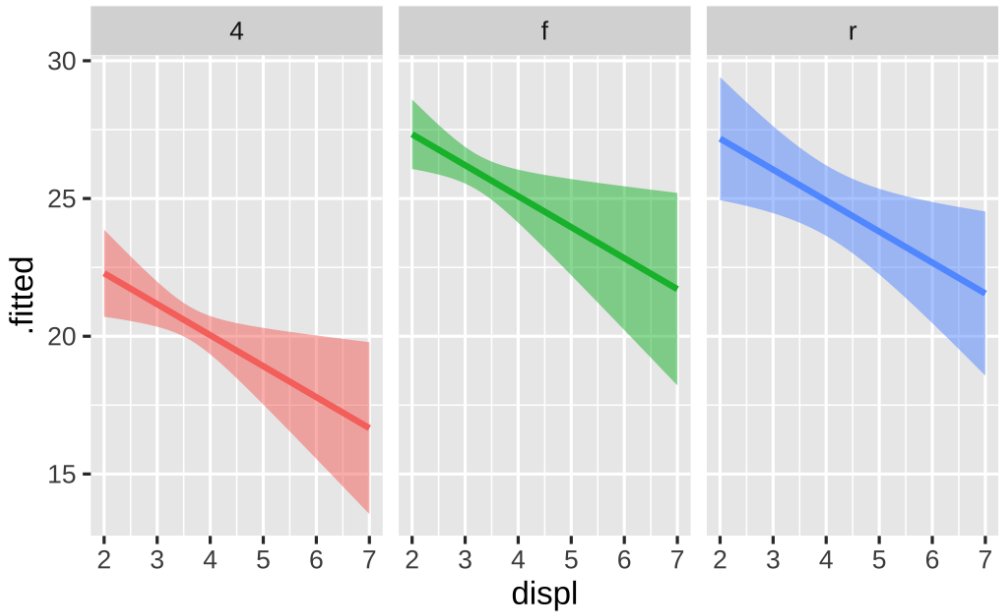


Note the different slopes!

Not just OLS!

These plots are for an OLS model built with `lm()`





Any type of statistical model

The same techniques work for pretty much any model R can run

OLS with high-dimensional fixed effects

Logistic, probit, and multinomial regression (ordered and unordered)

Multilevel (i.e., mixed and random effects) regression

Bayesian models

Machine learning models

If it has coefficients and/or makes predictions, you can (and should) visualize it!