Relational Algebra

Relational Query Languages

- Query languages: allow manipulation and retrieval of data from a database
- Relational model supports simple, powerful QLs:
  - Strong formal foundation
  - Allows for much optimization
- Query languages are NOT programming languages
  - QLs not expected to be "turing complete"
  - QLs not intended to be used for complex calculations
  - QLs support easy, efficient and sophisticated access to large data sets

Formal Relational Query Languages

- Mathematical query languages form the basis for "real" languages (e.g. SQL) and for implementation:
  - **Relational Algebra**:
    - Operational - a query is a sequence of operations on data
    - Very useful for representing execution plans, i.e., to describe how a SQL query is executed internally
  - **Relational Calculus:**
    - descriptive - a query describes how the data to be retrieved looks like

- Understanding Relational Algebra is key to understanding SQL, PigLatin, OQL, Xquery,….

Example Relations

Skaters

sid	sname	rating	age
28	yuppy	9	15
31	debby	7	10
22	conny	5	10
58	lilly	10	13

(age)
Used in all
Database courses
But really BAD

- difficult to maintain
- has to be updated annually
:(

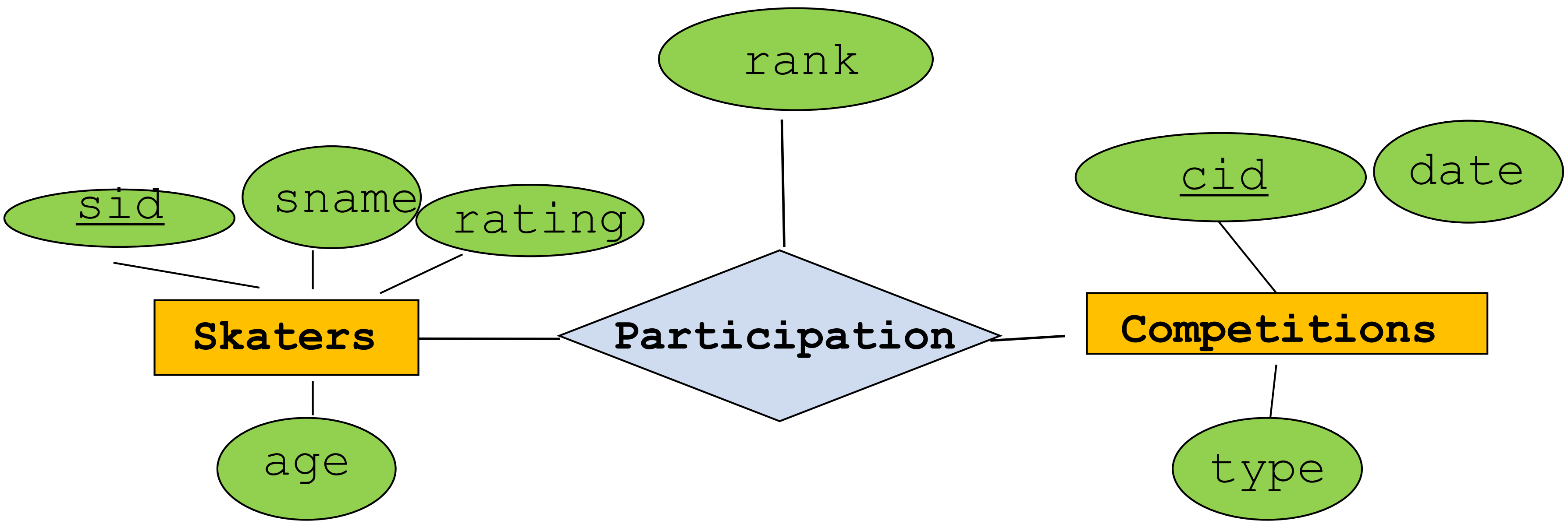
Participates

sid	cid	rank
31	101	2
58	103	7
58	101	7
58	104	1

Competitions

cid	date	type
101	12/13/2015	local
103	01/12/2016	regional
104	01/20/2016	local

Comes from… E/R



Relational Algebra: Basics

- RA consists of a set of basic *operators*
  - *Input:* one or two relations
    - schema of each relation is known
    - instance can be arbitrary
  - *Output:* a relation
    - schema of output relation depends on operator and input relations
- Relational algebra is closed:
  - since each operation has input relation(s), and returns a relation, operations can be composed
- Does not assume special primary key attributes in the relation
- Assumes a relation is a set — no two rows have all the same values
  - No two tuples have the same values in all attributes

(1-2 relations) → (1 relation)

if no key → combination of all attributes is the key

Relational Algebra: Operations

- Single relation as input
  - **Selection σ:** Selects a subset of tuples from a relation get rows
  - **Projection $\prod$:** projects to a subset of attributes from a relation get cols
  - **Renaming ρ:** of relations or attributes; useful when combining several operators
- Two relations as input
  - **Cross Product X:** Combines two relations
  - **Join $\bowtie$:** Combination of Cross product and selection
  - **(Division):** not covered in class
- Set operators with two relations as input
  - **Intersection ($\cap$)**
  - **Union ($\cup$)**
  - **Set Difference –:** Tuples that are in the first but not the second relation

Projection: $\prod_L(R_{in})$

- Returns the subset of the attributes of the input relation R_{in} that are in the projection list L
- Schema of result relation contains exactly the attributes of the projection list, with the same attribute names as in R_{in}

extract columns from table

Skaters

sid	sname	rating	age
28	yuppy	9	15
31	debby	7	10
22	conny	5	10
58	lilly	10	13

$\prod_{\texttt{sname,rating}}$ (**Skaters**)

sname	rating
yuppy	9
debby	7
conny	5
lilly	10

Projection: $\prod_L(R_{in})$

- Operational Semantics:
  - Imagine a tuple variable iterating over all tuples in the relation
  - for each tuple: extract the projected attributes and output the reduced tuple in result relation
  - eliminate duplicates
    - Note: real systems typically do NOT eliminate duplicates unless the user explicitly asks for it; eliminating duplicates is a very costly operation!

Skaters

sid	sname	rating	age
28	yuppy	9	15
31	debby	7	10
22	conny	5	10
58	lilly	10	13

$\prod_{age}$ **(Skaters)**

age
15
10
13

eliminate duplicates

Selection: $\sigma_C(R_{in})$

input table
condition

- *Selection:* $\sigma_C(R_{in})$
  - Schema of result relation identical to schema of R_{in}
  - Returns the subset of the rows of the input relation R_{in} that fulfill the condition C
  - Condition C involves the attributes of R_{in}
  - No duplicates (obviously)
- Operational Semantics
  - Imagine a tuple variable ranging over all tuples in the relation
  - for each tuple: check whether condition C is satisfied. If so, output the tuple into the result relation

extract rows

Skaters

sid	sname	rating	age
28	yuppy	9	15
31	debby	7	10
22	conny	5	10
58	lilly	10	13

$\sigma_{\texttt{rating > 8}}$**(Skaters)**

sid	sname	rating	age
28	yuppy	9	15
58	lilly	10	13

Operator Composition

- result relation of one operation can be input for another relational algebra operation

$$\pi_{sname,rating}(\sigma_{rating > 8}(Skaters))$$

sid	sname	rating	age
28	yuppy	9	15
31	debby	7	10
22	conny	5	10
58	lilly	10	13

sid	sname	rating	age
28	yuppy	9	15
58	lilly	10	13

sname	rating
yuppy	9
lilly	10

- Operational Semantics:
  - Stepwise one operator at a time:
    - build intermediate temporary relations

Operator Composition

- result relation of one operation can be input for another relational algebra operation

$$\Pi_{\texttt{sname,rating}}(\sigma_{\texttt{rating > 8}}(\texttt{Skaters}))$$

sid	sname	rating	age
28	yuppy	9	15
31	debby	7	10
22	conny	5	10
58	lilly	10	13

sname	rating
yuppy	9
lilly	10

pipelining through one row at a time

- Consecutive operators on the fly: one scan through the relation
  - Not always possible

no intermediate table

Union, Intersection, Set-Difference

- Notation:
  - $R_{in1} \cup R_{in2}$ (Union),
  - $R_{in1} \cap R_{in2}$ (Intersection),
  - R_{in1} - R_{in2} (Difference),
- Usual operations on sets
- R_{in1} and R_{in2} must be set-compatible,
  - same number of attributes
  - corresponding attributes must have the same type
  - no need for same name
- Result schema
  - same as the schema of the input relations
  - possibly renamed attributes

Example Tables

Skaters (Table of DB of the Glacier Club)

<u>sid</u>	sname	rating	age
28	yuppy	9	15
31	debby	7	10
22	conny	5	10
58	lilly	10	13

OurSkaters (Table of DB of the Icy Club)

<u>id</u>	name	rating	age
28	yuppy	9	15
25	debby	7	10
27	willy	8	8

Union

Skaters

sid	sname	rating	age
28	yuppy	9	15
31	debby	7	10
22	conny	5	10
58	lilly	10	13

dup

OurSkaters

id	name	rating	age
28	yuppy	9	15
22	debby	7	10
27	willy	8	8

Skaters ∪ OurSkaters

eliminate duplicates

		rating	age
28	yuppy	9	15
31	debby	7	10
22	conny	5	10
58	lilly	10	13
22	debby	7	10
27	willy	8	8

different

Intersection

Skaters

sid	sname	rating	age
28	yuppy	9	15
31	debby	7	10
22	conny	5	10
58	lilly	10	13

OurSkaters

id	name	rating	age
28	yuppy	9	15
25	debby	7	10
27	willy	8	8

exactly identical in both

Skaters ∩ OurSkaters

		rating	age
28	yuppy	9	15

Difference

Skaters

sid	sname	rating	age
28	yuppy	9	15
31	debby	7	10
22	conny	5	10
58	lilly	10	13

OurSkaters

id	name	rating	age
28	yuppy	9	15
25	debby	7	10
27	willy	8	8

in left but not right

Skaters - OurSkaters

		rating	age
31	debby	7	10
22	conny	5	10
58	lilly	10	13

Concatenation of operators

Skaters

sid	sname	rating	age
28	yuppy	9	15
31	debby	7	10
22	conny	5	10
58	lilly	10	13

OurSkaters

id	name	rating	age
28	yuppy	9	15
25	debby	7	10
27	willy	8	8

$$\left(\Pi_{\texttt{sname, rating, age}}(\texttt{Skaters})\right) \cap \left(\Pi_{\texttt{name, rating, age}}(\texttt{OurSkaters})\right)$$

	rating	age
yuppy	9	15
debby	7	10

implementations

Operational Semantics

- Take your favorite set-operator algorithm discussed in COMP 250/251, MATH 240
- Intersection
  - For each tuple in first relation
    - For each tuple in second relation
      - If tuples are equal: output
- Difference
  - For each tuple in first relation
    - For each tuple in second
      - If tuples are equal: exit loop / no output
    - If no early exit: output
- Union

Relational Algebra Quiz

all makers that don't make laptops:

$$\pi_{maker}(computer) - \pi_{maker}(\sigma_{category = \text{"laptop"}}(computer))$$

Cross-Product

- *Cross-Product*: R_{in1} **X** R_{in2}
- Each row of R_{in1} is paired with each row of R_{in2}
- Result schema
  - one attribute per attribute of R_{in1}
  - one attribute per attribute R_{in2}
  - field names inherited
    - if both have attribute with same name: prefix with relation name
- Operational semantics
  - Consider a tuple variable t1 for first relation;
  - Consider a tuple variable t2 for second relation

for each assignment of t1 {

for each assignment of t2 {

combine all attribute values of t1 and t2 and output them as one tuple into result relation; prefix attribute names with relation name

}

}

Cross-Product

Skaters

sid	sname	rating	age
28	yuppy	9	15
31	debby	7	10
22	conny	5	10

Participates

sid	cid	rank
31	101	2
58	103	7

Skaters X Participates

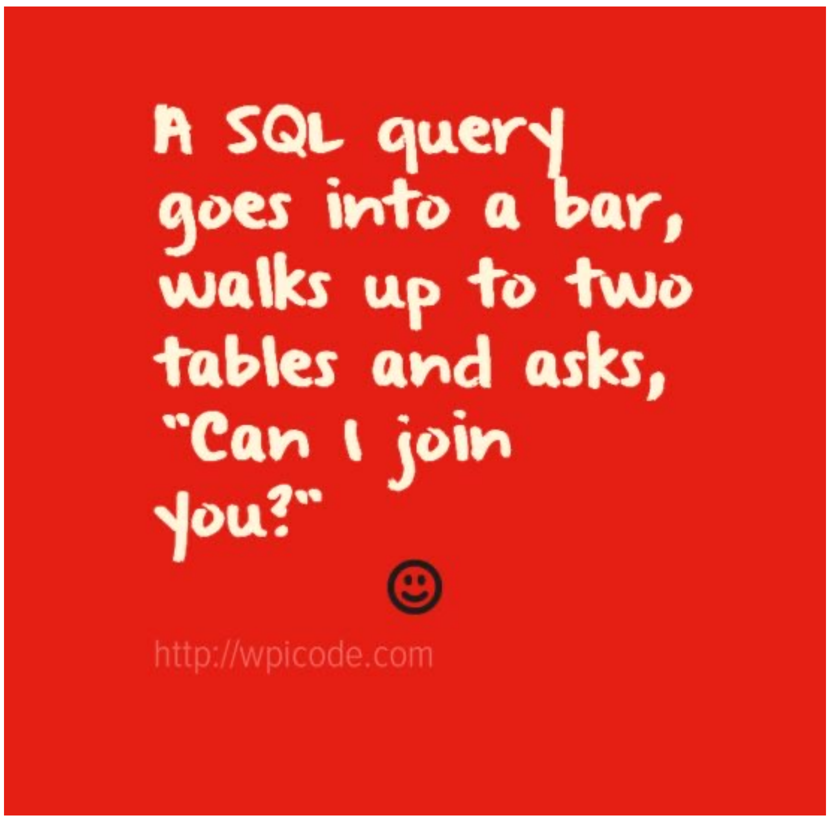
S.sid	sname	rating	age	P.sid	cid	rank
28	yuppy	9	15	31	101	2
28	yuppy	9	15	58	103	7
31	debby	7	10	31	101	2
31	debby	7	10	58	103	7
22	conny	5	10	31	101	2
22	conny	5	10	58	103	7

Joins

big one

Cross-Product +

Selection with attributes from both relations



It's what makes an advanced query language

-- the way to relate tables

Joins

- ***Condition Join (Theta-Join):*** $R_{out} = R_{in1} \bowtie_c R_{in2} = \sigma_C(R_{in1} \times R_{in2})$
- Result schema the same as for cross-product

cross product then selection on it

Skaters

sid	sname	rating	age
28	yuppy	9	15
31	debby	7	10
22	conny	5	10
58	lilly	10	13

OurSkaters

id	name	rating	age
28	yuppy	9	15
25	debby	7	10
27	willy	8	8

$$\texttt{Skaters} \bowtie_{\texttt{Skaters.rating > OurSkaters.rating}} \texttt{OurSkaters}$$

Joins

- *Condition Join (Theta-Join)*: $R_{out} = R_{in1} \bowtie_c R_{in2} = \sigma_C(R_{in1} \times R_{in2})$
- Result schema the same as for cross-product

Skaters

sid	sname	rating	age
28	yuppy	9	15
31	debby	7	10
22	conny	5	10
58	lilly	10	13

OurSkaters

id	name	rating	age
28	yuppy	9	15
25	debby	7	10
27	willy	8	8

sid	sname	Skaters .rating	Skaters. age	id	name	OurSkaters. rating	OurSkaters. age
28	yuppy	9	15	25	debby	7	10
28	yuppy	9	15	27	willy	8	8
58	lilly	10	13	28	yuppy	9	15
58	lilly	10	13	25	debby	7	10
58	lilly	10	13	27	willy	8	8

Operational Semantics

Consider a tuple variable t1 for first relation;

Consider a tuple variable t2 for second relation

for each assignment of t1

for each assignment of t2

if condition C is true, combine all attribute values of t1 and t2 and output them as one tuple into result relation; prefix attribute names with relation name

Equi-Join

- ***Equi-Join:*** $R_{out} = R_{in1} \bowtie_{Rin1.a1 = Rin2.b1 \wedge \ldots Rin1.an = Rin2.bn} R_{in2}$
- A special case of condition join where the condition C contains only *equalities*.
- Result schema similar to cross-product,
  - only *one copy of attributes* for which equality is specified

Equi-Join

Skaters

sid	sname	rating	age
28	yuppy	9	15
31	debby	7	10
22	conny	7	10
58	lilly	10	13

OurSkaters

id	name	rating	age
28	yuppy	9	15
25	debby	7	10
27	willy	8	8

$$\texttt{Skaters} \bowtie_{\texttt{Skaters.rating = OurSkaters.rating}} \texttt{OurSkaters}$$

sid	sname	rating	Skaters. age	id	name	OurSkaters. age
28	yuppy	9	15	28	yuppy	15
31	debby	7	10	25	debby	10
22	conny	7	10	25	debby	10

There is only one rating attribute in the output relation.

Natural Join

- ***Natural Join***: Equijoin on all common attributes, i.e., on all attributes with the same name
  – *Attributes do not need to be indicated* in index of join symbol

Skaters

sid	sname	rating	age
28	yuppy	9	15
31	debby	7	10
22	conny	5	10

Participates

sid	cid	rank
31	101	2
22	103	7
31	103	1
58	101	4

Skaters $\bowtie$ Participates

cross product retaining tuples with matching sid

sid	sname	rating	age	cid	rank
31	debby	7	10	101	2
31	debby	7	10	103	1
22	conny	5	10	103	7

syntactic sugar

Renaming

- ***Renaming :*** $\rho(R_{out}(B1,\ldots Bn), R_{in}(A1, \ldots An))$
  - Produces a relation identical to R_{in}
  - Output relation is named R_{out}
  - Attributes $A1, \ldots An$ of R_{in} renamed to $B1, \ldots Bn$

ρ(Temp, Skaters),

ρ(Temp1(sid1,rating1), Skaters(sid,rating)))

Examples (discussed in class)

- Relations
  - Skaters(sid,sname,rating,age)
  - Participates(sid,cid,day)
  - Competition(cid,date,type)
- Queries
  - Find names of skaters who have participated in competition #103 (three solutions)
  - Find names of skaters who have participated in a local competition (2 solutions)
  - Find sids of skaters who have participated in a local or regional competition (1 solution)
  - Find name of skaters who have participated in a local or regional competition
  - Find sids of skaters who have participated in a local and regional competition (2 solutions)

– Find names of skaters who have participated in competition #101 (three solutions)

S

sid	sname	rating	age
28	yuppy	9	15
31	debby	7	10
22	conny	5	10
58	lilly	10	13

P

sid	cid	rank
31	101	2
58	103	7
58	101	7
58	104	1

C

cid	date	type
101	12/13/2014	local
103	01/12/2015	regional
104	01/20/2015	local

$\Pi_{sname}(S \bowtie \sigma_{cid=101}(P))$

natural join so looks at attributes w/ same name

$$\Pi_{sname}(\sigma_{cid=101}(P) \bowtie S)$$
$$\Pi_{sname}(\sigma_{cid=101}(S \bowtie P))$$

want to select local get participants then name

– Find names of skaters who have participated in a local competition (2 solutions)

natural join

$\Pi_{name}(\sigma_{type=local}(C \bowtie P \bowtie S))$

S

sid	sname	rating	age
28	yuppy	9	15
31	debby	7	10
22	conny	5	10
58	lilly	10	13

P

sid	cid	rank
31	101	2
58	103	7
58	101	7
58	104	1

C

cid	date	type
101	12/13/2014	local
103	01/12/2015	regional
104	01/20/2015	local

$$\Pi_{sname}(\sigma_{type='local'}(C \bowtie P \bowtie S))$$

$$\Pi_{sname}(\sigma_{type='local'}(C) \bowtie P \bowtie S)$$

$$\rho(Temp, \sigma_{type='local'}(C) \bowtie P)$$
$$\Pi_{sname}(Temp \bowtie S)$$

– Find sids of skaters who have participated in a local or regional competition (1 solution)

S

sid	sname	rating	age
28	yuppy	9	15
31	debby	7	10
22	conny	5	10
58	lilly	10	13

P

sid	cid	rank
31	101	2
58	103	7
58	101	7
58	104	1

C

cid	date	type
101	12/13/2014	local
103	01/12/2015	regional
104	01/20/2015	local

$\Pi_{sid}(\sigma_{type=local \vee type=regional}(C \bowtie P))$

$$\Pi_{sid}(\sigma_{ctype='local' \vee ctype='regional'}(C) \bowtie P)$$

$$\Pi_{sid}(\sigma_{ctype='local'}(C) \bowtie P) \cup \Pi_{sid}(\sigma_{ctype='regional'}(C) \bowtie P)$$

– Find names of skaters who have participated in a local or regional competition (1 solution)

S

sid	sname	rating	age
28	yuppy	9	15
31	debby	7	10
22	conny	5	10
58	lilly	10	13

P

sid	cid	rank
31	101	2
58	103	7
58	101	7
58	104	1

C

cid	date	type
101	12/13/2014	local
103	01/12/2015	regional
104	01/20/2015	local

$\rho(Temp, \sigma_{type=local \vee type=regional}(C) \bowtie P)$

$\Pi_{name}(Temp \bowtie S)$

$\rho(Temp, \sigma_{ctype='local' \vee ctype='regional'}(C) \bowtie P)$

$\Pi_{sname}(Temp \bowtie S)$

(cant join S + C)

can only join tables w/ common attributes

– Find sids of skaters who have participated in a local AND a regional competition (2 solution)

P(locals, π_{sid} ($\sigma_{type=local}$(C) ⋈ P))
P(regionals, π_{sid} ($\sigma_{type=regional}$(c) ⋈ P))

locals ∩ regional

S

<u>sid</u>	sname	rating	age
28	yuppy	9	15
31	debby	7	10
22	conny	5	10
58	lilly	10	13

P

<u>sid</u>	<u>cid</u>	rank
31	101	2
58	103	7
58	101	7
58	104	1

C

<u>cid</u>	date	type
101	12/13/2014	local
103	01/12/2015	regional
104	01/20/2015	local

$\rho(Locals, \Pi_{sid}(\sigma_{ctype='local'}(C) \bowtie P))$

$\rho(Regionals, \Pi_{sid}(\sigma_{ctype='regional'}(C) \bowtie P))$

$Locals \cap Regionals$

get sids in both

WRONG !!

$\rho(Temp, \sigma_{ctype='local' \wedge ctype='regional'}(C) \bowtie P)$

Some rules and definitions

- Equivalence: Let R, S, T be relations; C, C1, C2 conditions; L projection lists of the relations R and S
  - Commutativity:
    - $\prod_L(\sigma_C(R)) = \sigma_C(\prod_L(R))$
      - But only if C only considers attributes of L
    - $R1 \bowtie R2 = R2 \bowtie R1$
  - Associativity:
    - $R1 \bowtie (R2 \bowtie R3) = (R1 \bowtie R2) \bowtie R3$
  - Idempotence:
    - $\prod_{L2} (\prod_{L1}(R)) = \prod_{L2} (R)$
      - Only if $L2 \subseteq L1$
    - $\sigma_{C2}(\sigma_{C1}(R)) = \sigma_{C1 \wedge C2}(R)$

Summary

- The relational model has rigorously defined query languages that are simple and powerful
- Relational algebra is operational; useful as internal representation for query evaluation plans
- Several ways of expressing a given query; a query optimizer should choose the most efficient version.
- *Relational Completeness* of a query language: A query language (e.g., SQL) can express every query that is expressible in relational algebra