

Load Balancing Unstructured Meshes for Massively Parallel Transport Sweeps

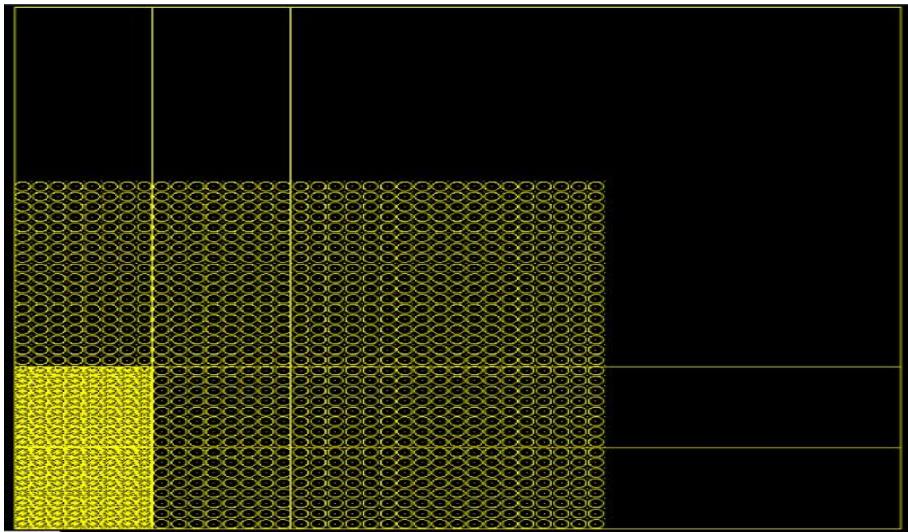
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- When running any massively parallel code, load balancing is a priority in order to achieve the best possible parallel efficiency.
- A load balanced problem has an equal number of degrees of freedom per processor.
- Load balancing a logically Cartesian mesh is “not difficult”, as the user specifies the number of cells being used.
- In an unstructured mesh, the user cannot always specify the number of cells they want per processor, and obtaining a load balanced problem is more difficult.
- A load balancing algorithm for unstructured meshes was implemented in PDT.

- The user inputs coordinates for cut lines in the X and Y directions.
- The cut lines will determine the number of “subsets” the problem is partitioned into.
- Optimizing the location of these cut lines is the basis of the load balancing algorithm.
- A “subset” is an orthogonal unit that is formed by intersecting cut lines.

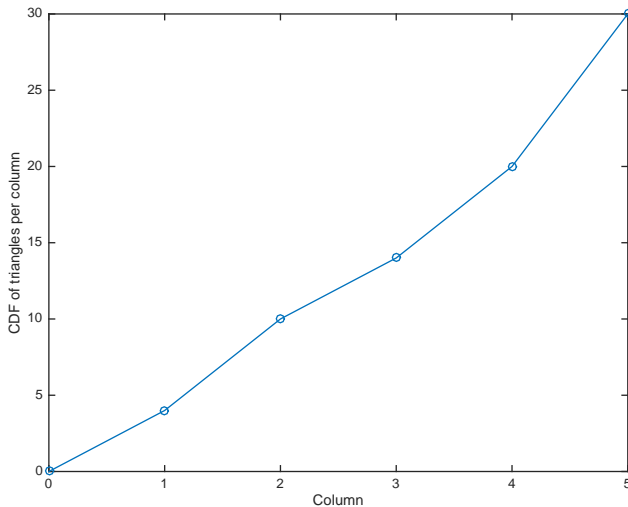
The Subset



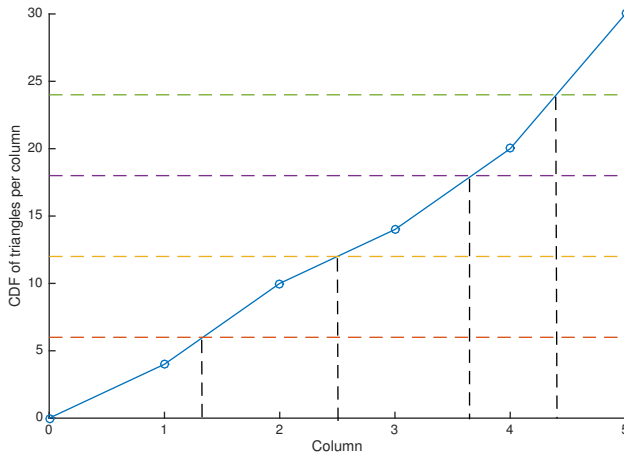
- **Goal:** Obtain an equal number of cells per processor, which for our purposes means an equal number of cells per subset.
- Achieved by optimizing the location of X_i and Y_j , the location of the cut lines.
- **Define:**
 - N_{ij} : The number of cells in subset i, j
 - $f = \frac{\max_{ij}(N_{ij})}{\frac{N_{tot}}{I \cdot J}}$
 - $f_I = \max_i [\sum_j N_{ij}] / \frac{N_{tot}}{I}$
 - $f_J = \max_j [\sum_i N_{ij}] / \frac{N_{tot}}{J}$

```
//I,J subsets specified by user
//Check if all subsets meet the tolerance
while (f > tol_subset)
{
    //Mesh all subsets
    if (f_I > tol_column)
    {
        Redistribute(X);
    }
    if (f_J > tol_row)
    {
        Redistribute(Y);
    }
}
```

Redistribution Function

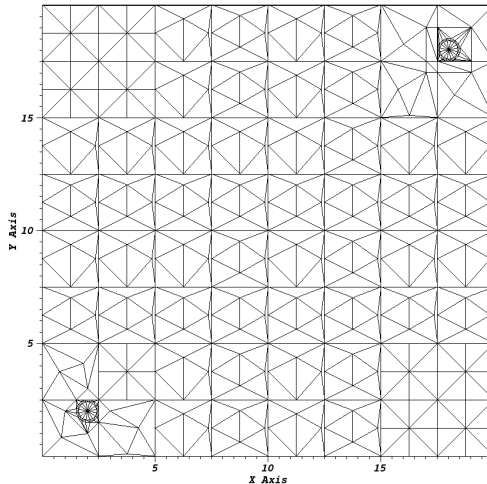


Redistribution Function



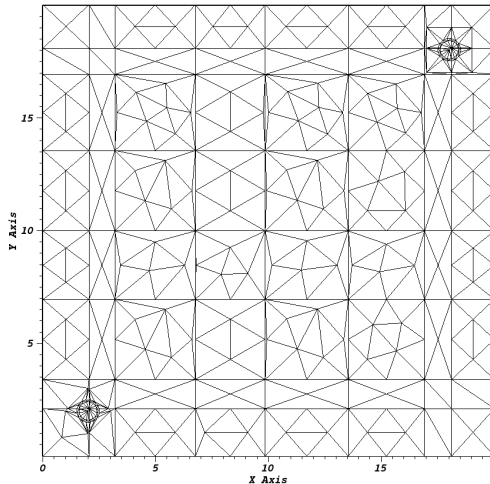
Example

Iter 0
 $f = 7.20583$



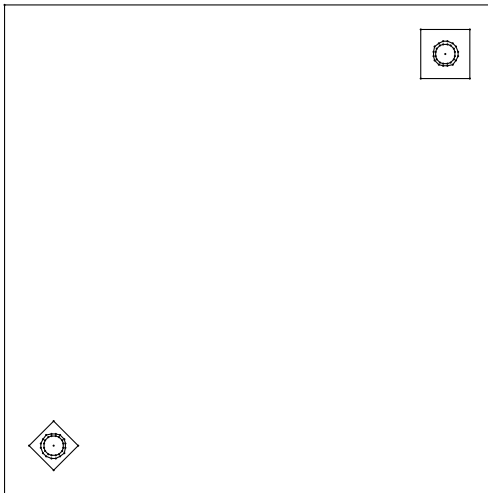
Example

Iter 1
 $f = 3.61695$



- Three test cases were used to study the behavior of the load balancing algorithm.
- For each test case, 162 inputs were constructed by varying:
 - The number of subsets
 - The spatial resolution of the mesh (maximum triangle area).

Test Case 1



1: The metric behavior of the first test case run with **no load balancing** iterations.

Area	N=4	N=9	N=16	N=25	N=36	N=49	N=64	N=81	N=100
Coarse	1.95	4.12	6.76	9.60	12.44	14.21	16.44	8.60	6.77
1.8	1.46	2.32	4.11	4.64	7.84	8.61	24.77	6.14	4.58
1.6	1.42	2.21	4.20	4.64	6.86	8.52	24.71	5.94	4.58
1.4	1.32	2.05	2.98	4.64	6.23	8.58	19.98	5.90	4.51
1.2	1.30	1.95	3.02	4.93	4.51	7.25	19.97	4.30	4.51
1	1.35	1.75	2.90	4.93	4.52	6.02	20.01	4.62	4.51
0.8	1.26	1.65	2.95	3.31	4.45	4.40	19.74	4.58	2.92
0.6	1.14	1.45	2.05	3.01	3.55	4.22	14.28	2.87	3.10
0.4	1.09	1.35	1.79	2.02	2.74	3.33	14.09	2.80	2.06
0.2	1.05	1.14	1.34	1.55	1.65	2.05	8.78	1.82	1.45
0.1	1.02	1.04	1.11	1.17	1.29	1.36	4.43	1.41	1.24
0.08	1.01	1.03	1.09	1.19	1.21	1.29	3.39	1.32	1.18
0.06	1.01	1.03	1.04	1.10	1.09	1.20	2.93	1.28	1.06
0.05	1.02	1.02	1.06	1.09	1.08	1.11	2.61	1.22	1.09
0.04	1.00	1.01	1.00	1.06	1.07	1.07	2.20	1.17	1.11
0.03	1.00	1.02	1.02	1.05	1.07	1.05	1.93	1.13	1.03
0.02	1.00	1.01	1.01	1.03	1.02	1.03	1.57	1.08	1.05
0.01	1.00	1.01	1.01	1.01	1.04	1.02	1.28	1.04	1.01

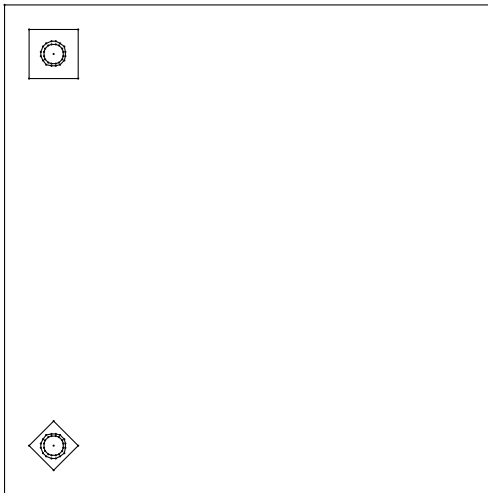
2: The metric behavior of the first test case after **10 load balancing iterations**.

Area	N=4	N=9	N=16	N=25	N=36	N=49	N=64	N=81	N=100
Coarse	1.95	1.60	3.37	2.10	2.28	2.68	2.53	2.81	3.05
1.8	1.46	1.94	2.81	2.59	2.98	2.89	2.97	4.50	4.33
1.6	1.42	1.95	2.43	2.42	3.00	3.05	2.71	4.11	4.09
1.4	1.32	1.87	2.65	3.13	2.45	3.03	4.14	4.39	4.15
1.2	1.30	1.77	2.46	2.66	2.59	3.18	4.02	4.28	5.05
1	1.35	1.64	2.26	2.33	2.35	3.01	3.93	3.67	4.34
0.8	1.26	1.51	2.02	2.79	2.02	2.61	3.27	3.37	3.63
0.6	1.14	1.45	1.79	2.41	2.81	2.09	2.90	2.87	3.63
0.4	1.09	1.35	1.45	1.87	2.40	1.84	1.96	2.35	2.26
0.2	1.05	1.14	1.34	1.55	1.65	2.05	1.40	1.79	1.71
0.1	1.02	1.04	1.11	1.17	1.29	1.36	1.32	1.41	1.22
0.08	1.01	1.03	1.09	1.19	1.21	1.29	1.20	1.32	1.38
0.06	1.01	1.03	1.04	1.10	1.09	1.20	1.15	1.28	1.07
0.05	1.02	1.02	1.06	1.09	1.08	1.11	1.14	1.22	1.18
0.04	1.00	1.01	1.00	1.06	1.07	1.07	1.16	1.17	1.17
0.03	1.00	1.02	1.02	1.05	1.07	1.05	1.93	1.13	1.04
0.02	1.00	1.01	1.01	1.03	1.02	1.03	1.57	1.08	1.09
0.01	1.00	1.01	1.01	1.01	1.04	1.02	1.28	1.04	1.02

3: The ratio of the metric with no iteration and 10 iterations. The closer the z-value to zero, the better the improvement.

Area	N=4	N=9	N=16	N=25	N=36	N=49	N=64	N=81	N=100
Coarse	1.00	0.39	0.50	0.22	0.18	0.19	0.15	0.33	0.45
1.8	1.00	0.83	0.68	0.56	0.38	0.34	0.12	0.73	0.95
1.6	1.00	0.88	0.58	0.52	0.44	0.36	0.11	0.69	0.89
1.4	1.00	0.91	0.89	0.67	0.39	0.35	0.21	0.74	0.92
1.2	1.00	0.90	0.81	0.54	0.58	0.44	0.20	1.00	1.12
1	1.00	0.93	0.78	0.47	0.52	0.50	0.20	0.79	0.96
0.8	1.00	0.92	0.68	0.84	0.45	0.59	0.17	0.74	1.24
0.6	1.00	1.00	0.87	0.80	0.79	0.50	0.20	1.00	1.17
0.4	1.00	1.00	0.81	0.93	0.88	0.55	0.14	0.84	1.10
0.2	1.00	1.00	1.00	1.00	1.00	1.00	0.16	0.99	1.19
0.1	1.00	1.00	1.00	1.00	1.00	1.00	0.30	1.00	0.98
0.08	1.00	1.00	1.00	1.00	1.00	1.00	0.35	1.00	1.17
0.06	1.00	1.00	1.00	1.00	1.00	1.00	0.39	1.00	1.00
0.05	1.00	1.00	1.00	1.00	1.00	1.00	0.44	1.00	1.08
0.04	1.00	1.00	1.00	1.00	1.00	1.00	0.52	1.00	1.05
0.03	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.01
0.02	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.04
0.01	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.01

Test Case 2



4: The metric behavior of the second test case after **no load balancing** iterations.

Area	N=4	N=9	N=16	N=25	N=36	N=49	N=64	N=81	N=100
Coarse	1.95	4.12	6.76	9.60	12.44	14.21	16.44	8.60	6.77
1.80	1.45	2.31	4.10	4.91	7.90	8.61	22.67	6.37	6.19
1.60	1.42	2.24	4.19	4.91	6.94	8.50	20.91	6.29	6.19
1.40	1.31	2.12	2.97	4.41	6.22	8.58	19.84	6.25	5.99
1.20	1.30	1.96	3.02	4.65	4.53	7.09	19.83	4.30	6.23
1.00	1.34	1.78	2.90	4.35	4.49	5.88	19.85	4.62	4.98
0.80	1.26	1.64	2.95	3.09	4.47	4.45	17.42	4.58	4.18
0.60	1.14	1.42	2.05	2.72	3.50	4.09	12.90	2.80	4.18
0.40	1.09	1.34	1.79	2.08	2.73	3.34	11.39	2.83	2.68
0.20	1.06	1.15	1.34	1.56	1.72	2.03	7.02	1.85	1.72
0.10	1.02	1.04	1.15	1.22	1.29	1.37	4.12	1.36	1.37
0.08	1.01	1.04	1.08	1.15	1.20	1.30	3.47	1.33	1.26
0.06	1.01	1.03	1.04	1.10	1.08	1.20	2.79	1.26	1.19
0.05	1.02	1.03	1.05	1.07	1.06	1.12	2.57	1.23	1.16
0.04	1.00	1.03	1.01	1.06	1.08	1.07	2.22	1.18	1.11
0.03	1.01	1.02	1.01	1.04	1.07	1.05	1.86	1.11	1.08
0.02	1.01	1.02	1.01	1.04	1.04	1.03	1.57	1.09	1.07
0.01	1.00	1.01	1.02	1.02	1.02	1.02	1.29	1.04	1.02

5: The metric behavior of the second test case after **10 load balancing iterations**.

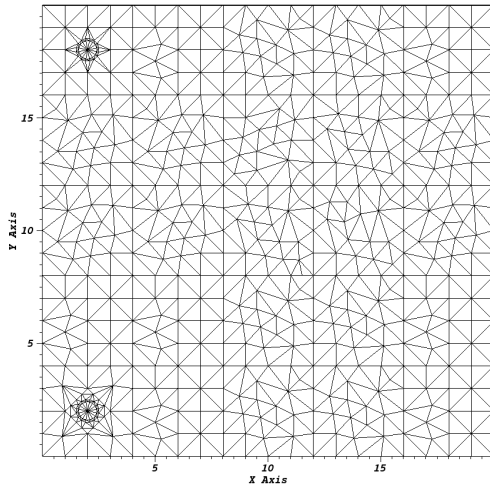
Area	N=4	N=9	N=16	N=25	N=36	N=49	N=64	N=81	N=100
Coarse	1.85	1.36	1.76	1.48	1.74	1.60	1.79	1.82	1.92
1.8	1.15	1.33	1.65	2.08	2.58	2.41	2.69	3.83	3.99
1.6	1.12	1.34	1.65	2.35	2.67	2.47	2.96	2.59	2.97
1.4	1.12	1.37	1.79	1.86	1.83	2.71	2.82	2.58	3.74
1.2	1.15	1.50	1.54	1.56	1.71	2.13	2.81	2.79	2.87
1	1.15	1.45	1.73	1.74	1.74	2.39	2.48	2.81	3.07
0.8	1.14	1.40	1.47	1.44	1.58	2.26	2.38	2.60	3.39
0.6	1.05	1.31	1.49	1.85	1.57	1.81	1.81	2.42	2.36
0.4	1.09	1.19	1.37	1.77	1.71	1.87	1.57	1.72	2.26
0.2	1.06	1.15	1.18	1.35	1.63	1.67	1.73	1.52	1.72
0.1	1.02	1.04	1.15	1.22	1.29	1.34	1.25	1.26	1.37
0.08	1.01	1.04	1.08	1.15	1.20	1.30	1.22	1.21	1.26
0.06	1.01	1.03	1.04	1.10	1.08	1.20	1.18	1.26	1.19
0.05	1.02	1.03	1.05	1.07	1.06	1.12	1.15	1.23	1.16
0.04	1.00	1.03	1.01	1.06	1.08	1.07	1.13	1.18	1.11
0.03	1.01	1.02	1.01	1.04	1.07	1.05	1.32	1.11	1.08
0.02	1.01	1.02	1.01	1.04	1.04	1.03	1.15	1.09	1.07
0.01	1.00	1.01	1.02	1.02	1.02	1.02	1.29	1.04	1.02

6: The ratio of the metric with no iteration and 10 iterations. The closer the z-value to zero, the better the improvement.

Area	N=4	N=9	N=16	N=25	N=36	N=49	N=64	N=81	N=100
Coarse	0.95	0.33	0.26	0.15	0.14	0.11	0.11	0.21	0.28
1.8	0.79	0.57	0.40	0.42	0.33	0.28	0.12	0.60	0.65
1.6	0.79	0.60	0.39	0.48	0.38	0.29	0.14	0.41	0.48
1.4	0.85	0.64	0.60	0.42	0.29	0.32	0.14	0.41	0.62
1.2	0.89	0.77	0.51	0.34	0.38	0.30	0.14	0.65	0.46
1	0.85	0.81	0.60	0.40	0.39	0.41	0.12	0.61	0.62
0.8	0.91	0.85	0.50	0.47	0.35	0.51	0.14	0.57	0.81
0.6	0.92	0.92	0.73	0.68	0.45	0.44	0.14	0.86	0.57
0.4	1.00	0.89	0.76	0.85	0.63	0.56	0.14	0.61	0.84
0.2	1.00	1.00	0.89	0.86	0.95	0.82	0.25	0.82	1.00
0.1	1.00	1.00	1.00	1.00	1.00	0.98	0.30	0.92	1.00
0.08	1.00	1.00	1.00	1.00	1.00	1.00	0.35	0.91	1.00
0.06	1.00	1.00	1.00	1.00	1.00	1.00	0.42	1.00	1.00
0.05	1.00	1.00	1.00	1.00	1.00	1.00	0.45	1.00	1.00
0.04	1.00	1.00	1.00	1.00	1.00	1.00	0.51	1.00	1.00
0.03	1.00	1.00	1.00	1.00	1.00	1.00	0.71	1.00	1.00
0.02	1.00	1.00	1.00	1.00	1.00	1.00	0.74	1.00	1.00
0.01	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00

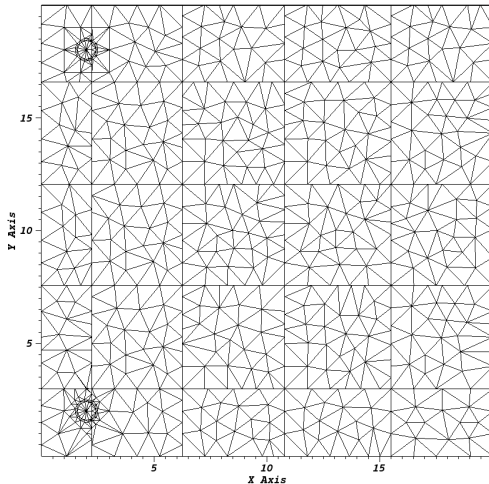
A Closer Look at Test Case 2

Iter 0
 $f = 2.72$

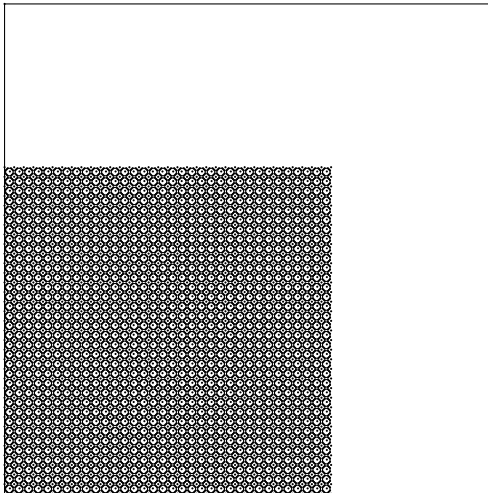


A Closer Look at Test Case 2

Iter 10
 $f = 1.85$



Test Case 3



7: The metric behavior of the third test case after **no load balancing iterations**.

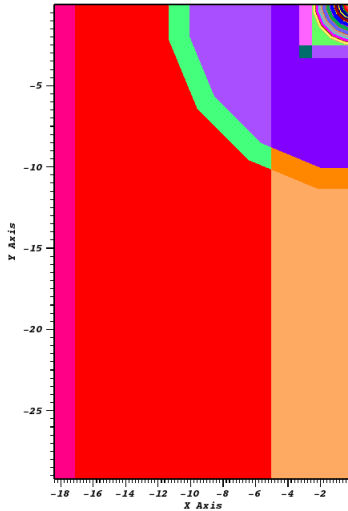
Area	N=4	N=9	N=16	N=25	N=36	N=49	N=64	N=81	N=100
Coarse	2.24	2.24	2.28	2.27	2.24	2.29	2.32	2.26	2.29
1.8	2.13	2.13	2.16	2.42	2.13	2.43	2.23	2.17	2.65
1.6	2.11	2.12	2.15	2.40	2.11	2.42	2.22	2.16	2.63
1.4	2.09	2.10	2.13	2.38	2.10	2.39	2.20	2.12	2.61
1.2	2.07	2.07	2.11	2.35	2.08	2.37	2.18	2.11	2.59
1	2.04	2.04	2.07	2.32	2.04	2.33	2.15	2.08	2.54
0.8	1.99	1.99	2.02	2.27	1.99	2.28	2.10	2.03	2.50
0.6	1.91	1.92	1.95	2.18	1.92	2.20	2.03	1.96	2.41
0.4	1.78	1.79	1.82	2.04	1.79	2.06	1.90	1.83	2.27
0.2	1.47	1.48	1.51	1.70	1.49	1.72	1.59	1.52	1.91
0.1	1.09	1.10	1.12	1.28	1.11	1.29	1.21	1.16	1.45
0.08	1.03	1.02	1.03	1.13	1.02	1.15	1.07	1.03	1.31
0.06	1.03	1.04	1.04	1.15	1.04	1.18	1.09	1.08	1.28
0.05	1.02	1.02	1.03	1.11	1.03	1.13	1.09	1.06	1.20
0.04	1.06	1.06	1.06	1.12	1.08	1.12	1.09	1.10	1.20
0.03	1.08	1.08	1.09	1.12	1.10	1.11	1.10	1.11	1.15
0.02	1.02	1.03	1.03	1.03	1.03	1.03	1.03	1.03	1.06
0.01	1.03	1.03	1.03	1.04	1.03	1.04	1.04	1.03	1.05

8: The metric behavior of the third test case after **10 load balancing iterations**.

Area	N=4	N=9	N=16	N=25	N=36	N=49	N=64	N=81	N=100
Coarse	1.00	1.01	1.04	1.05	1.01	1.06	1.06	1.06	1.08
1.8	1.02	1.03	1.15	1.21	1.20	1.23	1.36	1.42	1.54
1.6	1.03	1.04	1.08	1.20	1.18	1.23	1.54	1.69	1.58
1.4	1.02	1.06	1.09	1.25	1.32	1.39	1.37	1.52	1.62
1.2	1.03	1.06	1.24	1.24	1.30	1.32	1.48	1.56	1.84
1	1.02	1.05	1.15	1.25	1.31	1.35	1.49	1.80	2.15
0.8	1.04	1.06	1.10	1.23	1.27	1.53	1.79	1.84	1.95
0.6	1.03	1.11	1.13	1.38	1.51	1.61	1.79	1.96	2.17
0.4	1.04	1.19	1.26	1.39	1.66	1.47	1.90	1.83	2.27
0.2	1.06	1.17	1.16	1.33	1.49	1.62	1.59	1.52	1.78
0.1	1.09	1.10	1.12	1.14	1.11	1.19	1.21	1.16	1.19
0.08	1.03	1.02	1.03	1.13	1.02	1.15	1.07	1.03	1.14
0.06	1.03	1.04	1.04	1.15	1.04	1.18	1.09	1.08	1.28
0.05	1.02	1.02	1.03	1.11	1.03	1.13	1.09	1.06	1.20
0.04	1.06	1.06	1.06	1.12	1.08	1.12	1.09	1.10	1.20
0.03	1.08	1.08	1.09	1.12	1.10	1.11	1.10	1.11	1.15
0.02	1.02	1.03	1.03	1.03	1.03	1.03	1.03	1.03	1.06
0.01	1.03	1.03	1.03	1.04	1.03	1.04	1.04	1.03	1.05

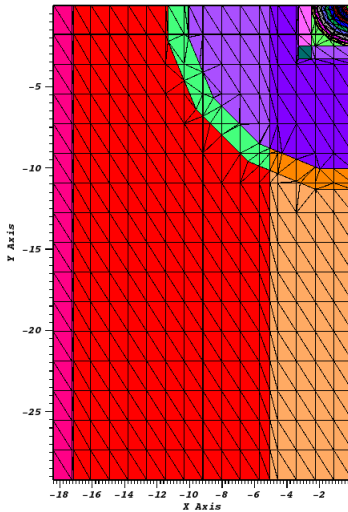
9: The ratio of the metric with no iteration and 10 iterations. The closer the z-value to zero, the better the improvement.

Area	N=4	N=9	N=16	N=25	N=36	N=49	N=64	N=81	N=100
Coarse	0.45	0.45	0.46	0.46	0.45	0.46	0.45	0.47	0.47
1.8	0.48	0.48	0.53	0.50	0.56	0.51	0.61	0.65	0.58
1.6	0.49	0.49	0.50	0.50	0.56	0.51	0.69	0.78	0.60
1.4	0.49	0.50	0.51	0.52	0.63	0.58	0.62	0.72	0.62
1.2	0.50	0.51	0.59	0.53	0.62	0.56	0.68	0.74	0.71
1	0.50	0.51	0.56	0.54	0.64	0.58	0.69	0.86	0.85
0.8	0.52	0.53	0.54	0.54	0.64	0.67	0.85	0.90	0.78
0.6	0.54	0.58	0.58	0.63	0.79	0.73	0.88	1.00	0.90
0.4	0.59	0.66	0.70	0.68	0.93	0.71	1.00	1.00	1.00
0.2	0.72	0.79	0.77	0.78	1.00	0.94	1.00	1.00	0.93
0.1	1.00	1.00	1.00	0.89	1.00	0.92	1.00	1.00	0.83
0.08	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	0.87
0.06	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
0.05	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
0.04	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
0.03	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
0.02	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
0.01	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00



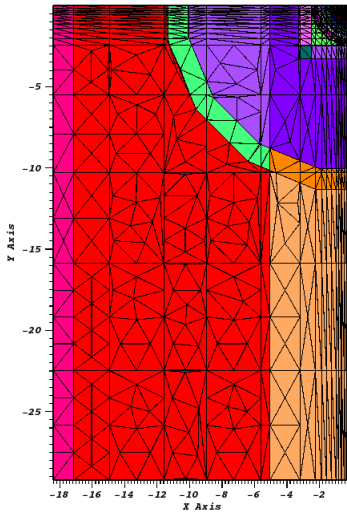
Iteration 0

$$f = 42.1526$$



Iteration 7

$$f = 2.99$$



- The effectiveness of the load balancing algorithm depends on the spatial distribution of fine geometric features, the maximum triangle area used, and the number of subsets the domain is decomposed into.
- Good improvement is seen for all test cases, particularly Test Cases 1,2, and IM1-B.
- Improvements to the algorithm must be made, as the user will often need to decide on the number of subsets based on how many processors are wanted.

- Three paths for improving the load balancing algorithm have been outlined.
 - Adaptively splitting the subsets that have large cell counts into smaller subsets, and redistributing subsets amongst processors.
 - Load balancing by column only (which our algorithm does effectively), then in each column individually load balancing rows.
 - Taking advantage of nested parallelism to assign more parallel processes at subsets that require more work to be done.

Initial Setup

P:2 4	P:5 16	P:8 4
P:1 16	P:4 4	P:7 16
P:0 4	P:3 4	P:6 4

P: Processor_ID
Number of Cells

$$f = 2$$

Adaptively Refine Subsets

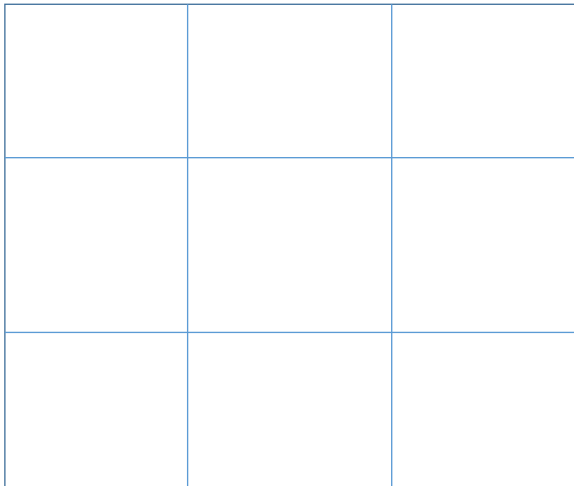
P:2 4		P:5 4	P:5 4	P:8 4	
		P:5 4	P:5 4		
P:1 4	P:1 4	P:4 4		P:7 4	P:7 4
P:1 4	P:1 4			P:7 4	P:7 4
P:0 4		P:3 4		P:6 4	

Subset Redistribution

$$f = 1$$

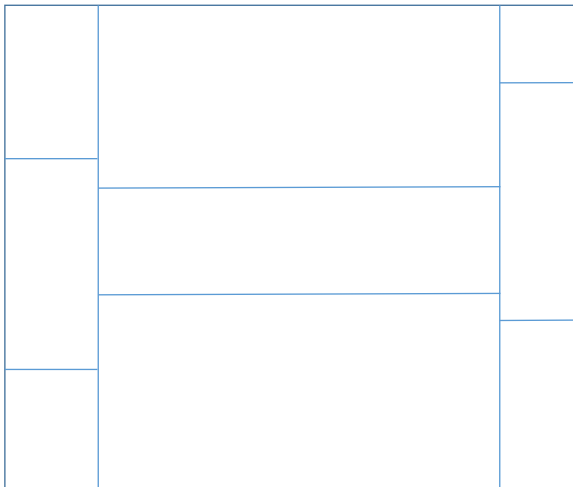
P:2 4		P:5 4	P:5 4	P:8 4	
		P:2 4	P:4 4		
P:1 4	P:1 4	P:4 4		P:8 4	P:7 4
P:0 4	P:3 4			P:6 4	P:7 4
P:0 4		P:3 4		P:6 4	

Load Balancing by Dimension



Load Balance Columns

Load Balancing by Dimension



- This algorithm will utilize the same machinery as the currently implemented load balancing algorithm.
- An initial dimension's cut lines will be redistributed.
- The second dimension's cut lines are redistributed per column/row.
- A new stitching algorithm had to be implemented in order to handle "hanging" subsets.

Nested Parallelism

P:2 4	P:5 16	P:8 4
P:1 16	P:4 4	P:7 16
P:0 4	P:3 4	P:6 4

P: Processor_ID
Number of Cells

$$f = 2$$