Exponential Distribution Analysis

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Overview

This report is an investigation of the exponential distribution, with focus on taking several sample sets and comparing theoretical and sample means and variances, as well as showing distributions.

1 Simulations

Set up simulation variables

```
lambda <- 0.2
#No. of simulations
nsim <- 1000
#No. of samples per simulation
n <- 40
```

Run 1000 simulations, take 40 samples from an exponential distribution for each one. The samples are arranged into a 1000x40 matrix, where each row represents a simulation with 40 samples.

```
simulations <- matrix(rexp(nsim * n,lambda), nsim)
```

Calculate the mean and variance of each simulation. Store them on a list of 1000 items, where each element represent the mean (or variance) of one simulation.

```
mns <- apply(simulations, 1, mean)
vars <- apply(simulations, 1, var)
```

2 Sample Mean vs Theoretical Mean

The theoretical mean for an exponential distribution is $1/\lambda$:

```
mean <- 1/lambda
mean
```

```
## [1] 5
```

To measure the sample mean, we'll get the list of means previously calculated and extract the mean of that list:

```
samplemean <- mean(mns)
samplemean
```

```
## [1] 4.97694
```

We can see that the sample mean taken from the means of 1000 simulations is very similar to the theoretical mean for the exponential distribution Theoretical mean: **5**
Sample mean: **4.9769398**

Figure 1 and **Figure 2** on the appendix give a more visual comparison of the theoretical and sample means.

3 Sample Variance vs Theoretical Variance

Since $sd = \sqrt{var(X)}$, then $var(X) = sd^2$, therefore, given that for an exponential distribution $sd = 1/\lambda$, the theoretical variance for an exponential distribution should be $1/\lambda^2$:

```
sd <- 1/lambda
var <- sd^2
var
```

```
## [1] 25
```

To measure the sample variance, we'll get the list of variances previously calculated and extract the mean of that list:

```
#Take an average of the sample vars
samplevar <- mean(vars)
samplevar
```

```
## [1] 24.7899
```

We can effectively see that the sample variance taken from the variances of 1000 simulations is very similar to the theoretical variance for the exponential distribution Theoretical variance: **25**
Sample variance: **24.7899019**

Figure 3 and **Figure 4** on the appendix give us a visual comparison of the theoretical and sample variance.

1.4 Distribution

Now we'll show that the distribution is approximately normal. If we take the 1000 simulations with 40 random samples each and consider it as just 1 simulation with 40000 random draws from en exponential distribution, we can see in **Figure 5** that the distribution is indeed exponential.
Finally, if we take the means of the 1000 simulations we ran and plot a histogram, **Figure 6** shows the distribution we get. Per the **Central Limi Theorem**, we can effectively see that the distribution of the means of 1000 simulations (with 40 random samples each) is **approximately normal**.

Appendix

Figures

```
hist(simulations,breaks = 50, xlab="Sample value", main="Figure 1: Theoretical mean")
abline(v = mean, col = "blue", lwd = 2)
text(mean+3, 5000 , paste("theoretical mean:", mean), pos = 4, col="blue")
```

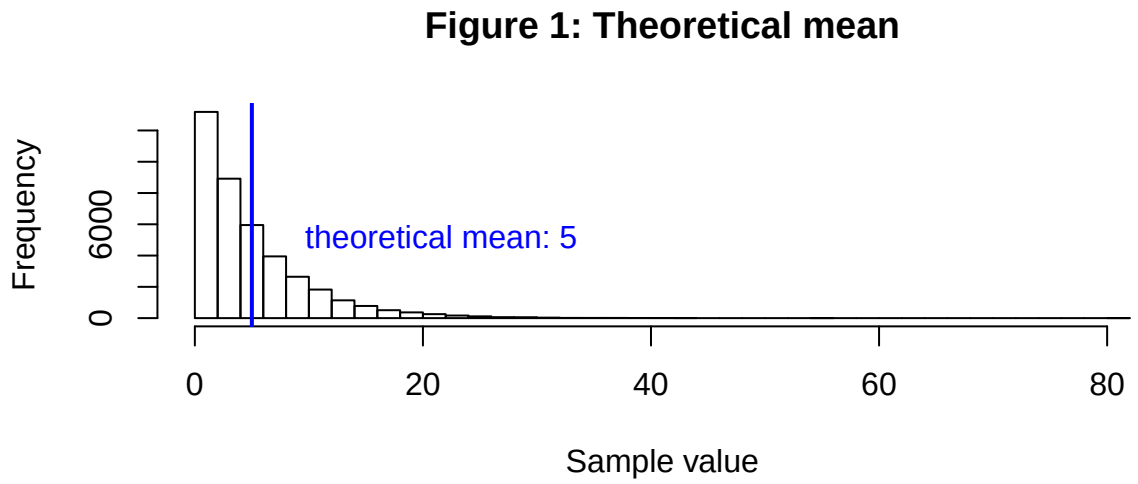

Figure 1. Histogram of random samples and theoretical mean taken from an exponential distribution

```
hist(simulations,breaks = 50, xlab="Sample value", main="Figure 2: Sample mean")
abline(v = samplemean, col = "red", lwd = 2)
text(samplemean+3, 5000 , paste("sample mean:", samplemean), pos = 4, col="red")
```

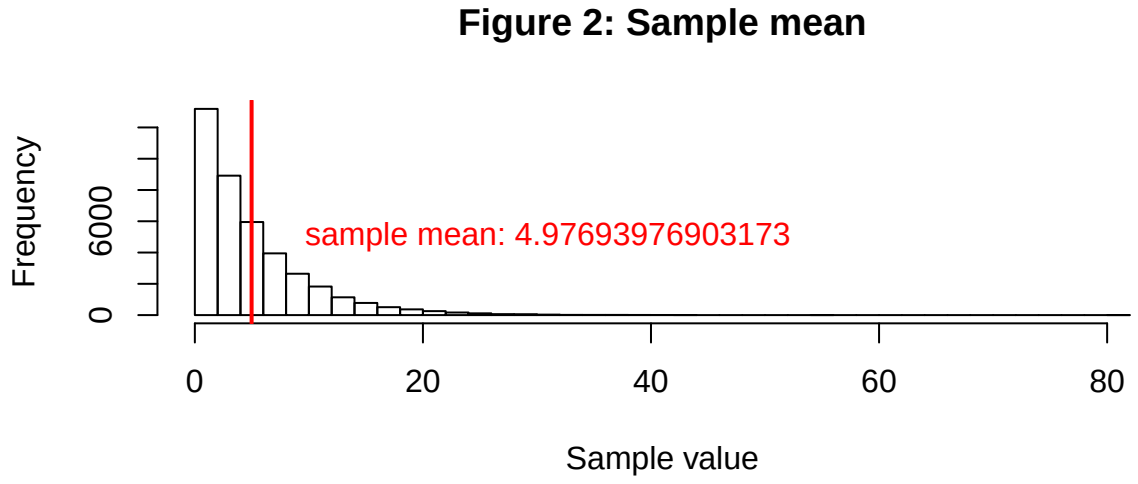

Figure 2. Histogram of random samples and sample mean taken from an exponential distribution

```
hist(simulations,breaks = 50, xlab="Sample value", main="Figure 3: Theoretical variance")
abline(v = mean, col = "blue", lwd = 1)
abline(v = var, col = "blue", lwd = 2)
text(var+3, 5000 , paste("theoretical var:", var), pos = 4, col="blue")
```

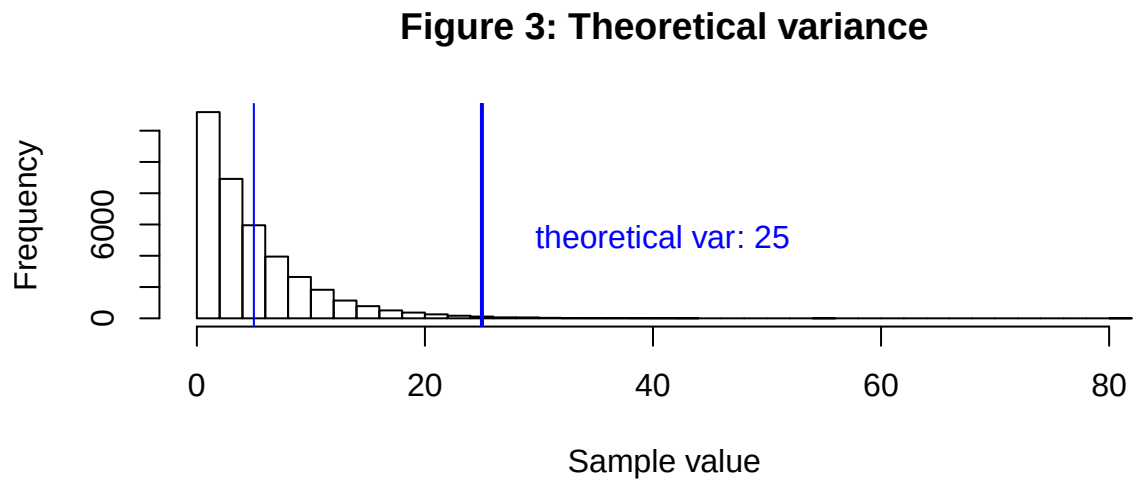

Figure 3. Histogram of random samples, theoretical mean and theoretical variance taken from an exponential distribution

```
hist(simulations,breaks = 50, xlab="Sample value", main="Figure 4: Sample variance")
abline(v = samplemean, col = "red", lwd = 1)
abline(v = samplevar, col = "red", lwd = 2)
text(samplevar+3, 5000 , paste("sample var:", samplevar), pos = 4, col="red")
```

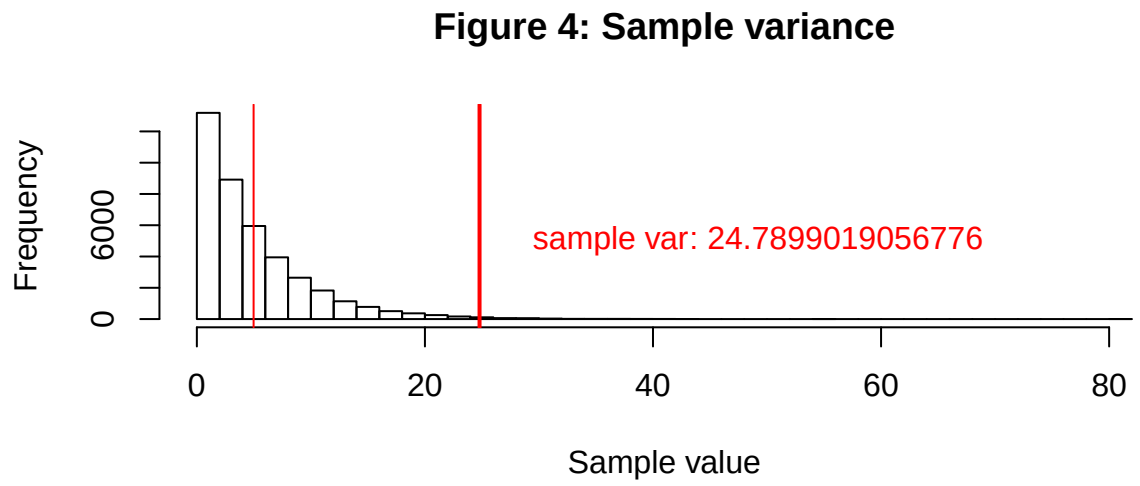

Figure 4. Histogram of random samples, sample mean and sample variance taken from an exponential distribution

```r
hist(simulations,breaks=50, xlab="Sample value",
     main="Figure 5: Exponential Distribution Random Samples")
```

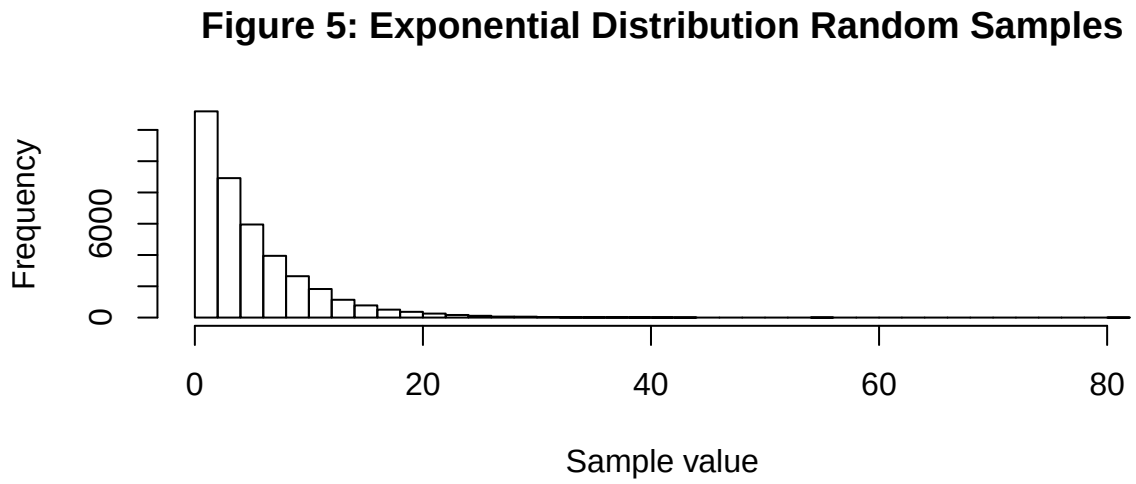

Figure 5. 40000 random samples drawn from an exponential distribution

```r
hist(mns,breaks=50, xlab="Sample value",
     main="Figure 6: Distribution of 1000 sample means")
```

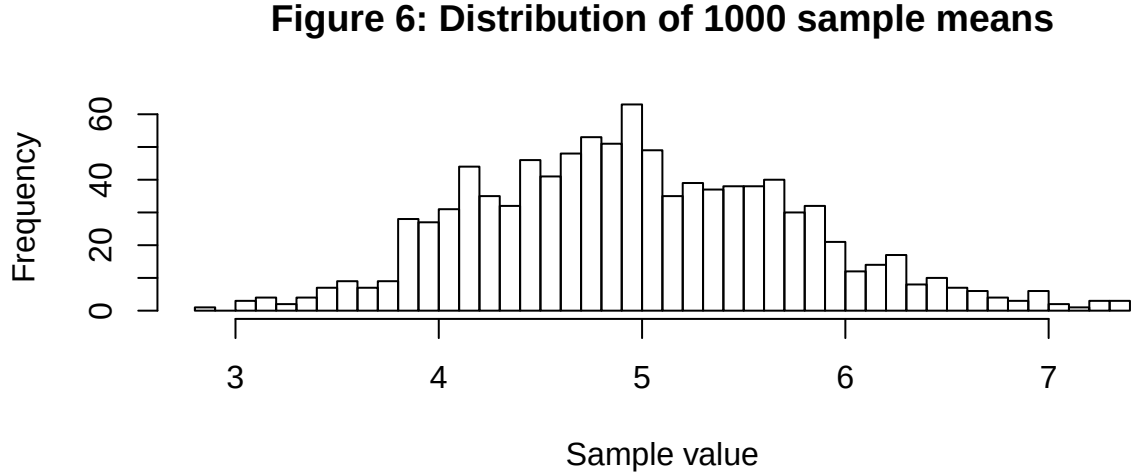

Figure 6. Distribution of the means of 1000 simulations