Load Balancing Unstructured Meshes for Massively Parallel Transport Sweeps

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Motivation

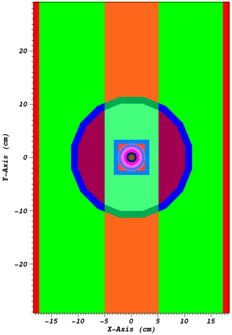
- When running any massively parallel code, load balancing is a priority in order to achieve the best possible parallel efficiency.
- A load balanced problem has an equal number of degrees of freedom per processor.
- Load balancing a logically Cartesian mesh is not difficult, as the user specifies the number of cells being used.
- In an unstructured mesh, the user cannot always specify the number of cells they want per processor, and obtaining a load balanced problem is more difficult.

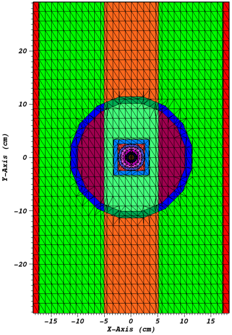
PDT

- All work presented in this thesis was implemented in Texas A&M's massively parallel deterministic transport code, PDT.
- It is capable of multi-group simulations and employs discrete ordinates for angular discretization.
- Features steady-state, time-dependent, criticality, and depletion simulations. It solves the transport equation for neutron, thermal, gamma, coupled neutron-gamma, electron, and coupled electron-photon radiation.
- PDT has been shown to scale on logically Cartesian grids out to 750,000 cores.

The Triangle Mesh Generator

- Unstructured meshes in PDT are generated using the Triangle Mesh Generator.





The Transport Equation

$$\vec{\Omega} \cdot \vec{\nabla} \psi(\vec{r}, E, \vec{\Omega}) + \Sigma_t(\vec{r}, E) \psi(\vec{r}, E, \vec{\Omega}) =$$
$$\int_0^\infty dE' \int_{4\pi} d\Omega' \Sigma_s(\vec{r}, E' \to E, \Omega' \to \Omega) \psi(\vec{r}, E', \vec{\Omega}') + S_{ext}(\vec{r}, E, \vec{\Omega})$$

$$\vec{\Omega} \cdot \vec{\nabla} \psi(\vec{r}, E, \vec{\Omega}) + \Sigma_t(\vec{r}, E) \psi(\vec{r}, E, \vec{\Omega}) =$$
$$\frac{1}{4\pi} \int_0^\infty dE' \Sigma_s(\vec{r}, E' \to E) \int_{4\pi} d\Omega' \psi(\vec{r}, E', \vec{\Omega}') + S_{ext}(\vec{r}, E, \vec{\Omega})$$
$$= \frac{1}{4\pi} \int_0^\infty dE' \Sigma_s(\vec{r}, E' \to E) \phi(\vec{r}, E') + S_{ext}(\vec{r}, E, \vec{\Omega})$$

The Multigroup Transport Equation

$$\phi(\vec{r}, E') = \int_{4\pi} d\Omega' \psi(\vec{r}, E', \vec{\Omega}')$$

$$\vec{\Omega} \cdot \vec{\nabla} \psi_g(\vec{r}, \vec{\Omega}) + \Sigma_{t,g}(\vec{r}) \psi_g(\vec{r}, \vec{\Omega}) = \frac{1}{4\pi} \sum_{g'} \Sigma_{s,g' \to g}(\vec{r}) \phi_{g'}(\vec{r}) + S_{ext,g}(\vec{r}, \vec{\Omega}),$$

$$\text{for } 1 \leq g \leq G$$

The Discrete Ordinates Transport Equation

$$\vec{\Omega}_m \cdot \vec{\nabla}\psi_{g,m}(\vec{r}) + \Sigma_{t,g}(\vec{r})\psi_{g,m}(\vec{r}) = \frac{1}{4\pi}\sum_{g'}\Sigma_{s,g'\to g}(\vec{r})\phi_{g'}(\vec{r}) + S_{ext,g,m}(\vec{r})$$

$$\phi_g(\vec{r}) \approx \sum_{m=1}^{m=M} w_m\psi_{g,m}(\vec{r}).$$

Source Iteration

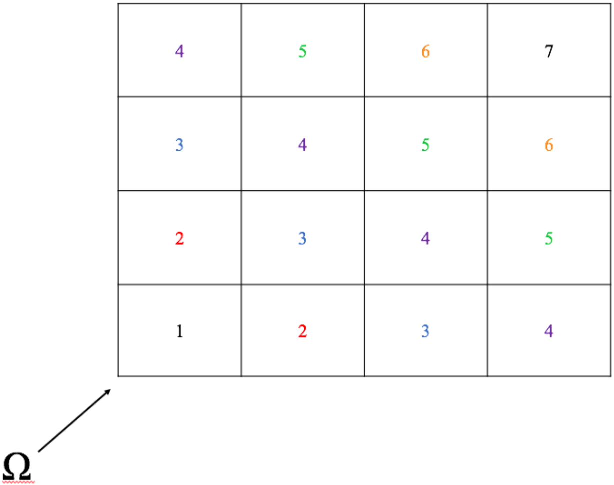
$$\vec{\Omega}_m \cdot \vec{\nabla}\psi_m^{(l+1)}(\vec{r}) + \Sigma_t \psi_m^{(l+1)}(\vec{r}) = q_m^{(l)}(\vec{r})$$

The Transport Sweep

A parallel sweep algorithm is defined by three properties:

- partitioning: dividing the spatial domain among available processors
- aggregation: grouping cells, directions, and energy groups into tasks
- scheduling: choosing which task to execute if more than one is available

The Sweep



Aggregation

- $A_x = \frac{N_x}{P_x}$, where N_x is the number of cells in x and P_x is the number of processors in x
- $A_y = \frac{N_y}{P_y}$, where N_y is the number of cells in y and P_y is the number of processors in y
- $N_g = \frac{G}{A_g}$
- $N_m = \frac{M}{A_m}$
- $N_k = \frac{N_z}{P_z A_z}$
- $N_k A_x A_y A_z = \frac{N_x N_y N_z}{P_x P_y P_z}$

Parallel Efficiency

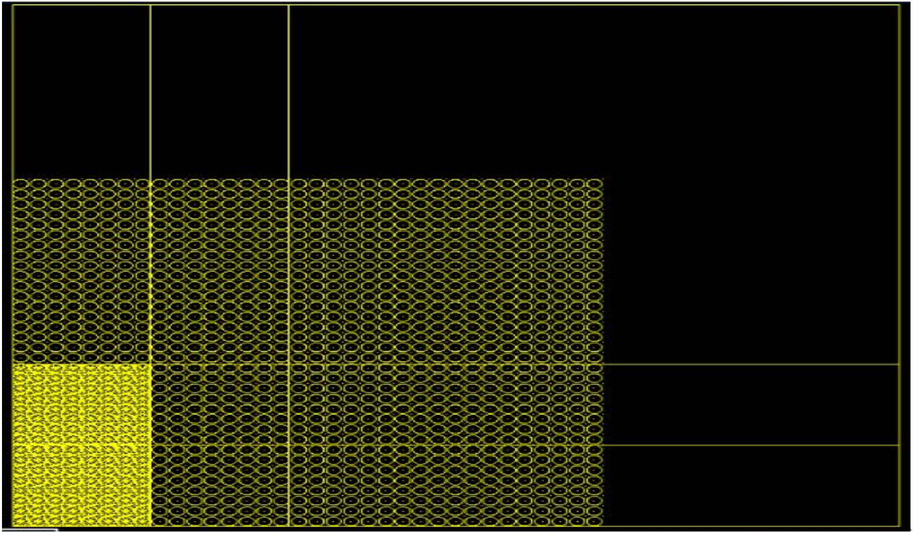
$$\epsilon = \frac{T_{\text{task}} N_{\text{tasks}}}{[N_{\text{stages}}][T_{\text{task}} + T_{\text{comm}}]}$$
$$= \frac{1}{[1 + \frac{N_{\text{idle}}}{N_{\text{tasks}}}][1 + \frac{T_{\text{comm}}}{T_{\text{task}}}]}$$

$$T_{\text{comm}} = M_L T_{\text{latency}} + T_{\text{byte}} N_{\text{bytes}}$$
$$T_{\text{task}} = A_x A_y A_z A_m A_g T_{\text{grind}}$$

Partitioning for an Unstructured Mesh

- The user inputs coordinates for cut lines in the X and Y directions.
- The cut lines will determine the number of subsets the problem is partitioned into.
- Optimizing the location of these cut lines is the basis of the load balancing algorithm.

The Subset



Metric Definitions

- $f = \dfrac{\max_{ij}(N_{ij})}{\frac{N_{tot}}{I \cdot J}}$
- $f_I = \max_{i}[\sum_j N_{ij}] / \frac{N_{tot}}{I}$
- $f_J = \max_{j}[\sum_i N_{ij}] / \frac{N_{tot}}{J}$

Load Balancing Algorithm

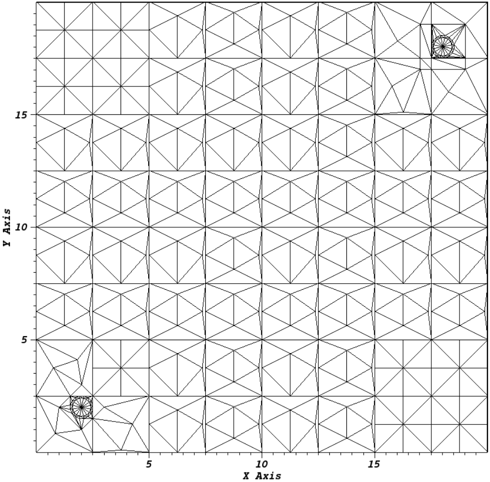
```
//I,J subsets specified by user
//Check if all subsets meet the tolerance
while (f > tol_subset)
{
  //Mesh all subsets
  if (f_I > tol_column)
  {
    Redistribute(X);
  }
  if (f_J > tol_row)
  {
    Redistribute(Y);
  }

}
```

Redistribution Function

```
//stores number of triangles for each row/col
num_tri_view
//stores the partial sum of num_tri_view
offset_view
//We now have a cumulative distribution stored in offset_view
for (i = 1:X.size()-1)
{
   pt1 = [X(i-1), offset_view(i-1)]
   pt2 = [x(i), offset_view(i)]
   ideal_num_triangles = i*(N_tot/num_subsets_X);
   x_val = X-intersect(pt1,pt2,ideal_value);
   //The cut line in question has been redistributed.
   X[i] = x_val;
}
```

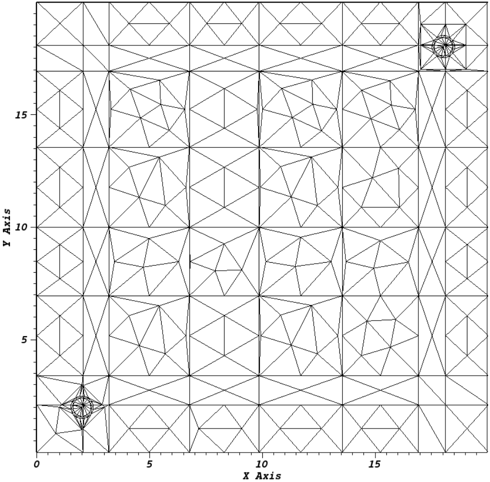
Example

$f = 7.20583$



Example

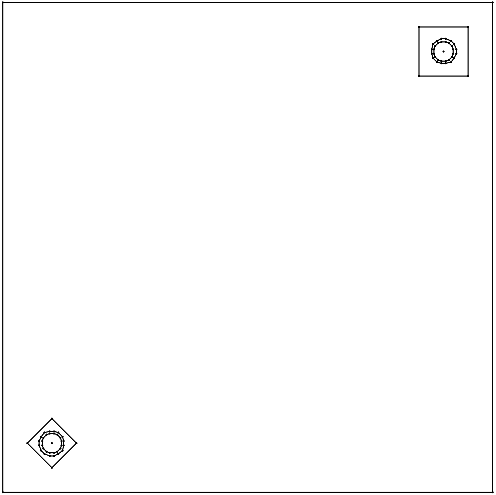
$f = 3.61695$



Load Balancing Results

- Three test cases were used to study the behavior of the load balancing algorithm.
- For each test case, 162 inputs were constructed by varying the number of subsets and the spatial resolution of the mesh (maximum triangle area).

Test Case 1

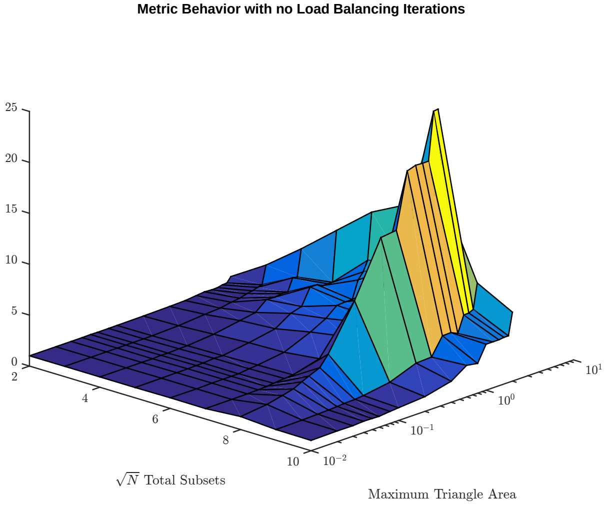


Test Case 1

1: The metric behavior of the first test case run with no load balancing iterations.

Area	N=4	N=9	N=16	N=25	N=36	N=49	N=64	N=81	N=100
Coarse	1.95	4.12	6.76	9.60	12.44	14.21	16.44	8.60	6.77
1.8	1.46	2.32	4.11	4.64	7.84	8.61	**24.77**	6.14	4.58
1.6	1.42	2.21	4.20	4.64	6.86	8.52	24.71	5.94	4.58
1.4	1.32	2.05	2.98	4.64	6.23	8.58	19.98	5.90	4.51
1.2	1.30	1.95	3.02	4.93	4.51	7.25	19.97	4.30	4.51
1	1.35	1.75	2.90	4.93	4.52	6.02	20.01	4.62	4.51
0.8	1.26	1.65	2.95	3.31	4.45	4.40	19.74	4.58	2.92
0.6	1.14	1.45	2.05	3.01	3.55	4.22	14.28	2.87	3.10
0.4	1.09	1.35	1.79	2.02	2.74	3.33	14.09	2.80	2.06
0.2	1.05	1.14	1.34	1.55	1.65	2.05	8.78	1.82	1.45
0.1	1.02	1.04	1.11	1.17	1.29	1.36	4.43	1.41	1.24
0.08	1.01	1.03	1.09	1.19	1.21	1.29	3.39	1.32	1.18
0.06	1.01	1.03	1.04	1.10	1.09	1.20	2.93	1.28	1.06
0.05	1.02	1.02	1.06	1.09	1.08	1.11	2.61	1.22	1.09
0.04	1.00	1.01	1.00	1.06	1.07	1.07	2.20	1.17	1.11
0.03	1.00	1.02	1.02	1.05	1.07	1.05	1.93	1.13	1.03
0.02	1.00	1.01	1.01	1.03	1.02	1.03	1.57	1.08	1.05
0.01	**1.00**	1.01	1.01	1.01	1.04	1.02	1.28	1.04	1.01

Test Case 1

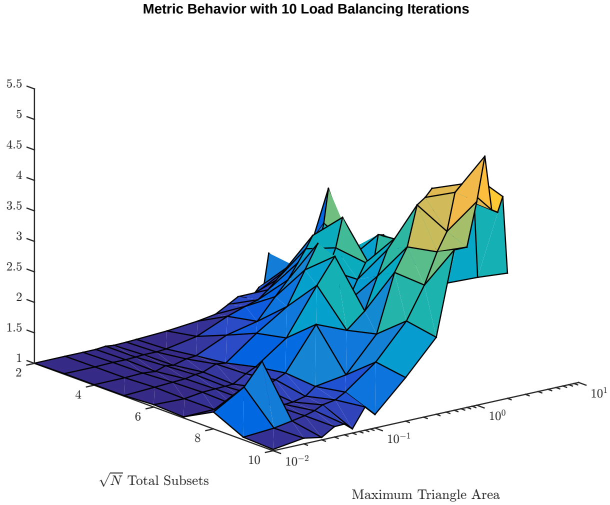

Metric Behavior with no Load Balancing Iterations
25
20
15
10
5
0
2
4
6
8
10
$\sqrt{N}$ Total Subsets
10^{-2}
10^{-1}
10^{0}
10^{1}
Maximum Triangle Area

Test Case 1

2: The metric behavior of the first test case after 10 load balancing iterations.

Area	N=4	N=9	N=16	N=25	N=36	N=49	N=64	N=81	N=100
Coarse	1.95	1.60	3.37	2.10	2.28	2.68	2.53	2.81	3.05
1.8	1.46	1.94	2.81	2.59	2.98	2.89	2.97	4.50	4.33
1.6	1.42	1.95	2.43	2.42	3.00	3.05	2.71	4.11	4.09
1.4	1.32	1.87	2.65	3.13	2.45	3.03	4.14	4.39	4.15
1.2	1.30	1.77	2.46	2.66	2.59	3.18	4.02	4.28	**5.05**
1	1.35	1.64	2.26	2.33	2.35	3.01	3.93	3.67	4.34
0.8	1.26	1.51	2.02	2.79	2.02	2.61	3.27	3.37	3.63
0.6	1.14	1.45	1.79	2.41	2.81	2.09	2.90	2.87	3.63
0.4	1.09	1.35	1.45	1.87	2.40	1.84	1.96	2.35	2.26
0.2	1.05	1.14	1.34	1.55	1.65	2.05	1.40	1.79	1.71
0.1	1.02	1.04	1.11	1.17	1.29	1.36	1.32	1.41	1.22
0.08	1.01	1.03	1.09	1.19	1.21	1.29	1.20	1.32	1.38
0.06	1.01	1.03	1.04	1.10	1.09	1.20	1.15	1.28	1.07
0.05	1.02	1.02	1.06	1.09	1.08	1.11	1.14	1.22	1.18
0.04	1.00	1.01	1.00	1.06	1.07	1.07	1.16	1.17	1.17
0.03	1.00	1.02	1.02	1.05	1.07	1.05	1.93	1.13	1.04
0.02	1.00	1.01	1.01	1.03	1.02	1.03	1.57	1.08	1.09
0.01	**1.00**	1.01	1.01	1.01	1.04	1.02	1.28	1.04	1.02

Test Case 1

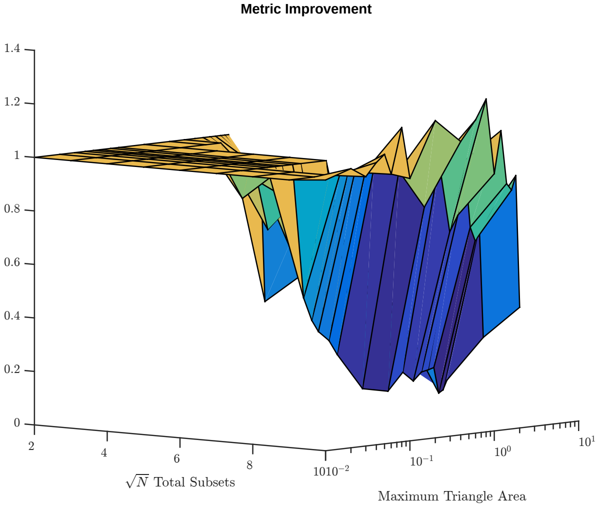

Metric Behavior with 10 Load Balancing Iterations
5.5
5
4.5
4
3.5
3
2.5
2
1.5
1
2
4
6
8
10
$\sqrt{N}$ Total Subsets
10^{-2}
10^{-1}
10^{0}
10^{1}
Maximum Triangle Area

Test Case 1

3: The difference in metric behavior between no iteration and 10 iterations. The closer the z-value to zero, the better the improvement.

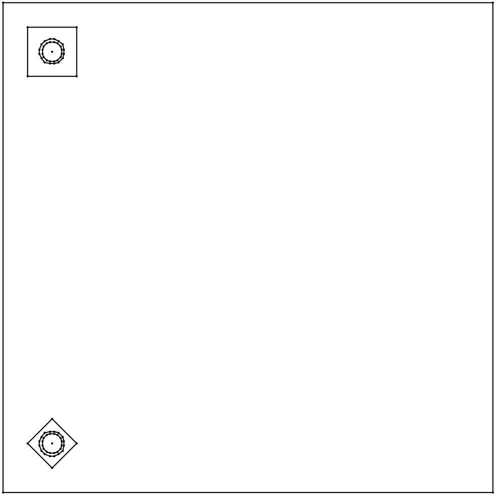
Area	N=4	N=9	N=16	N=25	N=36	N=49	N=64	N=81	N=100
Coarse	1.00	0.39	0.50	0.22	0.18	0.19	0.15	0.33	0.45
1.8	1.00	0.83	0.68	0.56	0.38	0.34	0.12	0.73	0.95
1.6	1.00	0.88	0.58	0.52	0.44	0.36	**0.11**	0.69	0.89
1.4	1.00	0.91	0.89	0.67	0.39	0.35	0.21	0.74	0.92
1.2	1.00	0.90	0.81	0.54	0.58	0.44	0.20	1.00	1.12
1	1.00	0.93	0.78	0.47	0.52	0.50	0.20	0.79	0.96
0.8	1.00	0.92	0.68	0.84	0.45	0.59	0.17	0.74	1.24
0.6	1.00	1.00	0.87	0.80	0.79	0.50	0.20	1.00	1.17
0.4	1.00	1.00	0.81	0.93	0.88	0.55	0.14	0.84	1.10
0.2	1.00	1.00	1.00	1.00	1.00	1.00	0.16	0.99	1.19
0.1	1.00	1.00	1.00	1.00	1.00	1.00	0.30	1.00	0.98
0.08	1.00	1.00	1.00	1.00	1.00	1.00	0.35	1.00	1.17
0.06	1.00	1.00	1.00	1.00	1.00	1.00	0.39	1.00	1.00
0.05	1.00	1.00	1.00	1.00	1.00	1.00	0.44	1.00	1.08
0.04	1.00	1.00	1.00	1.00	1.00	1.00	0.52	1.00	1.05
0.03	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.01
0.02	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.04
0.01	**1.00**	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.01

Test Case 1



Metric Improvement
1.4
1.2
1
0.8
0.6
0.4
0.2
0
2
4
6
8
10
$\sqrt{N}$ Total Subsets
10^{-2}
10^{-1}
10^{0}
10^{1}
Maximum Triangle Area

Test Case 2

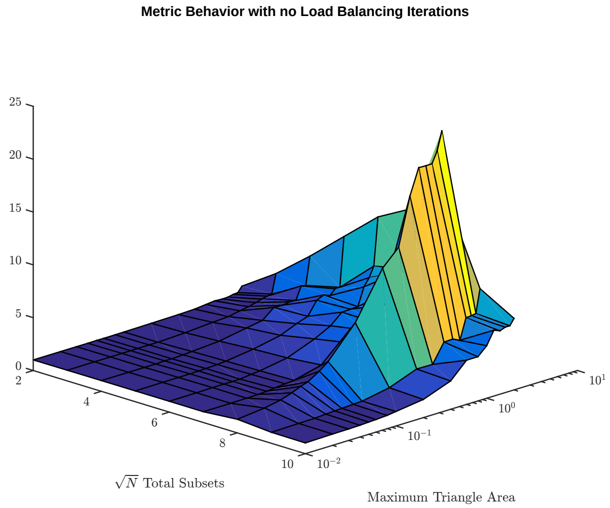


Test Case 2

4: The metric behavior of the second test case after no load balancing iterations.

Area	N=4	N=9	N=16	N=25	N=36	N=49	N=64	N=81	N=100
Coarse	1.95	4.12	6.76	9.60	12.44	14.21	16.44	8.60	6.77
1.80	1.45	2.31	4.10	4.91	7.90	8.61	**22.67**	6.37	6.19
1.60	1.42	2.24	4.19	4.91	6.94	8.50	20.91	6.29	6.19
1.40	1.31	2.12	2.97	4.41	6.22	8.58	19.84	6.25	5.99
1.20	1.30	1.96	3.02	4.65	4.53	7.09	19.83	4.30	6.23
1.00	1.34	1.78	2.90	4.35	4.49	5.88	19.85	4.62	4.98
0.80	1.26	1.64	2.95	3.09	4.47	4.45	17.42	4.58	4.18
0.60	1.14	1.42	2.05	2.72	3.50	4.09	12.90	2.80	4.18
0.40	1.09	1.34	1.79	2.08	2.73	3.34	11.39	2.83	2.68
0.20	1.06	1.15	1.34	1.56	1.72	2.03	7.02	1.85	1.72
0.10	1.02	1.04	1.15	1.22	1.29	1.37	4.12	1.36	1.37
0.08	1.01	1.04	1.08	1.15	1.20	1.30	3.47	1.33	1.26
0.06	1.01	1.03	1.04	1.10	1.08	1.20	2.79	1.26	1.19
0.05	1.02	1.03	1.05	1.07	1.06	1.12	2.57	1.23	1.16
0.04	1.00	1.03	1.01	1.06	1.08	1.07	2.22	1.18	1.11
0.03	1.01	1.02	1.01	1.04	1.07	1.05	1.86	1.11	1.08
0.02	1.01	1.02	1.01	1.04	1.04	1.03	1.57	1.09	1.07
0.01	**1.00**	1.01	1.02	1.02	1.02	1.02	1.29	1.04	1.02

Test Case 2

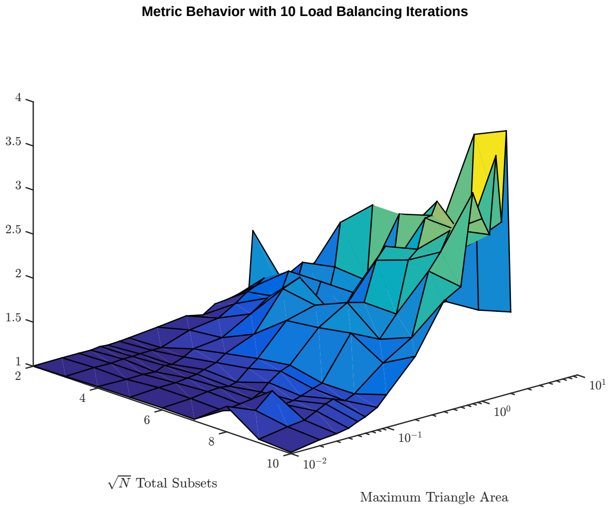

Metric Behavior with no Load Balancing Iterations
25
20
15
10
5
0
2
4
6
8
10
$\sqrt{N}$ Total Subsets
10^{-2}
10^{-1}
10^{0}
10^{1}
Maximum Triangle Area

Test Case 2

5: The metric behavior of the second test case after 10 load balancing iterations.

Area	N=4	N=9	N=16	N=25	N=36	N=49	N=64	N=81	N=100
Coarse	1.85	1.36	1.76	1.48	1.74	1.60	1.79	1.82	1.92
1.8	1.15	1.33	1.65	2.08	2.58	2.41	2.69	3.83	**3.99**
1.6	1.12	1.34	1.65	2.35	2.67	2.47	2.96	2.59	2.97
1.4	1.12	1.37	1.79	1.86	1.83	2.71	2.82	2.58	3.74
1.2	1.15	1.50	1.54	1.56	1.71	2.13	2.81	2.79	2.87
1	1.15	1.45	1.73	1.74	1.74	2.39	2.48	2.81	3.07
0.8	1.14	1.40	1.47	1.44	1.58	2.26	2.38	2.60	3.39
0.6	1.05	1.31	1.49	1.85	1.57	1.81	1.81	2.42	2.36
0.4	1.09	1.19	1.37	1.77	1.71	1.87	1.57	1.72	2.26
0.2	1.06	1.15	1.18	1.35	1.63	1.67	1.73	1.52	1.72
0.1	1.02	1.04	1.15	1.22	1.29	1.34	1.25	1.26	1.37
0.08	1.01	1.04	1.08	1.15	1.20	1.30	1.22	1.21	1.26
0.06	1.01	1.03	1.04	1.10	1.08	1.20	1.18	1.26	1.19
0.05	1.02	1.03	1.05	1.07	1.06	1.12	1.15	1.23	1.16
0.04	1.00	1.03	1.01	1.06	1.08	1.07	1.13	1.18	1.11
0.03	1.01	1.02	1.01	1.04	1.07	1.05	1.32	1.11	1.08
0.02	1.01	1.02	1.01	1.04	1.04	1.03	1.15	1.09	1.07
0.01	**1.00**	1.01	1.02	1.02	1.02	1.02	1.29	1.04	1.02

Test Case 2

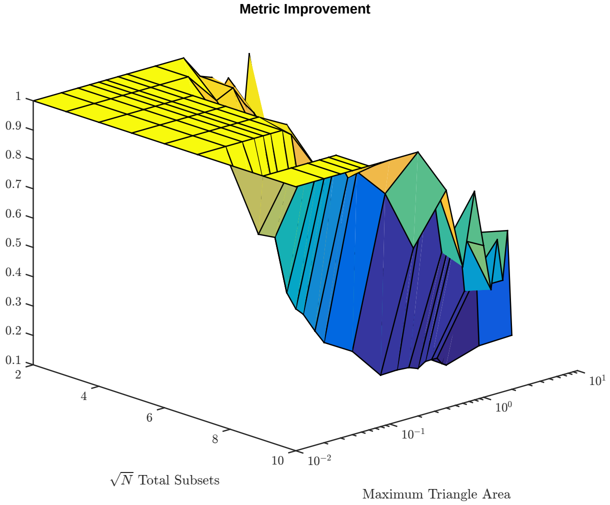

Metric Behavior with 10 Load Balancing Iterations
4
3.5
3
2.5
2
1.5
1
2
4
6
8
10
$\sqrt{N}$ Total Subsets
10^{-2}
10^{-1}
10^{0}
10^{1}
Maximum Triangle Area

Test Case 2

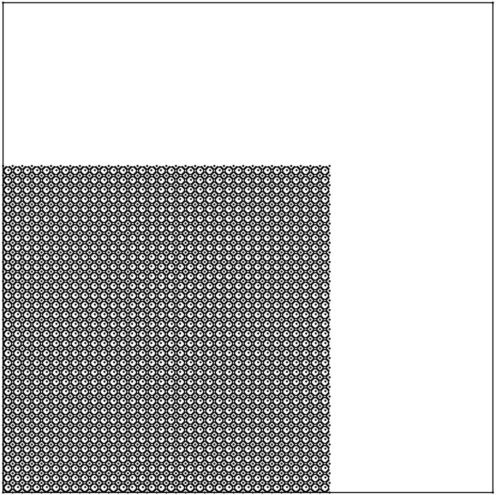
6: The difference in metric behavior between no iteration and 10 iterations. The closer the z-value to zero, the better the improvement.

Area	N=4	N=9	N=16	N=25	N=36	N=49	N=64	N=81	N=100
Coarse	0.95	0.33	0.26	0.15	0.14	0.11	**0.11**	0.21	0.28
1.8	0.79	0.57	0.40	0.42	0.33	0.28	0.12	0.60	0.65
1.6	0.79	0.60	0.39	0.48	0.38	0.29	0.14	0.41	0.48
1.4	0.85	0.64	0.60	0.42	0.29	0.32	0.14	0.41	0.62
1.2	0.89	0.77	0.51	0.34	0.38	0.30	0.14	0.65	0.46
1	0.85	0.81	0.60	0.40	0.39	0.41	0.12	0.61	0.62
0.8	0.91	0.85	0.50	0.47	0.35	0.51	0.14	0.57	0.81
0.6	0.92	0.92	0.73	0.68	0.45	0.44	0.14	0.86	0.57
0.4	1.00	0.89	0.76	0.85	0.63	0.56	0.14	0.61	0.84
0.2	1.00	1.00	0.89	0.86	0.95	0.82	0.25	0.82	1.00
0.1	1.00	1.00	1.00	1.00	1.00	0.98	0.30	0.92	1.00
0.08	1.00	1.00	1.00	1.00	1.00	1.00	0.35	0.91	1.00
0.06	1.00	1.00	1.00	1.00	1.00	1.00	0.42	1.00	1.00
0.05	1.00	1.00	1.00	1.00	1.00	1.00	0.45	1.00	1.00
0.04	1.00	1.00	1.00	1.00	1.00	1.00	0.51	1.00	1.00
0.03	1.00	1.00	1.00	1.00	1.00	1.00	0.71	1.00	1.00
0.02	1.00	1.00	1.00	1.00	1.00	1.00	0.74	1.00	1.00
0.01	**1.00**	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00

Test Case 2


Metric Improvement
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
2
4
6
8
10
$\sqrt{N}$ Total Subsets
10^{-2}
10^{-1}
10^{0}
10^{1}
Maximum Triangle Area

Test Case 3

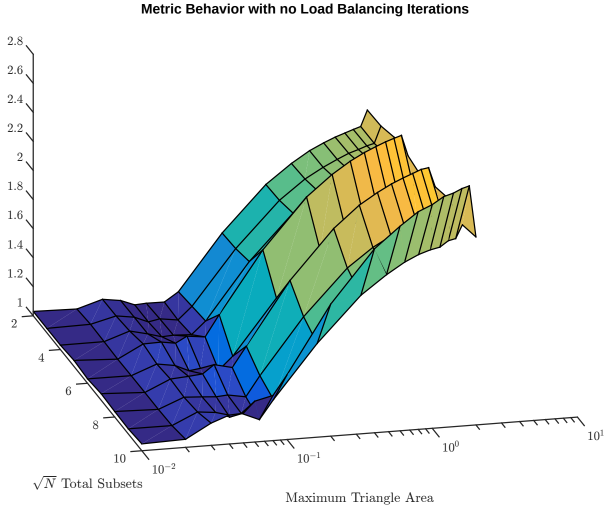


Test Case 3

7: The metric behavior of the third test case after no load balancing iterations.

Area	N=4	N=9	N=16	N=25	N=36	N=49	N=64	N=81	N=100
Coarse	2.24	2.24	2.28	2.27	2.24	2.29	2.32	2.26	2.29
1.8	2.13	2.13	2.16	2.42	2.13	2.43	2.23	2.17	**2.65**
1.6	2.11	2.12	2.15	2.40	2.11	2.42	2.22	2.16	2.63
1.4	2.09	2.10	2.13	2.38	2.10	2.39	2.20	2.12	2.61
1.2	2.07	2.07	2.11	2.35	2.08	2.37	2.18	2.11	2.59
1	2.04	2.04	2.07	2.32	2.04	2.33	2.15	2.08	2.54
0.8	1.99	1.99	2.02	2.27	1.99	2.28	2.10	2.03	2.50
0.6	1.91	1.92	1.95	2.18	1.92	2.20	2.03	1.96	2.41
0.4	1.78	1.79	1.82	2.04	1.79	2.06	1.90	1.83	2.27
0.2	1.47	1.48	1.51	1.70	1.49	1.72	1.59	1.52	1.91
0.1	1.09	1.10	1.12	1.28	1.11	1.29	1.21	1.16	1.45
0.08	1.03	1.02	1.03	1.13	1.02	1.15	1.07	1.03	1.31
0.06	1.03	1.04	1.04	1.15	1.04	1.18	1.09	1.08	1.28
0.05	1.02	1.02	1.03	1.11	1.03	1.13	1.09	1.06	1.20
0.04	1.06	1.06	1.06	1.12	1.08	1.12	1.09	1.10	1.20
0.03	1.08	1.08	1.09	1.12	1.10	1.11	1.10	1.11	1.15
0.02	**1.02**	1.03	1.03	1.03	1.03	1.03	1.03	1.03	1.06
0.01	1.03	1.03	1.03	1.04	1.03	1.04	1.04	1.03	1.05

Test Case 3

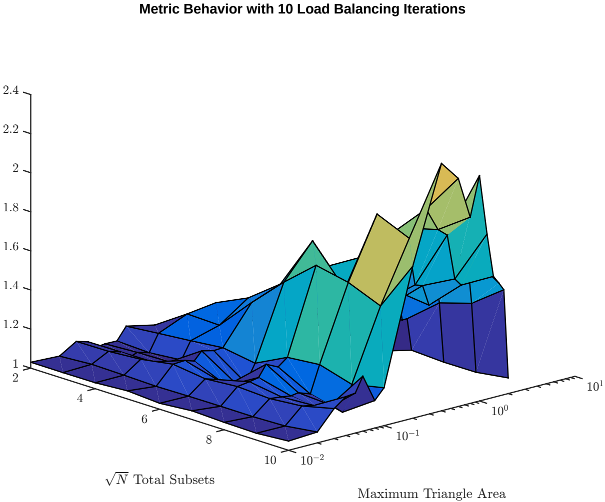

Metric Behavior with no Load Balancing Iterations
2.8
2.6
2.4
2.2
2
1.8
1.6
1.4
1.2
1
2
4
6
8
10
$\sqrt{N}$ Total Subsets
10^{-2}
10^{-1}
10^{0}
10^{1}
Maximum Triangle Area

Test Case 3

8: The metric behavior of the third test case after 10 load balancing iterations.

Area	N=4	N=9	N=16	N=25	N=36	N=49	N=64	N=81	N=100
Coarse	**1.00**	1.01	1.04	1.05	1.01	1.06	1.06	1.06	1.08
1.8	1.02	1.03	1.15	1.21	1.20	1.23	1.36	1.42	1.54
1.6	1.03	1.04	1.08	1.20	1.18	1.23	1.54	1.69	1.58
1.4	1.02	1.06	1.09	1.25	1.32	1.39	1.37	1.52	1.62
1.2	1.03	1.06	1.24	1.24	1.30	1.32	1.48	1.56	1.84
1	1.02	1.05	1.15	1.25	1.31	1.35	1.49	1.80	2.15
0.8	1.04	1.06	1.10	1.23	1.27	1.53	1.79	1.84	1.95
0.6	1.03	1.11	1.13	1.38	1.51	1.61	1.79	1.96	2.17
0.4	1.04	1.19	1.26	1.39	1.66	1.47	1.90	1.83	**2.27**
0.2	1.06	1.17	1.16	1.33	1.49	1.62	1.59	1.52	1.78
0.1	1.09	1.10	1.12	1.14	1.11	1.19	1.21	1.16	1.19
0.08	1.03	1.02	1.03	1.13	1.02	1.15	1.07	1.03	1.14
0.06	1.03	1.04	1.04	1.15	1.04	1.18	1.09	1.08	1.28
0.05	1.02	1.02	1.03	1.11	1.03	1.13	1.09	1.06	1.20
0.04	1.06	1.06	1.06	1.12	1.08	1.12	1.09	1.10	1.20
0.03	1.08	1.08	1.09	1.12	1.10	1.11	1.10	1.11	1.15
0.02	1.02	1.03	1.03	1.03	1.03	1.03	1.03	1.03	1.06
0.01	1.03	1.03	1.03	1.04	1.03	1.04	1.04	1.03	1.05

Test Case 3

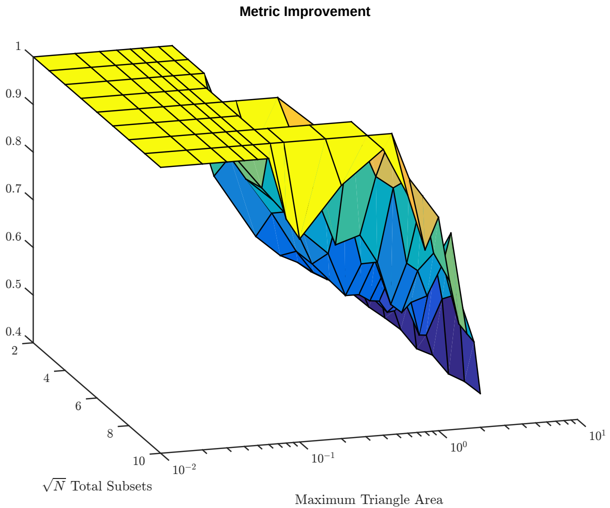

Metric Behavior with 10 Load Balancing Iterations
2.4
2.2
2
1.8
1.6
1.4
1.2
1
2
4
6
8
10
$\sqrt{N}$ Total Subsets
10^{-2}
10^{-1}
10^{0}
10^{1}
Maximum Triangle Area

Test Case 3

9: The difference in metric behavior between no iteration and 10 iterations. The closer the z-value to zero, the better the improvement.

Area	N=4	N=9	N=16	N=25	N=36	N=49	N=64	N=81	N=100
Coarse	**0.45**	0.45	0.46	0.46	0.45	0.46	0.45	0.47	0.47
1.8	0.48	0.48	0.53	0.50	0.56	0.51	0.61	0.65	0.58
1.6	0.49	0.49	0.50	0.50	0.56	0.51	0.69	0.78	0.60
1.4	0.49	0.50	0.51	0.52	0.63	0.58	0.62	0.72	0.62
1.2	0.50	0.51	0.59	0.53	0.62	0.56	0.68	0.74	0.71
1	0.50	0.51	0.56	0.54	0.64	0.58	0.69	0.86	0.85
0.8	0.52	0.53	0.54	0.54	0.64	0.67	0.85	0.90	0.78
0.6	0.54	0.58	0.58	0.63	0.79	0.73	0.88	1.00	0.90
0.4	0.59	0.66	0.70	0.68	0.93	0.71	1.00	1.00	1.00
0.2	0.72	0.79	0.77	0.78	1.00	0.94	1.00	1.00	0.93
0.1	1.00	1.00	1.00	0.89	1.00	0.92	1.00	1.00	0.83
0.08	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	0.87
0.06	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
0.05	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
0.04	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
0.03	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
0.02	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
0.01	**1.00**	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00

Test Case 3


Metric Improvement
1
0.9
0.8
0.7
0.6
0.5
0.4
2
4
6
8
10
$\sqrt{N}$ Total Subsets
10^{-2}
10^{-1}
10^{0}
10^{1}
Maximum Triangle Area

Solution Verification

- Two benchmark problems were set up to verify that the scalar flux was being computed correctly on unstructured meshes in PDT.
- Both problems utilized a 1 cm$\times$1 cm square domain, with opposing reflecting boundaries on the y boundaries, an incident isotropic angular flux on the left boundary, and a vacuum boundary on the right.

The error presented when comparing numerical to analytical solutions is defined as follows:

$$\epsilon = \frac{\|\text{Analytical} - \text{ Numerical}\|_{l2}}{\|\text{Analytical}\|_{l2}},$$

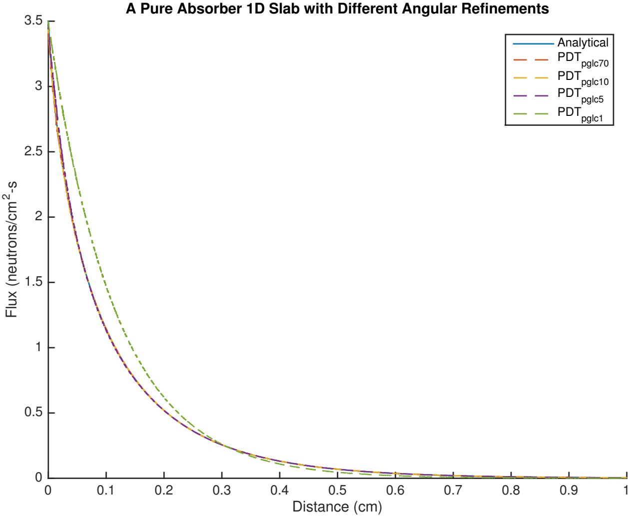
Pure Absorber

The analytical scalar flux solution of the 1D Pure Absorber is:

$$\phi(x) = \int_0^1 \psi(x, \mu > 0) d\mu$$
$$= \int_0^1 \psi_{inc} \exp(-\frac{\Sigma_a}{\mu} x) d\mu = \psi_{inc} E_2(\Sigma_a x),$$

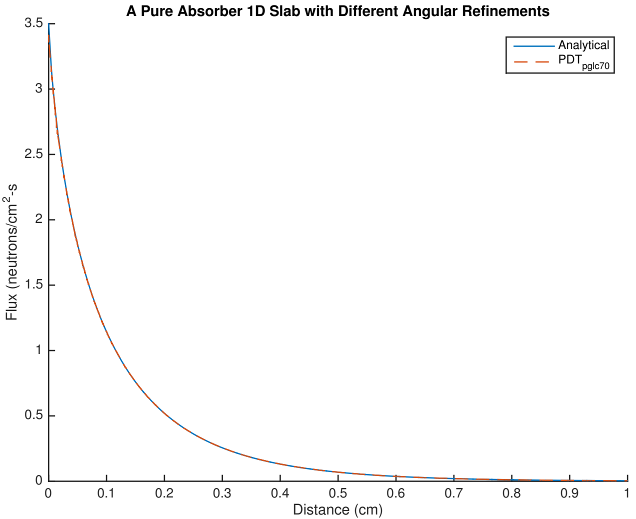
The pure absorber was run with $\psi_{inc} = 3.5 \frac{\text{n}}{\text{cm}^2\text{-s-ster}}$ and $\Sigma_a = 5 \text{ cm}^{-1}$.

PDT Results vs. Analytical for the Pure Absorber



Analysis with 70 Positive Polar Angles

$\epsilon = 0.012$



Pure Scatterer

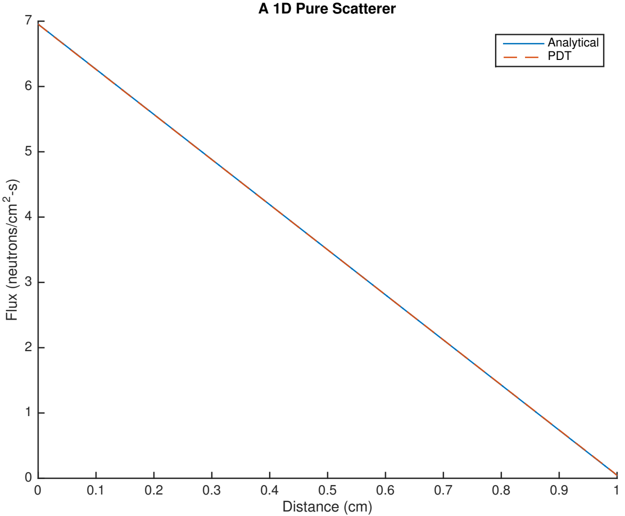
The transport solution for an optically thick pure scatterer reaches the diffusion limit, and the solution is:

$$\phi(x) = \frac{4j_{inc}}{1+4D}(-x + x_{\text{max}} + 2D).$$

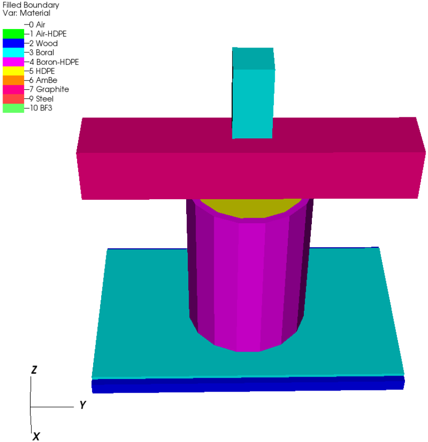
This problem was run with $\Sigma_t = 100\ \text{cm}^{-1}$ and $j_{inc} = \frac{7}{4}\frac{\text{n}}{\text{cm}^2\text{-s}}$.

PDT Results vs. Analytical for the Pure Absorber

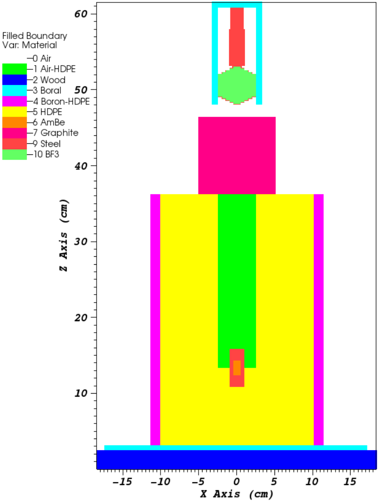
ϵ=4.25E-04



Extruded Mesh Capability



Extruded Mesh Capability



Conclusions

- The effectiveness of the load balancing algorithm depends on the maximum triangle are used, and the number of subsets the domain is decomposed into.
- Good improvement is seen for all test cases, particularly the first two.
- Improvements to the algorithm must be made, as the user will often need to decide on the number of subsets based on how many processors are wanted.

Future Work

- Improvements to the algorithm, moving portions of cut lines instead of moving the entire cutline.
- Domain overloading is the logical extension to the work presented in this thesis.
- Processors could own different numbers of subsets, with no restriction on these subsets being contiguous.

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