Bottom PDF with an intrinsic component

Here we describe the implementation of a PDF for the bottom quark which takes into account an intrinsic component. We have first derived an expanded expression for such a PDF at NLL, and then we have used that as input in APFEL to obtain another bottom PDF which takes into account the full evolution.

Exapanded bottom PDF

Considering the standard 5FS bottom PDF, we can write it in terms of the light flavours $q = u, d, s, c$ plus the gluon densities g in the 4FS:

$$f_b^{(5)}\left(x, Q^2\right) = \sum_{i=q,g} \int_x^1 \frac{dz}{z} K_{b,i}\left(z, Q^2\right) f_i^{(4)}\left(\frac{x}{z}, Q^2\right). \tag{1}$$

Writing this at NLL we get, according to notation of Ref.[1] and Ref.[3]

$$f_b^{(5)}\left(x, Q^2\right) = f^{(5),LO}\left(x, Q^2\right) + f_b^{(5),NLO}\left(x, Q^2\right) \tag{2}$$

with

$$f_b^{(5),LO}\left(x, Q^2\right) = \alpha_s\left(Q^2\right) \int_x^1 \frac{dz}{z} f_g^{(5)}\left(\frac{x}{z}, Q^2\right) A_{gb}^{(1)}\left(z, L\right)$$

$$f_b^{(5),NLO}\left(x, Q^2\right) = \left(\alpha_s\left(Q^2\right)\right)^2 \int_x^1 \frac{dz}{z} \left[f_g^{(5)}\left(\frac{x}{z}, Q^2\right) A_{gb}^{(2)}\left(z, L\right) + f_\Sigma^{(5)}\left(\frac{x}{z}, Q^2\right) A_{\Sigma b}^{(2)}\left(z, L\right)\right] \tag{3}$$

which is Eq.(16) of Ref.[2] and the coefficients A_{ij} are computed in Ref.[3]. If we want to compute an expanded bottom PDF $\tilde{f}_b\left(x, Q^2\right)$ which takes into account some intrinsic component then

$$\tilde{f}_b\left(x, Q^2\right) = f_b^{(5)}\left(x, Q^2\right) + f_b^{(I)}\left(x, Q^2\right), \tag{4}$$

with

$$f_b^{(I)}\left(x, Q^2\right) = \int_x^1 \frac{dz}{z} A_{bb}\left(z, L\right) f_b^{(4)}\left(\frac{x}{z}\right). \tag{5}$$

The coefficient $A_{bb}\left(z, L\right)$ is computed up to NLO in Ref.[4] and $f_b^{(4)}\left(x\right)$ is a parametrization for the intrinsic component of the bottom. Here we take

$$\begin{aligned} f_b^{(4)}\left(x\right) &= M * \frac{\alpha_s\left(m_b^2\right)}{2\pi} \log\frac{m_b^2}{Q_0^2} \int_x^1 \frac{dt}{t} P_{qg}\left(t\right) f_g\left(\frac{x}{t}, Q_0^2\right) \\ &= N_0 \int_x^1 \frac{dt}{t} P_{qg}\left(t\right) f_g\left(\frac{x}{t}, Q_0^2\right), \end{aligned} \tag{6}$$

where $Q_0 = 2\,GeV$ and M is a normalization constant such that

$$\int_0^1 dx\, x\left(f_b^{(4)}\left(x\right) + f_{\bar{b}}^{(4)}\left(x\right)\right) = 10^{-4}. \tag{7}$$

Since

$$A_{bb}\left(z,L\right)=\delta\left(z-1\right)+\alpha_s\left(Q^2\right)\left[A_{bb}^{(1)}\left(z,L\right)\right]_+, \tag{8}$$

we can write

$$f_b^{(I),LO}\left(x,Q^2\right)=f_b^{(4)}\left(x\right)=N_0\int_x^1\frac{dt}{t}P_{qg}\left(t\right)f_g\left(\frac{x}{t},Q_0^2\right) \tag{9}$$

and

$$f_b^{(I),NLO}\left(x,Q^2\right)=N_0\,\alpha_s\left(Q^2\right)\int_x^1\frac{dz}{z}\left[A_{bb}^{(1)}\left(z,L\right)\right]_+\int_{\frac{x}{z}}^1\frac{dt}{t}P_{qg}\left(\frac{x}{zt}\right)f_g\left(t\right). \tag{10}$$

Using

$$\int_x^1 dz\int_{\frac{x}{z}}^1 dt=\int_x^1 dt\int_{\frac{x}{t}}^1 dz \tag{11}$$

we get

$$\begin{aligned}\frac{f_b^{(I),NLO}\left(x,Q^2\right)}{N_0\,\alpha_s\left(Q^2\right)}&=\int_x^1\frac{dt}{t}f_g\left(t\right)\int_{\frac{x}{t}}^1\frac{dz}{z}\left[A_{bb}^{(1)}\left(z,L\right)\right]_+P_{qg}\left(\frac{x}{zt}\right)\\&=\int_x^1\frac{dt}{t}f_g\left(t\right)\int_0^1\frac{dz}{z}\left[A_{bb}^{(1)}\left(z,L\right)\right]_+P_{qg}\left(\frac{x}{zt}\right)\Theta\left(z-\frac{x}{t}\right)\\&=\int_x^1\frac{dt}{t}f_g\left(t\right)\int_0^1\frac{dz}{z}A_{bb}^{(1)}\left(z,L\right)\left[P_{qg}\left(\frac{x}{zt}\right)\Theta\left(z-\frac{x}{t}\right)-zP_{qg}\left(\frac{x}{t}\right)\Theta\left(1-\frac{x}{t}\right)\right]\end{aligned} \tag{12}$$

Since $t>x$ in the last line we can write $\Theta\left(1-\frac{x}{t}\right)=1=\Theta\left(z-\frac{x}{t}\right)+\Theta\left(\frac{x}{t}-z\right)$ getting

$$\begin{aligned}\frac{f_b^{(I),NLO}\left(x,Q^2\right)}{N_0\,\alpha_s\left(Q^2\right)}&=\int_x^1\frac{dt}{t}f_g\left(t\right)\int_{\frac{x}{t}}^1\frac{dz}{z}A_{bb}^{(1)}\left(z,L\right)\left[P_{qg}\left(\frac{x}{zt}\right)-zP_{qg}\left(\frac{x}{t}\right)\right]\\&-\int_x^1\frac{dt}{t}f_g\left(t\right)P_{qg}\left(\frac{x}{t}\right)\int_0^{\frac{x}{t}}A_{bb}^{(1)}\left(z,L\right)dz\end{aligned} \tag{13}$$

The last integral can be computed analytically getting $\int_0^{\frac{x}{t}}A_{bb}^{(1)}\left(z,L\right)dz=A_{bb,e}\left(\frac{x}{t},L\right)$. To sum up, we get a final result made by a single monodimensional integral, containing all the LO and part of the NLO contributions, and a bidimensional integral coming from the NLO part of the intrinsic component. The final expression is

$$\begin{aligned}\tilde{f}_b\left(x,Q^2\right)&=f^{(5)}\left(x,Q^2\right)+f_b^{(I)}\left(x,Q^2\right)\\&=\alpha_s\left(Q^2\right)\int_x^1\frac{dz}{z}\Bigg[f_g^{(5)}\left(\frac{x}{z},Q^2\right)A_{gb}^{(1)}\left(z,L\right)+N_0P_{qg}\left(z\right)f_g\left(\frac{x}{z},Q_0^2\right)\\&\quad+\alpha_s\left(Q^2\right)\Bigg(-N_0f_g\left(z\right)P_{qg}\left(\frac{x}{z}\right)A_{bb,e}\left(\frac{x}{z}\right)\\&\qquad+f_g^{(5)}\left(\frac{x}{z},Q^2\right)A_{gb}^{(2)}\left(z,L\right)+f_\Sigma^{(5)}\left(\frac{x}{z},Q^2\right)A_{\Sigma b}^{(2)}\left(z,L\right)\Bigg)\Bigg]\\&+N_0\alpha_s\left(Q^2\right)\int_x^1\frac{dt}{t}f_g\left(t\right)\int_{\frac{x}{t}}^1\frac{dz}{z}A_{bb}^{(1)}\left(z,L\right)\left[P_{qg}\left(\frac{x}{zt}\right)-zP_{qg}\left(\frac{x}{t}\right)\right]\end{aligned} \tag{14}$$

Full evolved PDF

We take the bottom PDF produced in the previous section and we evolved that using APFEL starting from a fixed threshold scale. Before performing the evolution, we impose the momentum sum rules at the threshold scale, modifying the gluon distribution.

References

[1] 1203.6393

[2] 1607.00389

[3] 9612398

[4] 1510.00009

[5] 1001.2312