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An Update to Mesh Generation and Load Balancing in PDT

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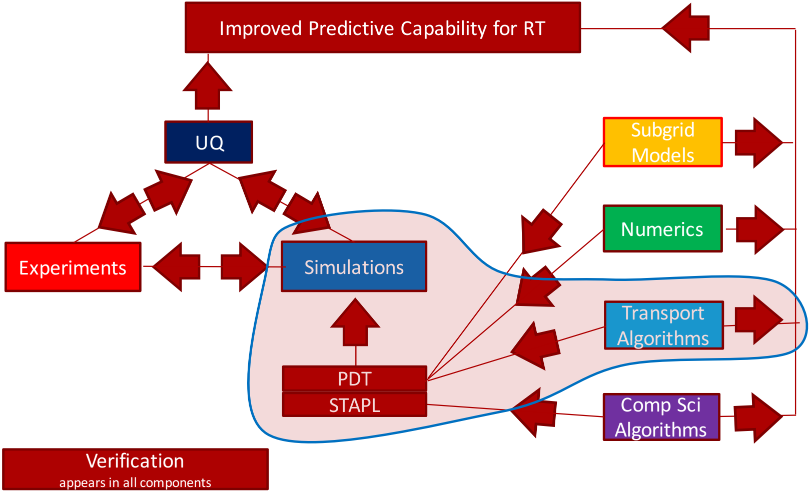
Project Components and Integration



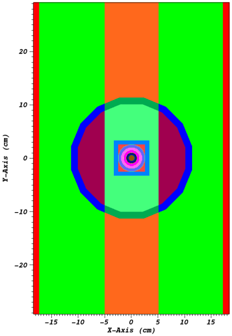
1. Introduction
2. Load Balance Algorithm
3. Load Balancing Results
4. Conclusions

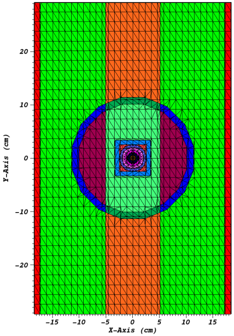
Motivation

- When running any massively parallel code, load balancing is a priority in order to achieve the best possible parallel efficiency.
- A load balanced problem has an equal number of degrees of freedom per processor.
- Load balancing a logically Cartesian mesh is "not difficult", as the user specifies the number of cells being used.
- In an unstructured mesh, the user cannot always specify the number of cells they want per processor, and obtaining a load balanced problem is more difficult.
- The goal is to implement a load balancing algorithm for unstructured meshes in PDT.
- Our problem is unique because we need our cellsets to be convex (bricks for now) because bricks are provably optimal.

The Triangle Mesh Generator

- Unstructured meshes in PDT are generated in 2D using the Triangle Mesh Generator.
- These can be extruded to create 3D meshes.
- Fixes by Daryl Hawkins within the Triangle source code have lead to much more stable mesh generation for complex geometries.





Partitioning for an Unstructured Mesh

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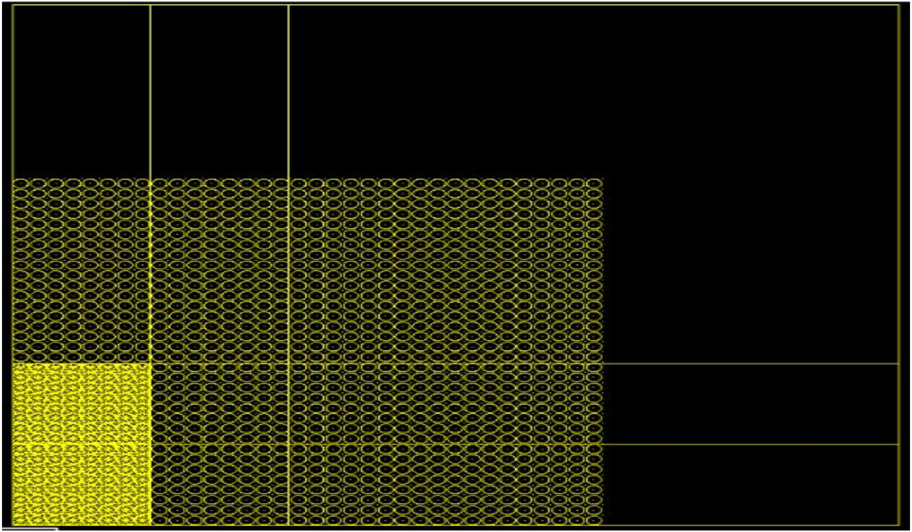
- The user inputs coordinates for cut lines in the X and Y directions.
- The cut lines will determine the number of "subsets" the problem is partitioned into.
- Optimizing the location of these cut lines is the basis of the load balancing algorithm.
- A "subset" is a rectangular unit that is formed by intersecting cut lines.

The Subset



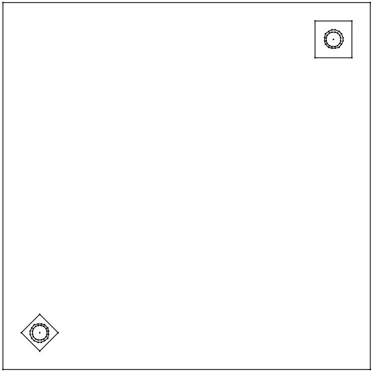
- **Goal:** Obtain an equal number of cells per processor, which for our purposes means an equal number of cells per subset.
- Achieved by optimizing the location of X_i and Y_j, the location of the cut lines.
- **Define**:
  - N_{ij}: The number of cells in subset i, j
  - $f = \frac{\max_{ij}(N_{ij})}{\frac{N_{tot}}{I \cdot J}}$
  - $f_I = \max_i [\sum_j N_{ij}] / \frac{N_{tot}}{I}$
  - $f_J = \max_j [\sum_i N_{ij}] / \frac{N_{tot}}{J}$
  - $f_{J_i} = \max [N_{ij}]_i / \frac{\sum_j N_{ij}}{J}$

Original Load Balancing Algorithm

```
//Check if all subsets meet the tolerance
while (f > tol_subset)
{
  if (f_I > tol_column)
  {
    Redistribute(X);
  }
  if (f_J > tol_row)
  {
    Redistribute(Y);
  }

  Remesh;
}
```

Why a new algorithm?



- Limited effectiveness if we don't move more cut lines into BOTH features.
- We allow one dimension's cut lines to be moved by row/column rather than cut all the way through the problem.

Challenges

- Altering and tinkering with the algorithm is not difficult.
- However, a completely new stitching algorithm had to be written.
- The old stitching algorithm assumed one neighboring subset to the north, south, east and west.
- We now have an arbitrary number of neighbors in the north/south or east/west directions.

Load Balancing By Dimension Algorithm

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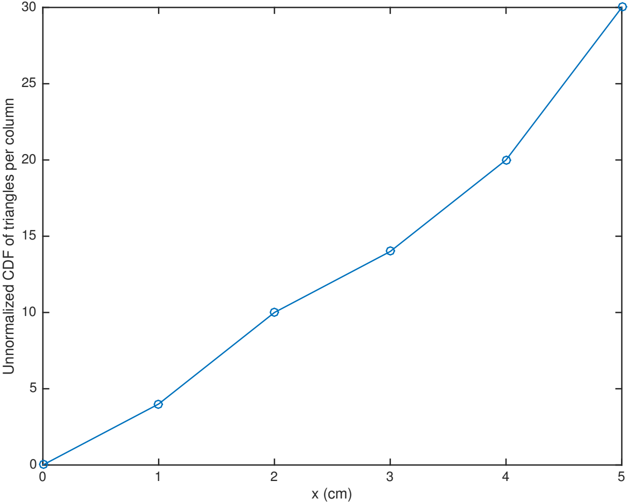
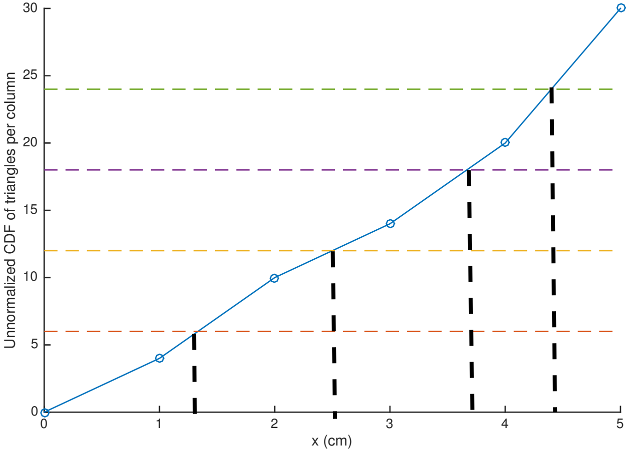
```
while (f_I > tol_column)
{
  if (f_I > tol_column)
  {
    Redistribute(X);
  }

  Remesh;
}

while (f > tol)
{
  for i < I
  {
    if (f_J[i] > tol_row)
    {
      Redistribute(Y_i);
    }
  }
  Remesh;
}
```

Redistribution Function



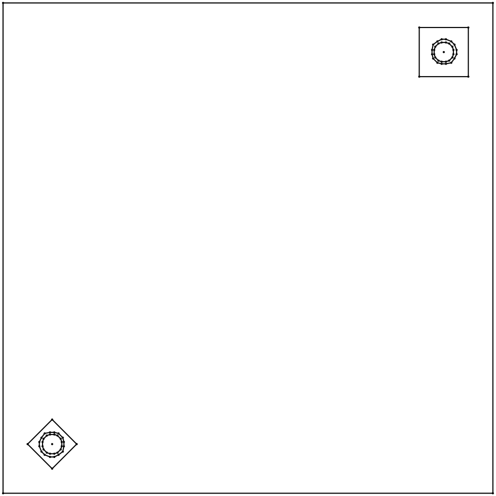
Redistribution Function



Load Balancing Results

- Two test cases were used to study the behavior of both load balancing algorithms.
- For each test case, 162 inputs were constructed by varying:
    - The number of subsets
    - The spatial resolution of the mesh (maximum triangle area).

Test Case 1



Test Case 1 - Original Load Balancing Method

1: The best metric behavior of the first test case after **10 load balancing iterations**.

Area	N=4	N=9	N=16	N=25	N=36	N=49	N=64	N=81	N=100
Coarse	1.949	1.598	3.368	2.098	2.278	2.678	2.535	2.805	3.053
1.8	1.458	1.940	2.449	2.590	2.980	3.441	2.957	4.657	3.434
1.6	1.423	1.949	2.427	2.411	3.004	3.053	3.579	4.107	4.105
1.4	1.316	1.871	2.654	3.130	2.451	3.030	3.473	4.040	3.898
1.2	1.298	1.765	2.462	2.656	2.592	3.178	3.144	4.282	**4.683**
1	1.348	1.638	2.260	2.327	2.347	3.013	3.357	3.841	4.245
0.8	1.257	1.513	2.017	2.792	2.018	2.617	2.884	3.423	3.629
0.6	1.142	1.452	1.788	2.408	2.332	2.092	2.669	2.874	3.629
0.4	1.095	1.353	1.449	1.872	2.397	1.836	2.153	2.351	2.262
0.2	1.046	1.136	1.336	1.545	1.648	2.049	1.678	1.790	1.714
0.1	1.020	1.043	1.109	1.170	1.287	1.357	1.297	1.409	1.221
0.08	1.011	1.029	1.094	1.190	1.209	1.290	1.268	1.318	1.381
0.06	1.005	1.031	1.037	1.105	1.087	1.189	1.177	1.283	1.068
0.05	1.021	1.022	1.058	1.092	1.079	1.115	1.157	1.218	1.176
0.04	1.004	1.013	**1.002**	1.061	1.074	1.073	1.158	1.171	1.171
0.03	1.003	1.016	1.021	1.050	1.065	1.048	1.928	1.132	1.041
0.02	1.004	1.008	1.010	1.034	1.024	1.028	1.574	1.075	1.094
0.01	1.003	1.010	1.008	1.009	1.039	1.018	1.276	1.043	1.022

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2: The best metric behavior of the first test case after **10 load balancing by dimension iterations**.

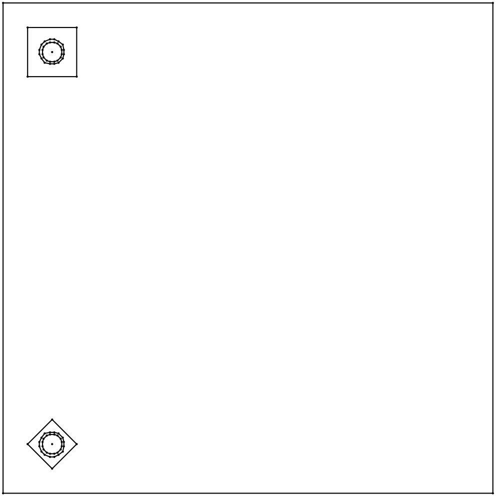
Area	N=4	N=9	N=16	N=25	N=36	N=49	N=64	N=81	N=100
Coarse	1.854	1.205	1.475	1.398	1.260	1.367	1.428	1.625	1.639
1.800	1.079	1.392	1.523	1.722	2.036	2.315	2.421	3.962	3.587
1.600	1.044	1.704	1.536	1.566	1.791	2.126	2.492	3.417	3.807
1.400	1.073	1.152	1.509	1.756	1.811	2.183	2.475	2.911	**3.974**
1.200	1.037	1.125	1.515	1.718	1.972	2.611	2.628	3.816	3.205
1.000	1.047	1.209	1.180	1.763	1.494	1.928	2.564	3.744	3.953
0.800	1.077	1.193	1.246	1.343	1.417	2.949	2.411	3.560	3.629
0.600	1.066	1.112	1.388	1.455	1.521	1.837	1.769	2.586	3.381
0.400	1.095	1.032	1.145	1.221	1.479	1.355	1.554	2.038	1.838
0.200	1.046	1.049	1.075	1.149	1.173	1.174	1.210	1.251	1.571
0.100	1.020	1.043	1.023	1.071	1.103	1.138	1.129	1.091	1.205
0.080	1.011	1.029	1.094	1.072	1.076	1.097	1.068	1.071	1.213
0.060	1.005	1.031	1.037	1.016	1.087	1.096	1.093	1.095	1.068
0.050	1.021	1.022	1.058	1.092	1.079	1.062	1.090	1.075	1.088
0.040	1.004	1.013	**1.002**	1.061	1.074	1.073	1.067	1.090	1.100
0.030	1.003	1.016	1.021	1.050	1.065	1.048	1.038	1.061	1.041
0.020	1.004	1.008	1.010	1.034	1.024	1.028	1.058	1.075	1.094
0.010	1.003	1.010	1.008	1.009	1.039	1.018	1.054	1.043	1.022

Test Case 1 - Improvement

3: The improvement with the load balancing by dimension method.

Area	N=4	N=9	N=16	N=25	N=36	N=49	N=64	N=81	N=100
Coarse	0.049	0.246	**0.562**	0.334	0.447	0.489	0.437	0.421	0.463
1.800	0.260	0.282	0.378	0.335	0.317	0.327	0.181	0.149	-0.045
1.600	0.266	0.126	0.367	0.350	0.404	0.304	0.304	0.168	0.072
1.400	0.185	0.384	0.431	0.439	0.261	0.279	0.287	0.279	-0.020
1.200	0.201	0.363	0.384	0.353	0.239	0.178	0.164	0.109	0.316
1.000	0.223	0.262	0.478	0.242	0.364	0.360	0.236	0.025	0.069
0.800	0.143	0.211	0.382	0.519	0.298	**-0.127**	0.164	-0.040	0.000
0.600	0.067	0.234	0.224	0.396	0.347	0.122	0.337	0.100	0.068
0.400	0.000	0.237	0.210	0.347	0.383	0.262	0.278	0.133	0.188
0.200	0.000	0.076	0.196	0.256	0.288	0.427	0.279	0.301	0.083
0.100	0.000	0.000	0.078	0.085	0.143	0.161	0.130	0.226	0.013
0.080	0.000	0.000	0.000	0.099	0.110	0.150	0.158	0.188	0.122
0.060	0.000	0.000	0.000	0.080	0.000	0.078	0.071	0.147	0.000
0.050	0.000	0.000	0.000	0.000	0.000	0.048	0.058	0.117	0.075
0.040	0.000	0.000	0.000	0.000	0.000	0.000	0.079	0.069	0.061
0.030	0.000	0.000	0.000	0.000	0.000	0.000	0.462	0.062	0.000
0.020	0.000	0.000	0.000	0.000	0.000	0.000	0.328	0.000	0.000
0.010	0.000	0.000	0.000	0.000	0.000	0.000	0.174	0.000	0.000

Test Case 2



4: The best metric behavior of the second test case after **10 load balancing iterations**.

Area	N=4	N=9	N=16	N=25	N=36	N=49	N=64	N=81	N=100
Coarse	1.854	1.361	1.765	1.479	1.742	1.595	1.792	1.820	1.923
1.800	1.176	1.336	1.882	2.375	2.269	2.359	2.544	3.841	**4.874**
1.600	1.109	1.482	1.783	1.701	1.990	2.421	2.848	3.345	2.989
1.400	1.133	1.366	1.854	1.746	1.882	2.600	2.833	3.617	2.692
1.200	1.153	1.506	1.575	1.599	2.162	2.270	2.562	3.355	3.771
1.000	1.132	1.418	1.729	1.694	1.581	2.452	2.491	3.231	3.902
0.800	1.139	1.355	1.437	1.610	1.940	2.167	2.152	2.250	2.936
0.600	1.053	1.360	1.604	1.705	1.687	1.960	1.902	2.458	2.500
0.400	1.095	1.176	1.401	1.534	1.771	1.797	1.841	1.792	2.262
0.200	1.043	1.140	1.183	1.364	1.561	1.741	1.587	1.495	1.714
0.100	1.028	1.042	1.114	1.193	1.284	1.335	1.268	1.227	1.283
0.080	1.013	1.037	1.091	1.190	1.210	1.293	1.197	1.236	1.178
0.060	1.007	1.033	1.037	1.105	1.087	1.205	1.183	1.281	1.101
0.050	1.021	1.026	1.050	1.088	1.061	1.115	1.182	1.215	1.176
0.040	1.005	1.019	**1.001**	1.061	1.075	1.073	1.098	1.173	1.171
0.030	1.005	1.013	1.021	1.045	1.060	1.050	1.265	1.101	1.041
0.020	1.006	1.017	1.008	1.034	1.022	1.024	1.186	1.076	1.097
0.010	1.003	1.010	1.008	1.009	1.039	1.018	1.276	1.043	1.022

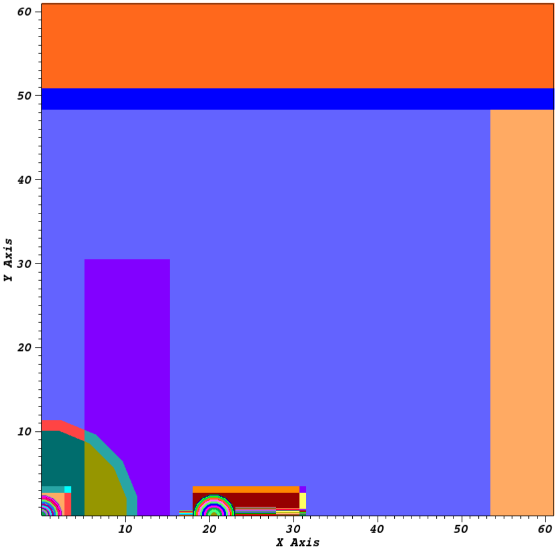
Test Case 2 - Load Balancing By Dimension

5: The best metric behavior of the second test case after **10 load balancing by dimension iterations**.

Area	N=4	N=9	N=16	N=25	N=36	N=49	N=64	N=81	N=100
Coarse	1.854	1.134	1.672	1.324	1.653	1.518	1.542	1.401	1.520
1.800	1.099	1.135	1.438	1.868	1.753	1.840	2.740	3.262	1.690
1.600	1.086	1.118	1.598	1.558	1.802	2.350	**4.700**	3.776	3.441
1.400	1.052	1.101	1.432	1.768	1.966	2.000	2.688	2.827	3.169
1.200	1.042	1.183	1.419	1.542	1.620	2.192	3.939	3.237	3.111
1.000	1.057	1.207	1.315	1.331	1.453	1.698	2.436	2.455	3.524
0.800	1.051	1.097	1.204	1.570	1.510	1.753	1.738	2.232	2.576
0.600	1.053	1.091	1.070	1.215	1.424	1.765	1.613	1.678	2.456
0.400	1.095	1.087	1.120	1.212	1.226	1.346	1.215	2.078	2.128
0.200	1.043	1.038	1.074	1.112	1.238	1.082	1.311	1.564	1.522
0.100	1.028	1.042	1.097	1.042	1.098	1.124	1.129	1.095	1.204
0.080	1.013	1.037	1.091	1.085	1.090	1.128	1.092	1.110	1.178
0.060	1.007	1.033	1.037	1.014	1.087	1.082	1.048	1.092	1.034
0.050	1.021	1.026	1.050	1.088	1.061	1.052	1.083	1.079	1.075
0.040	1.005	1.019	**1.001**	1.061	1.075	1.073	1.081	1.093	1.145
0.030	1.005	1.013	1.021	1.045	1.060	1.050	1.061	1.076	1.041
0.020	1.006	1.017	1.008	1.034	1.022	1.024	1.095	1.076	1.097
0.010	1.003	1.010	1.008	1.009	1.039	1.018	1.092	1.043	1.022

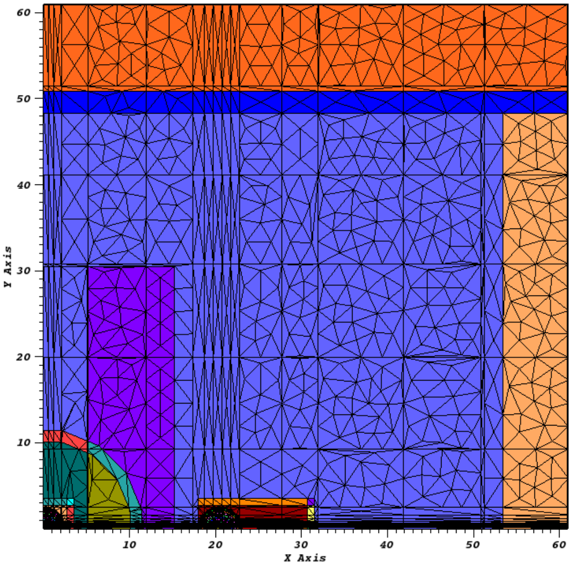
6: The improvement with the load balancing by dimension method.

Area	N=4	N=9	N=16	N=25	N=36	N=49	N=64	N=81	N=100
Coarse	0.000	0.167	0.053	0.105	0.051	0.049	0.139	0.230	0.210
1.800	0.066	0.151	0.236	0.214	0.228	0.220	-0.077	0.151	**0.653**
1.600	0.021	0.246	0.104	0.084	0.094	0.030	**-0.650**	-0.129	-0.151
1.400	0.072	0.193	0.227	-0.012	-0.045	0.231	0.051	0.218	-0.177
1.200	0.096	0.215	0.099	0.036	0.251	0.035	-0.537	0.035	0.175
1.000	0.066	0.149	0.239	0.214	0.081	0.308	0.022	0.240	0.097
0.800	0.078	0.191	0.162	0.025	0.221	0.191	0.192	0.008	0.123
0.600	0.000	0.198	0.333	0.288	0.156	0.099	0.152	0.318	0.018
0.400	0.000	0.075	0.201	0.210	0.308	0.251	0.340	-0.159	0.060
0.200	0.000	0.089	0.092	0.185	0.207	0.379	0.174	-0.046	0.112
0.100	0.000	0.000	0.015	0.126	0.145	0.158	0.109	0.108	0.062
0.080	0.000	0.000	0.000	0.089	0.099	0.128	0.088	0.102	0.000
0.060	0.000	0.000	0.000	0.082	0.000	0.102	0.115	0.148	0.061
0.050	0.000	0.000	0.000	0.000	0.000	0.057	0.084	0.112	0.086
0.040	0.000	0.000	0.000	0.000	0.000	0.000	0.016	0.069	0.022
0.030	0.000	0.000	0.000	0.000	0.000	0.000	0.162	0.022	0.000
0.020	0.000	0.000	0.000	0.000	0.000	0.000	0.077	0.000	0.000
0.010	0.000	0.000	0.000	0.000	0.000	0.000	0.144	0.000	0.000

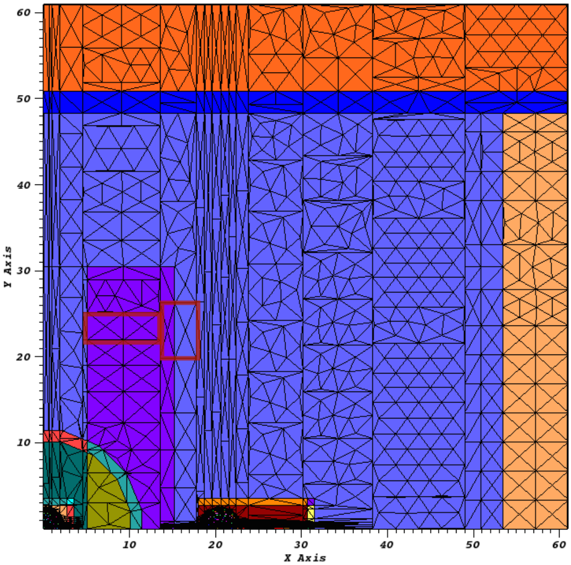


Y Axis
X Axis
60
50
40
30
20
10
10
20
30
40
50
60

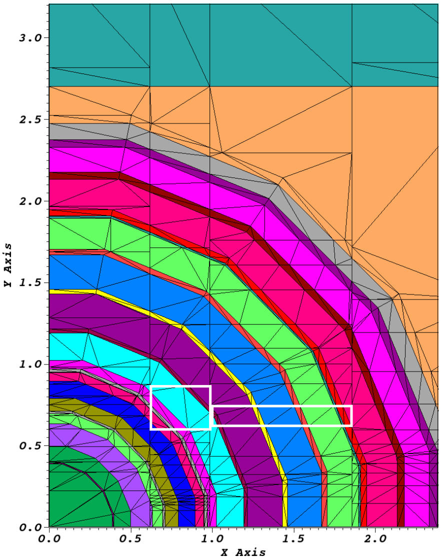
Original Load Balancing, f = 2.63



Load Balancing By Dimension, f = 1.76



A closer look



Conclusions

- The effectiveness of both load balancing algorithms depends on the spatial distribution of fine geometric features, the maximum triangle area used, and the number of subsets the domain is decomposed into.
- Some improvement is seen for Test Cases 1 and 2.
- More tinkering with the load balancing by dimension algorithm will be done to study its behavior and potential improvements.

Future Work - Meshing

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- Moving away from the Triangle Mesh Generator
  - Lack of support
  - Unable to enforce mesh quality consistently while load balancing problems
- Spiderweb and heterogeneous grids for 2D and 2D extruded cases.
- Extrusion based methods that avoid stair stepping.
  - Incredibly thin planes in Z that gives the diffusion solver problems.
- One path forward involves a collaborative effort with Richard Vega (TAMU/Sandia) using a combination of Cubit and OpenFoam.
  - Splitting the mesh into subsets rather than the problem geometry
  - Will allow us to mesh complex 3D problems

Future Work - Load Balancing

- Two more paths for improving the load balancing algorithm have been outlined.
  - Adaptively splitting the subsets that have large cell counts into smaller subsets, and redistributing subsets amongst processors.
  - Taking advantage of nested parallelism to assign more parallel processes at subsets that require more work to be done.
- Studying the behavior of the communication penalty while load balancing (brought up by Derek Gaston at M&C 2017).
- Begin the delevopment of a provably optimal sweep algorithm for "misaligned" cellsets.

Acknowledgments

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