Reaction-diffusion spatial modeling of COVID-19 in Chicago

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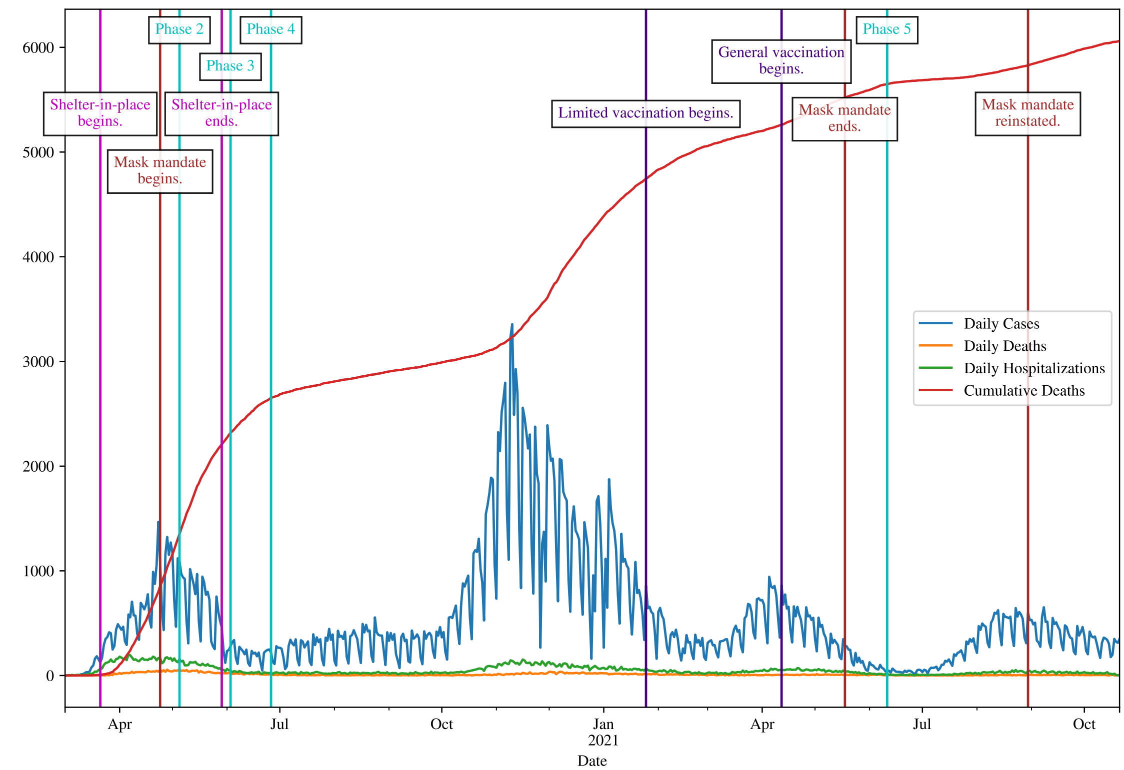

Figure 1: Timeline of the progression of COVID-19 in Chicago with key public policy events marked. The COVID-19 data was obtained from the City of Chicago Data Portal [3]. The dates of the policy events were gathered from the Illinois.gov press releases [7], [5], [6], [8], [4], the Chicago Tribune [2], and NBC Chicago [1].

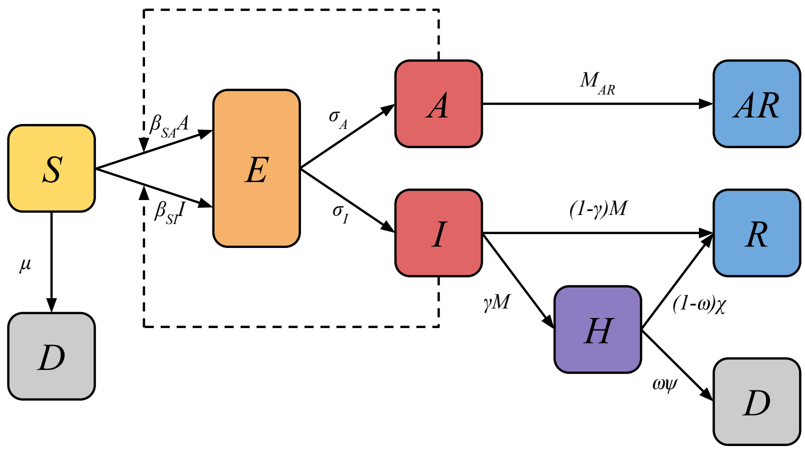

Figure 2: Schematic diagram of the model. The dashed lines indicate the interaction of the infected populations with the susceptible populations that leads to infection.

1 Model Setup

$$
\begin{aligned}
S_t &= \mathfrak{D}_S \Delta S - \beta_{SA} SA - \beta_{SI} SI - \mu S, && (1)\\
E_t &= \mathfrak{D}_E \Delta E + \beta_{SA} SA + \beta_{SI} SI - (\sigma_A + \sigma_I) E, && (2)\\
AR_t &= M_{AR} A, && (3)\\
A_t &= \mathfrak{D}_A \Delta A + \sigma_A E - M_{AR} A, && (4)\\
I_t &= \sigma_I E - MI, && (5)\\
H_t &= \gamma MI - (1-\omega)\chi H - \omega\psi H, && (6)\\
R_t &= (1-\gamma) MI + (1-\omega)\chi H, && (7)\\
D_t &= \omega\psi H. && (8)
\end{aligned}
$$

Table 1: Population values for Chicago. Initial populations are determined from March 13, 2020.

		Population
Total population	N	2,695,598
Initial infected	I_0	162
Initial hospitalized	H_0	38
Initial deceased	D_0	3

2 ODE Dynamics

We want to understand the trajectories of the dynamics of the ODE system under different initial conditions. To do this we first find the equilibrium points by solving

$$S_t = E_t = A_t = I_t = H_t = R_t = D_t = 0$$

simultaneously for $\mathbf{x} = (S, E, A, AR, I, H, R, D)$. The solutions of this system are of the form $\mathbf{x}^* = (0, 0, 0, AR, 0, 0, R, D)$. This implies there are infinitely many non-isolated equilibrium points.

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Table 2: Fitting parameters for Chicago: optimal (best-fitting), median and interquartile range, and variation range used in the optimization algorithm. Initial parameter guesses were uniformly sampled within these ranges.

		Median (interquartile range)	Initial value
Transmission rate, $S \to I$ [per day]	β_{SI}		$c \in \mathcal{U}[0,1]$
Transmission rate, $S \to A$ [per day]	β_{SA}		$c \in \mathcal{U}[0,1]$
Lockdown effect, $S \to I$	η_{SI}		$c \in \mathcal{U}[0,1]$
Lockdown effect, $S \to A$	η_{SA}		$c \in \mathcal{U}[0,1]$
Incubation period, $E \to I$ [days]	$1/\sigma_I$		$1/k, k \in \mathcal{U}[2,7]$
Latent period, $E \to A$ [days]	$1/\sigma_A$		$1/k, k \in \mathcal{U}[2,7]$
Infectivity period [days]	$1/M$		$1/k, k \in \mathcal{U}[5,12]$
Recovery period, $A \to AR$ [days]	$1/M_{AR}$		$1/k, k \in \mathcal{U}[5,12]$
Recovery period, $H \to R$ [days]	$1/\chi$		$1/k, k \in \mathcal{U}[5,20]$
Period to deceased, $H \to D$ [days]	$1/\psi$		$1/k, k \in \mathcal{U}[5,20]$
Conversion fraction ($I \xrightarrow{\gamma} H,\ I \xrightarrow{1-\gamma} R$)	γ		$c \in \mathcal{U}[0.25, 0.75]$
Conversion fraction ($H \xrightarrow{\omega} D,\ H \xrightarrow{1-\omega} R$)	ω		$c \in \mathcal{U}[0.1, 0.5]$
Initial population fraction, exposed	E_0/I_0		$c \in \mathcal{U}[1,5]$
Initial population fraction, asymptomatic	A_0/I_0		$c \in \mathcal{U}[1,5]$

We determine the stability of these equilibrium points by analyzing the linearized system near the points. The Jacobian of the system is

$$\mathsf{J} = \begin{pmatrix} -A\beta_{SA} - I\beta_{SI} - \mu & 0 & -S\beta_{SA} & 0 & -S\beta_{SI} & 0 & 0 & 0 \\ A\beta_{SA} + I\beta_{SI} & -\sigma_A - \sigma_I & S\beta_{SA} & 0 & S\beta_{SI} & 0 & 0 & 0 \\ 0 & \sigma_A & -M_{AR} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & M_{AR} & 0 & 0 & 0 & 0 & 0 \\ 0 & \sigma_I & 0 & 0 & -M & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & M\gamma & -\chi(1-\omega) - \psi\omega & 0 & 0 \\ 0 & 0 & 0 & 0 & M(1-\gamma) & \chi(1-\omega) & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \psi\omega & 0 & 0 \end{pmatrix} \tag{9}$$

Now evaluating J at the equilibrium point $\mathbf{x}^*$ and calculating the eigenvalues, we have

$$\lambda = \{0, 0, 0, -M, -M_{AR}, -\mu, -\sigma_A - \sigma_I, -\chi + \chi\omega - \psi\omega\}. \tag{10}$$

Note that the first three eigenvalues are 0, which implies the equilibrium points are non-isolated. This agrees with our earlier observation.

The equilibrium points are stable when $\lambda_i < 0$ for $4 \le i \le 8$. Since all the system parameters are positive, this implies $\lambda_i < 0$ for $4 \le i \le 7$. Thus the stability depends on the sign of λ_8. There are two cases when $\lambda_8 = -\chi + \chi\omega - \psi\omega < 0$ is true:

1. $0 < \omega \le 1$ implies $\lambda_8 < 0$, and
2. $\omega > 1$ and $\chi < \frac{\psi\omega}{\omega - 1}$ implies $\lambda_8 < 0$.

That is, whenever we have either of these conditions the equilibrium points are stable. We call this situation *endemic*. If $\lambda_8 > 0$, the equilibrium points are unstable and the situation is an *epidemic*.

References

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