The MU Puzzle

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Abstract

This document will attempt to prove the impossibility of the MU-Puzzle from Chapter 1 of AN ETERNAL GOLDEN BRAID.

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1 Introduction

The MU puzzle is a challenge to form the string MU from the starting string MI, following a set of rules:

1. **Rule I:** If you possess a string whose last letter is I, you can add on a U at the end.

2. **Rule II:** Suppose you have Mx. Then you may add Mxx to your collection.

3. **Rule III:** If III occurs in one of the strings in your collection, you may make a new string with U in place of III.

4. **Rule IV:** If UU occurs inside one of your strings, you can drop it.

This puzzle is an interesting exercise of formulating proofs and theorems, and as the book explains, is more about the process and thinking through it than the solution itself.

2 Analysis

From my understanding of the puzzle, the only way to achieve MU is by eliminating the starting I or getting a string of I's that is divisibile by 3, i.e. for $Mxx..x$, $x mod 3 = 0$.

Lemma 2.1. *Rule II allows the string after M to be doubled, but for any string of n I's, the prime factorization of n is always 2*2*...*2.*

Proof. Repeating Rule 2, we get:

1. **Step 1:** MI

2. **Step 2:** MII (2*2)

3. **Step 3:** $MIIII$ (2*2*2)

4. **Step 4:** $MIIIIIIII$ (2*2*2*2)

5. **etc...**

Rule III allows a U to be exchanged for a string of III, but there will always be an I leftover, as seen in the steps above.

Rules I and IV allow the number of U's to vary, but this will never affect the number of I's in the string.

Therefore, it is impossible to achieve MU from MI by the set of rules.

$\square$

3 Main Result

Theorem 3.1. *As shown, it is impossible to achieve MU from MI by the set of rules allowed. The string of I's will never be divisible by 3 without remainder, so there will always be at least 1 I in the string. Therefore MU, which has no I, is impossible to achieve.*

Remark 3.2. MU is only achievable by a different starting condition, a different set of rules, or both.

4 Conclusion

The goal of the puzzle was to achieve MU from a starting string MI which has been shown to be impossible. The key to this puzzle was the number of I's never being divisible by 3. Since MU is only achievable by eliminating all I's, the goal is impossible.

References

[1] D. Hofstadter, *Gödel, Escher, Bach: An Eternal Golden Braid*, Basic Books, 1979.