Reaction-diffusion spatial modeling of COVID-19 in Chicago

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Abstract

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1   Introduction

2   Materials and methods

2.1   Confirmed and death data

In this study, we used the publicly available data set of COVID-19 provided by the City of Chicago Data Portal. This data set includes daily counts of confirmed infected cases, hospitalizations, and deaths.

2.2   Mathematical model

We focus our study on four components of the epidemic flow (Figure 1). That is, the populations of Susceptible individuals (S), Asymptomatic infected individuals (A), symptomatic Infected individuals (I), and Removed individuals (R). Our model is known as the SAIR model [1], which
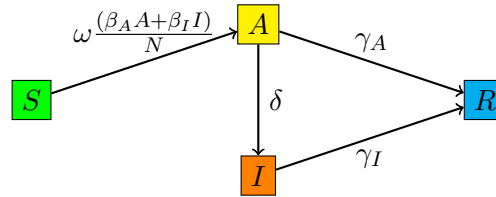

Figure 1: Compartmental representation of the SAIR model.

substitutes the E compartment of the SEIR model by the A compartment. This model is relevant when there are many undetected asymptomatic infectious individuals, which is known to be the case for COVID-19.

We motivate the use of this simple model for the following reasons:

- We do not consider the E group in this work because we feel this population can be merged with the A group.

- We do not distinguish between quarantined, hospitalized, or nursing-home populations. Including these additional compartments would over parameterize the model. Additionally, the

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way in which data has been collected and is provided by authorities has varied over time, making its usage rather difficult. For the same reasons, we do not distinguish between deceased and recovered.

To build the mathematical model, we followed the standard strategy developed in the literature concerning SIR models. We assume that individuals in S can be infected by both members of A and I. We suppose that the individuals in the A and I compartments may have different contact rates β_A and β_I, and different recovery rates γ_A and γ_I. Furthermore, we consider a rate δ at which individuals in A may develop symptoms or are otherwise detected and so will move to the I compartment. Lastly, we assume that only members of S and A are mobile.

The dynamics is governed by a system of two partial differential equations (PDE) and two ordinary differential equations (ODE) as follows, for $\mathbf{x} = (x, y) \in \Omega \subset \mathbf{R}^2, t > 0$,

$$
\begin{aligned}
\partial_t S - D(t)\Delta S &= -\omega(t)\frac{\beta_A A + \beta_I I}{N}S, \\
\partial_t A - D(t)\Delta A &= \omega(t)\frac{\beta_A A + \beta_I I}{N}S - (\gamma_A - \delta)A, \\
I' &= -\gamma_I I + \delta A, \\
R' &= \gamma_A A + \gamma_I.
\end{aligned}
\tag{1}
$$

Since travel into the city of Chicago was heavily restricted for the early stages of the pandemic, the homogeneous Neumann boundary condition is imposed [3]. The total population is $N = S + A + I + R$, and is considered a constant.

2.3   Parameter estimation

To account for the lockdown, the average number of of contacts is updated as follows [2]

$$
\omega(t) = \omega_0 \left[\eta + (1 - \eta)\frac{1 - \tanh\left[2(t - t_q)\right]}{2} \right],
\tag{2}
$$

while the diffusion coefficient is similarly

$$
D(t) = D_0 \left[\eta + (1 - \eta)\frac{1 - \tanh\left[2(t - t_q)\right]}{2} \right].
\tag{3}
$$

Here, $t_q = (t_{\mathrm{eol}} + t_{\mathrm{bol}})/2$, where bol denotes the beginning of the lockdown and eol denotes the end of the lockdown. The parameter $0 \leq \eta \leq 1$ is a varying coefficient translating respect for social distancing and other preventative measures. Note that ω_0 is the average number of contacts before any intervention, and is a constant.

2.4   Numerical discretization

3   Results

3.1   Existence of solutions and basic reproduction numbers

3.2   Model resolution

3.3   Spatial spread of COVID-19

4   Discussion

Conflicts of Interest: Author states no conflict of interest.

References

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