Reaction-diffusion spatial modeling of COVID-19 in Chicago

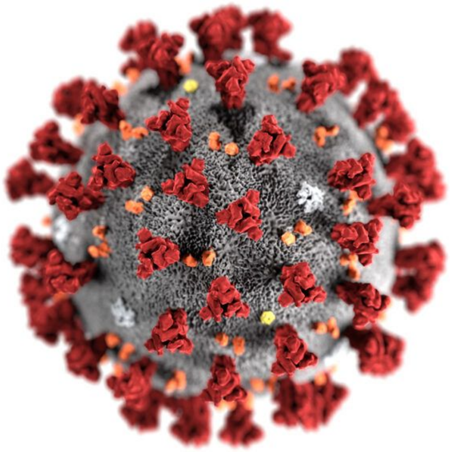
Trent Gerew

12/10/2021

ILLINOIS INSTITUTE OF TECHNOLOGY

The SoReMo Initiative

Motivation



- COVID-19 has now become a part of daily life.
- Many mathematical models exist to describe COVID-19, but few take into account spatial spreading.
- Models that account for diverse populations or demographics rapidly become complicated and computationally infeasible.

Goals

Objective

Build and verify a simple model with spatial dependence that recreates the known spreading patterns in Chicago.

A working model can…

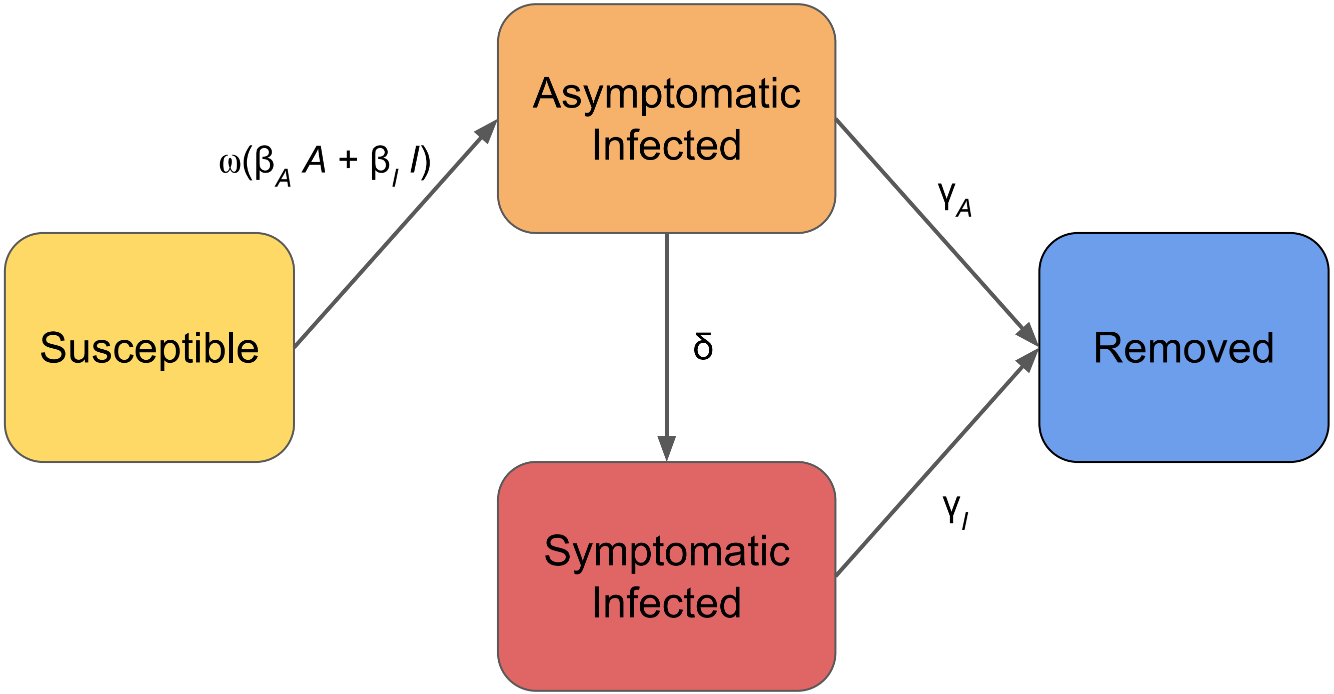
- test alternate lockdown strategies or travel restrictions.
- estimate a vaccination threshold.
- identify high risk or vulnerable locations.
- be applied to other locations and diseases.

Model Assumptions

Four populations: Susceptibles (S), Asymptomatic infected (A), symptomatic Infected (I), and Removed (R).

- Individuals in S can be infected by both members of A and I.
- Individuals in A and I have different contact rates and different recovery rates.
- Individuals in A may be detected and move to I.
- Only members of S and I are mobile.
- The total population remains constant.

Model Schematic



Reaction-Diffusion Equations

The dynamics is governed by three partial differential equations, for $(x,y) \in \Omega \subset \mathbf{R}^2$, $t > 0$.

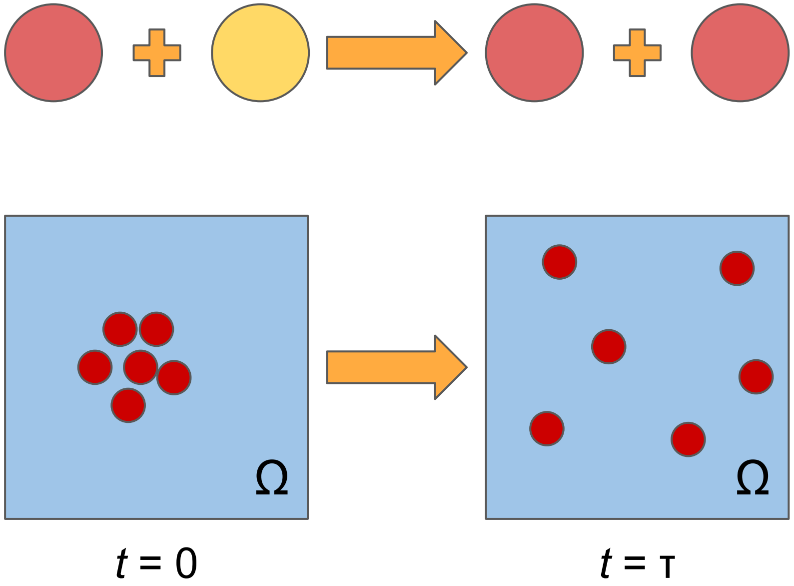
Equation

$$S_t - D(t)\,\Delta S = -\,\omega(t)\,(\beta_A A + \beta_I I)\,S$$

$$A_t - D(t)\,\Delta A = \omega(t)\,(\beta_A A + \beta_I I)\,S - (\gamma_A - \delta)\,A$$

$$I_t = -\,\gamma_I I + \delta_A A$$

Conservation Law: $S + A + I + R = 1$

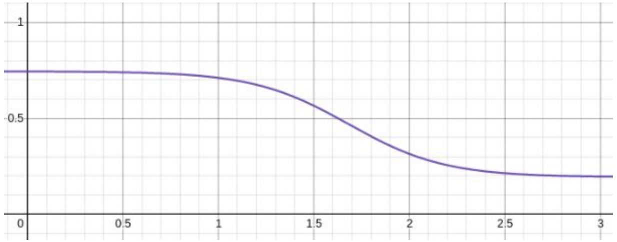


Lockdown Model

A lockdown reduces the contact rate and the traveling (diffusion) rate.

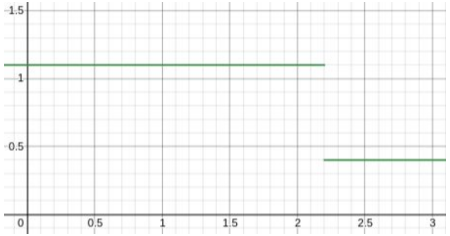
Equation

$$\omega(t) = \omega_0\,[\eta + (1 - \eta)(1 - \tanh[2(t - t_q)]) / 2]$$



Equation

$$D(t) = D_0 - \eta\,\theta(t - t_q)$$



Define $t_q = (t_{bol} + t_{eol}) / 5$ where bol and eol denote the beginning and end of the lockdown, respectively.

Existence of Solutions

Let $\boldsymbol{x} = (S, A, I, R)$ and $\boldsymbol{x_0} = (S_0, A_0, I_0, R_0)$.

Theorem

Let $0 \leq \boldsymbol{x_0} \leq 1$ be the initial datum. Then there exists a unique in time solution $\boldsymbol{x}$ of the initial value problem without diffusion over $C \subset \mathbf{R}^4 \times \mathbf{R}^1$ where C is a compact set that contains $(\boldsymbol{x_0}, t_0)$. Moreover, the solution is $\mathbf{C}^1$.

Conjecture

Let $0 \leq \boldsymbol{x_0} \leq 1$ be the initial datum. Then there exists a unique in time solution $\boldsymbol{x}$ of the initial boundary value problem.

Basic Reproduction Number

R_0 can be computed using the next generation matrix of the model without diffusion.

Definition

$$R_0 = \omega_0 S_0 (\beta_A + \beta_I \delta / \gamma_I) / (\gamma_A + \delta)$$

The first term represents the transmission by asymptomatics, and the second represents transmission by symptomatics.

Theorem

Let $(S_0, A_0, I_0, 0)$ be a nonnegative initial datum. If $R_0 > 1$, then (A, I) exponentially grows.

Parameter Estimation

Parameters are estimated according to the homogeneous (no diffusion) case.

ω_0, β_A, β_I are not independently identifiable. The problem reduces to optimizing six parameters: $\theta = (\omega_0\beta_A, \omega_0\beta_I, \eta, \gamma_A, \gamma_I, \delta)$.

For n days ($1 \leq i \leq n$), the cost function is the least squares function.

Equation

$$N = \sum[C_{\mathrm{num}}(t_i) - C_{\mathrm{obs}}(t_i)]^2$$

Let $C_{\mathrm{num}}(t_i) = I(t_i)$ and $C_{\mathrm{obs}}(t_i) = \mathrm{Cases}(t_i) / \mathrm{Population}$.

Optimization Details

Model is seeded with data from March 18, 2020.

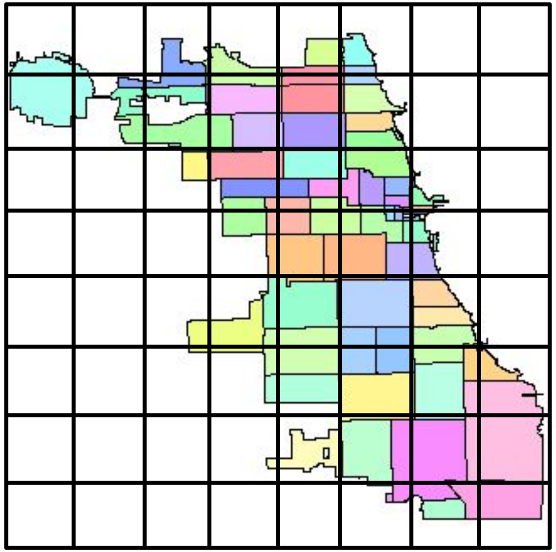
n	total population	2695598
c_0	initial cases	160
h_0	initial hospitalized	38
d_0	initial deceased	3

i_0	initial infected	$(c_0 + h_0) / n$
a_0	initial asymptomatic	$3\ i_0$
r_0	initial removed	$(d_0 + 8) / n$
s_0	initial susceptible	$1 - (a_0 + i_0 + r_0)$

Parameters sampled from uniform distribution [0,1] over 1000 optimization iterations.

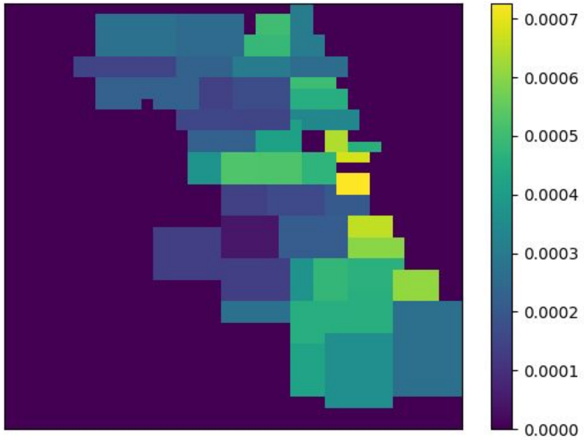
Median values are selected to parameterize the spatial model.

Spatial Discretization



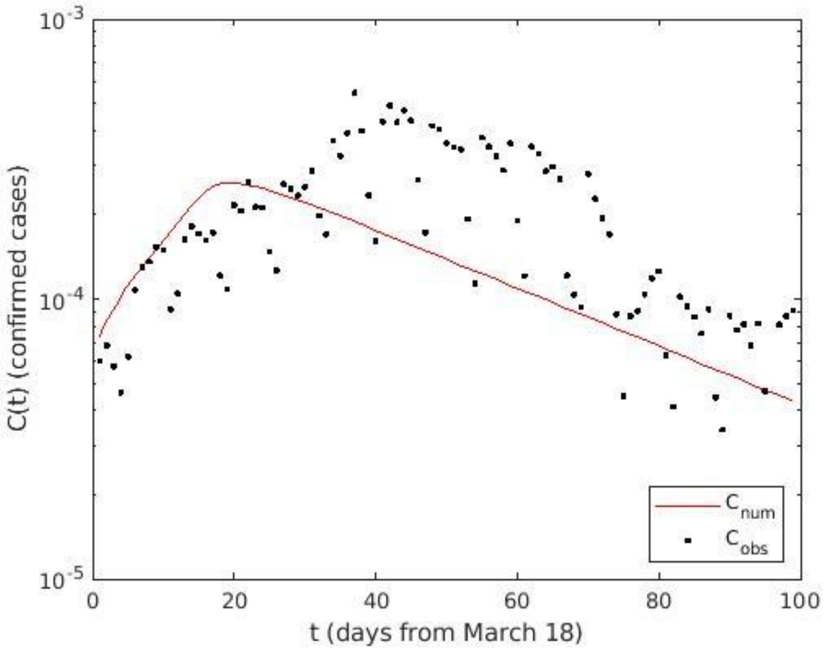
Computational domain Ω

Point	ZIP	ZIP Population	ZIP Cases
(1,7)	60666	0	0
(3,3)	60638	58797	8
(6,4)	60616	54464	11
(7,2)	60617	82534	22



Initial case distribution

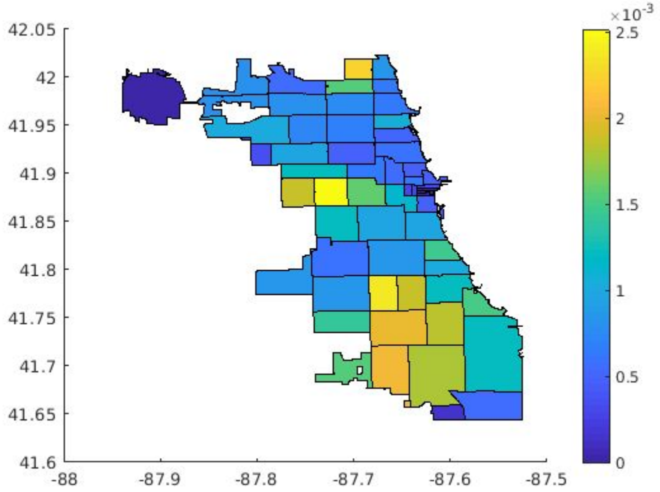
Parameter Results



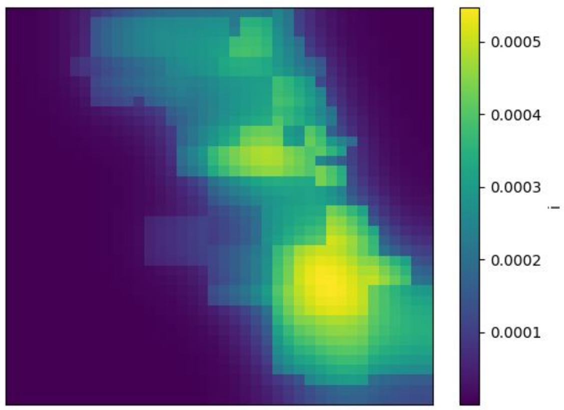
$\omega\beta_A$	transmission rate due to *A*	0.1969	days^{-1}
$\omega\beta_I$	transmission rate due to *I*	0.3061	days^{-1}
η	lockdown scale factor	0.5514	1
δ	symptom onset rate	0.0939	days^{-1}
γ_A	removal rate of *A*	0.3632	days^{-1}
γ_I	removal rate of *I*	0.1385	days^{-1}

Diffusion Results

Day 11 (t_q): March 29, 2020



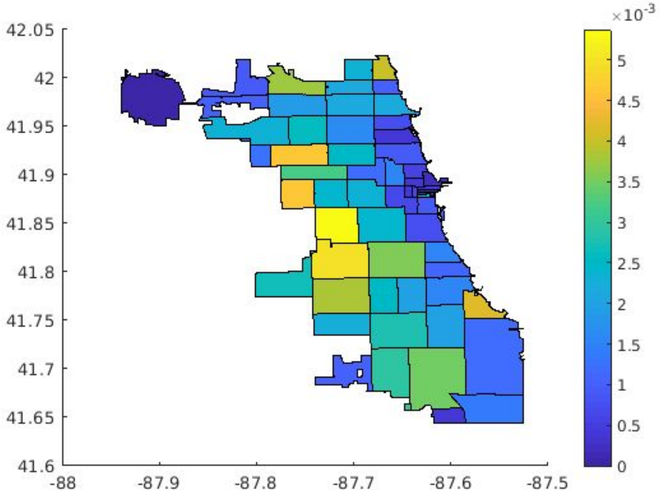
Confirmed COVID Cases by ZIP code



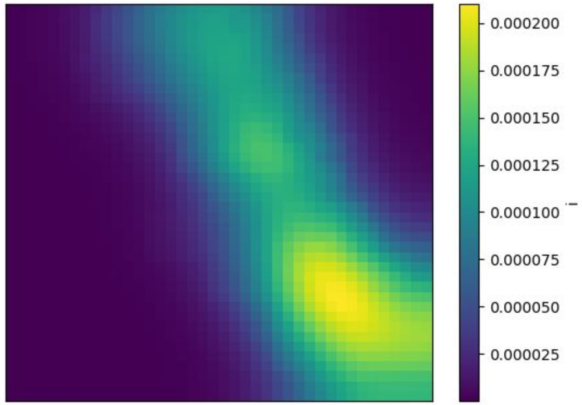
Model Infected

Diffusion Results

Day 35: April 22, 2020



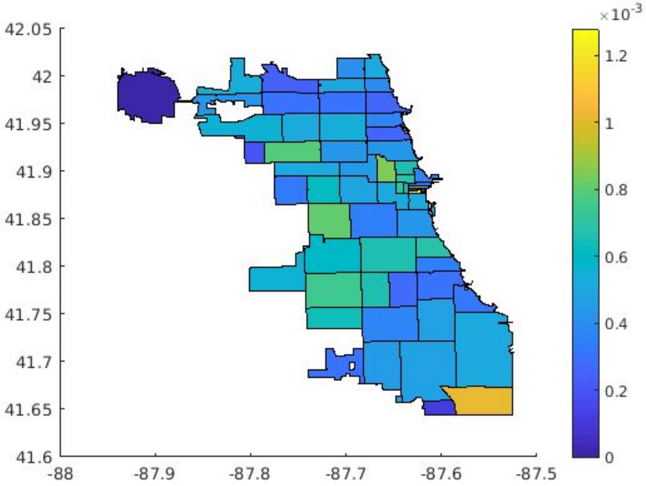
Confirmed COVID Cases by ZIP code



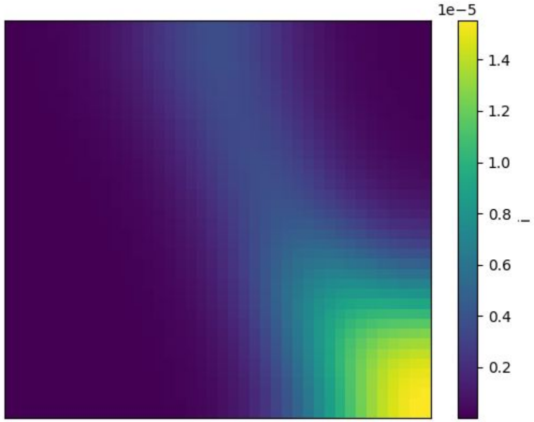
Model Infected

Diffusion Results

Day 99: June 24, 2020



Confirmed COVID Cases by ZIP code



Model Infected

Discussion

Observation

The homogenous model does not reproduce the city-wide data accurately.

- The initial ratio of symptomatic to asymptomatic is fixed.
- No compartment for a latent period.

Observation

Diffusion alone does not seem to properly account for the spread of the virus.

- The boundaries of the computational domain are not entirely representative.
- No account is made of any mass gatherings that occurred.
- The data isn't correct?!

Future Work

Opportunities to improve the model include...

- changing the compartment structure of the model.
- restructuring the cost function to fit directly to the spatial model.
- adding a forcing function to the diffusion.

Interesting applications for future study include...

- modeling over a different set of dates.
- incorporating vaccinations.
- applying the model to a different city.

Questions?