

Basic Probability supplementary material

Uniform Distribution

- A box of 5 red and 5 blue balls
 - Uniform distribution in color because all colors have the same percentage of balls.
- A box of 2 red, 2 blue, 2 green, 2 yellow, 2 white balls
 - Uniform distribution in color because all colors have the same percentage of balls.
- A box of 2 red, 2 blue, 2 green, 4 white balls
 - Not uniform distribution in color because all colors do not have the same percentage of balls.
- A box with unknown number of balls in 4 colors, where each color forms 25% of the total number of balls.
 - Uniform distribution in color because all colors have the same percentage of balls.
- A box of 2 red, 2 blue, 2 green, 4 white balls. All red, blue, and one green ball weigh 50g. The remaining balls all weigh 100g.
 - Not uniform distribution in color because all colors do not have the same percentage of balls.
 - Uniform distribution in weight, because both weights (50g and 100g) have same percentage of balls.

Independent distributions

- Example 1: A box contains 50 pairs of boots, 20 of them red, 30 of them black.
 - Color of the boot and the leg (left or right) are distributions independent of each other. This is because if you take all the red color boots, half of them (50%) will be left and other half will be right. The same applies for the black color boots.
 - Similarly, if you pick all the left leg boots, 20 of them (40%) will be red and 30 of them (60%) will be black. Similarly, if you pick the right leg boots, 20 of them will be red and 30 of them will be black.
 - Since in both the above scenarios, the proportions/percentages remain the same we can say that the color and leg distributions are independent of each other.

Dependent distributions

- Example 2: A box contains 50 pairs of boots, 20 of them red, 30 of them black. But 10 of the red pairs are left for both the boots (bad shoe maker).
 - Now if you take only the red colored shoes, there are 30 left boots (75%) and 10 right boots (25%). But if you take the black colored boots, there are 30 left boots (50%) and 30 right boots (50%). Clearly the percentages do not align. Hence in this case color and leg distributions are not independent of each other.

Working with independent distributions

- In Example 1, what is the percentage of boots that are both red and left ?
 - color and right/left distribution are independent in Ex. 1
 - Thus, we can calculate by multiplying the percentage of red boots and the percentage of left boots:
 - We know that half of all the boots are left(0.5), we also know that $20/(20+30) = 0.4$ fraction of boots are red.
 - Thus: $0.5 \times 0.4 = 0.2$ or 20% of the boots will be both red and left.
 - Not convinced ?
 - Number of red boot pairs = 20.
 - 20 left, 20 right boots.
 - Number of black boot pairs = 30.
 - 30 left, 30 right boots
 - So what is the percentage of left red boots ?
 - = number of left red boots / total number of boots.
 - = $20 / (20 + 20 + 30 + 30) = 0.20$, i.e 20 %

Working with dependent distributions

- In Example 2, what is the percentage of boots that are both red and left ?
 - The number of left boots is dependent on the color of the boots, thus not an independent distribution.
 - We know that number of red boot pairs is 20, which is 0.4 fraction of the total number of boots [$20 / (20 + 30)$]. We are also told that 10 of the red pairs are left on both boots due to manufacturing defect. This means we have 30 left boots and 10 right boots for red, i.e. a fraction of $30/(30+10) = 0.75$ (75%) of red boots are left. Therefore the total percentage of red left boots in this case will be $0.4 \times 0.75 = 0.3$ or 30%
 - Not convinced ?
 - Number of red boot pairs = 20
 - 30 left, 10 right
 - Number of black boot pairs = 30
 - 30 left, 30 right.
 - Percentage of left red boots ? Is same equation as before
 - = number of left red boots / total number of boots.
 - $30 / (30 + 10 + 30 + 30) = 0.30$ or 30 %
 - Notice how in this example we ignored the overall distribution of left boots but worked with the information of what percentage of red boots are left ? That is because this percentage was dependent on what color the boot is.