Reaction-diffusion spatial modeling of COVID-19 in Chicago

Trent Gerew*

Department of Applied Mathematics, Illinois Institute of Technology, Chicago, Illinois
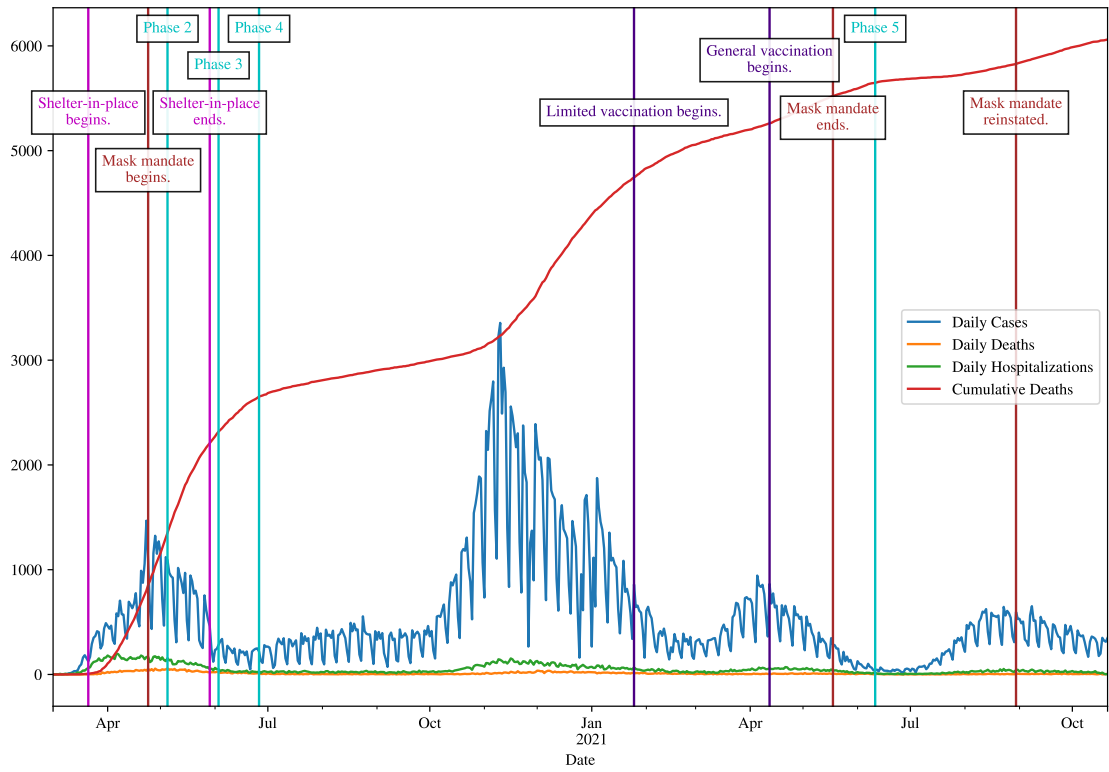
October 27, 2021

Figure 1: Timeline of the progression of COVID-19 in Chicago with key public policy events marked. The COVID-19 data was obtained from the City of Chicago Data Portal [3]. The dates of the policy events were gathered from the Illinois.gov press releases [7], [5], [6], [8], [4], the Chicago Tribune [2], NBC Chicago [1]
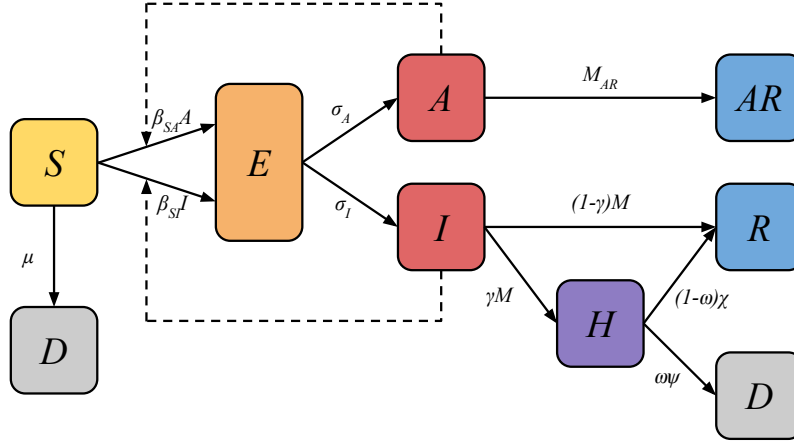
Figure 2: Schematic diagram of the model. The dashed lines indicate the interaction of the infected populations with the susceptible populations that leads to infection.

1 Model Setup

$$S_t = \mathfrak{D}_S \Delta S - \beta_{SA} SA - \beta_{SI} SI - \mu S, \tag{1}$$

$$E_t = \mathfrak{D}_E \Delta E + \beta_{SA} SA + \beta_{SI} SI - (\sigma_A + \sigma_I)E, \tag{2}$$

$$AR_t = M_{AR} A, \tag{3}$$

$$A_t = \mathfrak{D}_A \Delta A + \sigma_A E - M_{AR} A, \tag{4}$$

$$I_t = \sigma_I E - MI, \tag{5}$$

$$H_t = \gamma MI - (1 - \omega)\chi H - \omega\psi H, \tag{6}$$

$$R_t = (1 - \gamma)MI + (1 - \omega)\chi H, \tag{7}$$

$$D_t = \omega\psi H. \tag{8}$$

Table 1: Population values for Chicago.

		Population
Total population	N	2,695,598
Initial infected	I_0	127
Initial hospitalized	H_0	30
Initial deceased	D_0	6

2 ODE Dynamics

We want to understand the trajectories of the dynamics of the ODE system under different initial conditions. To do this we first find the equilibrium points by solving

$$S_t = E_t = A_t = I_t = H_t = R_t = D_t = 0$$

simultaneously for $\mathbf{x} = (S, E, A, AR, I, H, R, D)$. The solutions of this system are of the form $\mathbf{x}^* = (0, 0, 0, AR, 0, 0, R, D)$. This implies there are infinitely many non-isolated equilibrium points.

---

*tgerew@hawk.iit.edu

Table 2: Parameters for Chicago: optimal (best-fitting), median and interquartile range, and variation range used in the optimization algorithm. Initial parameter guesses were uniformly sampled within these ranges.

		Median (interquartile range)	Initial value
Population	N	2,695,598	
Initial population	(I_0, R_0)	(127, 2)	
Transmission rate, $S \to I$ [per day]	β^1	0.38206(0.38204-0.38209)	$c \in U[0, 1]$
Transition rate, $I \to R$ [per day]	γ	0.39656(0.39654-0.39659)	$c \in U[0.25, 0.75]$
Diffusivity, S [km^2/day]	$\mathfrak{D}_S$	10	
Diffusivity, I [km^2/day]	$\mathfrak{D}_I$	100	

We determine the stability of these equilibrium points by analyzing the linearized system near the points. The Jacobian of the system is

$$
\mathsf{J} = \begin{pmatrix}
-A\beta_{SA} - I\beta_{SI} - \mu & 0 & -S\beta_{SA} & 0 & -S\beta_{SI} & 0 & 0 & 0 \\
A\beta_{SA} + I\beta_{SI} & -\sigma_A - \sigma_I & S\beta_{SA} & 0 & S\beta_{SI} & 0 & 0 & 0 \\
0 & \sigma_A & -M_{AR} & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & M_{AR} & 0 & 0 & 0 & 0 & 0 \\
0 & \sigma_I & 0 & 0 & -M & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & M\gamma & -\chi(1-\omega) - \psi\omega & 0 & 0 \\
0 & 0 & 0 & 0 & M(1-\gamma) & \chi(1-\omega) & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & \psi\omega & 0 & 0
\end{pmatrix}
\tag{9}
$$

Now evaluating J at the equilibrium point $\mathbf{x}^*$ and calculating the eigenvalues, we have

$$
\lambda = \{0, 0, 0, -M, -M_{AR}, -\mu, -\sigma_A - \sigma_I, -\chi + \chi\omega - \psi\omega\}.
\tag{10}
$$

Note that the first three eigenvalues are 0, which implies the equilibrium points are non-isolated. This agrees with our earlier observation.

The equilibrium points are stable when $\lambda_i < 0$ for $4 \le i \le 8$. Since all the system parameters are positive, this implies $\lambda_i < 0$ for $4 \le i \le 7$. Thus the stability depends on the sign of λ_8. There are two cases when $\lambda_8 = -\chi + \chi\omega - \psi\omega < 0$ is true:

1. $0 < \omega \le 1$ implies $\lambda_8 < 0$, and

2. $\omega > 1$ and $\chi < \frac{\psi\omega}{\omega - 1}$ implies $\lambda_8 < 0$.

That is, whenever we have either of these conditions the equilibrium points are stable. We call this situation *endemic*. If $\lambda_8 > 0$, the equilibrium points are unstable and the situation is an *epidemic*.

References

[1] Read the full 'restore Illinois' plan aimed at reopening the state during coronavirus. *NBC Chicago*, May 2020. `https://www.nbcchicago.com/news/coronavirus/read-the-full-restore-illinois-plan-aimed-at-reopening-the-state-during-coronavirus/2267039/`.

[2] J. Byrne, D. Petrella, and A. Lukach. Mayor Lori Lightfoot says Chicago will move to phase 3 of her reopening plan on June 3 but warns: 'COVID-19 is still very much part of our present'. *Chicago Tribune*, May 2020. `https://www.chicagotribune.com/coronavirus/ct-coronavirus-chicago-lightfoot-reopening-20200528-cefwiuidwnfd7a57m25uavq6me-story.html`.

[3] City of Chicago. Daily chicago covid-19 cases, deaths, and hospitalizations, 2021. Data retrieved from Chicago Data Portal, `https://data.cityofchicago.org/Health-Human-Services/Daily-Chicago-COVID-19-Cases-Deaths-and-Hospitaliz/kxzd-kd6a`.

[4] Illinois.gov. Gov. Pritzker releases guidelines to safely reopen additional businesses and industries as state advances to next phase of restore Illinois. Press Release, June 2020. `https://www.illinois.gov/news/press-release.21714.html`.

[5] Illinois.gov. Gov. Pritzker aligns Illinois mask guidance with CDC for fully vaccinated people. Press Release, May 2021. `https://www.illinois.gov/news/press-release.23322.html`.

[6] Illinois.gov. Gov. Pritzker announces metrics-based pathway for Illinois to fully reopen; expands vaccine eligibility to all residents 16+ on April 12. Press Release, March 2021. `https://www.illinois.gov/news/press-release.22961.html`.

[7] Illinois.gov. Gov. Pritzker issues guidelines for Illinois reopening on June 11. Press Release, June 2021. `https://www.illinois.gov/news/press-release.23399.html8`.

[8] Illinois.gov. United Center vaccination appointments open Thursday for Illinois seniors. Press Release, March 2021. `https://www.illinois.gov/news/press-release.22868.html`.