

Reaction-diffusion spatial modeling of COVID-19 in Chicago

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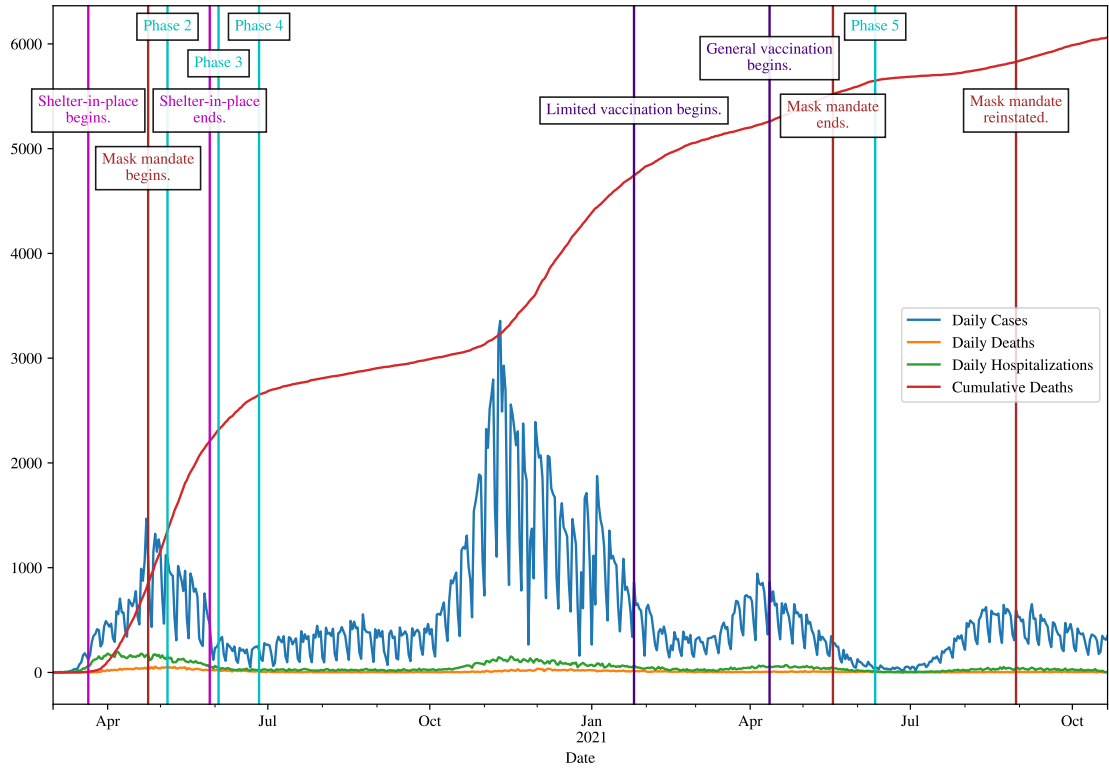


Figure 1: Timeline of the progression of COVID-19 in Chicago with key public policy events marked. The COVID-19 data was obtained from the City of Chicago Data Portal [3]. The dates of the policy events were gathered from the Illinois.gov press releases [7], [5], [6], [8], [4], the Chicago Tribune [2], and NBC Chicago [1].

1 Model Setup

[9]

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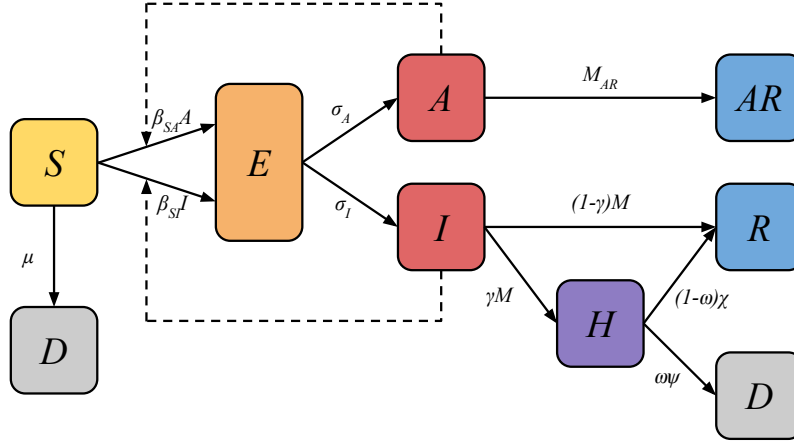


Figure 2: Schematic diagram of the model. The dashed lines indicate the interaction of the infected populations with the susceptible populations that leads to infection.

$$S_t = \mathfrak{D}_S \Delta S - \beta_{SA} S A - \beta_{SI} S I - \mu S, \quad (1)$$

$$E_t = \mathfrak{D}_E \Delta E + \beta_{SA} S A + \beta_{SI} S I - (\sigma_A + \sigma_I) E, \quad (2)$$

$$AR_t = M_{AR} A, \quad (3)$$

$$A_t = \mathfrak{D}_A \Delta A + \sigma_A E - M_{AR} A, \quad (4)$$

$$I_t = \sigma_I E - M I, \quad (5)$$

$$H_t = \gamma M I - (1 - \omega) \chi H - \omega \psi H, \quad (6)$$

$$R_t = (1 - \gamma) M I + (1 - \omega) \chi H, \quad (7)$$

$$D_t = \omega \psi H. \quad (8)$$

Table 1: Population values for Chicago. Initial populations are determined from March 13, 2020.

	Population	
Total population	N	2,695,598
Initial infected	I_0	162
Initial hospitalized	H_0	38
Initial deceased	D_0	3

2 ODE Dynamics

We want to understand the trajectories of the dynamics of the ODE system under different initial conditions. To do this we first find the equilibrium points by solving

$$S_t = E_t = A_t = I_t = H_t = R_t = D_t = 0$$

simultaneously for $\mathbf{x} = (S, E, A, AR, I, H, R, D)$. The solutions of this system are of the form $\mathbf{x}^* = (0, 0, 0, AR, 0, 0, R, D)$. This implies there are infinitely many non-isolated equilibrium points. We determine the stability of these equilibrium points by analyzing the linearized system near the

Table 2: Fitting parameters for Chicago: optimal (best-fitting), median and interquartile range, and variation range used in the optimization algorithm. Initial parameter guesses were uniformly sampled within these ranges.

	Median (interquartile range)	Initial value
Transmission rate, $S \rightarrow I$ [per day]	β_{SI}	$c \in \mathcal{U}[0, 1]$
Transmission rate, $S \rightarrow A$ [per day]	β_{SA}	$c \in \mathcal{U}[0, 1]$
Lockdown effect, $S \rightarrow I$	η_{SI}	$c \in \mathcal{U}[0, 1]$
Lockdown effect, $S \rightarrow A$	η_{SA}	$c \in \mathcal{U}[0, 1]$
Incubation period, $E \rightarrow I$ [days]	$1/\sigma_I$	$1/k, k \in \mathcal{U}[2, 7]$
Latent period, $E \rightarrow A$ [days]	$1/\sigma_A$	$1/k, k \in \mathcal{U}[2, 7]$
Infectivity period [days]	$1/M$	$1/k, k \in \mathcal{U}[5, 12]$
Recovery period, $A \rightarrow AR$ [days]	$1/M_{AR}$	$1/k, k \in \mathcal{U}[5, 12]$
Recovery period, $H \rightarrow R$ [days]	$1/\chi$	$1/k, k \in \mathcal{U}[5, 20]$
Period to deceased, $H \rightarrow D$ [days]	$1/\psi$	$1/k, k \in \mathcal{U}[5, 20]$
Conversion fraction ($I \xrightarrow{\gamma} H, I \xrightarrow{1-\gamma} R$)	γ	$c \in \mathcal{U}[0.25, 0.75]$
Conversion fraction ($H \xrightarrow{\omega} D, H \xrightarrow{1-\omega} R$)	ω	$c \in \mathcal{U}[0.1, 0.5]$
Initial population fraction, exposed	E_0/I_0	$c \in \mathcal{U}[1, 5]$
Initial population fraction, asymptomatic	A_0/I_0	$c \in \mathcal{U}[1, 5]$

points. The Jacobian of the system is

$$J = \begin{pmatrix} -A\beta_{SA} - I\beta_{SI} - \mu & 0 & -S\beta_{SA} & 0 & -S\beta_{SI} & 0 & 0 & 0 \\ A\beta_{SA} + I\beta_{SI} & -\sigma_A - \sigma_I & S\beta_{SA} & 0 & S\beta_{SI} & 0 & 0 & 0 \\ 0 & \sigma_A & -M_{AR} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & M_{AR} & 0 & 0 & 0 & 0 & 0 \\ 0 & \sigma_I & 0 & 0 & -M & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & M\gamma & -\chi(1-\omega) - \psi\omega & 0 & 0 \\ 0 & 0 & 0 & 0 & M(1-\gamma) & \chi(1-\omega) & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \psi\omega & 0 & 0 \end{pmatrix} \quad (9)$$

Now evaluating J at the equilibrium point \mathbf{x}^* and calculating the eigenvalues, we have

$$\lambda = \{0, 0, 0, -M, -M_{AR}, -\mu, -\sigma_A - \sigma_I, -\chi + \chi\omega - \psi\omega\}. \quad (10)$$

Note that the first three eigenvalues are 0, which implies the equilibrium points are non-isolated. This agrees with our earlier observation.

The equilibrium points are stable when $\lambda_i < 0$ for $4 \leq i \leq 8$. Since all the system parameters are positive, this implies $\lambda_i < 0$ for $4 \leq i \leq 7$. Thus the stability depends on the sign of λ_8 . There are two cases when $\lambda_8 = -\chi + \chi\omega - \psi\omega < 0$ is true:

1. $0 < \omega \leq 1$ implies $\lambda_8 < 0$, and
2. $\omega > 1$ and $\chi < \frac{\psi\omega}{\omega-1}$ implies $\lambda_8 < 0$.

That is, whenever we have either of these conditions the equilibrium points are stable. We call this situation *endemic*. If $\lambda_8 > 0$, the equilibrium points are unstable and the situation is an *epidemic*.

References

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