$$S_t = -\beta_{SA}SA - \beta_{SI}SI - \mu S, \tag{1}$$
$$E_t = \beta_{SA}SA - \beta_{SI} - (\sigma_A + \sigma_I)E, \tag{2}$$
$$A_t = \sigma_A E - M_{AR}A, \tag{3}$$
$$AR_t = M_{AR}A, \tag{4}$$
$$I_t = \sigma_I E - MI, \tag{5}$$
$$H_t = \gamma MI - (1-\omega)\chi H - \omega\psi H, \tag{6}$$
$$R_t = (1-\gamma)MI + (1-\omega)\chi H, \tag{7}$$
$$D_t = \omega\psi H. \tag{8}$$

We determine the optimal parameters by minimizing the Euclidean distance $\mathcal{N}$ between the time series generated by the model (num) and the corresponding observed time series (obs),

$$\mathcal{N} = \sum_i \left(|\log(C_{\text{num}}(t_i)) - \log(C_{\text{obs}}(t_i))|^2 + |\log(D_{\text{num}}(t_i)) - \log(D_{\text{obs}}(t_i))|^2 \right) \tag{9}$$

where the index i identifies a point in the time series. The optimization tries to reproduce the number of total reported cases ($C(t) = I(t) + H(t) + R(t) + D(t)$) and the total number of deceased ($D(t)$).

We can account for a change in the parameters due to the lockdown by imposing a time dependence on the transmission rates β:

$$\beta_{IS}(t) = \beta_{IS}\left[\eta_{IS} + (1-\eta_{IS})\frac{1-\tanh[2(t-t_q)]}{2}\right], \tag{10}$$
$$\beta_{AS}(t) = \beta_{AS}\left[\eta_{AS} + (1-\eta_{AS})\frac{1-\tanh[2(t-t_q)]}{2}\right]. \tag{11}$$

This causes the transmission rates β_{IS} and β_{AS} to decrease abruptly by a factor of η_{IS} and η_{AS} respectively at the time t_q when the lockdown was imposed.