

Reaction-diffusion spacial modeling of COVID-19 in Chicago

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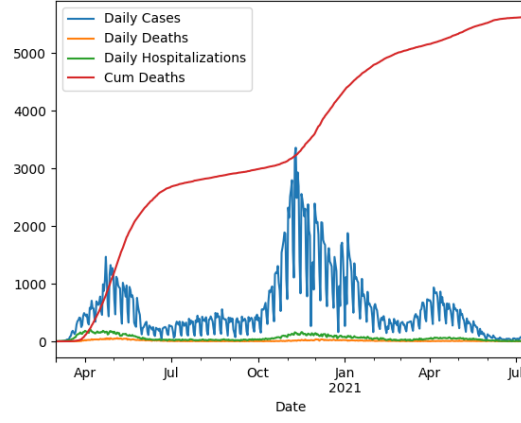


Figure 1: Reported cases in Chicago.

$$S_t = -\beta_{SA}SA - \beta_{SI}SI - \mu S, \quad (1)$$

$$E_t = \beta_{SA}SA - \beta_{SI}SI - (\sigma_A + \sigma_I)E, \quad (2)$$

$$A_t = \sigma_A E - M_{AR}A, \quad (3)$$

$$AR_t = M_{AR}A, \quad (4)$$

$$I_t = \sigma_I E - MI, \quad (5)$$

$$H_t = \gamma MI - (1 - \omega)\chi H - \omega\psi H, \quad (6)$$

$$R_t = (1 - \gamma)MI + (1 - \omega)\chi H, \quad (7)$$

$$D_t = \omega\psi H. \quad (8)$$

We determine the optimal parameters by minimizing the Euclidean distance \mathcal{N} between the time series generated by the model (num) and the corresponding observed time series (obs),

$$\mathcal{N} = \sum_i \left(|\log(C_{\text{num}}(t_i)) - \log(C_{\text{obs}}(t_i))|^2 + |\log(D_{\text{num}}(t_i)) - \log(D_{\text{obs}}(t_i))|^2 \right) \quad (9)$$

where the index i identifies a point in the time series. The optimization tries to reproduce the number of total reported cases ($C(t) = I(t) + H(t) + R(t) + D(t)$) and the total number of deceased ($D(t)$). [1]

We can account for a change in the parameters due to the lockdown by imposing a time dependence on the transmission rates β :

$$\beta_{IS}(t) = \beta_{IS} \left[\eta_{IS} + (1 - \eta_{IS}) \frac{1 - \tanh[2(t - t_q)]}{2} \right], \quad (10)$$

$$\beta_{AS}(t) = \beta_{AS} \left[\eta_{AS} + (1 - \eta_{AS}) \frac{1 - \tanh[2(t - t_q)]}{2} \right]. \quad (11)$$

This causes the transmission rates β_{IS} and β_{AS} to decrease abruptly by a factor of η_{IS} and η_{AS} respectively at the time t_q when the lockdown was imposed. [2]

		Median ($t_q = 3$)	Median ($t_q = 16$)	Initial value
Population	N		2,693,976	
Initial populations	(I_0, H_0, D_0)		(297, 134, 6)	
Non COVID-19 death rate [per day]	μ		5.79×10^{-7}	
Transmission rate, $S \rightarrow I$ [per day]	β_{IS}	0.35(0.28 – 0.41)	0.32(0.29 – 0.35)	$c \in U[0, 1]$

References

- [1] P. KEVREKIDIS, J. CUEVAS-MARAVER, Y. DROSSINOS, Z. RAPTİ, AND G. KEVREKIDIS, *Spatial modeling of covid-19: Greece and andalusia as case examples*, Physical Review E, (2021).
- [2] Y. WANG, C. LIU, P. LIU, AND B. EISENBERG, *Field theory of reaction-diffusion: Law of mass action with an energetic variational approach*, Physical Review E, 102 (2020).