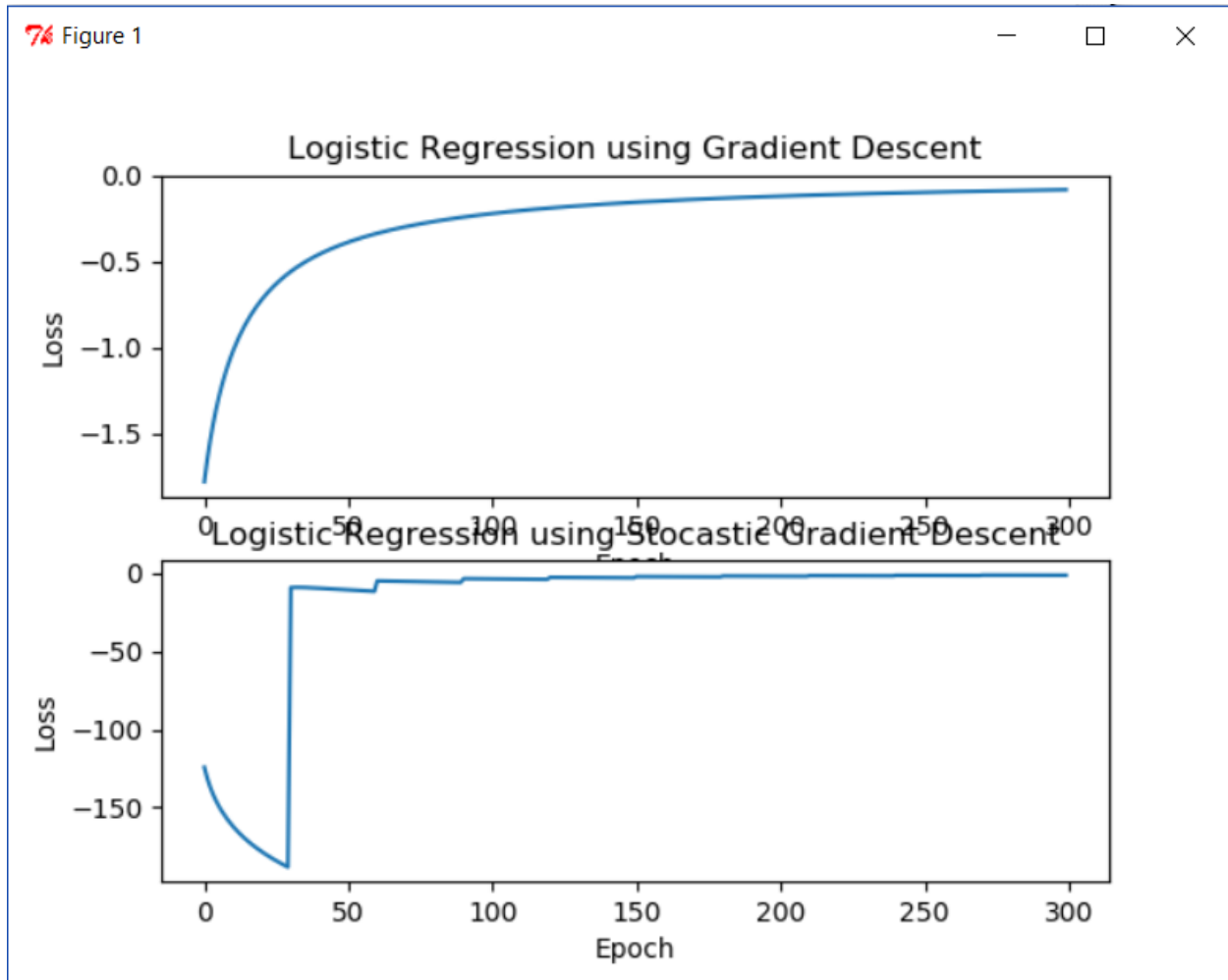


Q1.



Code:

```
# -*- coding: utf-8 -*-  
"""
```

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```
@author: tgore03  
"""
```

```
import numpy as np  
import matplotlib.pyplot as plt  
from sklearn.linear_model import SGDClassifier  
from sklearn.linear_model import LogisticRegression  
import random  
from sklearn.metrics import log_loss
```

```
#Generate Data  
mu, sigma = 0.5, 0.3
```

```
s1 = (np.random.randn(100, 2) / 10) + 0.5
```

```
s2 = (np.random.randn(100, 2) / 10) - 0.5
```

```
y1 = np.ones(100)
```

```
y2 = np.zeros(100)
```

```
x = np.vstack((s1, s2))
```

```
y = np.hstack((y1, y2))
```

```
#Train Logistic Regression Model
```

```
def sigmoid(scores):
```

```
    return 1 / (1 + np.exp(-scores))
```

```
def log_likelihood(features, target, weights):
```

```
    z = np.dot(features, weights)
```

```
    ll = np.sum( target*z - np.log(1 + np.exp(z)) )
```

```
    return ll
```

```
def logistic_regression(features, target, num_steps=30000, learning_rate=0.001, add_intercept = False):
```

```
    #Preprocess data
```

```
    if add_intercept:
```

```
        intercept = np.ones((features.shape[0], 1))
```

```
        features = np.hstack((intercept, features))
```

```
    weights = np.zeros(features.shape[1])
```

```
    #Initilize variables
```

```
    i=0
```

```
    loss = [None]*(num_steps)
```

```
    epoch = [None]*(num_steps)
```

```
#Train Model
```

```
for step in xrange(num_steps):
```

```
    #Predict based on current weights
```

```
    z = np.dot(features, weights)
```

```
    predictions = sigmoid(z)
```

```
    # Update weights with gradient
```

```
    output_error_signal = target - predictions
```

```
    gradient = np.dot(features.T, output_error_signal)
```

```
    weights += learning_rate * gradient
```

```
    # Print log-likelihood every so often
```

```
    loss[i] = log_likelihood(features, target, weights)
```

```
    epoch[i] = i;
```

```
    i+=1
```

```
#Plot Loss w.r.t. iteration
```

```
global plt
```

```
plt.subplot(211)
```

```
plt.plot(epoch, loss)
plt.xlabel('Epoch')
plt.ylabel('Loss')
plt.title('Logistic Regression using Gradient Descent')
return weights
```

```
def stochastic_logistic_regression(features, target, num_steps=30000, batch_size=10, learning_rate=0.1,
add_intercept=False):
```

```
    #Preprocess data
```

```
    if add_intercept:
```

```
        intercept = np.ones((features.shape[0], 1))
```

```
        features = np.hstack((intercept, features))
```

```
    weights = np.zeros(features.shape[1])
```

```
    #Initilize variables
```

```
    i=0
```

```
    data_size=len(features[:,0])
```

```
    steps_per_epoch = data_size/batch_size
```

```
    no_of_epoch = num_steps/steps_per_epoch
```

```
    print no_of_epoch
```

```
    loss = [None]*(no_of_epoch)
```

```
    epoch = [None]*(no_of_epoch)
```

```
    #Train Model
```

```
    index=0;
```

```
    for step in xrange(num_steps):
```

```
        x = features[index:index+batch_size]
```

```
        y = target[index:index+batch_size]
```

```
        index = index+batch_size
```

```
    #Predict based on current weights
```

```
    z = np.dot(x, weights)
```

```
    predictions = sigmoid(z)
```

```
    # Update weights with gradient
```

```
    output_error_signal = y - predictions
```

```
    gradient = np.dot(x.T, output_error_signal)
```

```
    weights += learning_rate * gradient
```

```
    # Print log-likelihood after each epoch (Obtained by len(features)/batch_size)
```

```
    if step % data_size/batch_size == 0:
```

```
        loss[i] = log_likelihood(features, target, weights)
```

```
        epoch[i] = i;
```

```
        i+=1
```

```
        index=0
```

```
    #Plot Loss w.r.t. iteration
```

```

global plt
plt.subplot(212)
plt.plot(epoch, loss)
plt.xlabel('Epoch')
plt.ylabel('Loss')
plt.title('Logistic Regression using Stochastic Gradient Descent')
plt.show()
return weights

```

```
#Define figure for plot
```

```
global plt
plt.figure(1)
```

```
#Train using Gradient Descent
```

```
print "Training using Gradient Descent"
```

```
weights = logistic_regression(x, y, num_steps = 300, learning_rate = 0.1, add_intercept=True)
```

```
#Train using Stochastic Gradient Descent
```

```
print "Training using Stochastic Gradient Descent"
```

```
weights = stochastic_logistic_regression(x, y, num_steps = 2000, batch_size=30, learning_rate = 0.1,
add_intercept=True)
```

Q2)

Q2)

a) For linear models

$$L(w) = \frac{1}{2} (y - \langle w, x \rangle)^2$$

Given the higher dimensional mapping  
 $x \rightarrow \phi(x)$

$$L(w) = \frac{1}{2} (y - \langle w, \phi(x) \rangle)^2$$

b)  $\nabla L(w) = 0$

The optimal closed form expression for  
 optimal linear predictor  $w$  is.

$$w = (X^T X)^{-1} X^T y$$

Given  $\phi$  where  $\phi(i) = \phi(z_i)$

The closed form expression becomes

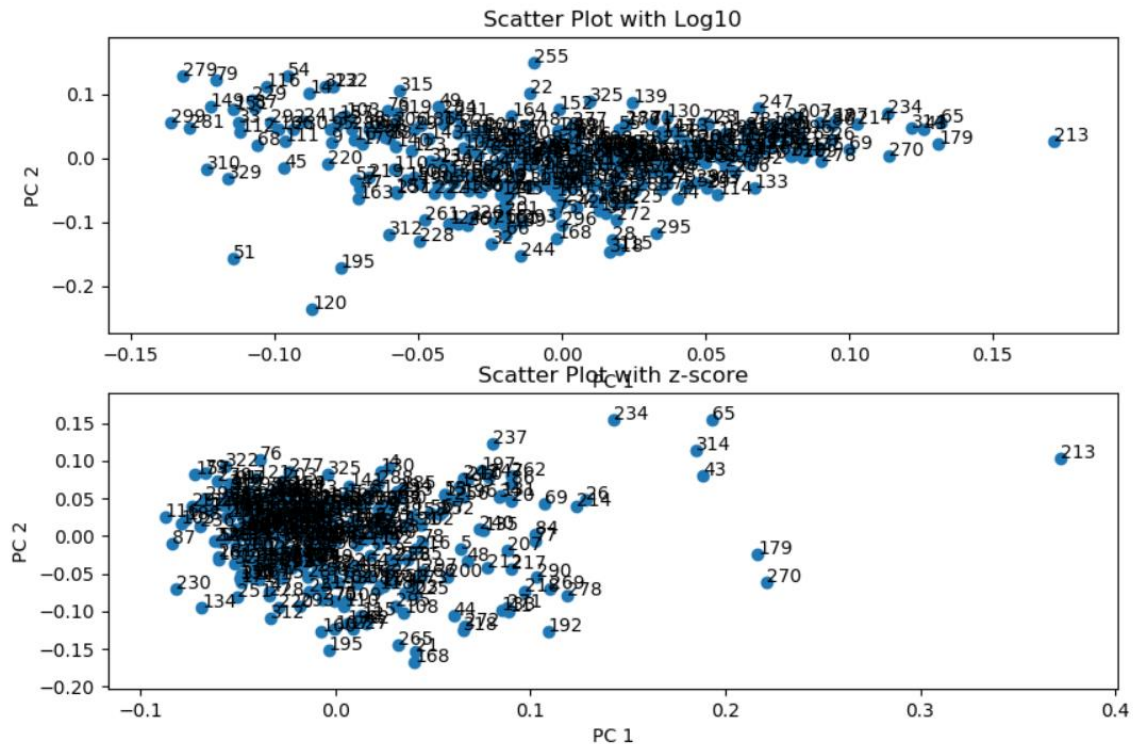
$$w = (\Phi^T \Phi)^{-1} \Phi^T y$$

c) Given  $f(z) = \langle w, \phi(z) \rangle$

From b, we get

$$f(z) = \langle (\Phi^T \Phi)^{-1} \Phi^T y, \phi(z) \rangle$$

Q3)



**With Log10 Normalization:**

Principle Directions:

1. [ 0.03507288 0.09335159 0.40776448 0.10044536 0.15009714 0.03215319 0.87434057 0.15899622 0.01949418]
2. [ 0.0088782 0.00923057 -0.85853187 0.22042372 0.05920111 -0.06058858 0.30380632 0.33399255 0.0561011 ]

The two components appear to correlate most with HealthCare and Arts

Variance of each features is :

[ 8.40161907 1.85948255 0.68742394 0.83689849 0.61121211 0.40852812  
0.26982829 0.13831459 0.06133763]

Outlier Cities:

1. 213 - New-Orleans,LA
2. 120 – Gary-Hammond,IN
3. 51 – Brockton,MA
4. 195 – Middletown,CT

**With Z-score Normalization:**

Principle Directions:

1. [ 0.20641395 0.35652161 0.46021465 0.28129838 0.35115078 0.27529264 0.46305449 0.32788791 0.13541225]
2. [ 0.21783531 0.250624 -0.29946528 0.35534227 -0.17960448 -0.48338209 -0.19478992 0.38447464 0.47128328]

Variance of each feature is:

```
[ 11.57517228  43.0323617  41.5675071  33.35398373  27.25678957  22.21711637
 18.02308595  11.5894271   4.40125211]
```

Since the variance of each feature does not vary much projection on 2d causes lot of data to be lost.  
Hence the 2d plot cannot be trusted to accurately represent the data.

Outlier Cities:

1. 213 – New-Orleans,LA
2. 270 – San-Diego,CA
3. 179 – Lorain-Elyria,OH
4. 43 – Boise-City,ID
5. 314 – Waco,TX
6. 65 – Chattanooga,TN-GA
7. 234 – Peoria,IL

**Code:**

```
import numpy as np
import math
import matplotlib.pyplot as plt
from sklearn.decomposition import PCA
from sklearn.decomposition import TruncatedSVD
```

```
f = open("places.txt", "r")
no_features = 9
no_records = 329
features = np.empty([no_records, no_features])
target = ["" for x in range(no_records)]
```

```
#Read the labels
f.readline()
```

```
#Read the file
lineno=-1
colno=-1
i=0
for line in f:
    lineno+=1
    for word in line.split():
        #Store label column
        if colno == -1:
            colno+=1
            target[lineno]=str(word)
            continue
        colno+=1
    #Skip last 5 columns
    if colno > no_features:
        break;
```

```
#Store word in matrix
features[[lineno,colno-1] = word
colno=-1
i+=1
f.close()

print "PCA using Log10"
#Taking log of matrix
x = np.log10(features)

#Taking mean of features and subtracting it from the features matrix
means = np.mean(x, axis=0)
std = np.std(x, axis=0)

for i in range(no_records):
    x[i]=(x[i] - means)

#SVD
u,s,v = np.linalg.svd(x, full_matrices=True)
d = np.diag(s[0:2])
scores = np.dot(u[:,0:2],d)
print "Principle Directions"
print v[0]
print v[1]

svd = TruncatedSVD(n_components=9)
svd.fit(x.T)
print "Variance of indivial features"
print svd.explained_variance_
print "Total Variance of Principle Components =", svd.explained_variance_ratio_.sum()

#Plot the 2 principle components
components = svd.components_.T

plt.subplot(211)
plt.scatter(components[:,0], components[:,1])
for row in range(no_records):
    plt.annotate(str(row+1), (components[row,0], components[row,1]))
plt.xlabel("PC 1")
plt.ylabel("PC 2")
plt.title("Scatter Plot with Log10")

print "\n\n PCA using z-score"
#using z-scores normalize data
means = np.mean(features, axis=0)
std = np.std(features, axis=0)
```

```
for i in range(no_records):
    for j in range(no_features):
        x[i][j]=(features[i][j] - means[j])/std[j]

u,s,v = np.linalg.svd(x, full_matrices=True)
print "Principle Directions:"
print v[0]
print v[1]

svd = TruncatedSVD(n_components=9)
svd.fit(x.T)
components = svd.components_.T
print "Variance of each features: \n",svd.explained_variance_
print "Total variance of Principle components =",svd.explained_variance_ratio_.sum()

plt.subplot(212)
plt.scatter(components[:,0], components[:,1])
for row in range(no_records):
    plt.annotate(str(row+1), (components[row,0], components[row,1]))
plt.xlabel("PC 1")
plt.ylabel("PC 2")
plt.title("Scatter Plot with z-score")
plt.show()
```

**Q4)**

I spent about 20hrs on this assignment.

**References:**

Discussed with Nitesh Gupta however I completed my assignment on my own.