Load Balancing Unstructured Meshes for Massively Parallel Transport Sweeps

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Motivation

- When running any massively parallel code, load balancing is a priority in order to achieve the best possible parallel efficiency.

- A load balanced problem has an equal number of degrees of freedom per processor.

- Load balancing a logically Cartesian mesh is "not difficult", as the user specifies the number of cells being used.

- In an unstructured mesh, the user cannot always specify the number of cells they want per processor, and obtaining a load balanced problem is more difficult.

- The goal is to implement a load balancing algorithm for unstructured meshes in PDT.

The Triangle Mesh Generator

- Unstructured meshes in PDT are generated in 2D using the Triangle Mesh Generator.
- These can be extruded to create 3D meshes.
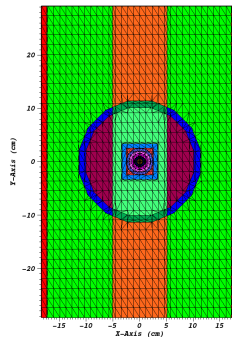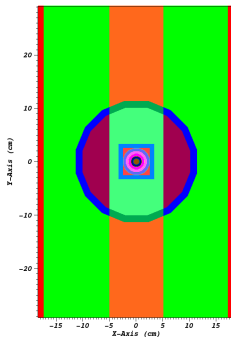
Partitioning for an Unstructured Mesh

- The user inputs coordinates for cut lines in the X and Y directions.
- The cut lines will determine the number of "subsets" the problem is partitioned into.
- Optimizing the location of these cut lines is the basis of the load balancing algorithm.
- A "subset" is an orthogonal unit that is formed by intersecting cut lines.

The Subset
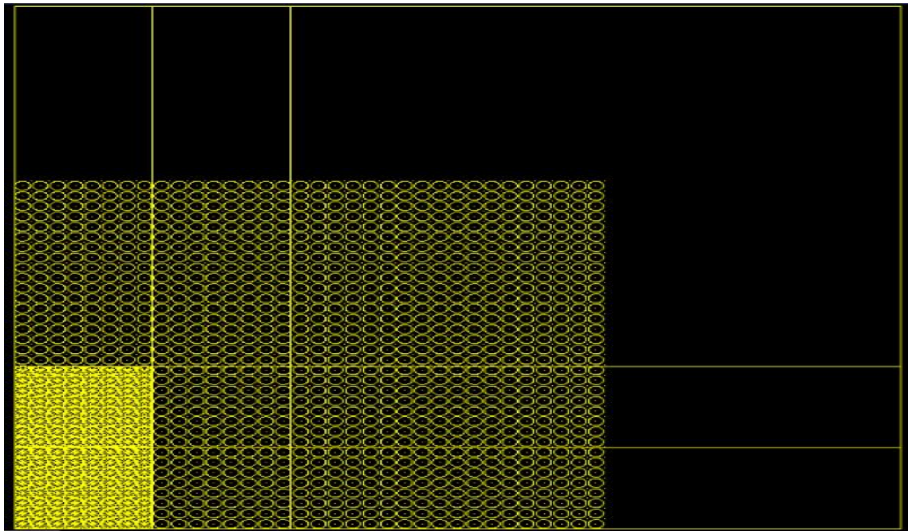
Goal and Definitions

- **Goal:** Obtain an equal number of cells per processor, which for our purposes means an equal number of cells per subset.
- Achieved by optimizing the location of X_i and Y_j, the location of the cut lines.
- N_{ij}: The number of cells in subset i, j
- $f = \dfrac{\max\limits_{ij}(N_{ij})}{\frac{N_{tot}}{I \cdot J}}$
- $f_I = \max\limits_{i}[\sum_j N_{ij}]/\frac{N_{tot}}{I}$
- $f_J = \max\limits_{j}[\sum_i N_{ij}]/\frac{N_{tot}}{J}$

Load Balancing Algorithm

```
//I,J subsets specified by user
//Check if all subsets meet the tolerance
while (f > tol_subset)
{
  //Mesh all subsets
  if (f_I > tol_column)
  {
    Redistribute(X);
  }
  if (f_J > tol_row)
  {
    Redistribute(Y);
  }

}
```

Redistribution Function
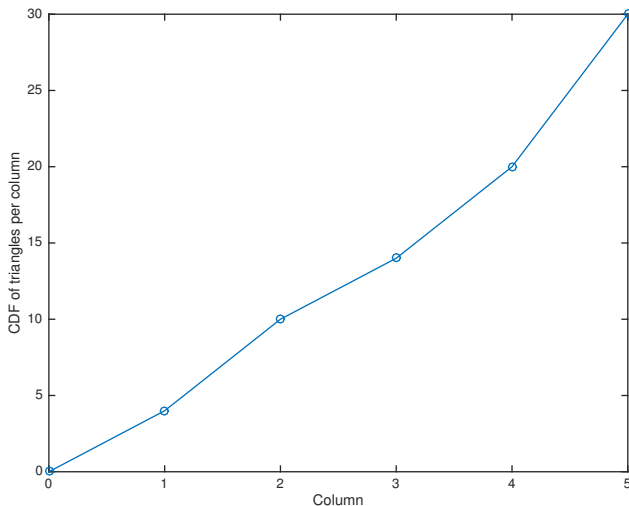
Redistribution Function
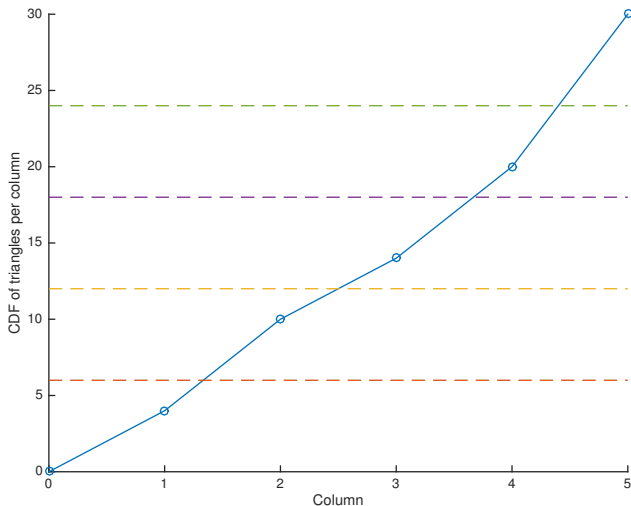
Example

$f = 7.20583$

Example

$f = 3.61695$

Load Balancing Results

- Three test cases were used to study the behavior of the load balancing algorithm.
- For each test case, 162 inputs were constructed by varying:
  - The number of subsets
  - The spatial resolution of the mesh (maximum triangle area).

Test Case 1

Test Case 1

1: The metric behavior of the first test case run with **no load balancing** iterations.

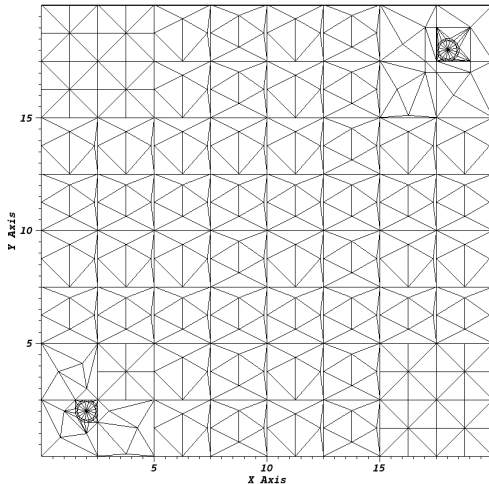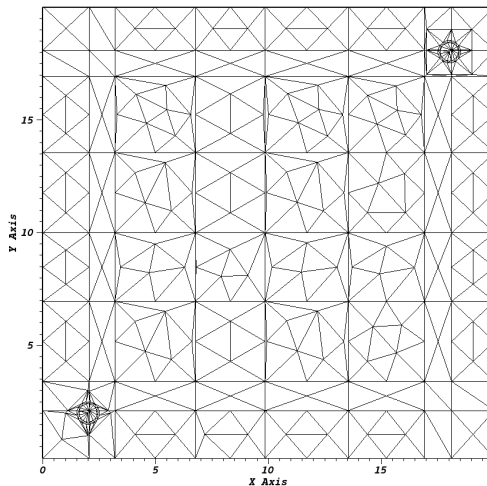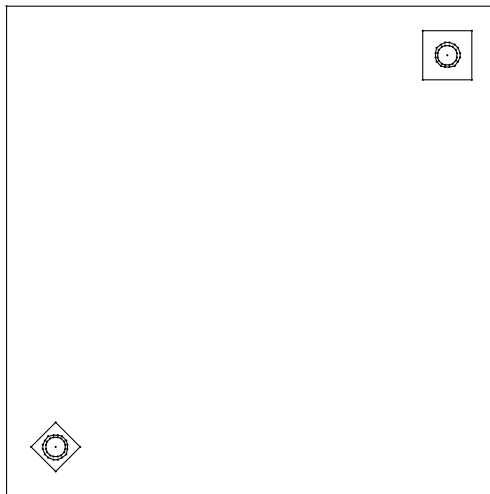| Area	N=4	N=9	N=16	N=25	N=36	N=49	N=64	N=81	N=100
Coarse	1.95	4.12	6.76	9.60	12.44	14.21	16.44	8.60	6.77
1.8	1.46	2.32	4.11	4.64	7.84	8.61	24.77	6.14	4.58
1.6	1.42	2.21	4.20	4.64	6.86	8.52	24.71	5.94	4.58
1.4	1.32	2.05	2.98	4.64	6.23	8.58	19.98	5.90	4.51
1.2	1.30	1.95	3.02	4.93	4.51	7.25	19.97	4.30	4.51
1	1.35	1.75	2.90	4.93	4.52	6.02	20.01	4.62	4.51
0.8	1.26	1.65	2.95	3.31	4.45	4.40	19.74	4.58	2.92
0.6	1.14	1.45	2.05	3.01	3.55	4.22	14.28	2.87	3.10
0.4	1.09	1.35	1.79	2.02	2.74	3.33	14.09	2.80	2.06
0.2	1.05	1.14	1.34	1.55	1.65	2.05	8.78	1.82	1.45
0.1	1.02	1.04	1.11	1.17	1.29	1.36	4.43	1.41	1.24
0.08	1.01	1.03	1.09	1.19	1.21	1.29	3.39	1.32	1.18
0.06	1.01	1.03	1.04	1.10	1.09	1.20	2.93	1.28	1.06
0.05	1.02	1.02	1.06	1.09	1.08	1.11	2.61	1.22	1.09
0.04	1.00	1.01	1.00	1.06	1.07	1.07	2.20	1.17	1.11
0.03	1.00	1.02	1.02	1.05	1.07	1.05	1.93	1.13	1.03
0.02	1.00	1.00	1.01	1.01	1.03	1.02	1.03	1.57	1.08	1.05
0.01	**1.00**	1.01	1.01	1.01	1.04	1.02	1.28	1.04	1.01

Test Case 1

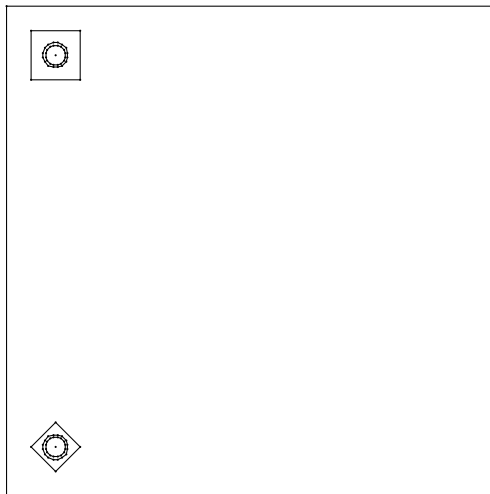2: The metric behavior of the first test case after **10 load balancing iterations**.

Area	N=4	N=9	N=16	N=25	N=36	N=49	N=64	N=81	N=100
Coarse	1.95	1.60	3.37	2.10	2.28	2.68	2.53	2.81	3.05
1.8	1.46	1.94	2.81	2.59	2.98	2.89	2.97	4.50	4.33
1.6	1.42	1.95	2.43	2.42	3.00	3.05	2.71	4.11	4.09
1.4	1.32	1.87	2.65	3.13	2.45	3.03	4.14	4.39	4.15
1.2	1.30	1.77	2.46	2.66	2.59	3.18	4.02	4.28	**5.05**
1	1.35	1.64	2.26	2.33	2.35	3.01	3.93	3.67	4.34
0.8	1.26	1.51	2.02	2.79	2.02	2.61	3.27	3.37	3.63
0.6	1.14	1.45	1.79	2.41	2.81	2.09	2.90	2.87	3.63
0.4	1.09	1.35	1.45	1.87	2.40	1.84	1.96	2.35	2.26
0.2	1.05	1.14	1.34	1.55	1.65	2.05	1.40	1.79	1.71
0.1	1.02	1.04	1.11	1.17	1.29	1.36	1.32	1.41	1.22
0.08	1.01	1.03	1.09	1.19	1.21	1.29	1.20	1.32	1.38
0.06	1.01	1.03	1.04	1.10	1.09	1.20	1.15	1.28	1.07
0.05	1.02	1.02	1.06	1.09	1.08	1.11	1.14	1.22	1.18
0.04	1.00	1.01	1.00	1.06	1.07	1.07	1.16	1.17	1.17
0.03	1.00	1.02	1.02	1.05	1.07	1.05	1.93	1.13	1.04
0.02	1.00	1.01	1.01	1.03	1.02	1.03	1.57	1.08	1.09
0.01	**1.00**	1.01	1.01	1.01	1.04	1.02	1.28	1.04	1.02

Test Case 1

3: The ratio of the metric with no iteration and 10 iterations. The closer the z-value to zero, the better the improvement.

Area	N=4	N=9	N=16	N=25	N=36	N=49	N=64	N=81	N=100
Coarse	1.00	0.39	0.50	0.22	0.18	0.19	0.15	0.33	0.45
1.8	1.00	0.83	0.68	0.56	0.38	0.34	0.12	0.73	0.95
1.6	1.00	0.88	0.58	0.52	0.44	0.36	**0.11**	0.69	0.89
1.4	1.00	0.91	0.89	0.67	0.39	0.35	0.21	0.74	0.92
1.2	1.00	0.90	0.81	0.54	0.58	0.44	0.20	1.00	1.12
1	1.00	0.93	0.78	0.47	0.52	0.50	0.20	0.79	0.96
0.8	1.00	0.92	0.68	0.84	0.45	0.59	0.17	0.74	**1.24**
0.6	1.00	1.00	0.87	0.80	0.79	0.50	0.20	1.00	1.17
0.4	1.00	1.00	0.81	0.93	0.88	0.55	0.14	0.84	1.10
0.2	1.00	1.00	1.00	1.00	1.00	1.00	0.16	0.99	1.19
0.1	1.00	1.00	1.00	1.00	1.00	1.00	0.30	1.00	0.98
0.08	1.00	1.00	1.00	1.00	1.00	1.00	0.35	1.00	1.17
0.06	1.00	1.00	1.00	1.00	1.00	1.00	0.39	1.00	1.00
0.05	1.00	1.00	1.00	1.00	1.00	1.00	0.44	1.00	1.08
0.04	1.00	1.00	1.00	1.00	1.00	1.00	0.52	1.00	1.05
0.03	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.01
0.02	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.04
0.01	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.01

Test Case 2

Test Case 2

4: The metric behavior of the second test case after **no load balancing** iterations.

Area	N=4	N=9	N=16	N=25	N=36	N=49	N=64	N=81	N=100
Coarse	1.95	4.12	6.76	9.60	12.44	14.21	16.44	8.60	6.77
1.80	1.45	2.31	4.10	4.91	7.90	8.61	22.67	6.37	6.19
1.60	1.42	2.24	4.19	4.91	6.94	8.50	20.91	6.29	6.19
1.40	1.31	2.12	2.97	4.41	6.22	8.58	19.84	6.25	5.99
1.20	1.30	1.96	3.02	4.65	4.53	7.09	19.83	4.30	6.23
1.00	1.34	1.78	2.90	4.35	4.49	5.88	19.85	4.62	4.98
0.80	1.26	1.64	2.95	3.09	4.47	4.45	17.42	4.58	4.18
0.60	1.14	1.42	2.05	2.72	3.50	4.09	12.90	2.80	4.18
0.40	1.09	1.34	1.79	2.08	2.73	3.34	11.39	2.83	2.68
0.20	1.06	1.15	1.34	1.56	1.72	2.03	7.02	1.85	1.72
0.10	1.02	1.04	1.15	1.22	1.29	1.37	4.12	1.36	1.37
0.08	1.01	1.04	1.08	1.15	1.20	1.30	3.47	1.33	1.26
0.06	1.01	1.03	1.04	1.10	1.08	1.20	2.79	1.26	1.19
0.05	1.02	1.03	1.05	1.07	1.06	1.12	2.57	1.23	1.16
0.04	1.00	1.03	1.01	1.06	1.08	1.07	2.22	1.18	1.11
0.03	1.01	1.02	1.01	1.04	1.07	1.05	1.86	1.11	1.08
0.02	1.01	1.01	1.01	1.04	1.04	1.03	1.57	1.09	1.07
0.01	1.00	1.01	1.02	1.02	1.02	1.02	1.29	1.04	1.02

Test Case 2

5: The metric behavior of the second test case after **10 load balancing iterations**.

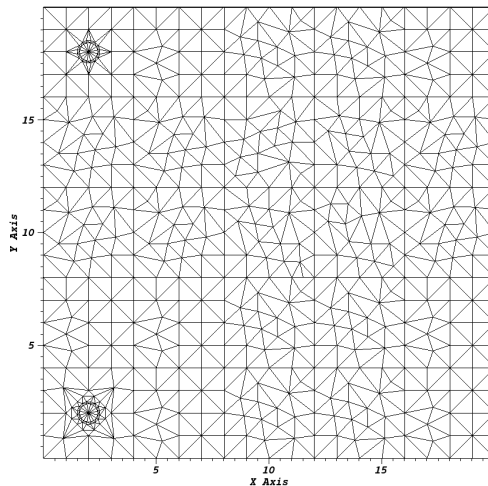| Area	N=4	N=9	N=16	N=25	N=36	N=49	N=64	N=81	N=100
Coarse	1.85	1.36	1.76	1.48	1.74	1.60	1.79	1.82	1.92
1.8	1.15	1.33	1.65	2.08	2.58	2.41	2.69	3.83	**3.99**
1.6	1.12	1.34	1.65	2.35	2.67	2.47	2.96	2.59	2.97
1.4	1.12	1.37	1.79	1.86	1.83	2.71	2.82	2.58	3.74
1.2	1.15	1.50	1.54	1.56	1.71	2.13	2.81	2.79	2.87
1	1.15	1.45	1.73	1.74	1.74	2.39	2.48	2.81	3.07
0.8	1.14	1.40	1.47	1.44	1.58	2.26	2.38	2.60	3.39
0.6	1.05	1.31	1.49	1.85	1.57	1.81	1.81	2.42	2.36
0.4	1.09	1.19	1.37	1.77	1.71	1.87	1.57	1.72	2.26
0.2	1.06	1.15	1.18	1.35	1.63	1.67	1.73	1.52	1.72
0.1	1.02	1.04	1.15	1.22	1.29	1.34	1.25	1.26	1.37
0.08	1.01	1.04	1.08	1.15	1.20	1.30	1.22	1.21	1.26
0.06	1.01	1.03	1.04	1.10	1.08	1.20	1.18	1.26	1.19
0.05	1.02	1.03	1.05	1.07	1.06	1.12	1.15	1.23	1.16
0.04	1.00	1.03	1.01	1.06	1.08	1.07	1.13	1.18	1.11
0.03	1.01	1.02	1.01	1.04	1.07	1.05	1.32	1.11	1.08
0.02	1.01	1.02	1.01	1.04	1.04	1.03	1.15	1.09	1.07
0.01	**1.00**	1.01	1.02	1.02	1.02	1.02	1.29	1.04	1.02

Test Case 2

6: The ratio of the metric with no iteration and 10 iterations. The closer the z-value to zero, the better the improvement.

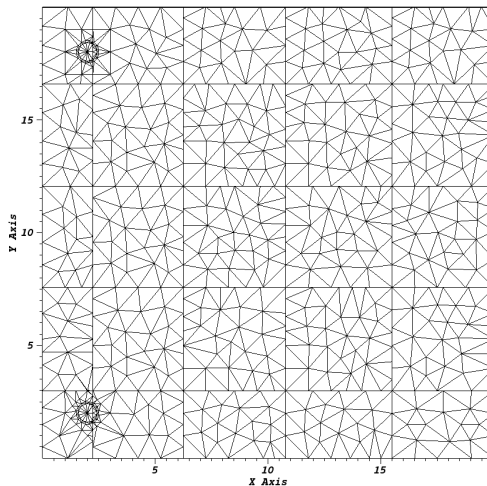| Area	N=4	N=9	N=16	N=25	N=36	N=49	N=64	N=81	N=100
Coarse	0.95	0.33	0.26	0.15	0.14	0.11	**0.11**	0.21	0.28
1.8	0.79	0.57	0.40	0.42	0.33	0.28	0.12	0.60	0.65
1.6	0.79	0.60	0.39	0.48	0.38	0.29	0.14	0.41	0.48
1.4	0.85	0.64	0.60	0.42	0.29	0.32	0.14	0.41	0.62
1.2	0.89	0.77	0.51	0.34	0.38	0.30	0.14	0.65	0.46
1	0.85	0.81	0.60	0.40	0.39	0.41	0.12	0.61	0.62
0.8	0.91	0.85	0.50	0.47	0.35	0.51	0.14	0.57	0.81
0.6	0.92	0.92	0.73	0.68	0.45	0.44	0.14	0.86	0.57
0.4	1.00	0.89	0.76	0.85	0.63	0.56	0.14	0.61	0.84
0.2	1.00	1.00	0.89	0.86	0.95	0.82	0.25	0.82	1.00
0.1	1.00	1.00	1.00	1.00	1.00	0.98	0.30	0.92	1.00
0.08	1.00	1.00	1.00	1.00	1.00	1.00	0.35	0.91	1.00
0.06	1.00	1.00	1.00	1.00	1.00	1.00	0.42	1.00	1.00
0.05	1.00	1.00	1.00	1.00	1.00	1.00	0.45	1.00	1.00
0.04	1.00	1.00	1.00	1.00	1.00	1.00	0.51	1.00	1.00
0.03	1.00	1.00	1.00	1.00	1.00	1.00	0.71	1.00	1.00
0.02	1.00	1.00	1.00	1.00	1.00	1.00	0.74	1.00	1.00
0.01	**1.00**	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00

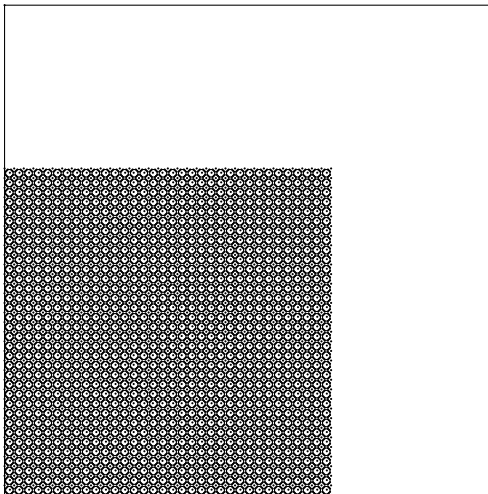# A Closer Look at Test Case 2

$f = 2.72$

A Closer Look at Test Case 2

$f = 1.85$

Test Case 3

Test Case 3

7: The metric behavior of the third test case after **no load balancing iterations**.

Area	N=4	N=9	N=16	N=25	N=36	N=49	N=64	N=81	N=100
Coarse	2.24	2.24	2.28	2.27	2.24	2.29	2.32	2.26	2.29
1.8	2.13	2.13	2.16	2.42	2.13	2.43	2.23	2.17	**2.65**
1.6	2.11	2.12	2.15	2.40	2.11	2.42	2.22	2.16	2.63
1.4	2.09	2.10	2.13	2.38	2.10	2.39	2.20	2.12	2.61
1.2	2.07	2.07	2.11	2.35	2.08	2.37	2.18	2.11	2.59
1	2.04	2.04	2.07	2.32	2.04	2.33	2.15	2.08	2.54
0.8	1.99	1.99	2.02	2.27	1.99	2.28	2.10	2.03	2.50
0.6	1.91	1.92	1.95	2.18	1.92	2.20	2.03	1.96	2.41
0.4	1.78	1.79	1.82	2.04	1.79	2.06	1.90	1.83	2.27
0.2	1.47	1.48	1.51	1.70	1.49	1.72	1.59	1.52	1.91
0.1	1.09	1.10	1.12	1.28	1.11	1.29	1.21	1.16	1.45
0.08	1.03	1.02	1.03	1.13	1.02	1.15	1.07	1.03	1.31
0.06	1.03	1.04	1.04	1.15	1.04	1.18	1.09	1.08	1.28
0.05	1.02	1.02	1.03	1.11	1.03	1.13	1.09	1.06	1.20
0.04	1.06	1.06	1.06	1.12	1.08	1.12	1.09	1.10	1.20
0.03	1.08	1.08	1.09	1.12	1.10	1.11	1.10	1.11	1.15
0.02	**1.02**	1.03	1.03	1.03	1.03	1.03	1.03	1.03	1.06
0.01	1.03	1.03	1.03	1.04	1.03	1.04	1.04	1.03	1.05

Test Case 3

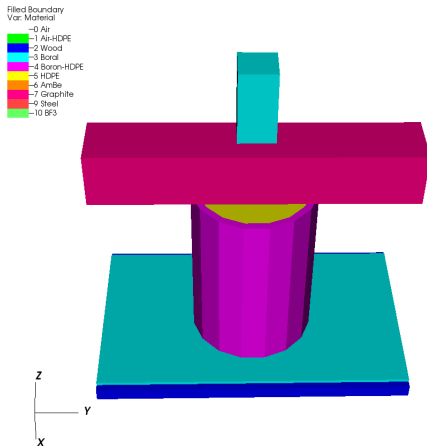8: The metric behavior of the third test case after **10 load balancing iterations**.

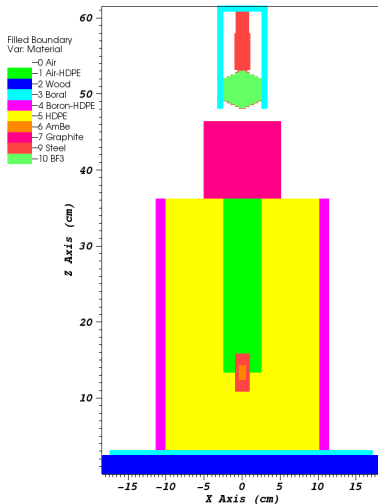| Area	N=4	N=9	N=16	N=25	N=36	N=49	N=64	N=81	N=100
Coarse	**1.00**	1.01	1.04	1.05	1.01	1.06	1.06	1.06	1.08
1.8	1.02	1.03	1.15	1.21	1.20	1.23	1.36	1.42	1.54
1.6	1.03	1.04	1.08	1.20	1.18	1.23	1.54	1.69	1.58
1.4	1.02	1.06	1.09	1.25	1.32	1.39	1.37	1.52	1.62
1.2	1.03	1.06	1.24	1.24	1.30	1.32	1.48	1.56	1.84
1	1.02	1.05	1.15	1.25	1.31	1.35	1.49	1.80	2.15
0.8	1.04	1.06	1.10	1.23	1.27	1.53	1.79	1.84	1.95
0.6	1.03	1.11	1.13	1.38	1.51	1.61	1.79	1.96	2.17
0.4	1.04	1.19	1.26	1.39	1.66	1.47	1.90	1.83	**2.27**
0.2	1.06	1.17	1.16	1.33	1.49	1.62	1.59	1.52	1.78
0.1	1.09	1.10	1.12	1.14	1.11	1.19	1.21	1.16	1.19
0.08	1.03	1.02	1.03	1.13	1.02	1.15	1.07	1.03	1.14
0.06	1.03	1.04	1.04	1.15	1.04	1.18	1.09	1.08	1.28
0.05	1.02	1.02	1.03	1.11	1.03	1.13	1.09	1.06	1.20
0.04	1.06	1.06	1.06	1.12	1.08	1.12	1.09	1.10	1.20
0.03	1.08	1.08	1.09	1.12	1.10	1.11	1.10	1.11	1.15
0.02	1.02	1.03	1.03	1.03	1.03	1.03	1.03	1.03	1.06
0.01	1.03	1.03	1.03	1.04	1.03	1.04	1.04	1.03	1.05

Test Case 3

9: The ratio of the metric with no iteration and 10 iterations. The closer the z-value to zero, the better the improvement.

Area	N=4	N=9	N=16	N=25	N=36	N=49	N=64	N=81	N=100
Coarse	**0.45**	0.45	0.46	0.46	0.45	0.46	0.45	0.47	0.47
1.8	0.48	0.48	0.53	0.50	0.56	0.51	0.61	0.65	0.58
1.6	0.49	0.49	0.50	0.50	0.56	0.51	0.69	0.78	0.60
1.4	0.49	0.50	0.51	0.52	0.63	0.58	0.62	0.72	0.62
1.2	0.50	0.51	0.59	0.53	0.62	0.56	0.68	0.74	0.71
1	0.50	0.51	0.56	0.54	0.64	0.58	0.69	0.86	0.85
0.8	0.52	0.53	0.54	0.54	0.64	0.67	0.85	0.90	0.78
0.6	0.54	0.58	0.58	0.63	0.79	0.73	0.88	1.00	0.90
0.4	0.59	0.66	0.70	0.68	0.93	0.71	1.00	1.00	1.00
0.2	0.72	0.79	0.77	0.78	1.00	0.94	1.00	1.00	0.93
0.1	1.00	1.00	1.00	0.89	1.00	0.92	1.00	1.00	0.83
0.08	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	0.87
0.06	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
0.05	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
0.04	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
0.03	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
0.02	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
0.01	**1.00**	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00

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Conclusions
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Extruded Mesh Capability

Extruded Mesh Capability

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MCNP vs. PDT

10: The results of MCNP and PDT Compared

Code and Setup	Abs. Rate w/Air (s^{-1})	Abs. Rate w/Graphite s^{-1}
MCNP	70.1	12.66
PDT Brick		
PDT Unstructured	68.93	12.75

Conclusions

- The effectiveness of the load balancing algorithm depends on the maximum triangle area used, and the number of subsets the domain is decomposed into.
- Good improvement is seen for all test cases, particularly the first two.
- Improvements to the algorithm must be made, as the user will often need to decide on the number of subsets based on how many processors are wanted.

Future Work

- Domain overloading (combined with some adaptive mesh refinement) is the logical extension to the work presented in this talk.
    - Processors could own different numbers of subsets, with no restriction on these subsets being contiguous.

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Initial Setup

AMR

Domain Overloading

Acknowledgements

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- Drs. Ragusa, Morel, Adams, and Popov
- Michael Adams, Daryl Hawkins, and Dr. Timmie Smith
- Dr. Andrew Till
- The CERT team and fellow grad students
- PSAAP-II

Backup Slides

Solution Verification

- Two benchmark problems were set up to verify that the scalar flux was being computed correctly on unstructured meshes in PDT.
- Both problems utilized a 1 cm×1 cm square domain, with opposing reflecting boundaries on the y boundaries, an incident isotropic angular flux on the left boundary, and a vacuum boundary on the right.

The error presented when comparing numerical to analytical solutions is defined as follows:

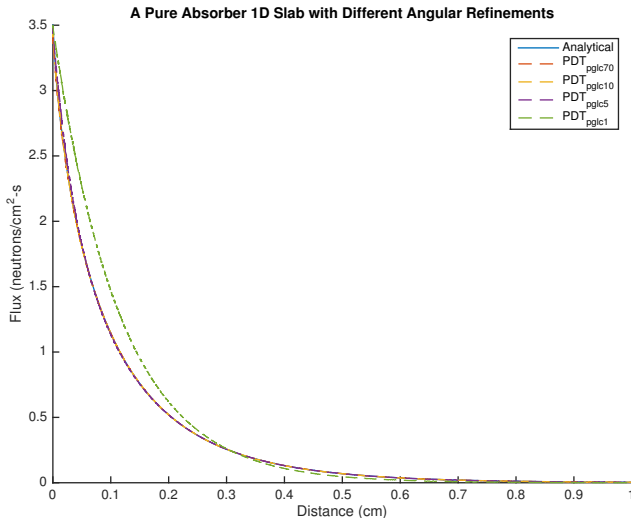$$\epsilon = \frac{\|\text{Analytical} - \text{Numerical}\|_{l2}}{\|\text{Analytical}\|_{l2}},$$

Pure Absorber

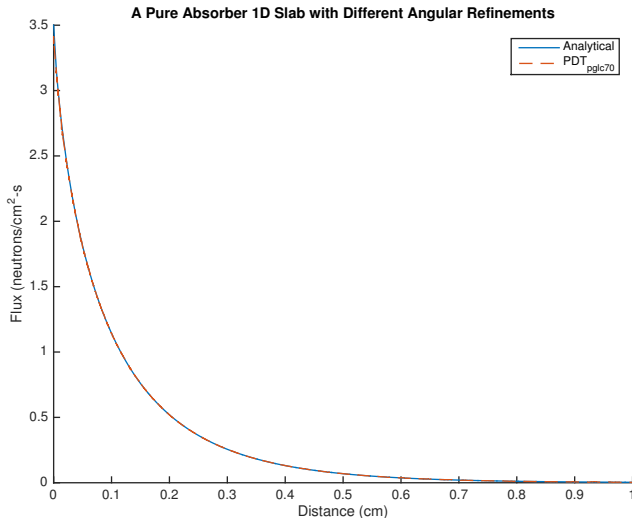The analytical scalar flux solution of the 1D Pure Absorber is:

$$\phi(x) = \int_0^1 \psi(x, \mu > 0) d\mu$$
$$= \int_0^1 \psi_{inc} \exp(-\frac{\Sigma_a}{\mu} x) d\mu = \psi_{inc} E_2(\Sigma_a x),$$

The pure absorber was run with $\psi_{inc} = 3.5 \frac{n}{cm^2\text{-s-ster}}$ and $\Sigma_a = 5$ cm^{-1}.

PDT Results vs. Analytical for the Pure Absorber

Analysis with 70 Positive Polar Angles

$\epsilon = 0.012$

A Pure Absorber 1D Slab with Different Angular Refinements

Pure Scatterer

The transport solution for a pure scatterer reaches the diffusion limit, and the solution is:

$$\phi(x) = \frac{4j_{inc}}{1 + 4D}(-x + x_{max} + 2D).$$

This problem was run with $\Sigma_t = 100$ cm^{-1} and $j_{inc} = \frac{7}{4}\frac{n}{cm^2\text{-s}}$.

PDT Results vs. Analytical for the Pure Scatterer

ϵ=4.25E-04