

Indexing

Cost Model for Execution

❑ How should we estimate the costs for executing a statement?

- ★ Number of I/Os

- ★ CPU Execution Cost

- ★ Network Cost in distributed system (ignore for now)

★ Assumption in this course

- ★ I/O cost >>> CPU cost

- ★ Real systems also consider CPU

❑ Simplifications

- ★ only consider disk reads (ignore writes -- assume read-only workload)

- ★ only consider number of I/Os and not the individual time for each read (ignores page pre-fetch)

- ★ Average-case analysis; based on several simplistic assumptions.

- ☞ Good enough to show the overall trends!

Typical Operations

- Scan over all records
 - `SELECT * FROM Students`
- Point Query
 - `SELECT * FROM Students WHERE sid = 100`
- Equality Query
 - `SELECT * FROM Students WHERE starty = 2015`
- Range Search
 - `SELECT * FROM Students`
`WHERE starty > 2012 and starty <= 2014`


Typical Operations

- Insert
 - `INSERT INTO Students VALUES (23, 'Bertino', 2016, ...)`
- Delete
 - `DELETE FROM Students WHERE sid = 100`
 - `DELETE FROM Students WHERE endyear < 1950`
- Update
 - Delete+insert

Indexes

❑ Even a sorted file only supports queries on sorted attributes.

❑ Solution: Build an index for any attribute (collection of attributes) that is frequently used in queries

☆ **Additional information / extra data structure** that helps finding specific tuples faster 

☆ We call the collection of attributes over which the index is built the **search key attributes** for the index.

☆ Any subset of the attributes of a relation can be the search key for an index on the relation.

☆ Search key is not the same as primary key / key candidate

Creating an index in DB2

□ Simple

★ CREATE INDEX ind1 ON Students (sid) ;
★ DROP INDEX ind1 ;

search key
attribute

helpful for
equality or
range queries
on sid

not free - extra data,
takes up space,
makes updates/
inserts/deletes
more expensive

B+ Tree: The Most Widely Used Index

❑ Each node/leaf represents one page

☆ Since the page is the transfer unit to disk

❑ Leafs contain **data entries** (denoted as k^*)

☆ For now, assume each data entry represents one tuple. The data entry consists of two parts

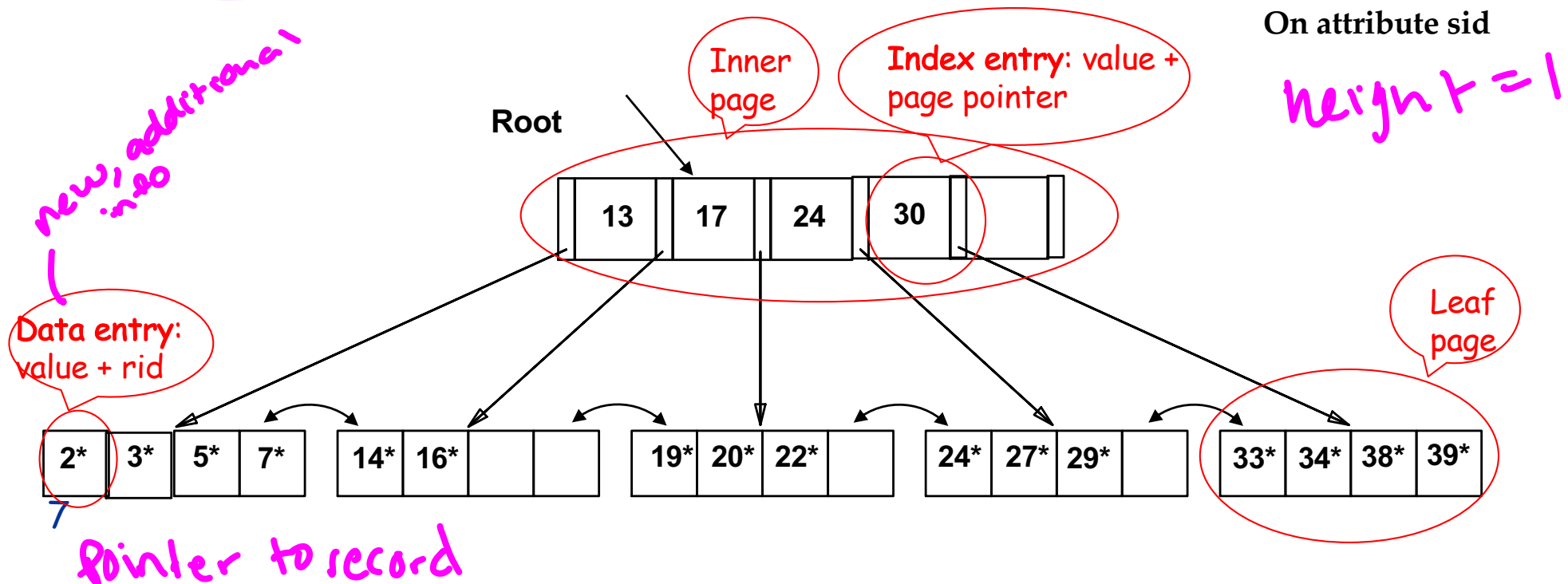
● Value of the search key (k) ←

● Record identifier ($rid = (page-id, slot)$) ←

☆ That is: data entry is NOT a tuple but a pointer to a tuple

❑ Root and inner nodes have auxiliary **index entries**

Index for skaters
On attribute sid



Disclaimer

- Size:
 - Example relation has only 16 tuples
 - You don't build an index for such a relation
 - Think of thousands, and million and more tuples
- Better Example:
 - McGill's student relations
 - Sid = student id = 15-digit number
- Index pages:
 - Inner pages contain hundreds of index entries
 - Leave pages contain hundreds of data entries

B+ Tree (contd.)

- ❑ *height-balanced.*

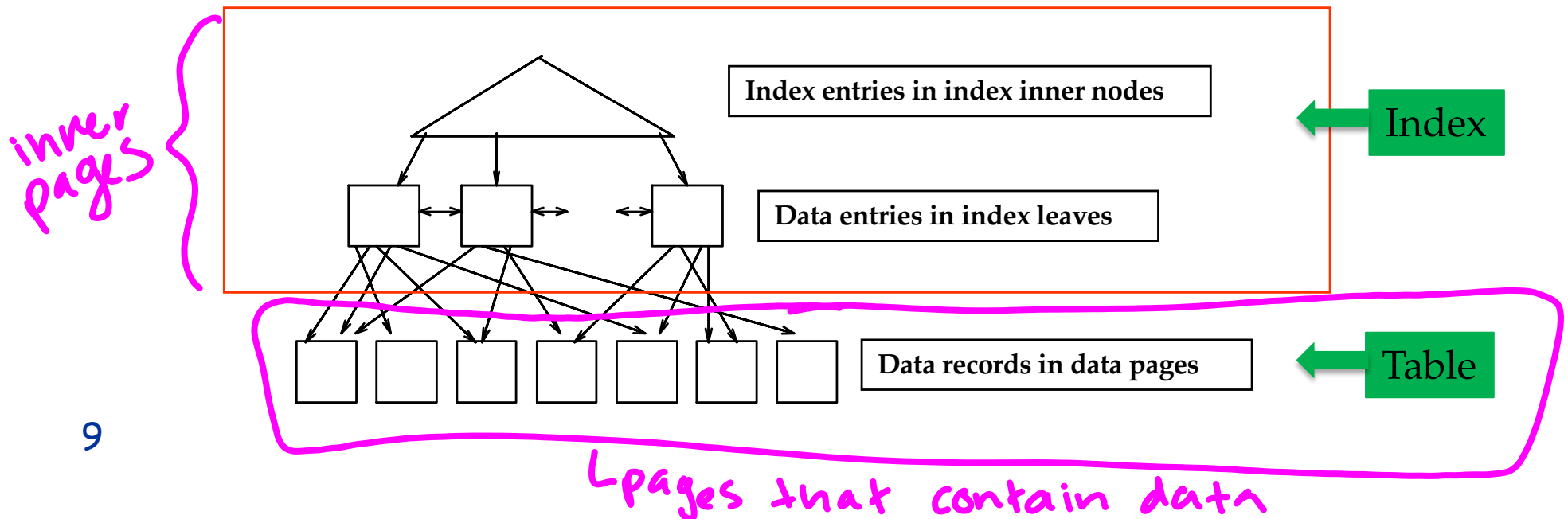
- ☆ Each path from root to tree has the same height

- ❑ F = fanout = number of children for each node (~ number of index entries stored in node)

- ❑ N = # leaf pages

- ❑ Insert/delete at $\log_F N$ cost;

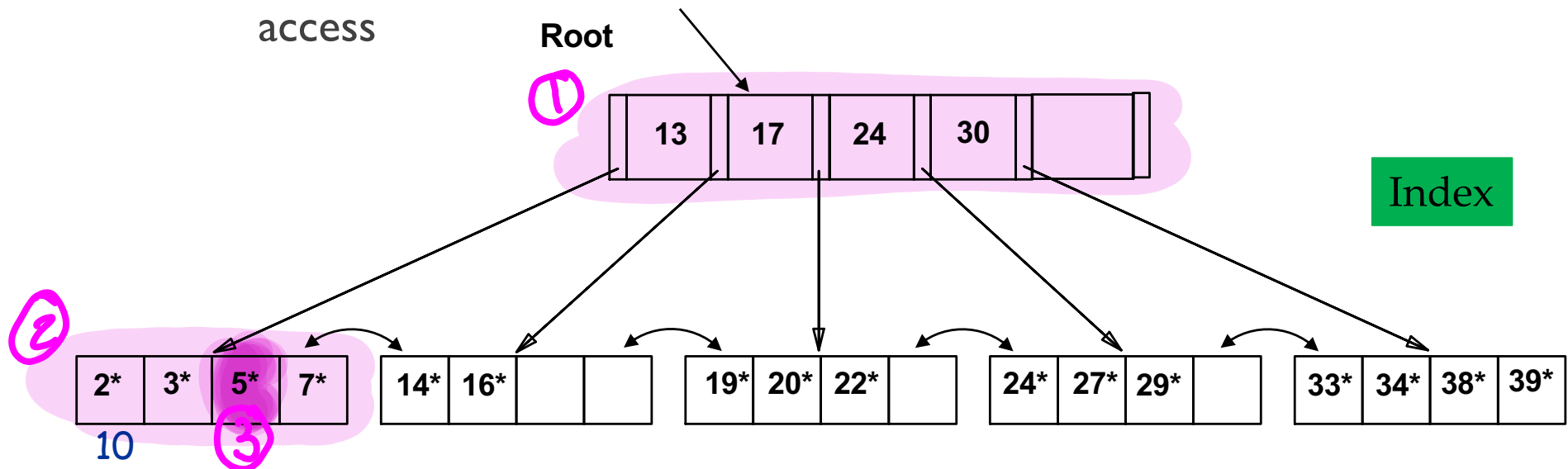
- ❑ Minimum 50% occupancy (except for root).



Example B+ Tree

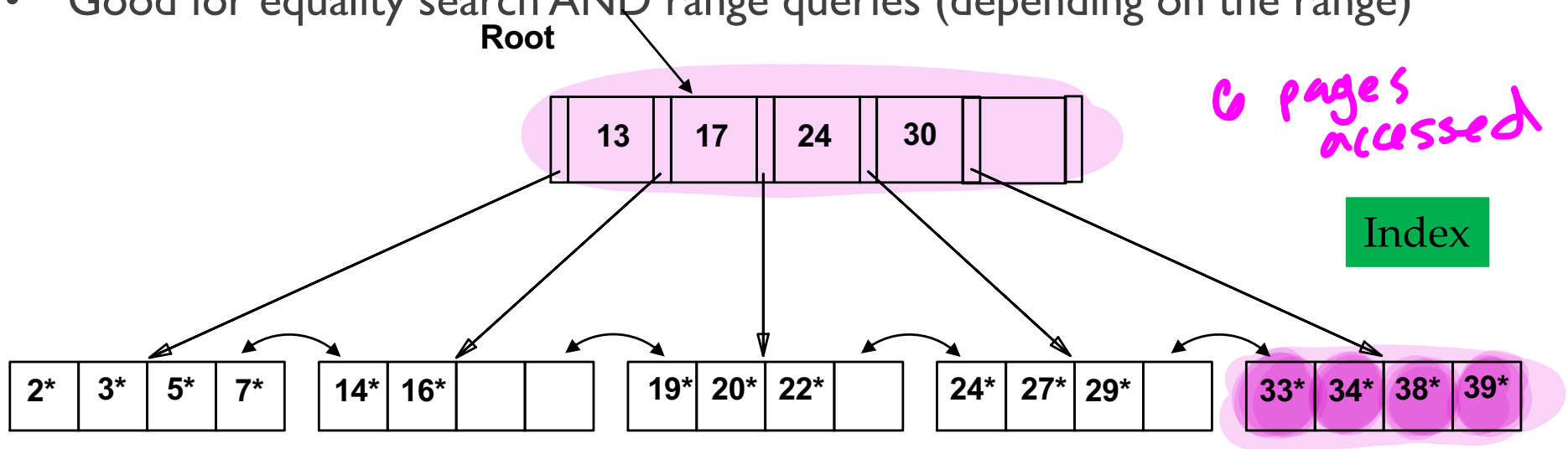
- Example tree has height of 1
- “**Select * from Skaters where sid = 5**”
 - Search begins at root, and key comparisons direct it to a leaf
 - Number of pages accessed:
 - three: root, leaf, data page with the corresponding record
 - Number of I/O: *likely 2*
 - depends of how much of tree is already in the buffer in main memory
 - rough assumption: root and intermediate nodes are always in main memory; index leaves and data pages not in main memory upon first access

then data page w/ record



Example B+ Tree

- “Select * from Skaters where sid = 5”
 - I/O costs:
 - one for leaf page with data entry, one for data page with data record
- “Select * from Skaters where sid >= 33”
 - I/O costs: *S* range query
 - one for leaf page
 - four for data pages with records
- Good for equality search AND range queries (depending on the range)

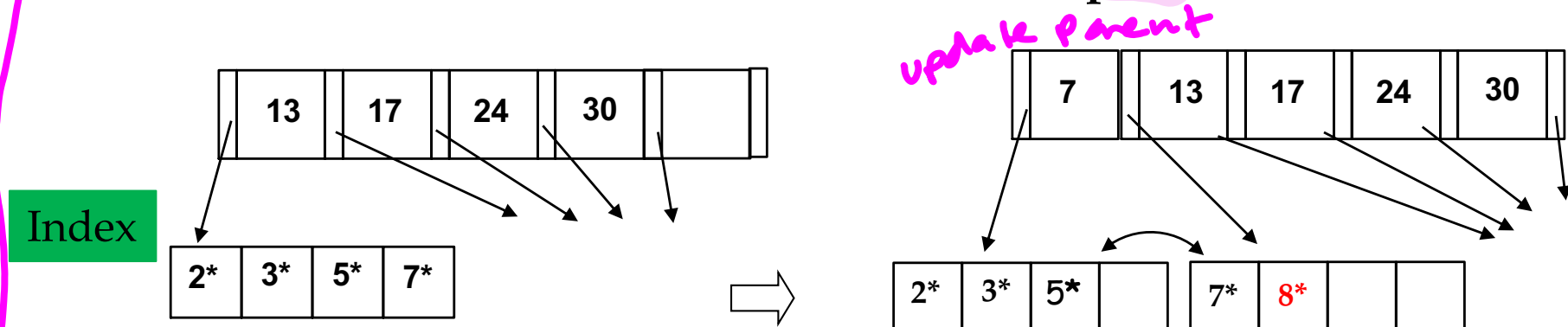


Inserting a Data Entry

- ① • Find correct leaf L .
- ② • Put data entry into L .
 - 2.1 – If L has enough space, *done!*
 - 2.2 – Else, must split L (into L and a new node $L2$)
 - Redistribute entries evenly, copy up middle key.
 - Insert index entry pointing to $L2$ into parent of L .
- This can happen recursively
 - To split index node, redistribute entries evenly, but push up middle key. (Contrast with leaf splits.)
- Splits “grow” tree; root split increases height.
 - Tree growth: gets wider or one level taller at top.

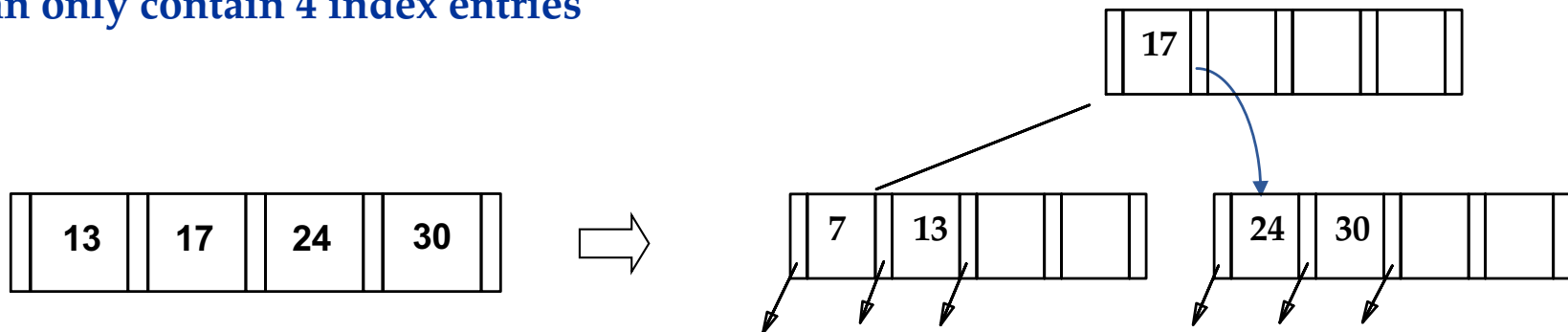
Inserting 8* into Example B+ Tree

Insert into Leaf with leaf split

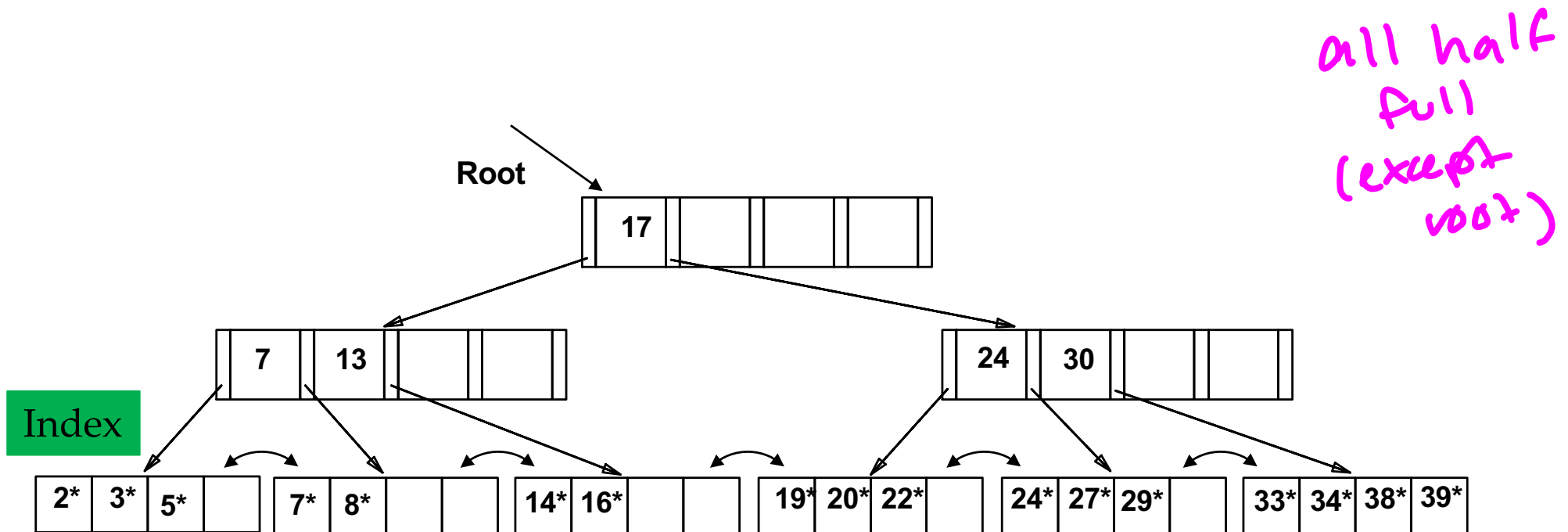


Insert into internal node with node split

Assume that inner pages
can only contain 4 index entries



Example: After Inserting 8*



- ❖ Notice that root was split, leading to increase in height.
- ❖ In this example, we can avoid split by redistributing entries; however, this is usually not done in practice.

height only increases when root splits

Data Entry Alternatives: indirect Indexing

pointer to record

- Indirect Indexing I

- so far: $k^* = \langle k, \text{rid of data record} \rangle$ with search key value k (indirect indexing)
- on non-primary key search key: (2015, rid1), (2015, rid2), (2015, rid3), ...
 - several entries with the same search key side by side

- Indirect indexing II *several tuples with the value*

- $\langle k, \text{list of rids of data records} \rangle$ with search key k (indirect indexing)
- on non-primary key search key: (2015, (rid1, rid2, rid3, ...)), (2016, (rid...

- Comparison:

don't repeat values

- first requires more space (search key repeated)
- second has variable length data entries
- second can have large data entries that span a page

list vs individual

Direct Indexing

❑ Instead of data-entries in index leaves containing rids, they could contain the entire tuple

★ data-entry = tuple

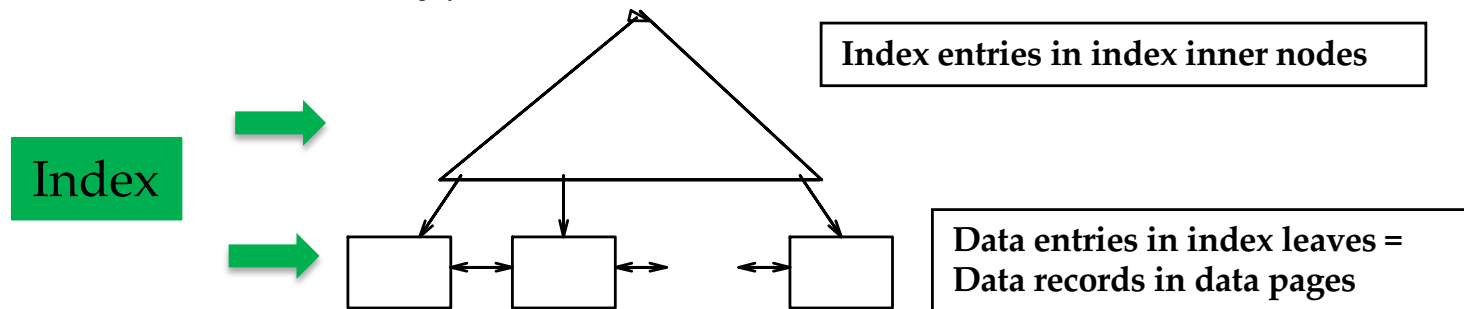
★ no extra data pages ←

❑ This is kind of a sorted file with an index on top

leaf pages = data pages

Data Entry Alternatives: Direct Index

- Structure
 - $\langle k, \text{full record} \rangle$
 - Leaf pages contain entire records
 - Data entries = records (not only pointers to them)
 - e.g., index on sid of Skaters
 - (1,lilly,10,16), (2,debby,8,10)...
 - Leaf pages represent sorted file for data records.
 - Inner nodes above leafs provide faster search
 - Typically at most one direct index per relation
 - (Otherwise, data records duplicated, leading to redundant storage and potential inconsistency.)



Index Classification

secondary - search key is not unique

- ❑ Primary vs. secondary: If search key contains primary key, then called primary index.

- ☆ Unique index: Search key is primary key or unique attribute.

- ❑ Clustered vs. unclustered:

- ☆ clustered:

- Relation in file sorted by the search key attributes of the index

- ☆ unclustered: (not sorted or sorted by a diff. attribute)

- Relation in heap file or sorted by an attribute different to the search key attribute of the index.

- ☆ A file can be clustered on at most one search key.

- ☆ Cost of retrieving data records through index varies greatly based on whether index is clustered or not!

direct index is a clustered index
indirect index is either clustered or non-clustered index

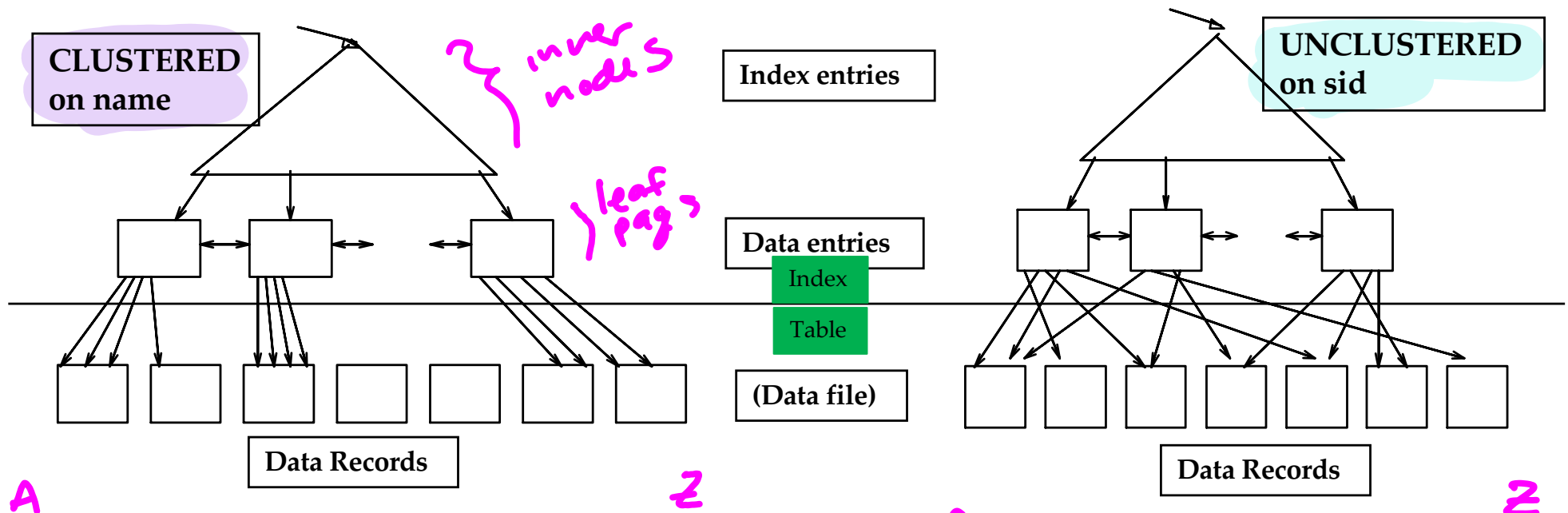
Clustered vs. Unclustered Index

Example for Students:

★ Indirect clustered index on name

★ Indirect unclustered on sid

leaf pages
contain rid
which points
to data page



19
data sorted by name

data sorted by name

① how big is my index, how many data pages

B+-tree cost example: **Given**

★ About the relation

- ★ Relation R(A,B,C,D,E,F)
- ★ A and B are int (each 4 Bytes), C-F is char[40] (160 Bytes)
- ★ Values of B are within [0;19999] uniform distribution
- ★ 200,000 tuples

★ About the heap file (unsorted)

- ★ Each data page has 4 K and is around 80% full (assume)
 - ★ I know, I know: with fixed sized records we don't need to leave space, but in real life records don't have fixed length and we should leave space....
- ★ The size of an rid = 10 Bytes

★ About the index to be built (on B) indirect index alternative !! - one data entry per different value of the search key

- ★ An index page has 4K
- ★ Index pages are filled between 50% - 100% (this always holds!!) - assume they are on average 75% full
- ★ The size of a pointer in intermediate pages: 8 Bytes

Search key is a value of B

Size of B+tree: To calculate

★ Number of data pages in the heap file

- ★ size of the tuple = 168 *fill 90%*
 - ★ $4000 * 0.15 / 168 \approx 18$ *tuples in a page on an average.*
 - ★ $200000 / 18 \approx 11111$ *pages in total.*
- index page size*
tuples
- or $200000 + 168 / 3000 = 11111$
75% of 4k = 3000*

★ The number of leaf pages

- ★ distinct values of B = 20000
- ★ $200000 / 20000 = 10$ records per val of B *total # tuples / # values of B*
- ★ $\text{sizeof}(B) + 10 \text{ rids} * \text{sizeof}(\text{rids})$
- ★ $4 + 10 * 10 = 104$ bytes/data entry *size of data entry*
- ★ $4000 * 0.75 / 104 \approx 28.846$ *data entries/page on an average.*
- ★ $20000 / 28.846 \approx 694$ *leaf pages.*

$$L_{\min}: 4000 * 0.30 / 104$$

$$L_{\max}: 4000 * 1.0 / 104$$

★ The height of the tree

- ★ $4 + 8 = 12$ *bytes intermediate node entry* *size of key + pointer size*
- ★ Can fit $4000 * 0.75 / 12 = 250$ *entries per intermediate node*
- ★ This requires 3 intermediate nodes right above the leaf pages (to manage 694 leaves).
- ★ 1 node above then to hold pointers to the 3 intermediate pages - root.
- ★ height = 2 (the number of edges from root to a leaf node)

B+ Trees in Practice

- ❑ Typical order d of inner nodes: 200 (I.e., an inner node has between 100 and 200 index entries)
 - ☆ Typical fill-factor: 67%.
 - ☆ average fanout = 133
- ❑ Leaf nodes have often less entries since data entries larger (rids)
- ❑ Typical capacities (roughly)
 - ☆ Height 3: $133^4 = 312,900,721$ records $133^3 \cdot 100 = 2,552,637,000$
 - ☆ Height 2: $133^3 = 2,352,637$ records $133^2 \cdot 100 = 176,890,000$
 - ☆ Height 1: $133^2 = 17,689$ records $133 \cdot 100 = 13,300$
- ❑ Can often hold top levels in buffer pool:
 - ☆ Level 1 (root) = 1 page = 4 Kbytes
 - ☆ Level 2 = 133 pages = 0.5 Mbyte
 - ☆ Level 3 = 17,689 pages = 70 MBytes

Multi-attribute index

- Index on Skaters (age, rating) ;
- Order is important:
 - Here data entries are first ordered by age
 - Skaters with the same age are then ordered by rating *within groups*
 - assume youngest skater is 3, oldest is 25 (list of rids: *):
 - the leaf pages then would look like:

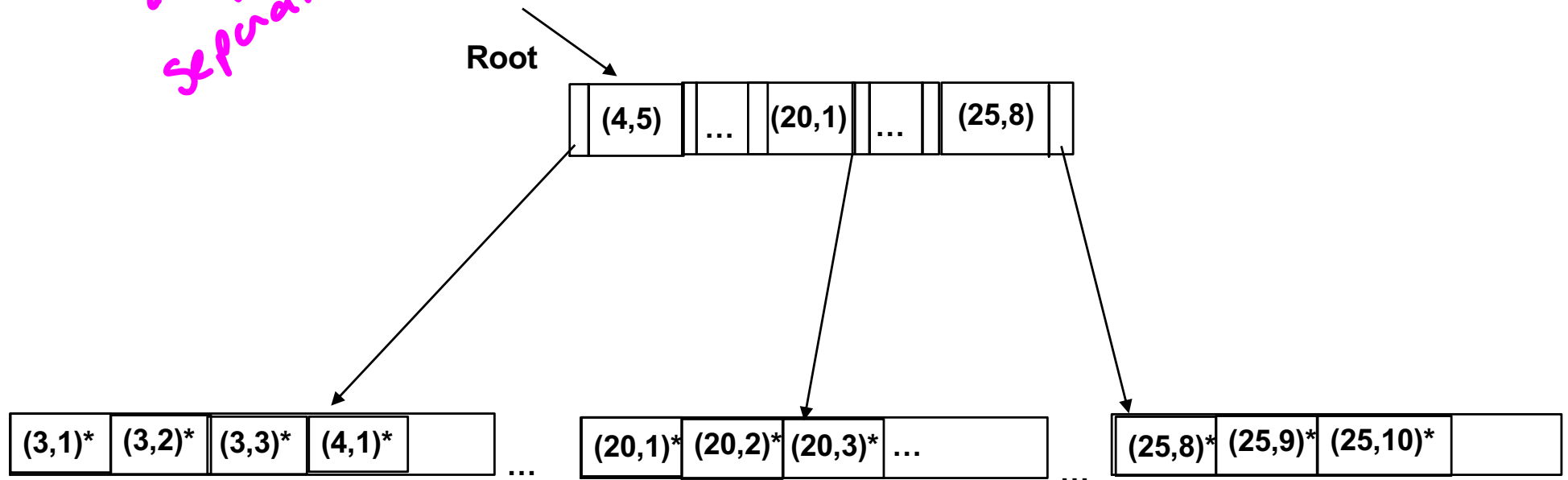
| | | | | | | | | | | | | | | |
|--------|--------|--------|--------|--|-----|---------|---------|---------|-----|-----|---------|---------|----------|--|
| (3,1)* | (3,2)* | (3,3)* | (4,1)* | | ... | (20,1)* | (20,2)* | (20,3)* | ... | ... | (25,8)* | (25,9)* | (25,10)* | |
|--------|--------|--------|--------|--|-----|---------|---------|---------|-----|-----|---------|---------|----------|--|

↑
youngest skater
w/ lowest
rating

↑
oldest skater
w/ highest
rating

why?
↳ not many
ages
or ratings
separately

What does it support



- What does it support?

- SELECT * FROM Skaters WHERE age = 20;

- yes

- SELECT * FROM Skaters WHERE age = 20 AND rating < 5;

- Yes

- SELECT * FROM Skaters WHERE rating < 5;

- no - would have to go through all entries

↳ index is not useful

– query only
on second
attribute

Index in DB2

❑ Simple

☆ CREATE INDEX ind1 ON Students(startyear);

☆ DROP INDEX ind1;

❑ Index also good for referential integrity (uniqueness)

☆ CREATE UNIQUE INDEX indemail ON Students(email)

❑ Additional attributes

☆ CREATE UNIQUE INDEX ind1 ON Students(sid)
INCLUDE (name)

☆ Index only on sid

☆ Data entry contains key value (sid) + name + rid

*include some of the
tuple in the page*

☆ SELECT name FROM Students WHERE sid = 100 ←

● Can be answered without accessing the real data pages of Students relation!

❑ Clustered index

☆ CREATE INDEX ind1 on Students(name) CLUSTER

25 *multi-attribute: create index ind2 on students(email, sid)*

Summary for B+-trees

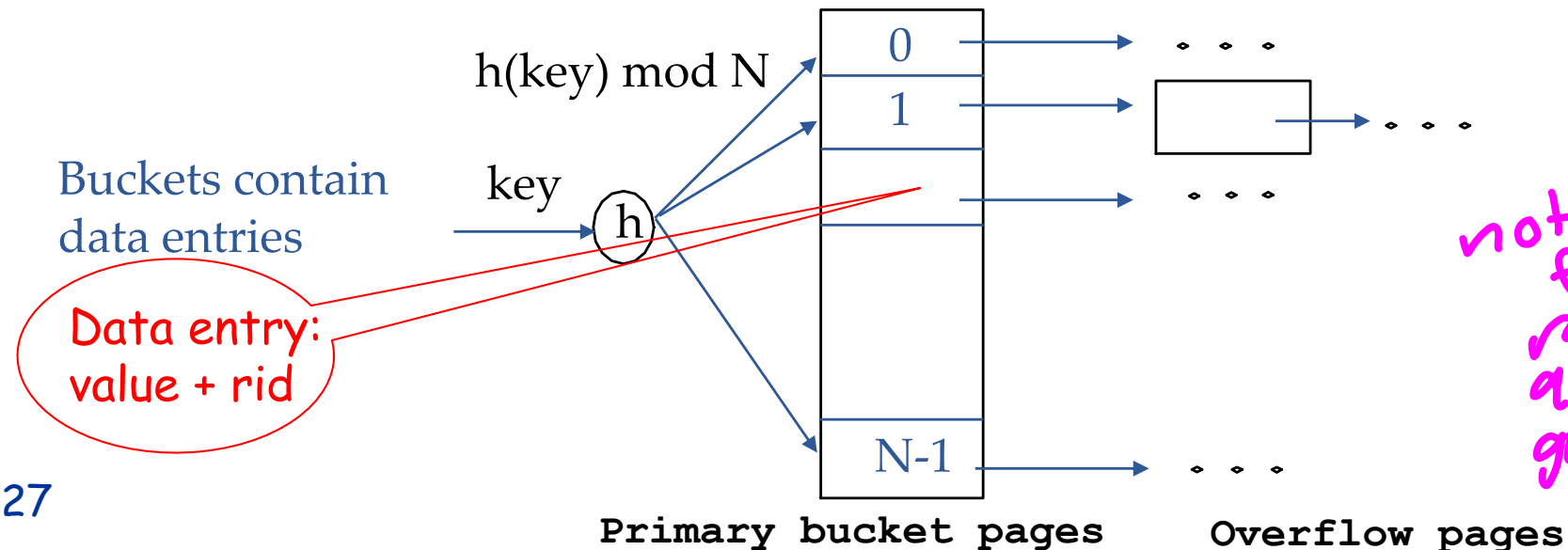
- ❑ Tree-structured indexes are ideal for range-searches, also good for equality searches.
 - ☆ High fanout (F) means depth rarely more than 3 or 4.
 - ☆ Almost always better than maintaining a sorted file.
- ❑ Can have several indices on same tables (over different attributes)
- ❑ Most widely used index in database management systems because. One of the most optimized components of a DBMS.

Other index structures besides B tree

Static Hashing

- ❖ Similar to standard hashing but with pages as the unit of storage (compare with array-based main memory implementation)
- ❖ Decide on a number N of buckets at index creation
- ❖ Allocate one primary page per bucket
- ❖ Overflow pages for individual buckets are created later as needed
- ❖ Buckets contain data entries (same as leaf pages in tree)
- ❖ let k be the search key of the index, h a hash function
 - ❖ $h(k) \bmod N =$ bucket to which data entry with key k belongs.

good for
point
queries



not good
for
range
queries
good for
equality
Search

contd

- ❖ **Buckets** contain *data entries*.
- ❖ **Hash fn** works on *search key* field of record *r*.
Must distribute values over range $0 \dots M-1$.
 - $h(\text{key}) = (a * \text{key} + b)$ usually works well.
 - *a* and *b* are constants; lots known about how to tune **h**.
- ❖ **Long overflow chains** can develop and degrade performance.
 - ❖ Several optimizations developed such to handle scale dynamically (e.g., extensible and linear hashing)

File Organizations

□ Hashed Files:

- ☆ File is a collection of buckets. Bucket = *primary* page plus zero or more *overflow* pages.
- ☆ *Hashing function h* : $h(r)$ = bucket in which record r belongs. h looks at only some of the fields of r , called the *search fields*.
- ☆ Best for equality search (only one page access and maybe access to overflow page)
- ☆ No advantage for range queries
- ☆ Fast insert
- ☆ Cost on delete depends on cost for WHERE clause

File Organization

assume each relation is a file:

❑ Heap files:

☆ Linked, unordered list of all pages of the file

☆ How does it do?

- scan retrieving all records (SELECT *)?
 - ▲ ok, you have to retrieve all pages anyways
- Point query on unique attributes
 - ▲ not great: have to read on avg. half the pages to return one record
- range search or equality search on non-primary key
 - ▲ not great: have to read all pages to return subset of records.
- insert
 - ▲ yes: Cost for insert low (insert anywhere)
- delete/update
 - ▲ same as for equality/range search -- depends on WHERE clause

File Organizations II

❏ Sorted Files:

- ☆ Records are ordered according to one or more attributes of the relation
- ☆ Is it good for:
 - scan retrieving all records (SELECT *)?
 - ▲ yes, you have to retrieve all pages anyways
 - equality search on sort attribute
 - ▲ good: find first qualifying page with binary search in $\log_2(\text{number-of-pages})$
 - range search on sort attribute
 - ▲ good: find first qualifying page with binary search in $\log_2(\text{number-of-pages})$; adjacent pages might have additional matching records
 - insert
 - ▲ not good: have to find proper page; overflow possible
 - delete/update
 - ▲ finding tuple same as equality/range search depending on WHERE clause
 - ▲ update itself might lead to restructuring of pages
- Sorted output: (ORDER BY)
 - ▲ good if on sorted attribute