

The Swap Curve

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ABSTRACT

A swap curve is a representation of the relationship of interest rates in the swaps market. It is constructed via a bootstrap procedure with benchmark deposit rates, Eurodollar futures prices, and swap rates as inputs. Described in the paper are the method for constructing the swap curve, and the numerous adjustments needed in the process.

INTRODUCTION

The interest rate swaps market has grown rapidly over the past decade due to the growing popularity and constant innovation of interest rate derivatives. Just as the Treasury yield curve is used to value the Treasury securities, the swaps market employs its own valuation vehicle, the *swap curve*. Not only is it used for swap valuation, it is also used for pricing swap-related products, including forward rate agreements, Eurodollar futures contracts, caps, swaptions, and their variants.

There are noticeable advantages to the swap curve over other curves. First, swaps are actively traded over a wide maturity range, with benchmark maturities of 1, 2, 3, 5, 7, 10, 15, 20, and 30 years in the U.S. Such a wide span assures that a swap curve constructed with them covers a large maturity spectrum of interest rates. Second, new swaps are being created constantly, which ensures that benchmark swap maturities do not vary over time. In contrast, the benchmark Treasury maturities decrease as time lapses and are updated only as new Treasuries are issued, a roll effect not desired in curve building. Third, benchmark swaps are zero-cost contracts, and their supply is nearly infinite, not limited by the amount of issuance. Because of this, new swaps do not carry any extra supply-demand premium, compared with the frequent richness of recently issued on-the-run Treasuries over older issues. The levels of swap rates are, therefore, not distorted due to financing issues such as the short squeeze present in the Treasuries.

A swap curve is a representation of the relationship of rates in the swap markets and their maturity, and is used for the valuations of swap-related instruments. It is usually constructed with so-called benchmark swaps and certain related contracts.

An interest rate swap is a contract to exchange fixed and floating coupon payments between two counterparties for a specified period. Both coupon payments are based on a notional amount, which is analogous to the principal amount of a bond and can be thought of as being exchanged at the swap maturity. These two streams of cash flows are referred to as the fixed leg and the floating leg of the swap, respectively. The fixed-leg payments resemble the cash flows of a regular bond. However, unlike a bond

that requires an up-front investment, a swap converts this initial amount into an equivalent set of floating coupon payments over the length of the contract, much like a floating coupon bond. In fact, a swap could be simply viewed as converting a fixed-rate bond into a floating bond. Receiving the fixed coupon on a swap, therefore, can be viewed as being long a fixed-rate bond and short a floating-rate one.

While the value of a bond is gauged in terms of its yield, a swap is measured in terms of its *swap rate*, which is defined as the fixed-leg coupon rate of a swap with which the present value of the fixed leg matches that of the floating leg. With S denoting the swap rate, the following equation holds when the swap is originated:

$$S \sum_{i=1}^N \theta_i DF(t_i) + DF(t_N) = 1, \quad (1)$$

where it is assumed that the swap in question has N fixed coupon payments, payable at time t_i . θ_i in the expression are the accrual factors of payment periods, and DF are the discount factors at payment dates. The right-hand side of the equation represents the floating bond value, which is par; the left-hand side represents the value of the fixed leg, which is a sum of the N coupon payments plus the principal at maturity.¹

Therefore, discount factors of various maturities are required to value a swap. Conversely, given swap rates of different maturities, corresponding discount factors could be deduced. Indeed, a swap curve can be described by discount factors as a function of maturity. But there are better representations of swap curves, for example, in terms of forward rates.

FORWARD RATE

Forward rates measure investment returns for periods starting at forward dates and are considered the most fundamental rates of the swaps market. Almost all variables used in measuring the time value of swaps can be traced back to forward rates. For instance, discount factors used in the swap equation (1) result from inverse compounding of spot rates, i.e.,

$$DF(t) = e^{-r(t)t}. \quad (2)$$

And the spot rate r is actually the time average of instantaneous forward rate f ,

$$r(t) = \frac{1}{t} \int_0^t f(\tau) d\tau, \quad (3)$$

In this chain, the variables r , DF , and S ultimately depend on forward rates f . More complex forms of dependence can be found in other products. In general, forward rates are considered the basic ingredients in analyzing swap-related instruments.

¹ For details on swaps markets and the valuation of swaps, please see the Lehman publication *The Lehman Brothers Swap Indices*, by Lev Dynkin, Yuri Greenfield, and Dev Joneja, 1992.

Unlike a spot rate that starts accruing at the same time as other spot rates and, therefore, overlaps with them in maturity, a forward rate starts at a future time, and an analysis in terms of forward rates isolates different segments of time horizon. This maturity separation is purposeful from a practical point of view. It allows market participants to have decoupled views on rates of different maturities, and this decomposition helps them interpret rate behavior better.

Furthermore, there is little empirical evidence or theoretical reasoning that a forward-rate curve conforms to any specific functional form. In contrast, a discount curve is intrinsically of an exponential decaying form, and a spot-rate curve is generally continuous, evident from equations (2) and (3), respectively. The lack of constraint on forward curves provides the desired freedom for mathematical modeling. Particularly, a forward-rate curve can be parameterized with polynomials of different orders, based on the preferred tradeoff between the curve's smoothness and adequacy of market representation. This flexibility will please curve users of different tastes and with different purposes.

Financial variables such as spot rates and discount factors have a well-defined dependence on forward rates. In particular, they are mathematically differentiable with respect to forward rates. A reliable forward-rate curve translates into reliable spot-rate and discount curves, but not necessarily vice versa. Even though it is acceptable to model curves in terms of discount factors or spot rates directly, modeling forward rates usually leads to robust valuation and is a preferred choice in building swap curves.

BUILDING BLOCKS

A swap curve is constructed using benchmark swap rates for a major part of the curve maturity. However, for short and intermediate maturities, the benchmark swaps, i.e., the 1-, 2- and 3-year swaps, are too far apart in length to describe any detail of the curve's shape. Over this maturity period, interest rates could display unique humps or aberrations distinctive to the short end. In addition, the curve could be highly sloped with visible curvature. The long maturity separation of benchmark swaps is just too coarse to reflect these shapes.

Fortunately, detailed information can be found in the prices of Eurodollar futures. These are forward-starting instruments, and each covers a three-month forward period. In particular, the futures contracts settle on the third Wednesday of the futures months: March, June, September, and December. Thus, these instruments mature sequentially every three months, and the embedded rates can be used as proxies of consecutive three-month forward rates. Because of their fine maturity structure, they contain desirable information on curve shapes and are preferred over swaps for building the front portion of swap curves. Furthermore, futures contracts are traded at major exchanges in large volume and are much more liquid than swaps of comparable maturity, which is another reason for including them in the swap curve construction. Unlike swaps, Eurodollar futures are quoted in terms of prices rather than rates. Specifically, a futures price P is the reverse of its rate F , scaled by ten thousand,

$$P = 10000(1 - F). \quad (4)$$

The contract quotes were originally designed in this way to make them consistent with price behaviors of bonds; that is, rates inversely affect price changes.

The closest futures period starts sometime in the next three months, the second closest futures starts three months after, and so on. The period between now and the start of the first futures is still not covered. To fill this gap, deposit rates are utilized. These are benchmark spot-rate agreements such as 1-, 3-, and 6-month LIBOR rates. These rates are market indices for setting swap floating rates. They are usually used to anchor the short end of swap curves.

In summary, three sets of market instruments covering different maturity regions are used for the construction of swap curves. For short maturities, typically within a year, prevailing market deposit rates are used; for intermediate maturities up to four years, prices of exchange-traded futures contracts are used; for long maturities beyond these futures contracts, benchmark interest-rate swaps rates are used. The one-year and four-year maturity separations are just a matter of choice. Depending on different views and market conditions, they may be chosen differently. For illustration, a sample set of data is displayed in Figure 1.

CURVE CONSTRUCTION

Building a swap curve is a process of decomposing the rates from input instruments and converting them to a function relating forward rates to maturity. Because of maturity

Figure 1. **A sample set of deposit, futures and swap inputs with settlement date on August 22, 2001 is shown below in order of their maturities. The inputs are identified in the column labeled "Instrument" with their market values displayed in the next column. The curve maturities represented by these inputs are listed in the last column.**

	Instrument	Quote	Maturity
Deposits	1M-LIBOR	3.580	9/24/01
	3M-LIBOR	3.529	11/22/01
Futures	Sept-01	9654.53	12/19/01
	Dec-01	9645.14	3/19/02
	Mar-02	9631.84	6/20/02
	Jun-02	9599.66	9/19/02
	Sept-02	9562.60	12/18/02
	Dec-02	9523.67	3/18/03
	Mar-03	9499.83	6/19/03
	Jun-03	9475.62	9/18/03
	Sept-03	9455.18	12/17/03
	Dec-03	9433.82	3/17/04
	Mar-04	9427.93	6/17/04
	Jun-04	9416.69	9/16/04
	Sept-04	9406.58	12/15/04
	Dec-04	9394.31	3/15/05
	Mar-05	9395.56	6/16/05
	Jun-05	9390.00	9/15/05
Swaps	5Y-Swap	5.272	8/22/06
	7Y-Swap	5.547	8/22/08
	10Y-Swap	5.773	8/22/11
	12Y-Swap	5.888	8/22/13
	15Y-Swap	6.016	8/22/16
	20Y-Swap	6.126	8/23/21
	30Y-Swap	6.161	8/22/31

overlaps, information contained in each input instrument is not independent. The goal, therefore, is to search for implied forward rates for maturity periods uniquely represented by individual instruments. To accomplish this, it is sensible to adopt the so-called bootstrap method. The instrument with the shortest maturity is used to initiate the curve, and then the curve is built out with instruments of longer maturities. With this approach, the input instruments are processed in order of increasing maturity, and at each stage, the forward rate not yet represented in the previous inputs is derived and attached to the end of the curve. Because these inputs are highly liquid market benchmarks, the bootstrapped curve must re-price them to preclude any arbitrage.

As noted earlier, piecewise polynomial functions are convenient choices to represent a forward curve. Illustrated here are the principles of applying the lowest order of polynomial representation—a piecewise flat forward curve. Higher order versions are viewed merely as mathematical extensions and will be discussed later. For uniformity across all maturities, continuously compounded instantaneous forward rates are assumed for the curve. Though not directly observable, the instantaneous rate is used for its simplicity. Other variables, such as discrete rates and discount factors, can be readily derived from it.

Deposit rates are spot rates with simple compounding over their accrual periods. From two such rates of different maturities, the forward rate for the period starting from the shorter maturity and ending at the longer maturity can be inferred. With this forward rate denoted as f_{sl} , the relation is established as

$$(1 + r_l \theta_l) = (1 + r_s \theta_s) e^{f_{sl} \theta_{sl}},$$

where the subscripts s and l indicate the short and long maturities, respectively, and r 's are deposit rates of corresponding maturities. As before, θ 's are accrual factors calculated according to the accrual conventions appropriate for that currency (for example, USD-LIBOR accrues on an actual-360 basis, in which the interest is calculated for the actual number of days of borrowing, but a year is assumed to be 360 days long). The exponential function is used according to the assumption of continuous compounding for forward rates. The forward rate for the period under consideration is then solved as

$$f_{sl} = \frac{1}{\theta_{sl}} \ln \left(\frac{1 + r_l \theta_l}{1 + r_s \theta_s} \right) \quad (5)$$

This formula can be applied to each consecutive pair of input deposit rates, and the piecewise forward rates can be obtained up to the maturity of the last deposit input.²

Bootstrapping forward rates from futures contracts is relatively straightforward since a futures rate is merely a rewrite of its quoted price according to the price-rate relationship (4). At the moment, a futures rate is assumed to be the same in value as its corresponding forward rate. A small gap between them usually exists due to their structural difference and volatile nature of interest rates—this is commonly referred to as the convexity adjustment and will be discussed in some detail later. Even though the rate extraction

² For the first period though, only one deposit is relevant and the expression (5) simplifies to $f = \ln(1 + r\theta)/\theta$.

from futures contracts is straightforward, the rate F is a simple interest that needs to be converted to a continuous forward rate f according to:

$$1 + F\theta_F = e^{f\theta_f}. \quad (6)$$

In building the curve in the futures region, formula (4), along with the compounding conversion (6), is applied to each futures period independently, and piecewise forward rates can be obtained for all the periods covered by futures inputs.

Compared with deposit rates and futures prices, swap rates have a much more complicated dependence on forward rates. As given earlier in equation (1) and shown here after a rearrangement

$$S = \frac{1 - DF(t_N)}{\sum_{i=1}^N \theta_i DF(t_i)}, \quad (7)$$

the swap rate S depends on forward rates implicitly, through discount factors, and in turn through spot rates. Solving equation (7) to determine the unknown forward rate for the current period in terms of the swap rate is more involved, not only because of the complex functional form but also because of the dependence on the forward rates for all the previous periods. However, numerical procedures are still applicable. For the sake of understanding, the embedded forward rate for the period uniquely represented by the swap under consideration can be found by a simple iterative approach. Assume a reasonable forward rate and evaluate the swap rate according to equation (7). Based on the mismatch between the evaluated and input swap rates, the guessed forward rate is then adjusted accordingly. The process is repeated until a close match is achieved. For practical purposes, however, there are many efficient and well-established algorithms that can be applied to this type of searching.

For illustration, some sample swap curves constructed with the bootstrap method described above are shown in Figure 2.

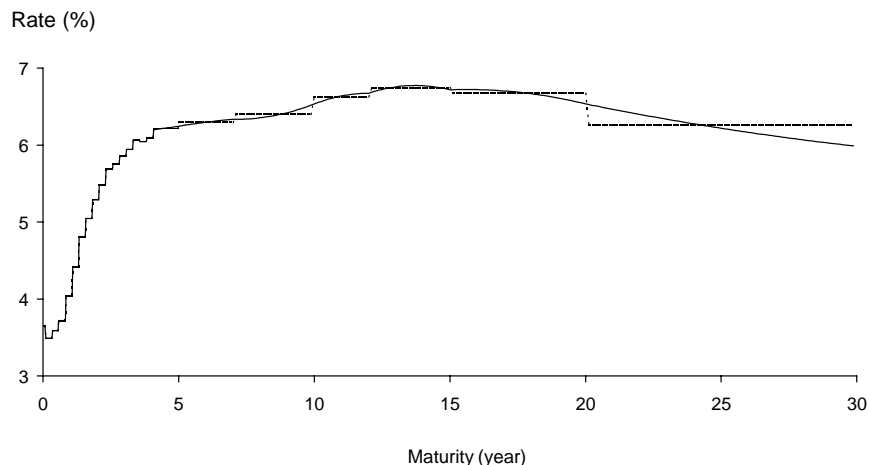
INTERPOLATION

The forward rate derivation detailed above for the three types of inputs may appear different algebraically from each other, but they all follow a common assumption; that is, for each maturity segment, the forward curve is modeled as a flat function of time,

$$f(t) = a_0, \quad (8)$$

where a_0 is a constant, the lowest order of polynomials. The resulting swap curve is actually a piecewise discontinuous step function. This functional form is reasonable considering that there is no additional information in the input market rates about the shape of the curve between maturity points. It is unsettling, though, to see that a so-called curve appears flat and discontinuous. Some curve smoothing is certainly desirable. However, any smoothing needs to be done carefully to avoid causing curve instability or introducing extra features not representative of interest rate market.

Figure 2. Illustrated below are the swap curves constructed with the market data displayed earlier for the settlement date August 22, 2001. For comparison, the piecewise flat curve and its quadratic interpolation (to be discussed below) are shown.



For short and intermediate maturities represented by deposit rates and futures contracts, the individual curve segments are very short—mostly three months—compared with the thirty-year curve horizon. For these periods, there do not seem to be other reliable and liquid market instruments that would provide finer structure. Smoothing curves over these periods may have only cosmetic consequences. Yet doing so can be misleading since it contradicts the market observation that forward rates do seem to trade discontinuously on a three-month scale. More seriously, artificial curve shaping over these periods may complicate risk analysis based on curve perturbation.

However, periods between input swap maturities are relatively long, 2-10 years. Because of these long stretches, the piecewise flat curve jumps considerably from period to period. On the other hand, it is reasonable to believe that forward rates should not remain constant for long periods and large discontinuity is unrealistic. Some form of curve smoothing is certainly needed to address these inadequacies. For example, the constant function (8) may be replaced by a quadratic version

$$f(t) = a_0 + a_1 t + a_2 t^2.$$

With three parameters to vary, the forward function can be made continuous at period boundaries. For example, an instantaneous forward rate at the left boundary of a period could be chosen somewhere between the flat rate of the left period and the flat rate of the current period; a forward rate at the right boundary could be chosen similarly. With these two boundary forward rates, the three parameters can be uniquely determined by requiring that the quadratic curve pass through these two chosen points and, in addition, that it still re-price the swap input. This interpolation smooths out the curve in each period and eliminates discontinuity at period boundaries.

The quadratic interpolation is only one of the possibilities for smoothing curves. With polynomials of higher orders, improvement in curve shape and continuity could be further explored. But any high-order curve manipulation should be handled with caution because excessive smoothing can cause spurious curve oscillation, as well as instability. Although ways of fitting polynomial, or other spline curves, are not limited, it is worth pointing out that the flat curve still stands out as the simplest and most stable. Despite its appearance, it is very robust in the sense that the movement of the curve is consistent with changing market inputs. This simple curve is able to accommodate a large number of practical needs effectively.

CURVE ADJUSTMENTS

The bootstrap procedure for building swap curves processes inputs sequentially in the order of ascending maturity. The process appears clean and orderly. Nevertheless, there are a number of adjustments that need to be taken into account along the way.

Convexity correction: As mentioned earlier, futures contracts are a better choice than forward-rate agreements in constructing swap curves because they are very liquid. However, unlike forward-rate agreements that price-settle at maturity, exchange-traded futures are marked to market. The parties have to settle the profit or loss on a daily basis. The rates implied by their prices differ from real forward rates because the daily cash settlement is reinvested or margined at going interest rates that are not known beforehand. This uncertainty causes a futures rate to be priced higher than its forward counterpart to compensate for this additional risk. The difference is usually small, yet large enough in view of arbitrage opportunities, and it is commonly referred to as the *convexity adjustment* of futures. Theoretically, the magnitude of convexity corrections can be evaluated given a dynamic interest rate model. For example, based on a single factor, normal interest-rate model with a flat volatility surface, the size of this effect can be approximated as³

$$F - f \approx \frac{1}{2} \sigma^2 t^2, \quad (9)$$

where σ is the annualized basis-point volatility and t is the maturity of the futures contract. As can be seen, the convexity correction increases with increasing maturity and varies with rate volatility. Approximation (9) is appreciated for its qualitative simplicity; the value calculated based on (9), though, is viewed as a rough estimate only. The accurate computation of convexity values requires more elaborate interest-rate models, involving multi-factor dynamics and a volatility structure calibrated to prices of interest rate options, typically caps and floors. For building swap curves, the input futures rates should be adjusted down by proper convexity corrections because the curve is modeled in terms of forward rates.

³ In fact, under such a simple model, the convexity correction is known exactly and can be written as

$$F - f = \frac{DF(t)}{DF(t+\theta)\theta} \left(e^{\frac{1}{2}\sigma^2 t^2} - 1 \right)$$

Approximation (9) is obtained by Taylor expanding the exponential term and also by neglecting the difference between the two discount factors in the expression.

Overlap or gap: So far in the discussion on futures, we have assumed that the underlying accrual periods line up perfectly, with no overlaps or gaps between neighboring pairs. In reality, this is not the case. According to the contract's official specification, a Eurodollar futures contract covers a period that always starts on the third Wednesday of the futures months and ends at a date three months out from the Wednesday start. Thus, there will be either overlaps or gaps between adjacent futures periods. Depending on the actual calendar, the mismatch can be substantial. For example, the Dec-05 contract that ends on Mar. 21, 2006 overlaps seven days with the Mar-06 contract that starts on Mar. 15, 2006. Because of this type of mismatch the calculated forward rates have to be modified. In case of gaps, the preceding forward rate can be simply extended to the start of the following period without causing any mispricing. But in case of overlaps, simply ignoring the rate difference of the two overlapping periods is not acceptable. For example, if the rate of the following period is used as the rate for the overlapping region, the preceding futures rate would be mispriced, and to a good degree of accuracy the mispricing can be estimated as

$$\Delta f \approx \frac{n}{90} (f_R - f_L).$$

where n is the number of overlapping days and the subscripts L and R denote preceding and following rates respectively. The mispricing might not be important if the curve is more or less flat; but it could be intolerable in a steep curve environment. To prevent this mispricing, the above adjustment can be made to the preceding rate and at the same time the start of the following period is used as the end of the preceding period in the curve representation.

Year-end effect: It is a well-known fact that deposit rates, as well as futures rates, with accrual periods bridging two years are noticeably higher than they would have been normally priced. This is because there is certain degree of uncertainty over the turn of years. Lenders usually require a rate premium from borrowers to compensate for any unexpected event as a year draws to a close. Although the risk applies only to a few days at year-end, its effect on rates usually gets averaged over accrual periods of a relevant instrument, say, the December futures contracts. Overlooking this price differential in curve construction could result in rates for dates both before and after the New Year being overpriced by the curve. To avoid this mispricing, the instantaneous forward rates during the New Year holidays need to be raised above those on surrounding dates. Normally, this increase is about a couple hundreds of basis points. However, for major turn-of-year events, the effect could be much bigger. For instance, approaching the year 2000, financial markets anticipated excessive risks associated with the Y2K computer breakdowns, and the year-end premium was over ten hundreds of basis points. Therefore, to assume accurate security pricing and risk management for maturities around year-ends, swap curves should be constructed by incorporating this effect.

Par value assumption: In the definition of swap rate given earlier in equation (1), it is assumed that the floating leg of a swap is priced at par. This is true only if for each floating-leg cash flow the period of index rate (typically three-month LIBOR) is identical to the period over which this interest accrues. Although this assumption is always taken for granted, especially in research, actual swaps traded in the market do not always agree with it. For real swaps, there are adjustments on payment dates and index periods, mostly to avoid weekends and holidays. As a result, an index period does not always match its

accrual period, which invalidates the assumption for par value. Therefore, the left side of the swap balancing equation (1) should be replaced with actual floating-leg value, and the swap rate then calculated accordingly. This adjustment usually results in small changes in swap rates, less than one-tenth of a basis point, on average. The effect is bigger, though, for short-dated swaps.

Asymptotic behavior: Usually reliable swap rate inputs reach up to thirty years in maturity. Extrapolation is required if a swap curve is used to price securities with longer maturities. Unless there is convincing evidence regarding the long-term curve shape, a swap curve is normally extrapolated in terms of a flat forward rate; that is, the last reliable forward rate is assumed to be the curve's asymptotic value. Specifically, the last forward rate of a piecewise flat curve can be extended out for maturity beyond the last curve piece. Theoretically, the flat rate extrapolation is consistent with the assumption that interest-rate evolution is mean reverting. For example, with the Vasicek model of interest rates, the forward curve can be shown explicitly to approach a constant level asymptotically.

REMARKS

Use of Futures

In building the front part of a swap curve, Eurodollar futures contracts are used instead of short-maturity swaps. This is because the futures contracts reveal information about the shape of the curve at short maturities. But this is not without cost: as discussed earlier, the convexity correction has to be considered at the same time. Since the convexity value depends on interest rate volatility, which is not directly observable, there is no market standard for its estimate. Practitioners usually employ their own proprietary models to calculate convexity values, which often leads to a range for convexity adjustments.

This non-uniformity may seem to provide arbitrage opportunities among market players. For instance, a particular swap curve may reprice convexity-corrected futures prices, but it may misprice swaps rates of comparable maturity. Even though arbitrage profit may be suggested, typically it would not be trivial to realize. Most of the time, the argument runs into complication when hedging difficulties, exchange fees, or model risks are being considered. Because of these inefficiencies, convexity values cannot be determined precisely and are usually confined in a small range. This should be kept in mind when building swap curves. Although it is preferable to use futures over swaps up to a certain maturity, there is always a trade-off in view of swap mispricing.

Curve Methodology

As to swap curve methodology, a bootstrap method is selected in constructing swap curves. This is done mainly because there are only a handful of benchmark swap inputs covering the maturity horizon. Since these swaps are highly liquid, they should be priced exactly. The bootstrap procedure processes consecutive swap instruments individually to ensure exact fits. Furthermore, the method, along with low-order polynomial parameterization, ensures that the constructed curve does not have excessive wiggles due to large spacing between swap maturities.

In contrast, the yield curve for Treasuries is usually built with least-square optimization of cubic splines. This is because in the U.S. Treasury market there are over a hundred bonds densely distributed over the maturity horizon, and the supply-demand difference

of these bonds causes them to be priced rich or cheap relative to each other. The least-squares method searches for the best fit, not for exact pricing of input bonds but rather for overall accuracy and stability. The stability is achievable because of the large number of bond inputs. In comparison, the method would not be effective for constructing swap curves due to the limited number of swap inputs. Spline-fitting a small number of inputs usually leads to wavy and unstable curves.

SUMMARY

The swap curve construction has been an integral part of interest rate derivatives business. A benchmark swap curve is constructed using the prices of benchmark instruments. Deposits, futures, and swaps are employed as inputs because of their high liquidity and representative maturities. A swap curve is constructed in terms of piecewise continuous forward rates, and a bootstrap scheme is used to build curve pieces in order of increasing maturity. Meanwhile, various adjustments have to be considered to make the curve more realistic and more representative of actual market.

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