Reaction-diffusion spatial modeling of COVID-19 in Chicago

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1 Model Setup

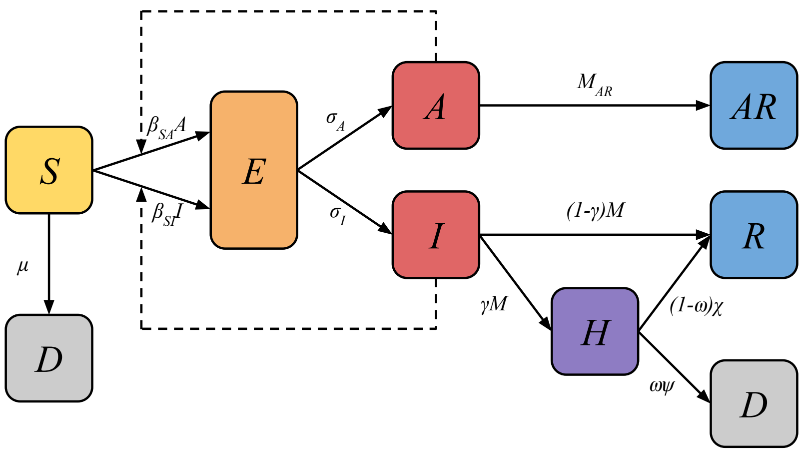

Figure 1: Schematic diagram of the model. The dashed lines indicate the interaction of the infected populations with the susceptible populations that leads to infection.

$$S_t = \mathfrak{D}_S \Delta S - \beta_{SA} SA - \beta_{SI} SI - \mu S, \tag{1}$$
$$E_t = \mathfrak{D}_E \Delta E + \beta_{SA} SA + \beta_{SI} SI - (\sigma_A + \sigma_I)E, \tag{2}$$
$$A_t = \mathfrak{D}_A \Delta A + \sigma_A E - M_{AR} A, \tag{3}$$
$$I_t = \sigma_I E - MI, \tag{4}$$
$$H_t = \gamma MI - (1-\omega)\chi H - \omega\chi H, \tag{5}$$
$$R_t = (1-\gamma)MI + (1-\omega)\chi H, \tag{6}$$
$$D_t = \omega\chi H. \tag{7}$$

2 ODE Dynamics

We want to understand the trajectories of the dynamics of the ODE system under different initial conditions. To do this we first find the equilibrium points by solving

$$S_t = E_t = A_t = I_t = H_t = R_t = D_t = 0$$

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Table 1: Population values for Chicago.

		Population
Total population	N	2,695,598
Initial infected	I_0	127
Initial hospitalized	H_0	30
Initial deceased	D_0	6

Table 2: Parameters for Chicago: optimal (best-fitting), median and interquartile range, and variation range used in the optimization algorithm. Initial parameter guesses were uniformly sampled within these ranges.

		Median (interquartile range)	Initial value
Population	N	2,695,598	
Initial population	(I_0, R_0)	(127, 2)	
Transmission rate, $S \to I$ [per day]	β^1	0.38206(0.38204-0.38209)	$c \in U[0,1]$
Transition rate, $I \to R$ [per day]	γ	0.39656(0.39654-0.39659)	$c \in U[0.25, 0.75]$
Diffusivity, S [km^2/day]	$\mathfrak{D}_S$	10	
Diffusivity, I [km^2/day]	$\mathfrak{D}_I$	100	

simultaneously for $\mathbf{x} = (S, E, A, AR, I, H, R, D)$. The solutions of this system are of the form $\mathbf{x}^* = (0, 0, 0, AR, 0, 0, R, D)$. This implies there are infinitely many non-isolated equilibrium points. We determine the stability of these equilibrium points by analyzing the linearized system near the points. The Jacobian of the system is

$$\mathsf{J} = \begin{pmatrix} -A\beta_{SA} - I\beta_{SI} - \mu & 0 & -S\beta_{SA} & 0 & -S\beta_{SI} & 0 & 0 & 0 \\ A\beta_{SA} + I\beta_{SI} & -\sigma_A - \sigma_I & S\beta_{SA} & 0 & S\beta_{SI} & 0 & 0 & 0 \\ 0 & \sigma_A & -M_{AR} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & M_{AR} & 0 & 0 & 0 & 0 & 0 \\ 0 & \sigma_I & 0 & 0 & -M & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & M\gamma & -\chi(1-\omega) - \psi\omega & 0 & 0 \\ 0 & 0 & 0 & 0 & M(1-\gamma) & \chi(1-\omega) & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \psi\omega & 0 & 0 \end{pmatrix} \tag{8}$$

Now evaluating J at the equilibrium point $\mathbf{x}^*$ and calculating the eigenvalues, we have

$$\lambda = \{0, 0, 0, -M, -M_{AR}, -\mu, -\sigma_A - \sigma_I, -\chi + \chi\omega - \psi\omega\}. \tag{9}$$

Note that the first three eigenvalues are 0, which implies the equilibrium points are non-isolated. This agrees with our earlier observation.

The equilibrium points are stable when $\lambda_i < 0$ for $4 \leq i \leq 8$. Since all the system parameters are positive, this implies $\lambda_i < 0$ for $4 \leq i \leq 7$. Thus the stability depends on the sign of λ_8. There are two cases when $\lambda_8 = -\chi + \chi\omega - \psi\omega < 0$ is true:

1. $0 < \omega \leq 1$ implies $\lambda_8 < 0$, and
2. $\omega > 1$ and $\chi < \frac{\psi\omega}{\omega - 1}$ implies $\lambda_8 < 0$.

That is, whenever we have either of these conditions the equilibrium points are stable. We call this situation *endemic*. If $\lambda_8 > 0$, the equilibrium points are unstable and the situation is an *epidemic*.

References