

```
In[ ]:= ClearAll["Global`*"]
```

```
(*
   $\vec{v} = \{\alpha_1, \alpha_1 + \alpha_2, \alpha_1 + 2\alpha_2, \alpha_2, -\alpha_1, -(\alpha_1 + \alpha_2), -(\alpha_1 + 2\alpha_2), -\alpha_2\}$ 
*)
```

```
In[ ]:= Inverse[{{2, -2}, {-1, 2}}]
```

```
Out[ ]:=  $\left\{ \left\{ 1, 1 \right\}, \left\{ \frac{1}{2}, 1 \right\} \right\}$ 
```

```
In[ ]:= (* Derived from the FR of  $\lambda_1$  *)
```

```
H1 =
```

```
{ {1, 0, 0, 0, 0}, {0, -1, 0, 0, 0}, {0, 0, 0, 0, 0}, {0, 0, 0, 1, 0}, {0, 0, 0, 0, -1}};
```

```
H2 = { {0, 0, 0, 0, 0}, {0, 2, 0, 0, 0}, {0, 0, 0, 0, 0}, {0, 0, 0, -2, 0}, {0, 0, 0, 0, 0}};
```

```
X1 = { {0, 1, 0, 0, 0}, {0, 0, 0, 0, 0}, {0, 0, 0, 0, 0}, {0, 0, 0, 0, 1}, {0, 0, 0, 0, 0}};
```

```
X2 = { {0, 0, 0, 0, 0}, {0, 0,  $\sqrt{2}$ , 0, 0}, {0, 0, 0,  $\sqrt{2}$ , 0}, {0, 0, 0, 0, 0}, {0, 0, 0, 0, 0}};
```

```
X3 = { {0, 0,  $\sqrt{2}$ , 0, 0}, {0, 0, 0, 0, 0},
  {0, 0, 0, 0, - $\sqrt{2}$ }, {0, 0, 0, 0, 0}, {0, 0, 0, 0, 0}};
```

```
X4 = { {0, 0, 0, - $\sqrt{2}$ , 0}, {0, 0, 0, 0, - $\sqrt{2}$ }, {0, 0, 0, 0, 0},
  {0, 0, 0, 0, 0}, {0, 0, 0, 0, 0}};
```

```
Y1 = { {0, 0, 0, 0, 0}, {1, 0, 0, 0, 0}, {0, 0, 0, 0, 0}, {0, 0, 0, 0, 0}, {0, 0, 0, 1, 0}};
```

```
Y2 = { {0, 0, 0, 0, 0}, {0, 0, 0, 0, 0}, {0,  $\sqrt{2}$ , 0, 0, 0}, {0, 0,  $\sqrt{2}$ , 0, 0}, {0, 0, 0, 0, 0}};
```

```
Y3 = { {0, 0, 0, 0, 0}, {0, 0, 0, 0, 0},
  { $\sqrt{2}$ , 0, 0, 0, 0}, {0, 0, 0, 0, 0}, {0, 0, - $\sqrt{2}$ , 0, 0}};
```

```
Y4 = { {0, 0, 0, 0, 0}, {0, 0, 0, 0, 0}, {0, 0, 0, 0, 0},
  {- $\sqrt{2}$ , 0, 0, 0, 0}, {0, - $\sqrt{2}$ , 0, 0, 0}};
```

```
In[ ]:= LB[x_, y_] := x.y - y.x;
```

```
LB[X3, Y3] // MatrixForm
```

```
Out[ ]//MatrixForm=
```

```

$$\begin{pmatrix} 2 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -2 \end{pmatrix}$$

```

```
In[ ]:= H2 // MatrixForm
```

```
Out[ ]//MatrixForm=
```

```

$$\begin{pmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -2 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

```

```
In[ ]:= (H1 + 2 H2) // MatrixForm
```

```
Out[ ]//MatrixForm=
```

$$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 3 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -3 & 0 \\ 0 & 0 & 0 & 0 & -1 \end{pmatrix}$$

```
In[ ]:= M = (i h1 H1 + i h2 H2) + x1 (X1 - Y1) + i y1 (X1 + Y1) + x2 (X2 - Y2) +
            i y2 (X2 + Y2) + x3 (X3 - Y3) + i y3 (X3 + Y3) + x4 (X4 - Y4) + i y4 (X4 + Y4) ;
M // MatrixForm
```

```
Out[ ]//MatrixForm=
```

$$\begin{pmatrix} i h_1 & x_1 + i y_1 & \sqrt{2} x_3 + i \sqrt{2} y_3 & -\sqrt{2} x_4 - i \sqrt{2} y_4 & 0 \\ -x_1 + i y_1 & -i h_1 + 2 i h_2 & \sqrt{2} x_2 + i \sqrt{2} y_2 & 0 & -\sqrt{2} x_4 - i \sqrt{2} y_4 \\ -\sqrt{2} x_3 + i \sqrt{2} y_3 & -\sqrt{2} x_2 + i \sqrt{2} y_2 & 0 & \sqrt{2} x_2 + i \sqrt{2} y_2 & -\sqrt{2} x_3 - i \sqrt{2} y_3 \\ \sqrt{2} x_4 - i \sqrt{2} y_4 & 0 & -\sqrt{2} x_2 + i \sqrt{2} y_2 & i h_1 - 2 i h_2 & x_1 + i y_1 \\ 0 & \sqrt{2} x_4 - i \sqrt{2} y_4 & \sqrt{2} x_3 - i \sqrt{2} y_3 & -x_1 + i y_1 & -i h_1 \end{pmatrix}$$

```
In[ ]:= Eigenvalues[M] // MatrixForm
```

```
Out[ ]//MatrixForm=
```

$$\begin{pmatrix} -\sqrt{-h_1^2 + 2 h_1 h_2 - 2 h_2^2 - x_1^2 - 2 x_2^2 - 2 x_3^2 - 2 x_4^2 - y_1^2 - 2 y_2^2 - 2 y_3^2 - 2 y_4^2 - 2 \sqrt{h_1^2 h_2^2 - 2 h_1 h_2^3 + h_2^4 + h_2^2 x_1^2 - 2 h_1 x_2^2 - 2 h_1 x_3^2 - 2 h_1 x_4^2 - 2 h_1 y_1^2 - 2 h_1 y_2^2 - 2 h_1 y_3^2 - 2 h_1 y_4^2}} \\ \sqrt{-h_1^2 + 2 h_1 h_2 - 2 h_2^2 - x_1^2 - 2 x_2^2 - 2 x_3^2 - 2 x_4^2 - y_1^2 - 2 y_2^2 - 2 y_3^2 - 2 y_4^2 - 2 \sqrt{h_1^2 h_2^2 - 2 h_1 h_2^3 + h_2^4 + h_2^2 x_1^2 - 2 h_1 x_2^2 - 2 h_1 x_3^2 - 2 h_1 x_4^2 - 2 h_1 y_1^2 - 2 h_1 y_2^2 - 2 h_1 y_3^2 - 2 h_1 y_4^2}} \\ -\sqrt{-h_1^2 + 2 h_1 h_2 - 2 h_2^2 - x_1^2 - 2 x_2^2 - 2 x_3^2 - 2 x_4^2 - y_1^2 - 2 y_2^2 - 2 y_3^2 - 2 y_4^2 + 2 \sqrt{h_1^2 h_2^2 - 2 h_1 h_2^3 + h_2^4 + h_2^2 x_1^2 - 2 h_1 x_2^2 - 2 h_1 x_3^2 - 2 h_1 x_4^2 - 2 h_1 y_1^2 - 2 h_1 y_2^2 - 2 h_1 y_3^2 - 2 h_1 y_4^2}} \\ \sqrt{-h_1^2 + 2 h_1 h_2 - 2 h_2^2 - x_1^2 - 2 x_2^2 - 2 x_3^2 - 2 x_4^2 - y_1^2 - 2 y_2^2 - 2 y_3^2 - 2 y_4^2 + 2 \sqrt{h_1^2 h_2^2 - 2 h_1 h_2^3 + h_2^4 + h_2^2 x_1^2 - 2 h_1 x_2^2 - 2 h_1 x_3^2 - 2 h_1 x_4^2 - 2 h_1 y_1^2 - 2 h_1 y_2^2 - 2 h_1 y_3^2 - 2 h_1 y_4^2}} \end{pmatrix}$$

```
In[ ]:= M1 = (i h1 H1) + x1 (X1 - Y1) + i y1 (X1 + Y1) ;
M1 // MatrixForm
```

```
Out[ ]//MatrixForm=
```

$$\begin{pmatrix} i h_1 & x_1 + i y_1 & 0 & 0 & 0 \\ -x_1 + i y_1 & -i h_1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & i h_1 & x_1 + i y_1 \\ 0 & 0 & 0 & -x_1 + i y_1 & -i h_1 \end{pmatrix}$$

```
In[ ]:= M2 = (i h2 H2) + x2 (X2 - Y2) + i y2 (X2 + Y2) ;
M2 // MatrixForm
```

```
Out[ ]//MatrixForm=
```

$$\begin{pmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 2 i h_2 & \sqrt{2} x_2 + i \sqrt{2} y_2 & 0 & 0 \\ 0 & -\sqrt{2} x_2 + i \sqrt{2} y_2 & 0 & \sqrt{2} x_2 + i \sqrt{2} y_2 & 0 \\ 0 & 0 & -\sqrt{2} x_2 + i \sqrt{2} y_2 & -2 i h_2 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

```
In[ ]:= M3 = (i h3 (2 H1 + H2)) + x3 (X3 - Y3) + i y3 (X3 + Y3);
M3 // MatrixForm
```

Out[ ]//MatrixForm=

$$\begin{pmatrix} 2 i h_3 & 0 & \sqrt{2} x_3 + i \sqrt{2} y_3 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ -\sqrt{2} x_3 + i \sqrt{2} y_3 & 0 & 0 & 0 & -\sqrt{2} x_3 - i \sqrt{2} y_3 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \sqrt{2} x_3 - i \sqrt{2} y_3 & 0 & -2 i h_3 \end{pmatrix}$$

```
In[ ]:= M4 = (i 2 h4 (H1 + H2)) + x4 (X4 - Y4) + i y4 (X4 + Y4);
M4 // MatrixForm
```

Out[ ]//MatrixForm=

$$\begin{pmatrix} 2 i h_4 & 0 & 0 & -\sqrt{2} x_4 - i \sqrt{2} y_4 & 0 \\ 0 & 2 i h_4 & 0 & 0 & -\sqrt{2} x_4 - i \sqrt{2} y_4 \\ 0 & 0 & 0 & 0 & 0 \\ \sqrt{2} x_4 - i \sqrt{2} y_4 & 0 & 0 & -2 i h_4 & 0 \\ 0 & \sqrt{2} x_4 - i \sqrt{2} y_4 & 0 & 0 & -2 i h_4 \end{pmatrix}$$

```
In[ ]:= M5 = (i h1 H1 + i h2 H2);
M5 // MatrixForm
```

Out[ ]//MatrixForm=

$$\begin{pmatrix} i h_1 & 0 & 0 & 0 & 0 \\ 0 & -i h_1 + 2 i h_2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & i h_1 - 2 i h_2 & 0 \\ 0 & 0 & 0 & 0 & -i h_1 \end{pmatrix}$$

```
In[ ]:= f[p_] := Sum[Binomial[p, q] (-1)^(p-q) (p-q)! (dY) * If[q == 0, 1, f[q-1]]; i = 5;
```

```
n = 5;
```

```
A = Sum[k=0 to n-1] Sum[j=0 to n-k-1] Sum[m=0 to j] (-1)^(n-k-j-1) * Binomial[j, m] *
  ((n-1)! / ((n-j+m-1)!)) * If[k == 0, IdentityMatrix[i], MatrixPower[M, k]] *
  Tr[Minors[M, n-j-k-1]] * If[m == 0, 1, f[m-1]];
```

```
In[ ]:= Tr[A] // ExpandAll
```

```
Out[ ]:= 120 + 250 dY + 175 dY^2 + 50 dY^3 + 5 dY^4 + 72 h1^2 + 42 dY h1^2 + 6 dY^2 h1^2 + h1^4 - 144 h1 h2 - 84 dY h1 h2 -
  12 dY^2 h1 h2 - 4 h1^3 h2 + 144 h2^2 + 84 dY h2^2 + 12 dY^2 h2^2 + 4 h1^2 h2^2 + 72 x1^2 + 42 dY x1^2 + 6 dY^2 x1^2 +
  2 h1^2 x1^2 - 4 h1 h2 x1^2 + x1^4 + 144 x2^2 + 84 dY x2^2 + 12 dY^2 x2^2 + 4 h1^2 x2^2 + 4 x1^2 x2^2 + 144 x3^2 + 84 dY x3^2 +
  12 dY^2 x3^2 + 4 h1^2 x3^2 - 16 h1 h2 x3^2 + 16 h2^2 x3^2 + 4 x1^2 x3^2 - 8 sqrt(2) x1 x2^2 x4 - 8 sqrt(2) x1 x3^2 x4 + 144 x4^2 +
  84 dY x4^2 + 12 dY^2 x4^2 - 4 h1^2 x4^2 + 8 h1 h2 x4^2 - 4 x1^2 x4^2 + 8 x2^2 x4^2 + 8 x3^2 x4^2 + 4 x4^4 - 16 h2 x2 x3 y1 +
  72 y1^2 + 42 dY y1^2 + 6 dY^2 y1^2 + 2 h1^2 y1^2 - 4 h1 h2 y1^2 + 2 x1^2 y1^2 + 4 x2^2 y1^2 + 4 x3^2 y1^2 - 4 x4^2 y1^2 + y1^4 -
  16 h2 x1 x3 y2 + 16 sqrt(2) h1 x3 x4 y2 - 16 sqrt(2) h2 x3 x4 y2 + 16 sqrt(2) x2 x4 y1 y2 + 144 y2^2 + 84 dY y2^2 +
  12 dY^2 y2^2 + 4 h1^2 y2^2 + 4 x1^2 y2^2 + 8 sqrt(2) x1 x4 y2^2 + 8 x4^2 y2^2 + 4 y1^2 y2^2 + 16 h2 x1 x2 y3 + 16 sqrt(2) h1 x2 x4 y3 -
  16 sqrt(2) h2 x2 x4 y3 - 16 sqrt(2) x3 x4 y1 y3 - 16 h2 y1 y2 y3 + 144 y3^2 + 84 dY y3^2 + 12 dY^2 y3^2 +
  4 h1^2 y3^2 - 16 h1 h2 y3^2 + 16 h2^2 y3^2 + 4 x1^2 y3^2 + 8 sqrt(2) x1 x4 y3^2 + 8 x4^2 y3^2 + 4 y1^2 y3^2 - 16 sqrt(2) h1 x2 x3 y4 +
  16 sqrt(2) h2 x2 x3 y4 - 8 sqrt(2) x2^2 y1 y4 + 8 sqrt(2) x3^2 y1 y4 - 16 sqrt(2) x1 x2 y2 y4 + 8 sqrt(2) y1 y2^2 y4 -
  16 sqrt(2) x1 x3 y3 y4 + 16 sqrt(2) h1 y2 y3 y4 - 16 sqrt(2) h2 y2 y3 y4 - 8 sqrt(2) y1 y3^2 y4 + 144 y4^2 + 84 dY y4^2 +
  12 dY^2 y4^2 - 4 h1^2 y4^2 + 8 h1 h2 y4^2 - 4 x1^2 y4^2 + 8 x2^2 y4^2 + 8 x3^2 y4^2 + 8 x4^2 y4^2 - 4 y1^2 y4^2 + 8 y2^2 y4^2 + 8 y3^2 y4^2 + 4 y4^4
```

In[\*]:= A[[1, 1]] // Expand

$$\begin{aligned} \text{Out[*]} = & 24 + 50 d\gamma + 35 d\gamma^2 + 10 d\gamma^3 + d\gamma^4 + 24 \, \mathfrak{h}_1 + 26 \, \mathfrak{h}_1 d\gamma + 9 \, \mathfrak{h}_1 d\gamma^2 + \mathfrak{h}_1 d\gamma^3 + 12 h_1^2 + 7 d\gamma h_1^2 + \\ & d\gamma^2 h_1^2 + 4 \, \mathfrak{h}_1^3 + \mathfrak{h}_1 d\gamma h_1^2 - 48 h_1 h_2 - 28 d\gamma h_1 h_2 - 4 d\gamma^2 h_1 h_2 - 16 \, \mathfrak{h}_1^2 h_2 - 4 \, \mathfrak{h}_1 d\gamma h_1^2 h_2 + \\ & 48 h_2^2 + 28 d\gamma h_2^2 + 4 d\gamma^2 h_2^2 + 16 \, \mathfrak{h}_1 h_2^2 + 4 \, \mathfrak{h}_1 d\gamma h_1 h_2^2 + 12 x_1^2 + 7 d\gamma x_1^2 + d\gamma^2 x_1^2 + 4 \, \mathfrak{h}_1 x_1^2 + \\ & \mathfrak{h}_1 d\gamma h_1 x_1^2 - 8 \, \mathfrak{h}_2 x_1^2 - 2 \, \mathfrak{h}_1 d\gamma h_2 x_1^2 + 48 x_2^2 + 28 d\gamma x_2^2 + 4 d\gamma^2 x_2^2 + 16 \, \mathfrak{h}_1 x_2^2 + 4 \, \mathfrak{h}_1 d\gamma h_1 x_2^2 + \\ & 2 x_1^2 x_2^2 + 24 x_3^2 + 14 d\gamma x_3^2 + 2 d\gamma^2 x_3^2 + 2 \, \mathfrak{h}_1^2 x_3^2 - 8 h_1 h_2 x_3^2 + 8 h_2^2 x_3^2 - 4 \sqrt{2} x_1 x_2^2 x_4 + 24 x_4^2 + \\ & 14 d\gamma x_4^2 + 2 d\gamma^2 x_4^2 - 8 \, \mathfrak{h}_1 x_4^2 - 2 \, \mathfrak{h}_1 d\gamma h_1 x_4^2 + 16 \, \mathfrak{h}_2 x_4^2 + 4 \, \mathfrak{h}_1 d\gamma h_2 x_4^2 + 4 x_2^2 x_4^2 - 16 \, \mathfrak{h}_2 x_2 x_3 y_1 - \\ & 4 \, \mathfrak{h}_1 d\gamma x_2 x_3 y_1 + 4 h_1 x_2 x_3 y_1 - 8 h_2 x_2 x_3 y_1 + 12 y_1^2 + 7 d\gamma y_1^2 + d\gamma^2 y_1^2 + 4 \, \mathfrak{h}_1 y_1^2 + \mathfrak{h}_1 d\gamma h_1 y_1^2 - \\ & 8 \, \mathfrak{h}_2 y_1^2 - 2 \, \mathfrak{h}_1 d\gamma h_2 y_1^2 + 2 x_2^2 y_1^2 - 16 \, \mathfrak{h}_1 x_1 x_3 y_2 - 4 \, \mathfrak{h}_1 d\gamma x_1 x_3 y_2 + 4 h_1 x_1 x_3 y_2 - 8 h_2 x_1 x_3 y_2 + \\ & 16 \, \mathfrak{h}_1 \sqrt{2} x_3 x_4 y_2 + 4 \, \mathfrak{h}_1 \sqrt{2} d\gamma x_3 x_4 y_2 + 4 \sqrt{2} h_1 x_3 x_4 y_2 - 8 \sqrt{2} h_2 x_3 x_4 y_2 + 8 \sqrt{2} x_2 x_4 y_1 y_2 + \\ & 48 y_2^2 + 28 d\gamma y_2^2 + 4 d\gamma^2 y_2^2 + 16 \, \mathfrak{h}_1 y_2^2 + 4 \, \mathfrak{h}_1 d\gamma h_1 y_2^2 + 2 x_1^2 y_2^2 + 4 \sqrt{2} x_1 x_4 y_2^2 + 4 x_4^2 y_2^2 + 2 y_1^2 y_2^2 + \\ & 16 \, \mathfrak{h}_1 x_1 x_2 y_3 + 4 \, \mathfrak{h}_1 d\gamma x_1 x_2 y_3 - 4 h_1 x_1 x_2 y_3 + 8 h_2 x_1 x_2 y_3 + 16 \, \mathfrak{h}_1 \sqrt{2} x_2 x_4 y_3 + 4 \, \mathfrak{h}_1 \sqrt{2} d\gamma x_2 x_4 y_3 + \\ & 4 \sqrt{2} h_1 x_2 x_4 y_3 - 8 \sqrt{2} h_2 x_2 x_4 y_3 - 16 \, \mathfrak{h}_1 y_1 y_2 y_3 - 4 \, \mathfrak{h}_1 d\gamma y_1 y_2 y_3 + 4 h_1 y_1 y_2 y_3 - \\ & 8 h_2 y_1 y_2 y_3 + 24 y_3^2 + 14 d\gamma y_3^2 + 2 d\gamma^2 y_3^2 + 2 \, \mathfrak{h}_1^2 y_3^2 - 8 h_1 h_2 y_3^2 + 8 h_2^2 y_3^2 - 16 \, \mathfrak{h}_1 \sqrt{2} x_2 x_3 y_4 - \\ & 4 \, \mathfrak{h}_1 \sqrt{2} d\gamma x_2 x_3 y_4 - 4 \sqrt{2} h_1 x_2 x_3 y_4 + 8 \sqrt{2} h_2 x_2 x_3 y_4 - 4 \sqrt{2} x_2^2 y_1 y_4 - 8 \sqrt{2} x_1 x_2 y_2 y_4 + \\ & 4 \sqrt{2} y_1 y_2^2 y_4 + 16 \, \mathfrak{h}_1 \sqrt{2} y_2 y_3 y_4 + 4 \, \mathfrak{h}_1 \sqrt{2} d\gamma y_2 y_3 y_4 + 4 \sqrt{2} h_1 y_2 y_3 y_4 - 8 \sqrt{2} h_2 y_2 y_3 y_4 + \\ & 24 y_4^2 + 14 d\gamma y_4^2 + 2 d\gamma^2 y_4^2 - 8 \, \mathfrak{h}_1 y_4^2 - 2 \, \mathfrak{h}_1 d\gamma h_1 y_4^2 + 16 \, \mathfrak{h}_2 y_4^2 + 4 \, \mathfrak{h}_1 d\gamma h_2 y_4^2 + 4 x_2^2 y_4^2 + 4 y_2^2 y_4^2 \end{aligned}$$

In[\*]:= A[[3, 3]] // Expand

$$\begin{aligned} \text{Out[*]} = & 24 + 50 d\gamma + 35 d\gamma^2 + 10 d\gamma^3 + d\gamma^4 + 24 h_1^2 + 14 d\gamma h_1^2 + 2 d\gamma^2 h_1^2 + h_1^4 - 48 h_1 h_2 - \\ & 28 d\gamma h_1 h_2 - 4 d\gamma^2 h_1 h_2 - 4 h_1^3 h_2 + 48 h_2^2 + 28 d\gamma h_2^2 + 4 d\gamma^2 h_2^2 + 4 h_1^2 h_2^2 + 24 x_1^2 + 14 d\gamma x_1^2 + \\ & 2 d\gamma^2 x_1^2 + 2 h_1^2 x_1^2 - 4 h_1 h_2 x_1^2 + x_1^4 + 48 x_4^2 + 28 d\gamma x_4^2 + 4 d\gamma^2 x_4^2 - 4 h_1^2 x_4^2 + 8 h_1 h_2 x_4^2 - \\ & 4 x_1^2 x_4^2 + 4 x_4^4 + 24 y_1^2 + 14 d\gamma y_1^2 + 2 d\gamma^2 y_1^2 + 2 h_1^2 y_1^2 - 4 h_1 h_2 y_1^2 + 2 x_1^2 y_1^2 - 4 x_4^2 y_1^2 + \\ & y_1^4 + 48 y_4^2 + 28 d\gamma y_4^2 + 4 d\gamma^2 y_4^2 - 4 h_1^2 y_4^2 + 8 h_1 h_2 y_4^2 - 4 x_1^2 y_4^2 + 8 x_4^2 y_4^2 - 4 y_1^2 y_4^2 + 4 y_4^4 \end{aligned}$$

(\*

G - induced differential operators:

$$X_1 : 2y_1 dh_1 + (2h_2 - 2h_1) dy_1 + x_2 dx_3 + y_2 dy_3 - x_3 dx_2 - y_3 dy_2 ;$$

$$Y_1 : -2x_1 dh_1 - (2h_2 - 2h_1) dx_1 - y_2 dx_3 + x_2 dy_3 - y_3 dx_2 + x_3 dy_2 ;$$

$$\begin{aligned} X_2 : & 2y_2 dh_2 + (h_1 - 2h_2) dy_2 + 2x_3 dx_1 + 2y_3 dy_1 + \\ & (-x_1 - \sqrt{2} x_4) dx_3 + (-y_1 - \sqrt{2} y_4) dy_3 - \sqrt{2} x_3 dx_4 - \sqrt{2} y_3 dy_4 ; \end{aligned}$$

$$\begin{aligned} Y_2 : & -2x_2 dh_2 + (-h_1 + 2h_2) dx_2 + 2y_3 dx_1 - 2x_3 dy_1 + \\ & (y_1 - \sqrt{2} y_4) dx_3 + (-x_1 + \sqrt{2} x_4) dy_3 + \sqrt{2} y_3 dx_4 - \sqrt{2} x_3 dy_4 ; \end{aligned}$$

$$Y_{\alpha_1}^G = -\frac{1}{2} (X_1^G + \mathfrak{h}_1 Y_1^G),$$

$$\text{we know } Y_{\alpha_1} \cdot \pi_{1,1} = \pi_{1,2} \text{ and } -Y_{\alpha_1}^G \cdot \pi_{1,1}(\exp) = \pi_{1,2}(\exp);$$

$$\text{This holds true for all } \lambda = \sum_i c_i \lambda_i$$

\*)

$$\text{In}[*]:= - \left( - (y_1 - i x_1) D[A[[1, 1]], h_1] - \frac{1}{2} * (2 h_2 - 2 h_1) D[A[[1, 1]], y_1] + i \frac{1}{2} * (2 h_2 - 2 h_1) \right. \\ \left. D[A[[1, 1]], x_1] - \frac{1}{2} * (x_2 - i y_2) D[A[[1, 1]], x_3] - \frac{1}{2} * (y_2 + i x_2) D[A[[1, 1]], y_3] - \right. \\ \left. \frac{1}{2} * (-x_3 - i y_3) D[A[[1, 1]], x_2] - \frac{1}{2} * (-y_3 + i x_3) D[A[[1, 1]], y_2] \right) // \text{Expand}$$

**In[\*]:= (\* Apply  $X_{\alpha 1}$  to  $t^{\lambda_{15}}$  \*)**

**1 = 2; k = 5;**

$$\frac{1}{2} \left( (2 y_1 + i 2 x_1) D[A[[1, k]], h_1] + (2 h_2 - 2 h_1) D[A[[1, k]], y_1] + \right. \\ \left. i (2 h_2 - 2 h_1) D[A[[1, k]], x_1] + (x_2 + i y_2) D[A[[1, k]], x_3] + (y_2 - i x_2) D[A[[1, k]], y_3] + \right. \\ \left. (-x_3 + i y_3) D[A[[1, k]], x_2] + (-y_3 - i x_3) D[A[[1, k]], y_2] \right) // \text{Expand}$$

$$\text{In}[*]:= -24 x_2^2 - 14 d_{\gamma} x_2^2 - 2 d_{\gamma}^2 x_2^2 - 2 h_1^2 x_2^2 + 4 i h_1 x_1 x_2 x_3 + 2 x_1^2 x_3^2 - 24 \sqrt{2} x_1 x_4 - 14 \sqrt{2} d_{\gamma} x_1 x_4 - \\ 2 \sqrt{2} d_{\gamma}^2 x_1 x_4 - 4 i \sqrt{2} h_1 x_2 x_3 x_4 - 4 \sqrt{2} x_1 x_3^2 x_4 + 4 x_3^2 x_4^2 + 4 h_1 x_2 x_3 y_1 - 4 i x_1 x_3^2 y_1 + \\ 24 i \sqrt{2} x_4 y_1 + 14 i \sqrt{2} d_{\gamma} x_4 y_1 + 2 i \sqrt{2} d_{\gamma}^2 x_4 y_1 + 4 i \sqrt{2} x_3^2 x_4 y_1 - 2 x_3^2 y_1^2 - 48 i x_2 y_2 - \\ 28 i d_{\gamma} x_2 y_2 - 4 i d_{\gamma}^2 x_2 y_2 - 4 i h_1^2 x_2 y_2 - 4 h_1 x_1 x_3 y_2 + 4 \sqrt{2} h_1 x_3 x_4 y_2 + 4 i h_1 x_3 y_1 y_2 + \\ 24 y_2^2 + 14 d_{\gamma} y_2^2 + 2 d_{\gamma}^2 y_2^2 + 2 h_1^2 y_2^2 - 4 h_1 x_1 x_2 y_3 + 4 i x_1^2 x_3 y_3 - 4 \sqrt{2} h_1 x_2 x_4 y_3 - \\ 8 i x_3 x_4^2 y_3 + 4 i h_1 x_2 y_1 y_3 + 8 x_1 x_3 y_1 y_3 - 4 i x_3 y_1^2 y_3 - 4 i h_1 x_1 y_2 y_3 - 4 i \sqrt{2} h_1 x_4 y_2 y_3 - \\ 4 h_1 y_1 y_2 y_3 - 2 x_1^2 y_3^2 - 4 \sqrt{2} x_1 x_4 y_3^2 - 4 x_4^2 y_3^2 + 4 i x_1 y_1 y_3^2 + 4 i \sqrt{2} x_4 y_1 y_3^2 + 2 y_1^2 y_3^2 - \\ 24 i \sqrt{2} x_1 y_4 - 14 i \sqrt{2} d_{\gamma} x_1 y_4 - 2 i \sqrt{2} d_{\gamma}^2 x_1 y_4 + 4 \sqrt{2} h_1 x_2 x_3 y_4 - 4 i \sqrt{2} x_1 x_3^2 y_4 + \\ 8 i x_3^2 x_4 y_4 - 24 \sqrt{2} y_1 y_4 - 14 \sqrt{2} d_{\gamma} y_1 y_4 - 2 \sqrt{2} d_{\gamma}^2 y_1 y_4 - 4 \sqrt{2} x_3^2 y_1 y_4 + \\ 4 i \sqrt{2} h_1 x_3 y_2 y_4 - 4 i \sqrt{2} h_1 x_2 y_3 y_4 + 16 x_3 x_4 y_3 y_4 + 4 \sqrt{2} h_1 y_2 y_3 y_4 - 4 i \sqrt{2} x_1 y_3^2 y_4 - \\ 8 i x_4 y_3^2 y_4 - 4 \sqrt{2} y_1 y_3^2 y_4 - 4 x_3^2 y_4^2 + 8 i x_3 y_3 y_4^2 + 4 y_3^2 y_4^2 + A[[2, 4]] // \text{Expand}$$

**Out[\*]= 0**

**In[\*]= A[[2, 4]] // Expand**

$$\text{Out}[*]:= 24 x_2^2 + 14 d_{\gamma} x_2^2 + 2 d_{\gamma}^2 x_2^2 + 2 h_1^2 x_2^2 - 4 i h_1 x_1 x_2 x_3 - 2 x_1^2 x_3^2 + 24 \sqrt{2} x_1 x_4 + 14 \sqrt{2} d_{\gamma} x_1 x_4 + \\ 2 \sqrt{2} d_{\gamma}^2 x_1 x_4 + 4 i \sqrt{2} h_1 x_2 x_3 x_4 + 4 \sqrt{2} x_1 x_3^2 x_4 - 4 x_3^2 x_4^2 - 4 h_1 x_2 x_3 y_1 + 4 i x_1 x_3^2 y_1 - \\ 24 i \sqrt{2} x_4 y_1 - 14 i \sqrt{2} d_{\gamma} x_4 y_1 - 2 i \sqrt{2} d_{\gamma}^2 x_4 y_1 - 4 i \sqrt{2} x_3^2 x_4 y_1 + 2 x_3^2 y_1^2 + 48 i x_2 y_2 + \\ 28 i d_{\gamma} x_2 y_2 + 4 i d_{\gamma}^2 x_2 y_2 + 4 i h_1^2 x_2 y_2 + 4 h_1 x_1 x_3 y_2 - 4 \sqrt{2} h_1 x_3 x_4 y_2 - 4 i h_1 x_3 y_1 y_2 - \\ 24 y_2^2 - 14 d_{\gamma} y_2^2 - 2 d_{\gamma}^2 y_2^2 - 2 h_1^2 y_2^2 + 4 h_1 x_1 x_2 y_3 - 4 i x_1^2 x_3 y_3 + 4 \sqrt{2} h_1 x_2 x_4 y_3 + 8 i x_3 x_4^2 y_3 - \\ 4 i h_1 x_2 y_1 y_3 - 8 x_1 x_3 y_1 y_3 + 4 i x_3 y_1^2 y_3 + 4 i h_1 x_1 y_2 y_3 + 4 i \sqrt{2} h_1 x_4 y_2 y_3 + 4 h_1 y_1 y_2 y_3 + 2 x_1^2 y_3^2 + \\ 4 \sqrt{2} x_1 x_4 y_3^2 + 4 x_4^2 y_3^2 - 4 i x_1 y_1 y_3^2 - 4 i \sqrt{2} x_4 y_1 y_3^2 - 2 y_1^2 y_3^2 + 24 i \sqrt{2} x_1 y_4 + 14 i \sqrt{2} d_{\gamma} x_1 y_4 + \\ 2 i \sqrt{2} d_{\gamma}^2 x_1 y_4 - 4 \sqrt{2} h_1 x_2 x_3 y_4 + 4 i \sqrt{2} x_1 x_3^2 y_4 - 8 i x_3^2 x_4 y_4 + 24 \sqrt{2} y_1 y_4 + 14 \sqrt{2} d_{\gamma} y_1 y_4 + \\ 2 \sqrt{2} d_{\gamma}^2 y_1 y_4 + 4 \sqrt{2} x_3^2 y_1 y_4 - 4 i \sqrt{2} h_1 x_3 y_2 y_4 + 4 i \sqrt{2} h_1 x_2 y_3 y_4 - 16 x_3 x_4 y_3 y_4 - \\ 4 \sqrt{2} h_1 y_2 y_3 y_4 + 4 i \sqrt{2} x_1 y_3^2 y_4 + 8 i x_4 y_3^2 y_4 + 4 \sqrt{2} y_1 y_3^2 y_4 + 4 x_3^2 y_4^2 - 8 i x_3 y_3 y_4^2 - 4 y_3^2 y_4^2$$

```

In[ ]:= -4 \[ImaginaryI] h1 x1 x2^2 - 48 x2 x3 - 28 dY x2 x3 - 4 dY^2 x2 x3 + 4 h1^2 x2 x3 - 8 h1 h2 x2 x3 - 4 x1^2 x2 x3 - 4 \[ImaginaryI] h1 x1 x3^2 +
      8 \[ImaginaryI] h2 x1 x3^2 + 48 \[ImaginaryI] \[Sqrt][2] h1 x4 + 28 \[ImaginaryI] \[Sqrt][2] dY h1 x4 + 4 \[ImaginaryI] \[Sqrt][2] dY^2 h1 x4 - 48 \[ImaginaryI] \[Sqrt][2] h2 x4 -
      28 \[ImaginaryI] \[Sqrt][2] dY h2 x4 - 4 \[ImaginaryI] \[Sqrt][2] dY^2 h2 x4 + 4 \[ImaginaryI] \[Sqrt][2] h1 x2^2 x4 + 8 \[Sqrt][2] x1 x2 x3 x4 + 4 \[ImaginaryI] \[Sqrt][2] h1 x3^2 x4 -
      8 \[ImaginaryI] \[Sqrt][2] h2 x3^2 x4 - 8 x2 x3 x4^2 + 4 h1 x2^2 y1 - 4 h1 x3^2 y1 + 8 h2 x3^2 y1 - 4 x2 x3 y1^2 + 8 h1 x1 x2 y2 -
      48 \[ImaginaryI] x3 y2 - 28 \[ImaginaryI] dY x3 y2 - 4 \[ImaginaryI] dY^2 x3 y2 + 4 \[ImaginaryI] h1^2 x3 y2 - 8 \[ImaginaryI] h1 h2 x3 y2 - 4 \[ImaginaryI] x1^2 x3 y2 + 8 \[ImaginaryI] x3 x4^2 y2 +
      8 \[ImaginaryI] h1 x2 y1 y2 - 8 \[Sqrt][2] x3 x4 y1 y2 - 4 \[ImaginaryI] x3 y1^2 y2 + 4 \[ImaginaryI] h1 x1 y2^2 + 4 \[ImaginaryI] \[Sqrt][2] h1 x4 y2^2 - 4 h1 y1 y2^2 -
      48 \[ImaginaryI] x2 y3 - 28 \[ImaginaryI] dY x2 y3 - 4 \[ImaginaryI] dY^2 x2 y3 + 4 \[ImaginaryI] h1^2 x2 y3 - 8 \[ImaginaryI] h1 h2 x2 y3 - 4 \[ImaginaryI] x1^2 x2 y3 + 8 h1 x1 x3 y3 -
      16 h2 x1 x3 y3 + 8 \[ImaginaryI] x2 x4^2 y3 - 8 \[ImaginaryI] h1 x3 y1 y3 + 16 \[ImaginaryI] h2 x3 y1 y3 + 8 \[Sqrt][2] x2 x4 y1 y3 - 4 \[ImaginaryI] x2 y1^2 y3 +
      48 y2 y3 + 28 dY y2 y3 + 4 dY^2 y2 y3 - 4 h1^2 y2 y3 + 8 h1 h2 y2 y3 + 4 x1^2 y2 y3 + 8 \[Sqrt][2] x1 x4 y2 y3 +
      8 x4^2 y2 y3 + 4 y1^2 y2 y3 + 4 \[ImaginaryI] h1 x1 y3^2 - 8 \[ImaginaryI] h2 x1 y3^2 + 4 \[ImaginaryI] \[Sqrt][2] h1 x4 y3^2 - 8 \[ImaginaryI] \[Sqrt][2] h2 x4 y3^2 +
      4 h1 y1 y3^2 - 8 h2 y1 y3^2 - 48 \[Sqrt][2] h1 y4 - 28 \[Sqrt][2] dY h1 y4 - 4 \[Sqrt][2] dY^2 h1 y4 + 48 \[Sqrt][2] h2 y4 +
      28 \[Sqrt][2] dY h2 y4 + 4 \[Sqrt][2] dY^2 h2 y4 - 4 \[Sqrt][2] h1 x2^2 y4 + 8 \[ImaginaryI] \[Sqrt][2] x1 x2 x3 y4 - 4 \[Sqrt][2] h1 x3^2 y4 +
      8 \[Sqrt][2] h2 x3^2 y4 - 16 \[ImaginaryI] x2 x3 x4 y4 - 16 x3 x4 y2 y4 - 8 \[ImaginaryI] \[Sqrt][2] x3 y1 y2 y4 - 4 \[Sqrt][2] h1 y2^2 y4 -
      16 x2 x4 y3 y4 + 8 \[ImaginaryI] \[Sqrt][2] x2 y1 y3 y4 + 8 \[ImaginaryI] \[Sqrt][2] x1 y2 y3 y4 + 16 \[ImaginaryI] x4 y2 y3 y4 - 4 \[Sqrt][2] h1 y3^2 y4 +
      8 \[Sqrt][2] h2 y3^2 y4 + 8 x2 x3 y4^2 - 8 \[ImaginaryI] x3 y2 y4^2 - 8 \[ImaginaryI] x2 y3 y4^2 - 8 y2 y3 y4^2 - (A[[2, 5]] - A[[1, 4]]) // Expand

```

```
Out[ ]:= 0
```

```
In[ ]:= A[[2, 5]] - A[[1, 4]] // Expand
```

```

Out[ ]:= -4 \[ImaginaryI] h1 x1 x2^2 - 48 x2 x3 - 28 dY x2 x3 - 4 dY^2 x2 x3 + 4 h1^2 x2 x3 - 8 h1 h2 x2 x3 - 4 x1^2 x2 x3 -
      4 \[ImaginaryI] h1 x1 x3^2 + 8 \[ImaginaryI] h2 x1 x3^2 + 48 \[ImaginaryI] \[Sqrt][2] h1 x4 + 28 \[ImaginaryI] \[Sqrt][2] dY h1 x4 + 4 \[ImaginaryI] \[Sqrt][2] dY^2 h1 x4 - 48 \[ImaginaryI] \[Sqrt][2] h2 x4 -
      28 \[ImaginaryI] \[Sqrt][2] dY h2 x4 - 4 \[ImaginaryI] \[Sqrt][2] dY^2 h2 x4 + 4 \[ImaginaryI] \[Sqrt][2] h1 x2^2 x4 + 8 \[Sqrt][2] x1 x2 x3 x4 + 4 \[ImaginaryI] \[Sqrt][2] h1 x3^2 x4 -
      8 \[ImaginaryI] \[Sqrt][2] h2 x3^2 x4 - 8 x2 x3 x4^2 + 4 h1 x2^2 y1 - 4 h1 x3^2 y1 + 8 h2 x3^2 y1 - 4 x2 x3 y1^2 + 8 h1 x1 x2 y2 -
      48 \[ImaginaryI] x3 y2 - 28 \[ImaginaryI] dY x3 y2 - 4 \[ImaginaryI] dY^2 x3 y2 + 4 \[ImaginaryI] h1^2 x3 y2 - 8 \[ImaginaryI] h1 h2 x3 y2 - 4 \[ImaginaryI] x1^2 x3 y2 + 8 \[ImaginaryI] x3 x4^2 y2 +
      8 \[ImaginaryI] h1 x2 y1 y2 - 8 \[Sqrt][2] x3 x4 y1 y2 - 4 \[ImaginaryI] x3 y1^2 y2 + 4 \[ImaginaryI] h1 x1 y2^2 + 4 \[ImaginaryI] \[Sqrt][2] h1 x4 y2^2 - 4 h1 y1 y2^2 -
      48 \[ImaginaryI] x2 y3 - 28 \[ImaginaryI] dY x2 y3 - 4 \[ImaginaryI] dY^2 x2 y3 + 4 \[ImaginaryI] h1^2 x2 y3 - 8 \[ImaginaryI] h1 h2 x2 y3 - 4 \[ImaginaryI] x1^2 x2 y3 + 8 h1 x1 x3 y3 -
      16 h2 x1 x3 y3 + 8 \[ImaginaryI] x2 x4^2 y3 - 8 \[ImaginaryI] h1 x3 y1 y3 + 16 \[ImaginaryI] h2 x3 y1 y3 + 8 \[Sqrt][2] x2 x4 y1 y3 - 4 \[ImaginaryI] x2 y1^2 y3 +
      48 y2 y3 + 28 dY y2 y3 + 4 dY^2 y2 y3 - 4 h1^2 y2 y3 + 8 h1 h2 y2 y3 + 4 x1^2 y2 y3 + 8 \[Sqrt][2] x1 x4 y2 y3 + 8 x4^2 y2 y3 +
      4 y1^2 y2 y3 + 4 \[ImaginaryI] h1 x1 y3^2 - 8 \[ImaginaryI] h2 x1 y3^2 + 4 \[ImaginaryI] \[Sqrt][2] h1 x4 y3^2 - 8 \[ImaginaryI] \[Sqrt][2] h2 x4 y3^2 + 4 h1 y1 y3^2 - 8 h2 y1 y3^2 -
      48 \[Sqrt][2] h1 y4 - 28 \[Sqrt][2] dY h1 y4 - 4 \[Sqrt][2] dY^2 h1 y4 + 48 \[Sqrt][2] h2 y4 + 28 \[Sqrt][2] dY h2 y4 + 4 \[Sqrt][2] dY^2 h2 y4 -
      4 \[Sqrt][2] h1 x2^2 y4 + 8 \[ImaginaryI] \[Sqrt][2] x1 x2 x3 y4 - 4 \[Sqrt][2] h1 x3^2 y4 + 8 \[Sqrt][2] h2 x3^2 y4 - 16 \[ImaginaryI] x2 x3 x4 y4 - 16 x3 x4 y2 y4 -
      8 \[ImaginaryI] \[Sqrt][2] x3 y1 y2 y4 - 4 \[Sqrt][2] h1 y2^2 y4 - 16 x2 x4 y3 y4 + 8 \[ImaginaryI] \[Sqrt][2] x2 y1 y3 y4 + 8 \[ImaginaryI] \[Sqrt][2] x1 y2 y3 y4 +
      16 \[ImaginaryI] x4 y2 y3 y4 - 4 \[Sqrt][2] h1 y3^2 y4 + 8 \[Sqrt][2] h2 y3^2 y4 + 8 x2 x3 y4^2 - 8 \[ImaginaryI] x3 y2 y4^2 - 8 \[ImaginaryI] x2 y3 y4^2 - 8 y2 y3 y4^2

```

```
In[ ]:= A // Expand
```

$$\begin{aligned}
In[ ] := & \left\{ \left\{ 24 + 50 d\gamma + 35 d\gamma^2 + 10 d\gamma^3 + d\gamma^4 + 24 \, i \, h_1 + 26 \, i \, d\gamma h_1 + 9 \, i \, d\gamma^2 h_1 + i \, d\gamma^3 h_1 + 12 h_1^2 + \right. \right. \\
& 7 d\gamma h_1^2 + d\gamma^2 h_1^2 + 4 \, i \, h_1^3 + i \, d\gamma h_1^3 + 12 x_1^2 + 7 d\gamma x_1^2 + d\gamma^2 x_1^2 + 4 \, i \, h_1 x_1^2 + i \, d\gamma h_1 x_1^2 + \\
& 12 y_1^2 + 7 d\gamma y_1^2 + d\gamma^2 y_1^2 + 4 \, i \, h_1 y_1^2 + i \, d\gamma h_1 y_1^2, 24 x_1 + 26 d\gamma x_1 + 9 d\gamma^2 x_1 + d\gamma^3 x_1 + \\
& 4 h_1^2 x_1 + d\gamma h_1^2 x_1 + 4 x_1^3 + d\gamma x_1^3 + 24 \, i \, y_1 + 26 \, i \, d\gamma y_1 + 9 \, i \, d\gamma^2 y_1 + i \, d\gamma^3 y_1 + 4 \, i \, h_1^2 y_1 + \\
& i \, d\gamma h_1^2 y_1 + 4 \, i \, x_1^2 y_1 + i \, d\gamma x_1^2 y_1 + 4 x_1 y_1^2 + d\gamma x_1 y_1^2 + 4 \, i \, y_1^3 + i \, d\gamma y_1^3, 0, 0, 0 \left. \right\}, \\
& \left\{ -24 x_1 - 26 d\gamma x_1 - 9 d\gamma^2 x_1 - d\gamma^3 x_1 - 4 h_1^2 x_1 - d\gamma h_1^2 x_1 - 4 x_1^3 - d\gamma x_1^3 + 24 \, i \, y_1 + \right. \\
& 26 \, i \, d\gamma y_1 + 9 \, i \, d\gamma^2 y_1 + i \, d\gamma^3 y_1 + 4 \, i \, h_1^2 y_1 + i \, d\gamma h_1^2 y_1 + 4 \, i \, x_1^2 y_1 + i \, d\gamma x_1^2 y_1 - 4 x_1 y_1^2 - \\
& d\gamma x_1 y_1^2 + 4 \, i \, y_1^3 + i \, d\gamma y_1^3, 24 + 50 d\gamma + 35 d\gamma^2 + 10 d\gamma^3 + d\gamma^4 - 24 \, i \, h_1 - 26 \, i \, d\gamma h_1 - \\
& 9 \, i \, d\gamma^2 h_1 - i \, d\gamma^3 h_1 + 12 h_1^2 + 7 d\gamma h_1^2 + d\gamma^2 h_1^2 - 4 \, i \, h_1^3 - i \, d\gamma h_1^3 + 12 x_1^2 + 7 d\gamma x_1^2 + \\
& d\gamma^2 x_1^2 - 4 \, i \, h_1 x_1^2 - i \, d\gamma h_1 x_1^2 + 12 y_1^2 + 7 d\gamma y_1^2 + d\gamma^2 y_1^2 - 4 \, i \, h_1 y_1^2 - i \, d\gamma h_1 y_1^2, 0, 0, 0 \left. \right\}, \\
& \left\{ 0, 0, 24 + 50 d\gamma + 35 d\gamma^2 + 10 d\gamma^3 + d\gamma^4 + 24 h_1^2 + 14 d\gamma h_1^2 + 2 d\gamma^2 h_1^2 + h_1^4 + 24 x_1^2 + \right. \\
& 14 d\gamma x_1^2 + 2 d\gamma^2 x_1^2 + 2 h_1^2 x_1^2 + x_1^4 + 24 y_1^2 + 14 d\gamma y_1^2 + 2 d\gamma^2 y_1^2 + 2 h_1^2 y_1^2 + 2 x_1^2 y_1^2 + y_1^4, 0, 0 \left. \right\}, \\
& \left\{ 0, 0, 0, 24 + 50 d\gamma + 35 d\gamma^2 + 10 d\gamma^3 + d\gamma^4 + 24 \, i \, h_1 + 26 \, i \, d\gamma h_1 + 9 \, i \, d\gamma^2 h_1 + i \, d\gamma^3 h_1 + \right. \\
& 12 h_1^2 + 7 d\gamma h_1^2 + d\gamma^2 h_1^2 + 4 \, i \, h_1^3 + i \, d\gamma h_1^3 + 12 x_1^2 + 7 d\gamma x_1^2 + d\gamma^2 x_1^2 + 4 \, i \, h_1 x_1^2 + \\
& i \, d\gamma h_1 x_1^2 + 12 y_1^2 + 7 d\gamma y_1^2 + d\gamma^2 y_1^2 + 4 \, i \, h_1 y_1^2 + i \, d\gamma h_1 y_1^2, 24 x_1 + 26 d\gamma x_1 + 9 d\gamma^2 x_1 + \\
& d\gamma^3 x_1 + 4 h_1^2 x_1 + d\gamma h_1^2 x_1 + 4 x_1^3 + d\gamma x_1^3 + 24 \, i \, y_1 + 26 \, i \, d\gamma y_1 + 9 \, i \, d\gamma^2 y_1 + i \, d\gamma^3 y_1 + \\
& 4 \, i \, h_1^2 y_1 + i \, d\gamma h_1^2 y_1 + 4 \, i \, x_1^2 y_1 + i \, d\gamma x_1^2 y_1 + 4 x_1 y_1^2 + d\gamma x_1 y_1^2 + 4 \, i \, y_1^3 + i \, d\gamma y_1^3 \left. \right\}, \\
& \left\{ 0, 0, 0, -24 x_1 - 26 d\gamma x_1 - 9 d\gamma^2 x_1 - d\gamma^3 x_1 - 4 h_1^2 x_1 - d\gamma h_1^2 x_1 - 4 x_1^3 - d\gamma x_1^3 + 24 \, i \, y_1 + \right. \\
& 26 \, i \, d\gamma y_1 + 9 \, i \, d\gamma^2 y_1 + i \, d\gamma^3 y_1 + 4 \, i \, h_1^2 y_1 + i \, d\gamma h_1^2 y_1 + 4 \, i \, x_1^2 y_1 + i \, d\gamma x_1^2 y_1 - 4 x_1 y_1^2 - \\
& d\gamma x_1 y_1^2 + 4 \, i \, y_1^3 + i \, d\gamma y_1^3, 24 + 50 d\gamma + 35 d\gamma^2 + 10 d\gamma^3 + d\gamma^4 - 24 \, i \, h_1 - 26 \, i \, d\gamma h_1 - \\
& 9 \, i \, d\gamma^2 h_1 - i \, d\gamma^3 h_1 + 12 h_1^2 + 7 d\gamma h_1^2 + d\gamma^2 h_1^2 - 4 \, i \, h_1^3 - i \, d\gamma h_1^3 + 12 x_1^2 + 7 d\gamma x_1^2 + d\gamma^2 x_1^2 - \\
& 4 \, i \, h_1 x_1^2 - i \, d\gamma h_1 x_1^2 + 12 y_1^2 + 7 d\gamma y_1^2 + d\gamma^2 y_1^2 - 4 \, i \, h_1 y_1^2 - i \, d\gamma h_1 y_1^2 \left. \right\} // \text{MatrixForm}
\end{aligned}$$

In[ ] := (\* Apply  $Y_{\alpha 1}$  to  $t^{\lambda_{11}}$  \*)

$$\begin{aligned}
& - \left( - (y_1 - i x_1) D[A[[1, 1]], h_1] - \frac{1}{2} * (2 h_2 - 2 h_1) D[A[[1, 1]], y_1] + i \frac{1}{2} * (2 h_2 - 2 h_1) \right. \\
& D[A[[1, 1]], x_1] - \frac{1}{2} * (x_2 - i y_2) D[A[[1, 1]], x_3] - \frac{1}{2} * (y_2 + i x_2) D[A[[1, 1]], y_3] - \\
& \left. \frac{1}{2} * (-x_3 - i y_3) D[A[[1, 1]], x_2] - \frac{1}{2} * (-y_3 + i x_3) D[A[[1, 1]], y_2] \right) // \text{Expand}
\end{aligned}$$

```

In[ ]:= 24 x1 + 26 dY x1 + 9 dY^2 x1 + dY^3 x1 + 4 h1^2 x1 + dY h1^2 x1 + 24 i h2 x1 + 14 i dY h2 x1 + 2 i dY^2 h2 x1 -
      8 h1 h2 x1 - 2 dY h1 h2 x1 + 4 x1^3 + dY x1^3 + 8 x1 x2^2 + 2 dY x1 x2^2 + 2 i h1 x1 x2^2 - 24 x2 x3 - 14 dY x2 x3 -
      2 dY^2 x2 x3 - 2 h1^2 x2 x3 - 16 i h2 x2 x3 - 4 i dY h2 x2 x3 + 4 h1 h2 x2 x3 - 2 x1^2 x2 x3 + 8 x1 x3^2 +
      2 dY x1 x3^2 - 2 i h1 x1 x3^2 + 4 i h2 x1 x3^2 - 8 sqrt(2) x2^2 x4 - 2 sqrt(2) dY x2^2 x4 - 2 i sqrt(2) h1 x2^2 x4 +
      4 sqrt(2) x1 x2 x3 x4 - 8 sqrt(2) x3^2 x4 - 2 sqrt(2) dY x3^2 x4 + 2 i sqrt(2) h1 x3^2 x4 - 4 i sqrt(2) h2 x3^2 x4 - 8 x1 x4^2 -
      2 dY x1 x4^2 - 4 x2 x3 x4^2 + 24 i y1 + 26 i dY y1 + 9 i dY^2 y1 + i dY^3 y1 + 4 i h1^2 y1 + i dY h1^2 y1 -
      24 h2 y1 - 14 dY h2 y1 - 2 dY^2 h2 y1 - 8 i h1 h2 y1 - 2 i dY h1 h2 y1 + 4 i x1^2 y1 + i dY x1^2 y1 + 8 i x2^2 y1 +
      2 i dY x2^2 y1 - 2 h1 x2^2 y1 - 4 i x1 x2 x3 y1 + 8 i x3^2 y1 + 2 i dY x3^2 y1 + 2 h1 x3^2 y1 - 4 h2 x3^2 y1 +
      4 i sqrt(2) x2 x3 x4 y1 - 8 i x4^2 y1 - 2 i dY x4^2 y1 + 4 x1 y1^2 + dY x1 y1^2 + 2 x2 x3 y1^2 + 4 i y1^3 + i dY y1^3 +
      24 i x3 y2 + 14 i dY x3 y2 + 2 i dY^2 x3 y2 + 2 i h1^2 x3 y2 - 16 h2 x3 y2 - 4 dY h2 x3 y2 - 4 i h1 h2 x3 y2 -
      2 i x1^2 x3 y2 + 16 i sqrt(2) x2 x4 y2 + 4 i sqrt(2) dY x2 x4 y2 - 4 sqrt(2) h1 x2 x4 y2 + 4 i x3 x4^2 y2 +
      4 x1 x3 y1 y2 + 2 i x3 y1^2 y2 + 8 x1 y2^2 + 2 dY x1 y2^2 + 2 i h1 x1 y2^2 + 8 sqrt(2) x4 y2^2 + 2 sqrt(2) dY x4 y2^2 +
      2 i sqrt(2) h1 x4 y2^2 + 8 i y1 y2^2 + 2 i dY y1 y2^2 - 2 h1 y1 y2^2 - 24 i x2 y3 - 14 i dY x2 y3 - 2 i dY^2 x2 y3 -
      2 i h1^2 x2 y3 + 16 h2 x2 y3 + 4 dY h2 x2 y3 + 4 i h1 h2 x2 y3 + 2 i x1^2 x2 y3 - 16 i sqrt(2) x3 x4 y3 -
      4 i sqrt(2) dY x3 x4 y3 - 4 sqrt(2) h1 x3 x4 y3 + 8 sqrt(2) h2 x3 x4 y3 - 4 i x2 x4^2 y3 - 4 x1 x2 y1 y3 -
      2 i x2 y1^2 y3 - 24 y2 y3 - 14 dY y2 y3 - 2 dY^2 y2 y3 - 2 h1^2 y2 y3 - 16 i h2 y2 y3 - 4 i dY h2 y2 y3 +
      4 h1 h2 y2 y3 - 2 x1^2 y2 y3 - 4 sqrt(2) x1 x4 y2 y3 - 4 x4^2 y2 y3 - 4 i x1 y1 y2 y3 - 4 i sqrt(2) x4 y1 y2 y3 +
      2 y1^2 y2 y3 + 8 x1 y3^2 + 2 dY x1 y3^2 - 2 i h1 x1 y3^2 + 4 i h2 x1 y3^2 + 8 sqrt(2) x4 y3^2 + 2 sqrt(2) dY x4 y3^2 -
      2 i sqrt(2) h1 x4 y3^2 + 4 i sqrt(2) h2 x4 y3^2 + 8 i y1 y3^2 + 2 i dY y1 y3^2 + 2 h1 y1 y3^2 - 4 h2 y1 y3^2 - 8 i sqrt(2) x2^2 y4 -
      2 i sqrt(2) dY x2^2 y4 + 2 sqrt(2) h1 x2^2 y4 + 8 i sqrt(2) x3^2 y4 + 2 i sqrt(2) dY x3^2 y4 + 2 sqrt(2) h1 x3^2 y4 -
      4 sqrt(2) h2 x3^2 y4 - 16 sqrt(2) x2 y2 y4 - 4 sqrt(2) dY x2 y2 y4 - 4 i sqrt(2) h1 x2 y2 y4 + 4 sqrt(2) x1 x3 y2 y4 +
      4 i sqrt(2) x3 y1 y2 y4 + 8 i sqrt(2) y2^2 y4 + 2 i sqrt(2) dY y2^2 y4 - 2 sqrt(2) h1 y2^2 y4 + 4 sqrt(2) x1 x2 y3 y4 -
      16 sqrt(2) x3 y3 y4 - 4 sqrt(2) dY x3 y3 y4 + 4 i sqrt(2) h1 x3 y3 y4 - 8 i sqrt(2) h2 x3 y3 y4 + 4 i sqrt(2) x2 y1 y3 y4 -
      8 i sqrt(2) y3^2 y4 - 2 i sqrt(2) dY y3^2 y4 - 2 sqrt(2) h1 y3^2 y4 + 4 sqrt(2) h2 y3^2 y4 - 8 x1 y4^2 - 2 dY x1 y4^2 -
      4 x2 x3 y4^2 - 8 i y1 y4^2 - 2 i dY y1 y4^2 + 4 i x3 y2 y4^2 - 4 i x2 y3 y4^2 - 4 y2 y3 y4^2 - A[[1, 2]] // Expand

```

Out[ ]:= 0



In[\*]:= A[[1, 2]] // Expand

Out[\*]= 
$$\begin{aligned} & 24 x_1 + 26 d_\gamma x_1 + 9 d_\gamma^2 x_1 + d_\gamma^3 x_1 + 4 h_1^2 x_1 + d_\gamma h_1^2 x_1 + 24 i h_2 x_1 + 14 i d_\gamma h_2 x_1 + 2 i d_\gamma^2 h_2 x_1 - \\ & 8 h_1 h_2 x_1 - 2 d_\gamma h_1 h_2 x_1 + 4 x_1^3 + d_\gamma x_1^3 + 8 x_1 x_2^2 + 2 d_\gamma x_1 x_2^2 + 2 i h_1 x_1 x_2^2 - 24 x_2 x_3 - 14 d_\gamma x_2 x_3 - \\ & 2 d_\gamma^2 x_2 x_3 - 2 h_1^2 x_2 x_3 - 16 i h_2 x_2 x_3 - 4 i d_\gamma h_2 x_2 x_3 + 4 h_1 h_2 x_2 x_3 - 2 x_1^2 x_2 x_3 + 8 x_1 x_3^2 + \\ & 2 d_\gamma x_1 x_3^2 - 2 i h_1 x_1 x_3^2 + 4 i h_2 x_1 x_3^2 - 8 \sqrt{2} x_2^2 x_4 - 2 \sqrt{2} d_\gamma x_2^2 x_4 - 2 i \sqrt{2} h_1 x_2^2 x_4 + \\ & 4 \sqrt{2} x_1 x_2 x_3 x_4 - 8 \sqrt{2} x_3^2 x_4 - 2 \sqrt{2} d_\gamma x_3^2 x_4 + 2 i \sqrt{2} h_1 x_3^2 x_4 - 4 i \sqrt{2} h_2 x_3^2 x_4 - 8 x_1 x_4^2 - \\ & 2 d_\gamma x_1 x_4^2 - 4 x_2 x_3 x_4^2 + 24 i y_1 + 26 i d_\gamma y_1 + 9 i d_\gamma^2 y_1 + i d_\gamma^3 y_1 + 4 i h_1^2 y_1 + i d_\gamma h_1^2 y_1 - \\ & 24 h_2 y_1 - 14 d_\gamma h_2 y_1 - 2 d_\gamma^2 h_2 y_1 - 8 i h_1 h_2 y_1 - 2 i d_\gamma h_1 h_2 y_1 + 4 i x_1^2 y_1 + i d_\gamma x_1^2 y_1 + \\ & 8 i x_2^2 y_1 + 2 i d_\gamma x_2^2 y_1 - 2 h_1 x_2^2 y_1 - 4 i x_1 x_2 x_3 y_1 + 8 i x_3^2 y_1 + 2 i d_\gamma x_3^2 y_1 + 2 h_1 x_3^2 y_1 - \\ & 4 h_2 x_3^2 y_1 + 4 i \sqrt{2} x_2 x_3 x_4 y_1 - 8 i x_4^2 y_1 - 2 i d_\gamma x_4^2 y_1 + 4 x_1 y_1^2 + d_\gamma x_1 y_1^2 + 2 x_2 x_3 y_1^2 + 4 i y_1^3 + \\ & i d_\gamma y_1^3 + 24 i x_3 y_2 + 14 i d_\gamma x_3 y_2 + 2 i d_\gamma^2 x_3 y_2 + 2 i h_1^2 x_3 y_2 - 16 h_2 x_3 y_2 - 4 d_\gamma h_2 x_3 y_2 - \\ & 4 i h_1 h_2 x_3 y_2 - 2 i x_1^2 x_3 y_2 + 16 i \sqrt{2} x_2 x_4 y_2 + 4 i \sqrt{2} d_\gamma x_2 x_4 y_2 - 4 \sqrt{2} h_1 x_2 x_4 y_2 + \\ & 4 i x_3 x_4^2 y_2 + 4 x_1 x_3 y_1 y_2 + 2 i x_3 y_1^2 y_2 + 8 x_1 y_2^2 + 2 d_\gamma x_1 y_2^2 + 2 i h_1 x_1 y_2^2 + 8 \sqrt{2} x_4 y_2^2 + \\ & 2 \sqrt{2} d_\gamma x_4 y_2^2 + 2 i \sqrt{2} h_1 x_4 y_2^2 + 8 i y_1 y_2^2 + 2 i d_\gamma y_1 y_2^2 - 2 h_1 y_1 y_2^2 - 24 i x_2 y_3 - 14 i d_\gamma x_2 y_3 - \\ & 2 i d_\gamma^2 x_2 y_3 - 2 i h_1^2 x_2 y_3 + 16 h_2 x_2 y_3 + 4 d_\gamma h_2 x_2 y_3 + 4 i h_1 h_2 x_2 y_3 + 2 i x_1^2 x_2 y_3 - \\ & 16 i \sqrt{2} x_3 x_4 y_3 - 4 i \sqrt{2} d_\gamma x_3 x_4 y_3 - 4 \sqrt{2} h_1 x_3 x_4 y_3 + 8 \sqrt{2} h_2 x_3 x_4 y_3 - 4 i x_2 x_4^2 y_3 - \\ & 4 x_1 x_2 y_1 y_3 - 2 i x_2 y_1^2 y_3 - 24 y_2 y_3 - 14 d_\gamma y_2 y_3 - 2 d_\gamma^2 y_2 y_3 - 2 h_1^2 y_2 y_3 - 16 i h_2 y_2 y_3 - \\ & 4 i d_\gamma h_2 y_2 y_3 + 4 h_1 h_2 y_2 y_3 - 2 x_1^2 y_2 y_3 - 4 \sqrt{2} x_1 x_4 y_2 y_3 - 4 x_4^2 y_2 y_3 - 4 i x_1 y_1 y_2 y_3 - \\ & 4 i \sqrt{2} x_4 y_1 y_2 y_3 + 2 y_1^2 y_2 y_3 + 8 x_1 y_3^2 + 2 d_\gamma x_1 y_3^2 - 2 i h_1 x_1 y_3^2 + 4 i h_2 x_1 y_3^2 + 8 \sqrt{2} x_4 y_3^2 + \\ & 2 \sqrt{2} d_\gamma x_4 y_3^2 - 2 i \sqrt{2} h_1 x_4 y_3^2 + 4 i \sqrt{2} h_2 x_4 y_3^2 + 8 i y_1 y_3^2 + 2 i d_\gamma y_1 y_3^2 + 2 h_1 y_1 y_3^2 - \\ & 4 h_2 y_1 y_3^2 - 8 i \sqrt{2} x_2^2 y_4 - 2 i \sqrt{2} d_\gamma x_2^2 y_4 + 2 \sqrt{2} h_1 x_2^2 y_4 + 8 i \sqrt{2} x_3^2 y_4 + 2 i \sqrt{2} d_\gamma x_3^2 y_4 + \\ & 2 \sqrt{2} h_1 x_3^2 y_4 - 4 \sqrt{2} h_2 x_3^2 y_4 - 16 \sqrt{2} x_2 y_2 y_4 - 4 \sqrt{2} d_\gamma x_2 y_2 y_4 - 4 i \sqrt{2} h_1 x_2 y_2 y_4 + \\ & 4 \sqrt{2} x_1 x_3 y_2 y_4 + 4 i \sqrt{2} x_3 y_1 y_2 y_4 + 8 i \sqrt{2} y_2^2 y_4 + 2 i \sqrt{2} d_\gamma y_2^2 y_4 - 2 \sqrt{2} h_1 y_2^2 y_4 + \\ & 4 \sqrt{2} x_1 x_2 y_3 y_4 - 16 \sqrt{2} x_3 y_3 y_4 - 4 \sqrt{2} d_\gamma x_3 y_3 y_4 + 4 i \sqrt{2} h_1 x_3 y_3 y_4 - 8 i \sqrt{2} h_2 x_3 y_3 y_4 + \\ & 4 i \sqrt{2} x_2 y_1 y_3 y_4 - 8 i \sqrt{2} y_3^2 y_4 - 2 i \sqrt{2} d_\gamma y_3^2 y_4 - 2 \sqrt{2} h_1 y_3^2 y_4 + 4 \sqrt{2} h_2 y_3^2 y_4 - \\ & 8 x_1 y_4^2 - 2 d_\gamma x_1 y_4^2 - 4 x_2 x_3 y_4^2 - 8 i y_1 y_4^2 - 2 i d_\gamma y_1 y_4^2 + 4 i x_3 y_2 y_4^2 - 4 i x_2 y_3 y_4^2 - 4 y_2 y_3 y_4^2 \end{aligned}$$

In[\*]:= (\* Apply  $Y_{\alpha 1}$  to  $t^{\lambda_{14}}$  \*)

k = 4;

$$\begin{aligned} & - \left( - (y_1 - i x_1) D[A[[1, k]], h_1] - \frac{1}{2} * (2 h_2 - 2 h_1) D[A[[1, k]], y_1] + i \frac{1}{2} * (2 h_2 - 2 h_1) \right. \\ & \quad D[A[[1, k]], x_1] - \frac{1}{2} * (x_2 - i y_2) D[A[[1, k]], x_3] - \frac{1}{2} * (y_2 + i x_2) D[A[[1, k]], y_3] - \\ & \quad \left. \frac{1}{2} * (-x_3 - i y_3) D[A[[1, k]], x_2] - \frac{1}{2} * (-y_3 + i x_3) D[A[[1, k]], y_2] \right) // \text{Expand} \end{aligned}$$

```

In[ ]:= 2 x1^2 x2^2 + 4 i h1 x1 x2 x3 - 8 i h2 x1 x2 x3 - 24 x3^2 - 14 dY x3^2 - 2 dY^2 x3^2 - 2 h1^2 x3^2 + 8 h1 h2 x3^2 - 8 h2^2 x3^2 -
24 sqrt(2) x1 x4 - 14 sqrt(2) dY x1 x4 - 2 sqrt(2) dY^2 x1 x4 - 4 sqrt(2) x1 x2^2 x4 - 4 i sqrt(2) h1 x2 x3 x4 +
8 i sqrt(2) h2 x2 x3 x4 + 4 x2^2 x4^2 + 4 i x1 x2^2 y1 - 4 h1 x2 x3 y1 + 8 h2 x2 x3 y1 - 24 i sqrt(2) x4 y1 -
14 i sqrt(2) dY x4 y1 - 2 i sqrt(2) dY^2 x4 y1 - 4 i sqrt(2) x2^2 x4 y1 - 2 x2^2 y1^2 + 4 i x1^2 x2 y2 - 4 h1 x1 x3 y2 +
8 h2 x1 x3 y2 - 4 sqrt(2) h1 x3 x4 y2 + 8 sqrt(2) h2 x3 x4 y2 - 8 i x2 x4^2 y2 - 8 x1 x2 y1 y2 -
4 i h1 x3 y1 y2 + 8 i h2 x3 y1 y2 - 4 i x2 y1^2 y2 - 2 x1^2 y2^2 - 4 sqrt(2) x1 x4 y2^2 - 4 x4^2 y2^2 - 4 i x1 y1 y2^2 -
4 i sqrt(2) x4 y1 y2^2 + 2 y1^2 y2^2 - 4 h1 x1 x2 y3 + 8 h2 x1 x2 y3 - 48 i x3 y3 - 28 i dY x3 y3 - 4 i dY^2 x3 y3 -
4 i h1^2 x3 y3 + 16 i h1 h2 x3 y3 - 16 i h2^2 x3 y3 + 4 sqrt(2) h1 x2 x4 y3 - 8 sqrt(2) h2 x2 x4 y3 -
4 i h1 x2 y1 y3 + 8 i h2 x2 y1 y3 - 4 i h1 x1 y2 y3 + 8 i h2 x1 y2 y3 - 4 i sqrt(2) h1 x4 y2 y3 +
8 i sqrt(2) h2 x4 y2 y3 + 4 h1 y1 y2 y3 - 8 h2 y1 y2 y3 + 24 y3^2 + 14 dY y3^2 + 2 dY^2 y3^2 + 2 h1^2 y3^2 -
8 h1 h2 y3^2 + 8 h2^2 y3^2 - 24 i sqrt(2) x1 y4 - 14 i sqrt(2) dY x1 y4 - 2 i sqrt(2) dY^2 x1 y4 - 4 i sqrt(2) x1 x2^2 y4 +
4 sqrt(2) h1 x2 x3 y4 - 8 sqrt(2) h2 x2 x3 y4 + 8 i x2^2 x4 y4 + 24 sqrt(2) y1 y4 + 14 sqrt(2) dY y1 y4 +
2 sqrt(2) dY^2 y1 y4 + 4 sqrt(2) x2^2 y1 y4 - 4 i sqrt(2) h1 x3 y2 y4 + 8 i sqrt(2) h2 x3 y2 y4 + 16 x2 x4 y2 y4 -
4 i sqrt(2) x1 y2^2 y4 - 8 i x4 y2^2 y4 + 4 sqrt(2) y1 y2^2 y4 + 4 i sqrt(2) h1 x2 y3 y4 - 8 i sqrt(2) h2 x2 y3 y4 +
4 sqrt(2) h1 y2 y3 y4 - 8 sqrt(2) h2 y2 y3 y4 - 4 x2^2 y4^2 + 8 i x2 y2 y4^2 + 4 y2^2 y4^2 - A[[1, 5]] // Expand

```

Out[ ]:= 0

```

In[ ]:= x1^2 x2^2 + 2 i h1 x1 x2 x3 - 4 i h2 x1 x2 x3 - 12 x3^2 - 7 dY x3^2 - dY^2 x3^2 - h1^2 x3^2 + 4 h1 h2 x3^2 - 4 h2^2 x3^2 - 24 x1 x4 -
14 dY x1 x4 - 2 dY^2 x1 x4 - 2 x1 x2^2 x4 - 2 i h1 x2 x3 x4 + 4 i h2 x2 x3 x4 + x2^2 x4^2 + 2 i x1 x2^2 y1 -
2 h1 x2 x3 y1 + 4 h2 x2 x3 y1 - 24 i x4 y1 - 14 i dY x4 y1 - 2 i dY^2 x4 y1 - 2 i x2^2 x4 y1 - x2^2 y1^2 +
2 i x1^2 x2 y2 - 2 h1 x1 x3 y2 + 4 h2 x1 x3 y2 - 2 h1 x3 x4 y2 + 4 h2 x3 x4 y2 - 2 i x2 x4^2 y2 - 4 x1 x2 y1 y2 -
2 i h1 x3 y1 y2 + 4 i h2 x3 y1 y2 - 2 i x2 y1^2 y2 - x1^2 y2^2 - 2 x1 x4 y2^2 - x4^2 y2^2 - 2 i x1 y1 y2^2 - 2 i x4 y1 y2^2 +
y1^2 y2^2 - 2 h1 x1 x2 y3 + 4 h2 x1 x2 y3 - 24 i x3 y3 - 14 i dY x3 y3 - 2 i dY^2 x3 y3 - 2 i h1^2 x3 y3 +
8 i h1 h2 x3 y3 - 8 i h2^2 x3 y3 + 2 h1 x2 x4 y3 - 4 h2 x2 x4 y3 - 2 i h1 x2 y1 y3 + 4 i h2 x2 y1 y3 -
2 i h1 x1 y2 y3 + 4 i h2 x1 y2 y3 - 2 i h1 x4 y2 y3 + 4 i h2 x4 y2 y3 + 2 h1 y1 y2 y3 - 4 h2 y1 y2 y3 + 12 y3^2 +
7 dY y3^2 + dY^2 y3^2 + h1^2 y3^2 - 4 h1 h2 y3^2 + 4 h2^2 y3^2 - 24 i x1 y4 - 14 i dY x1 y4 - 2 i dY^2 x1 y4 - 2 i x1 x2^2 y4 +
2 h1 x2 x3 y4 - 4 h2 x2 x3 y4 + 2 i x2^2 x4 y4 + 24 y1 y4 + 14 dY y1 y4 + 2 dY^2 y1 y4 + 2 x2^2 y1 y4 -
2 i h1 x3 y2 y4 + 4 i h2 x3 y2 y4 + 4 x2 x4 y2 y4 - 2 i x1 y2^2 y4 - 2 i x4 y2^2 y4 + 2 y1 y2^2 y4 + 2 i h1 x2 y3 y4 -
4 i h2 x2 y3 y4 + 2 h1 y2 y3 y4 - 4 h2 y2 y3 y4 - x2^2 y4^2 + 2 i x2 y2 y4^2 + y2^2 y4^2 - A[[1, 5]] // Expand

```

Out[ ]:= 0

```

In[ ]:= (* Apply Y_{\alpha 2} to t^{\lambda_{11}} *)

```

k = 2;

```

- ( - 1/2 ( (2 y2 - i 2 x2) D[A[[1, k]], h2] + (h1 - 2 h2) D[A[[1, k]], y2] +
i (-h1 + 2 h2) D[A[[1, k]], x2] + (2 x3 + i 2 y3) D[A[[1, k]], x1] +
(2 y3 - i 2 x3) D[A[[1, k]], y1] + ((-x1 - sqrt(2) x4) + i (y1 - sqrt(2) y4)) D[A[[1, k]], x3] +
((-y1 - sqrt(2) y4) + i (-x1 + sqrt(2) x4)) D[A[[1, k]], y3] +
(sqrt(2) x3 - i sqrt(2) y3) D[A[[1, k]], x4] + (sqrt(2) y3 + i sqrt(2) x3) D[A[[1, k]], y4] ) // Expand

```

```

In[ ]:= 24 x1 x2 + 14 dY x1 x2 + 2 dY^2 x1 x2 + 2 h1^2 x1 x2 + 16 i h2 x1 x2 + 4 i dY h2 x1 x2 - 4 h1 h2 x1 x2 + 2 x1^3 x2 +
48 x3 + 52 dY x3 + 18 dY^2 x3 + 2 dY^3 x3 + 24 i h1 x3 + 14 i dY h1 x3 + 2 i dY^2 h1 x3 + 8 h1^2 x3 +
2 dY h1^2 x3 + 2 i h1^3 x3 - 32 h1 h2 x3 - 8 dY h1 h2 x3 - 8 i h1^2 h2 x3 + 32 h2^2 x3 + 8 dY h2^2 x3 + 8 i h1 h2^2 x3 +
8 x1^2 x3 + 2 dY x1^2 x3 + 2 i h1 x1^2 x3 - 4 i h2 x1^2 x3 + 24 sqrt(2) x2 x4 + 14 sqrt(2) dY x2 x4 + 2 sqrt(2) dY^2 x2 x4 +
16 i sqrt(2) h1 x2 x4 + 4 i sqrt(2) dY h1 x2 x4 - 2 sqrt(2) h1^2 x2 x4 - 16 i sqrt(2) h2 x2 x4 - 4 i sqrt(2) dY h2 x2 x4 +
4 sqrt(2) h1 h2 x2 x4 - 2 sqrt(2) x1^2 x2 x4 - 16 sqrt(2) x1 x3 x4 - 4 sqrt(2) dY x1 x3 x4 - 4 x1 x2 x4^2 + 16 x3 x4^2 +
4 dY x3 x4^2 - 4 i h1 x3 x4^2 + 8 i h2 x3 x4^2 + 4 sqrt(2) x2 x4^3 + 24 i x2 y1 + 14 i dY x2 y1 + 2 i dY^2 x2 y1 +
2 i h1^2 x2 y1 - 16 h2 x2 y1 - 4 dY h2 x2 y1 - 4 i h1 h2 x2 y1 + 2 i x1^2 x2 y1 - 16 i sqrt(2) x3 x4 y1 -
4 i sqrt(2) dY x3 x4 y1 - 4 i x2 x4^2 y1 + 2 x1 x2 y1^2 + 8 x3 y1^2 + 2 dY x3 y1^2 + 2 i h1 x3 y1^2 - 4 i h2 x3 y1^2 -
2 sqrt(2) x2 x4 y1^2 + 2 i x2 y1^3 + 24 i x1 y2 + 14 i dY x1 y2 + 2 i dY^2 x1 y2 + 2 i h1^2 x1 y2 - 16 h2 x1 y2 -
4 dY h2 x1 y2 - 4 i h1 h2 x1 y2 + 2 i x1^3 y2 - 24 i sqrt(2) x4 y2 - 14 i sqrt(2) dY x4 y2 - 2 i sqrt(2) dY^2 x4 y2 +
16 sqrt(2) h1 x4 y2 + 4 sqrt(2) dY h1 x4 y2 + 2 i sqrt(2) h1^2 x4 y2 - 16 sqrt(2) h2 x4 y2 - 4 sqrt(2) dY h2 x4 y2 -
4 i sqrt(2) h1 h2 x4 y2 + 2 i sqrt(2) x1^2 x4 y2 - 4 i x1 x4^2 y2 - 4 i sqrt(2) x4^3 y2 - 24 y1 y2 - 14 dY y1 y2 -
2 dY^2 y1 y2 - 2 h1^2 y1 y2 - 16 i h2 y1 y2 - 4 i dY h2 y1 y2 + 4 h1 h2 y1 y2 - 2 x1^2 y1 y2 + 4 x4^2 y1 y2 +
2 i x1 y1^2 y2 + 2 i sqrt(2) x4 y1^2 y2 - 2 y1^3 y2 + 48 i y3 + 52 i dY y3 + 18 i dY^2 y3 + 2 i dY^3 y3 - 24 h1 y3 -
14 dY h1 y3 - 2 dY^2 h1 y3 + 8 i h1^2 y3 + 2 i dY h1^2 y3 - 2 h1^3 y3 - 32 i h1 h2 y3 - 8 i dY h1 h2 y3 +
8 h2^2 h2 y3 + 32 i h2^2 y3 + 8 i dY h2^2 y3 - 8 h1 h2^2 y3 + 8 i x1^2 y3 + 2 i dY x1^2 y3 - 2 h1 x1^2 y3 +
4 h2 x1^2 y3 + 16 i sqrt(2) x1 x4 y3 + 4 i sqrt(2) dY x1 x4 y3 + 16 i x4^2 y3 + 4 i dY x4^2 y3 + 4 h1 x4^2 y3 -
8 h2 x4^2 y3 - 16 sqrt(2) x4 y1 y3 - 4 sqrt(2) dY x4 y1 y3 + 8 i y1^2 y3 + 2 i dY y1^2 y3 - 2 h1 y1^2 y3 + 4 h2 y1^2 y3 +
24 i sqrt(2) x2 y4 + 14 i sqrt(2) dY x2 y4 + 2 i sqrt(2) dY^2 x2 y4 - 16 sqrt(2) h1 x2 y4 - 4 sqrt(2) dY h1 x2 y4 -
2 i sqrt(2) h1^2 x2 y4 + 16 sqrt(2) h2 x2 y4 + 4 sqrt(2) dY h2 x2 y4 + 4 i sqrt(2) h1 h2 x2 y4 - 2 i sqrt(2) x1^2 x2 y4 -
16 i sqrt(2) x1 x3 y4 - 4 i sqrt(2) dY x1 x3 y4 + 4 i sqrt(2) x2 x4^2 y4 + 16 sqrt(2) x3 y1 y4 + 4 sqrt(2) dY x3 y1 y4 -
2 i sqrt(2) x2 y1^2 y4 + 24 sqrt(2) y2 y4 + 14 sqrt(2) dY y2 y4 + 2 sqrt(2) dY^2 y2 y4 + 16 i sqrt(2) h1 y2 y4 +
4 i sqrt(2) dY h1 y2 y4 - 2 sqrt(2) h1^2 y2 y4 - 16 i sqrt(2) h2 y2 y4 - 4 i sqrt(2) dY h2 y2 y4 + 4 sqrt(2) h1 h2 y2 y4 -
2 sqrt(2) x1^2 y2 y4 + 4 sqrt(2) x4^2 y2 y4 - 2 sqrt(2) y1^2 y2 y4 - 16 sqrt(2) x1 y3 y4 - 4 sqrt(2) dY x1 y3 y4 -
16 i sqrt(2) y1 y3 y4 - 4 i sqrt(2) dY y1 y3 y4 - 4 x1 x2 y4^2 + 16 x3 y4^2 + 4 dY x3 y4^2 - 4 i h1 x3 y4^2 +
8 i h2 x3 y4^2 + 4 sqrt(2) x2 x4 y4^2 - 4 i x2 y1 y4^2 - 4 i x1 y2 y4^2 - 4 i sqrt(2) x4 y2 y4^2 + 4 y1 y2 y4^2 + 16 i y3 y4^2 +
4 i dY y3 y4^2 + 4 h1 y3 y4^2 - 8 h2 y3 y4^2 + 4 i sqrt(2) x2 y4^3 + 4 sqrt(2) y2 y4^3 - sqrt(2) A[[1, 3]] // Expand

```

Out[ ]:= 0

```

In[ ]:= (* Apply Y_{\alpha 2} to t^{\lambda_{11}} *)

```

```

k = 3;

```

```

- ( - 1/2 ( (2 y2 - i 2 x2) D[A[[1, k]], h2] + (h1 - 2 h2) D[A[[1, k]], y2] +
i (-h1 + 2 h2) D[A[[1, k]], x2] + (2 x3 + i 2 y3) D[A[[1, k]], x1] +
(2 y3 - i 2 x3) D[A[[1, k]], y1] + ((-x1 - sqrt(2) x4) + i (y1 - sqrt(2) y4)) D[A[[1, k]], x3] +
((-y1 - sqrt(2) y4) + i (-x1 + sqrt(2) x4)) D[A[[1, k]], y3] +
(sqrt(2) x3 - i sqrt(2) y3) D[A[[1, k]], x4] + (sqrt(2) y3 + i sqrt(2) x3) D[A[[1, k]], y4] ) // Expand

```

```

In[ ]:= 8  $\sqrt{2}$  x1 x22 + 2  $\sqrt{2}$  dY x1 x22 + 2  $\sqrt{2}$  h1 x1 x22 + 24  $\sqrt{2}$  x2 x3 + 14  $\sqrt{2}$  dY x2 x3 + 2  $\sqrt{2}$  dY2 x2 x3 +
16  $\sqrt{2}$  h1 x2 x3 + 4  $\sqrt{2}$  dY h1 x2 x3 - 2  $\sqrt{2}$  h12 x2 x3 - 16  $\sqrt{2}$  h2 x2 x3 - 4  $\sqrt{2}$  dY h2 x2 x3 +
4  $\sqrt{2}$  h1 h2 x2 x3 + 2  $\sqrt{2}$  x12 x2 x3 + 8  $\sqrt{2}$  x1 x22 + 2  $\sqrt{2}$  dY x1 x22 + 2  $\sqrt{2}$  h1 x1 x22 -
4  $\sqrt{2}$  h2 x1 x22 - 48 x4 - 52 dY x4 - 18 dY2 x4 - 2 dY3 x4 - 48  $\sqrt{2}$  h1 x4 - 28  $\sqrt{2}$  dY h1 x4 -
4  $\sqrt{2}$  dY2 h1 x4 + 8 h12 x4 + 2 dY h12 x4 + 48  $\sqrt{2}$  h2 x4 + 28  $\sqrt{2}$  dY h2 x4 + 4  $\sqrt{2}$  dY2 h2 x4 - 16 h1 h2 x4 -
4 dY h1 h2 x4 + 8 x12 x4 + 2 dY x12 x4 - 16 x22 x4 - 4 dY x22 x4 - 4  $\sqrt{2}$  h1 x22 x4 - 8 x1 x2 x3 x4 - 16 x32 x4 -
4 dY x32 x4 - 4  $\sqrt{2}$  h1 x32 x4 + 8  $\sqrt{2}$  h2 x32 x4 + 4  $\sqrt{2}$  x2 x3 x42 - 16 x43 - 4 dY x43 + 8  $\sqrt{2}$  x22 y1 +
2  $\sqrt{2}$  dY x22 y1 - 2  $\sqrt{2}$  h1 x22 y1 - 8  $\sqrt{2}$  x32 y1 - 2  $\sqrt{2}$  dY x32 y1 + 2  $\sqrt{2}$  h1 x32 y1 -
4  $\sqrt{2}$  h2 x32 y1 + 2  $\sqrt{2}$  x2 x3 y12 + 8 x4 y12 + 2 dY x4 y12 + 16  $\sqrt{2}$  x1 x2 y2 + 4  $\sqrt{2}$  dY x1 x2 y2 -
4  $\sqrt{2}$  h1 x1 x2 y2 + 24  $\sqrt{2}$  x3 y2 + 14  $\sqrt{2}$  dY x3 y2 + 2  $\sqrt{2}$  dY2 x3 y2 - 16  $\sqrt{2}$  h1 x3 y2 -
4  $\sqrt{2}$  dY h1 x3 y2 - 2  $\sqrt{2}$  h12 x3 y2 + 16  $\sqrt{2}$  h2 x3 y2 + 4  $\sqrt{2}$  dY h2 x3 y2 + 4  $\sqrt{2}$  h1 h2 x3 y2 +
2  $\sqrt{2}$  x12 x3 y2 - 4  $\sqrt{2}$  x3 x42 y2 - 16  $\sqrt{2}$  x2 y1 y2 - 4  $\sqrt{2}$  dY x2 y1 y2 - 4  $\sqrt{2}$  h1 x2 y1 y2 +
8 x3 x4 y1 y2 + 2  $\sqrt{2}$  x3 y12 y2 - 8  $\sqrt{2}$  x1 y22 - 2  $\sqrt{2}$  dY x1 y22 - 2  $\sqrt{2}$  h1 x1 y22 - 16 x4 y22 -
4 dY x4 y22 - 4  $\sqrt{2}$  h1 x4 y22 - 8  $\sqrt{2}$  y1 y22 - 2  $\sqrt{2}$  dY y1 y22 + 2  $\sqrt{2}$  h1 y1 y22 + 24  $\sqrt{2}$  x2 y3 +
14  $\sqrt{2}$  dY x2 y3 + 2  $\sqrt{2}$  dY2 x2 y3 - 16  $\sqrt{2}$  h1 x2 y3 - 4  $\sqrt{2}$  dY h1 x2 y3 - 2  $\sqrt{2}$  h12 x2 y3 +
16  $\sqrt{2}$  h2 x2 y3 + 4  $\sqrt{2}$  dY h2 x2 y3 + 4  $\sqrt{2}$  h1 h2 x2 y3 + 2  $\sqrt{2}$  x12 x2 y3 + 16  $\sqrt{2}$  x1 x3 y3 +
4  $\sqrt{2}$  dY x1 x3 y3 - 4  $\sqrt{2}$  h1 x1 x3 y3 + 8  $\sqrt{2}$  h2 x1 x3 y3 - 4  $\sqrt{2}$  x2 x42 y3 + 16  $\sqrt{2}$  x3 y1 y3 +
4  $\sqrt{2}$  dY x3 y1 y3 + 4  $\sqrt{2}$  h1 x3 y1 y3 - 8  $\sqrt{2}$  h2 x3 y1 y3 - 8 x2 x4 y1 y3 + 2  $\sqrt{2}$  x2 y12 y3 -
24  $\sqrt{2}$  y2 y3 - 14  $\sqrt{2}$  dY y2 y3 - 2  $\sqrt{2}$  dY2 y2 y3 - 16  $\sqrt{2}$  h1 y2 y3 - 4  $\sqrt{2}$  dY h1 y2 y3 +
2  $\sqrt{2}$  h12 y2 y3 + 16  $\sqrt{2}$  h2 y2 y3 + 4  $\sqrt{2}$  dY h2 y2 y3 - 4  $\sqrt{2}$  h1 h2 y2 y3 - 2  $\sqrt{2}$  x12 y2 y3 -
8 x1 x4 y2 y3 - 4  $\sqrt{2}$  x42 y2 y3 - 2  $\sqrt{2}$  y12 y2 y3 - 8  $\sqrt{2}$  x1 y32 - 2  $\sqrt{2}$  dY x1 y32 - 2  $\sqrt{2}$  h1 x1 y32 +
4  $\sqrt{2}$  h2 x1 y32 - 16 x4 y32 - 4 dY x4 y32 - 4  $\sqrt{2}$  h1 x4 y32 + 8  $\sqrt{2}$  h2 x4 y32 + 8  $\sqrt{2}$  y1 y32 + 2  $\sqrt{2}$  dY y1 y32 -
2  $\sqrt{2}$  h1 y1 y32 + 4  $\sqrt{2}$  h2 y1 y32 - 48  $\sqrt{2}$  y4 - 52  $\sqrt{2}$  dY y4 - 18  $\sqrt{2}$  dY2 y4 - 2  $\sqrt{2}$  dY3 y4 + 48 h1 y4 +
28 dY h1 y4 + 4 dY2 h1 y4 + 8  $\sqrt{2}$  h12 y4 + 2  $\sqrt{2}$  dY h12 y4 - 48 h2 y4 - 28 dY h2 y4 - 4 dY2 h2 y4 -
16  $\sqrt{2}$  h1 h2 y4 - 4  $\sqrt{2}$  dY h1 h2 y4 + 8  $\sqrt{2}$  x12 y4 + 2  $\sqrt{2}$  dY x12 y4 - 16  $\sqrt{2}$  x22 y4 - 4  $\sqrt{2}$  dY x22 y4 + 4 h1 x22 y4 -
8  $\sqrt{2}$  x1 x2 x3 y4 - 16  $\sqrt{2}$  x32 y4 - 4  $\sqrt{2}$  dY x32 y4 + 4 h1 x32 y4 - 8 h2 x32 y4 + 8  $\sqrt{2}$  x2 x3 x4 y4 -
16  $\sqrt{2}$  x42 y4 - 4  $\sqrt{2}$  dY x42 y4 + 8  $\sqrt{2}$  y12 y4 + 2  $\sqrt{2}$  dY y12 y4 + 8  $\sqrt{2}$  x3 x4 y2 y4 + 8  $\sqrt{2}$  x3 y1 y2 y4 - 16  $\sqrt{2}$  y22 y4 -
4  $\sqrt{2}$  dY y22 y4 + 4 h1 y22 y4 + 8  $\sqrt{2}$  x2 x4 y3 y4 - 8  $\sqrt{2}$  x2 y1 y3 y4 - 8  $\sqrt{2}$  x1 y2 y3 y4 - 8  $\sqrt{2}$  x4 y2 y3 y4 -
16  $\sqrt{2}$  y32 y4 - 4  $\sqrt{2}$  dY y32 y4 + 4 h1 y32 y4 - 8 h2 y32 y4 - 4  $\sqrt{2}$  x2 x3 y42 - 16 x4 y42 - 4 dY x4 y42 +
4  $\sqrt{2}$  x3 y2 y42 + 4  $\sqrt{2}$  x2 y3 y42 + 4  $\sqrt{2}$  y2 y3 y42 - 16  $\sqrt{2}$  y43 - 4  $\sqrt{2}$  dY y43 -  $\sqrt{2}$  A[[1, 4]] // Expand

```

Out[ ]:= 0

(\*\*\*\*\*)

```

In[ ]:= MatrixExp[t * X1] // MatrixForm

```

Out[ ]:= MatrixForm=

$$\begin{pmatrix} 1 & t & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & -t \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

```
In[ ]:= Temp = {{a1, a2, a3, a4, a5}, {b1, b2, b3, b4, b5},
               {c1, c2, c3, c4, c5}, {d1, d2, d3, d4, d5}, {e1, e2, e3, e4, e5}};
Temp // MatrixForm
```

Out[ ]//MatrixForm=

$$\begin{pmatrix} a1 & a2 & a3 & a4 & a5 \\ b1 & b2 & b3 & b4 & b5 \\ c1 & c2 & c3 & c4 & c5 \\ d1 & d2 & d3 & d4 & d5 \\ e1 & e2 & e3 & e4 & e5 \end{pmatrix}$$

```
In[ ]:= D[Temp.MatrixExp[t * Y1], t] // MatrixForm
```

Out[ ]//MatrixForm=

$$\begin{pmatrix} a2 & 0 & 0 & -a5 & 0 \\ b2 & 0 & 0 & -b5 & 0 \\ c2 & 0 & 0 & -c5 & 0 \\ d2 & 0 & 0 & -d5 & 0 \\ e2 & 0 & 0 & -e5 & 0 \end{pmatrix}$$

```
In[ ]:= D[MatrixExp[t * X1].
          \begin{pmatrix} a2 & 0 & 0 & -a5 & 0 \\ b2 & 0 & 0 & -b5 & 0 \\ c2 & 0 & 0 & -c5 & 0 \\ d2 & 0 & 0 & -d5 & 0 \\ e2 & 0 & 0 & -e5 & 0 \end{pmatrix}, t] // MatrixForm
```

Out[ ]//MatrixForm=

$$\begin{pmatrix} b2 & 0 & 0 & -b5 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ -e2 & 0 & 0 & e5 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

```
In[ ]:= (* Calculate G-induced  $Y_\alpha$  *)
```

```
X11 = t (X1 - Y1);
```

```
Y11 = i t (X1 + Y1);
```

```
X22 = t (X2 - Y2);
```

```
Y22 = i t (X2 + Y2);
```

```
MatrixExp[Y11].M.Inverse[MatrixExp[Y11]] // MatrixForm
```

Out[ ]//MatrixForm=

$$\begin{pmatrix} \frac{(\cos[t]^3 + \cos[t] \sin[t]^2) (i \cos[t] h_1 + i \sin[t] (-x_1 + i y_1))}{\cos[t]^4 + 2 \cos[t]^2 \sin[t]^2 + \sin[t]^4} + \frac{(-i \cos[t]^2 \sin[t] - i \sin[t]^3) (i \sin[t] (-h_1 + 2 i h_2) + \cos[t] (x_1 + i y_1))}{\cos[t]^4 + 2 \cos[t]^2 \sin[t]^2 + \sin[t]^4} \\ \frac{(\cos[t]^3 + \cos[t] \sin[t]^2) (-\sin[t] h_1 + \cos[t] (-x_1 + i y_1))}{\cos[t]^4 + 2 \cos[t]^2 \sin[t]^2 + \sin[t]^4} + \frac{(-i \cos[t]^2 \sin[t] - i \sin[t]^3) (\cos[t] (-h_1 + 2 i h_2) + i \sin[t] (x_1 + i y_1))}{\cos[t]^4 + 2 \cos[t]^2 \sin[t]^2 + \sin[t]^4} \\ \frac{(-i \cos[t]^2 \sin[t] - i \sin[t]^3) (-\sqrt{2} x_2 + i \sqrt{2} y_2)}{\cos[t]^4 + 2 \cos[t]^2 \sin[t]^2 + \sin[t]^4} + \frac{(\cos[t]^3 + \cos[t] \sin[t]^2) (-\sqrt{2} x_3 + i \sqrt{2} y_3)}{\cos[t]^4 + 2 \cos[t]^2 \sin[t]^2 + \sin[t]^4} \\ \frac{\cos[t] (\cos[t]^3 + \cos[t] \sin[t]^2) (\sqrt{2} x_4 - i \sqrt{2} y_4)}{\cos[t]^4 + 2 \cos[t]^2 \sin[t]^2 + \sin[t]^4} + \frac{i \sin[t] (-i \cos[t]^2 \sin[t] - i \sin[t]^3) (\sqrt{2} x_4 - i \sqrt{2} y_4)}{\cos[t]^4 + 2 \cos[t]^2 \sin[t]^2 + \sin[t]^4} \\ \frac{i \sin[t] (\cos[t]^3 + \cos[t] \sin[t]^2) (\sqrt{2} x_4 - i \sqrt{2} y_4)}{\cos[t]^4 + 2 \cos[t]^2 \sin[t]^2 + \sin[t]^4} + \frac{\cos[t] (-i \cos[t]^2 \sin[t] - i \sin[t]^3) (\sqrt{2} x_4 - i \sqrt{2} y_4)}{\cos[t]^4 + 2 \cos[t]^2 \sin[t]^2 + \sin[t]^4} \end{pmatrix}$$

$$In[ ] := D \left[ \begin{array}{l} \frac{(\cos[t]^3 + \cos[t] \sin[t]^2) (\sin[t] h_1 + \sin[t] (-x_1 + \sin[t] y_1))}{\cos[t]^4 + 2 \cos[t]^2 \sin[t]^2 + \sin[t]^4} + \frac{(-\sin[t]^2 \sin[t] - \sin[t]^3) (\sin[t] (-\sin[t] h_1 + 2 \sin[t] h_2) + \cos[t] (-\sin[t] h_1 + 2 \sin[t] h_2) + \sin[t] (-\sin[t] h_1 + 2 \sin[t] h_2))}{\cos[t]^4 + 2 \cos[t]^2 \sin[t]^2 + \sin[t]^4} \\ \frac{(\cos[t]^3 + \cos[t] \sin[t]^2) (-\sin[t] h_1 + \cos[t] (-x_1 + \sin[t] y_1))}{\cos[t]^4 + 2 \cos[t]^2 \sin[t]^2 + \sin[t]^4} + \frac{(-\sin[t]^2 \sin[t] - \sin[t]^3) (\cos[t] (-\sin[t] h_1 + 2 \sin[t] h_2) + \sin[t] (-\sin[t] h_1 + 2 \sin[t] h_2))}{\cos[t]^4 + 2 \cos[t]^2 \sin[t]^2 + \sin[t]^4} \\ \frac{(-\sin[t]^2 \sin[t] - \sin[t]^3) (-\sqrt{2} x_2 + \sqrt{2} y_2)}{\cos[t]^4 + 2 \cos[t]^2 \sin[t]^2 + \sin[t]^4} + \frac{(\cos[t]^3 + \cos[t] \sin[t]^2) (-\sqrt{2} x_3 + \sqrt{2} y_3)}{\cos[t]^4 + 2 \cos[t]^2 \sin[t]^2 + \sin[t]^4} \\ \frac{\cos[t] (\cos[t]^3 + \cos[t] \sin[t]^2) (\sqrt{2} x_4 - \sqrt{2} y_4)}{\cos[t]^4 + 2 \cos[t]^2 \sin[t]^2 + \sin[t]^4} + \frac{\sin[t] (-\sin[t]^2 \sin[t] - \sin[t]^3) (\sqrt{2} x_4 - \sqrt{2} y_4)}{\cos[t]^4 + 2 \cos[t]^2 \sin[t]^2 + \sin[t]^4} \\ \frac{\sin[t] (\cos[t]^3 + \cos[t] \sin[t]^2) (\sqrt{2} x_4 - \sqrt{2} y_4)}{\cos[t]^4 + 2 \cos[t]^2 \sin[t]^2 + \sin[t]^4} + \frac{\cos[t] (-\sin[t]^2 \sin[t] - \sin[t]^3) (\sqrt{2} x_4 - \sqrt{2} y_4)}{\cos[t]^4 + 2 \cos[t]^2 \sin[t]^2 + \sin[t]^4} \end{array} \right]$$

Out[ ] := MatrixForm =

$$\left( \begin{array}{l} \frac{(\cos[t]^3 + \cos[t] \sin[t]^2) (-\sin[t] h_1 + \sin[t] (-x_1 + \sin[t] y_1))}{\cos[t]^4 + 2 \cos[t]^2 \sin[t]^2 + \sin[t]^4} + \frac{(-\cos[t]^2 \sin[t] - \sin[t]^3) (\sin[t] h_1 + \sin[t] (-x_1 + \sin[t] y_1))}{\cos[t]^4 + 2 \cos[t]^2 \sin[t]^2 + \sin[t]^4} + \frac{(-\cos[t]^2 \sin[t] - \sin[t]^3) (-\sin[t] h_1 + \cos[t] (-x_1 + \sin[t] y_1))}{\cos[t]^4 + 2 \cos[t]^2 \sin[t]^2 + \sin[t]^4} + \frac{(\cos[t]^3 + \cos[t] \sin[t]^2) (-\cos[t] h_1 - \sin[t] (-x_1 + \sin[t] y_1))}{\cos[t]^4 + 2 \cos[t]^2 \sin[t]^2 + \sin[t]^4} + \frac{(-\cos[t]^2 \sin[t] - \sin[t]^3) (-\sqrt{2} x_2 + \sqrt{2} y_2)}{\cos[t]^4 + 2 \cos[t]^2 \sin[t]^2 + \sin[t]^4} \\ \frac{\sin[t] (-\cos[t]^3 - \cos[t] \sin[t]^2) (\sqrt{2} x_4 - \sqrt{2} y_4)}{\cos[t]^4 + 2 \cos[t]^2 \sin[t]^2 + \sin[t]^4} - \frac{\sin[t] (\cos[t]^3 + \cos[t] \sin[t]^2) (\sqrt{2} x_4 - \sqrt{2} y_4)}{\cos[t]^4 + 2 \cos[t]^2 \sin[t]^2 + \sin[t]^4} \\ \frac{\cos[t] (-\cos[t]^3 - \cos[t] \sin[t]^2) (\sqrt{2} x_4 - \sqrt{2} y_4)}{\cos[t]^4 + 2 \cos[t]^2 \sin[t]^2 + \sin[t]^4} + \frac{\sin[t] (\cos[t]^3 + \cos[t] \sin[t]^2) (\sqrt{2} x_4 - \sqrt{2} y_4)}{\cos[t]^4 + 2 \cos[t]^2 \sin[t]^2 + \sin[t]^4} \end{array} \right)$$

$$In[ ] := f[t_] := \left( \begin{array}{l} \frac{(\cos[t]^3 + \cos[t] \sin[t]^2) (-\sin[t] h_1 + \sin[t] (-x_1 + \sin[t] y_1))}{\cos[t]^4 + 2 \cos[t]^2 \sin[t]^2 + \sin[t]^4} + \frac{(-\cos[t]^2 \sin[t] - \sin[t]^3) (\sin[t] h_1 + \sin[t] (-x_1 + \sin[t] y_1))}{\cos[t]^4 + 2 \cos[t]^2 \sin[t]^2 + \sin[t]^4} \\ \frac{(-\cos[t]^2 \sin[t] - \sin[t]^3) (-\sin[t] h_1 + \cos[t] (-x_1 + \sin[t] y_1))}{\cos[t]^4 + 2 \cos[t]^2 \sin[t]^2 + \sin[t]^4} + \frac{(\cos[t]^3 + \cos[t] \sin[t]^2) (-\cos[t] h_1 - \sin[t] (-x_1 + \sin[t] y_1))}{\cos[t]^4 + 2 \cos[t]^2 \sin[t]^2 + \sin[t]^4} \\ \frac{(-\cos[t]^2 \sin[t] - \sin[t]^3) (-\sqrt{2} x_2 + \sqrt{2} y_2)}{\cos[t]^4 + 2 \cos[t]^2 \sin[t]^2 + \sin[t]^4} \\ \frac{\sin[t] (-\cos[t]^3 - \cos[t] \sin[t]^2) (\sqrt{2} x_4 - \sqrt{2} y_4)}{\cos[t]^4 + 2 \cos[t]^2 \sin[t]^2 + \sin[t]^4} - \frac{\sin[t] (\cos[t]^3 + \cos[t] \sin[t]^2) (\sqrt{2} x_4 - \sqrt{2} y_4)}{\cos[t]^4 + 2 \cos[t]^2 \sin[t]^2 + \sin[t]^4} \\ \frac{\cos[t] (-\cos[t]^3 - \cos[t] \sin[t]^2) (\sqrt{2} x_4 - \sqrt{2} y_4)}{\cos[t]^4 + 2 \cos[t]^2 \sin[t]^2 + \sin[t]^4} + \frac{\sin[t] (\cos[t]^3 + \cos[t] \sin[t]^2) (\sqrt{2} x_4 - \sqrt{2} y_4)}{\cos[t]^4 + 2 \cos[t]^2 \sin[t]^2 + \sin[t]^4} \end{array} \right)$$

In[ ] := f[0] // Expand

$$In[ ] := \left\{ \left\{ -2 \sin x_1, 2 h_1 - 2 h_2, \sin \sqrt{2} x_2 - \sqrt{2} y_2, 0, 0 \right\}, \left\{ -2 h_1 + 2 h_2, 2 \sin x_1, \sin \sqrt{2} x_3 - \sqrt{2} y_3, 0, 0 \right\}, \right. \\ \left. \left\{ \sin \sqrt{2} x_2 + \sqrt{2} y_2, \sin \sqrt{2} x_3 + \sqrt{2} y_3, 0, \sin \sqrt{2} x_3 - \sqrt{2} y_3, -\sin \sqrt{2} x_2 + \sqrt{2} y_2 \right\}, \right. \\ \left. \left\{ 0, 0, \sin \sqrt{2} x_3 + \sqrt{2} y_3, -2 \sin x_1, 2 h_1 - 2 h_2 \right\}, \right. \\ \left. \left\{ 0, 0, -\sin \sqrt{2} x_2 - \sqrt{2} y_2, -2 h_1 + 2 h_2, 2 \sin x_1 \right\} \right\} // \text{MatrixForm}$$

Out[ ] := MatrixForm =

$$\left( \begin{array}{ccccc} -2 \sin x_1 & 2 h_1 - 2 h_2 & \sin \sqrt{2} x_2 - \sqrt{2} y_2 & 0 & 0 \\ -2 h_1 + 2 h_2 & 2 \sin x_1 & \sin \sqrt{2} x_3 - \sqrt{2} y_3 & 0 & 0 \\ \sin \sqrt{2} x_2 + \sqrt{2} y_2 & \sin \sqrt{2} x_3 + \sqrt{2} y_3 & 0 & \sin \sqrt{2} x_3 - \sqrt{2} y_3 & -\sin \sqrt{2} x_2 + \sqrt{2} y_2 \\ 0 & 0 & \sin \sqrt{2} x_3 + \sqrt{2} y_3 & -2 \sin x_1 & 2 h_1 - 2 h_2 \\ 0 & 0 & -\sin \sqrt{2} x_2 - \sqrt{2} y_2 & -2 h_1 + 2 h_2 & 2 \sin x_1 \end{array} \right) \\ \left( \begin{array}{ccccc} 2 \sin y_1 & -2 \sin h_1 + 2 \sin h_2 & \sqrt{2} x_2 + \sin \sqrt{2} y_2 & 0 & 0 \\ -2 \sin h_1 + 2 \sin h_2 & -2 \sin y_1 & -\sqrt{2} x_3 - \sin \sqrt{2} y_3 & 0 & 0 \\ -\sqrt{2} x_2 + \sin \sqrt{2} y_2 & \sqrt{2} x_3 - \sin \sqrt{2} y_3 & 0 & -\sqrt{2} x_3 - \sin \sqrt{2} y_3 & -\sqrt{2} x_2 - \sin \sqrt{2} y_2 \\ 0 & 0 & \sqrt{2} x_3 - \sin \sqrt{2} y_3 & 2 \sin y_1 & -2 \sin h_1 + 2 \sin h_2 \\ 0 & 0 & \sqrt{2} x_2 - \sin \sqrt{2} y_2 & -2 \sin h_1 + 2 \sin h_2 & -2 \sin y_1 \end{array} \right)$$

In[ ]:= M // MatrixForm

Out[ ]//MatrixForm=

$$\begin{pmatrix} \mathbf{i} h_1 & x_1 + \mathbf{i} y_1 & \sqrt{2} x_3 + \mathbf{i} \sqrt{2} y_3 & -\sqrt{2} x_4 - \mathbf{i} \sqrt{2} y_4 & 0 \\ -x_1 + \mathbf{i} y_1 & -\mathbf{i} h_1 + 2 \mathbf{i} h_2 & \sqrt{2} x_2 + \mathbf{i} \sqrt{2} y_2 & 0 & -\sqrt{2} x_4 - \mathbf{i} \sqrt{2} y_4 \\ -\sqrt{2} x_3 + \mathbf{i} \sqrt{2} y_3 & -\sqrt{2} x_2 + \mathbf{i} \sqrt{2} y_2 & 0 & \sqrt{2} x_2 + \mathbf{i} \sqrt{2} y_2 & -\sqrt{2} x_3 - \mathbf{i} \sqrt{2} y_3 \\ \sqrt{2} x_4 - \mathbf{i} \sqrt{2} y_4 & 0 & -\sqrt{2} x_2 + \mathbf{i} \sqrt{2} y_2 & \mathbf{i} h_1 - 2 \mathbf{i} h_2 & x_1 + \mathbf{i} y_1 \\ 0 & \sqrt{2} x_4 - \mathbf{i} \sqrt{2} y_4 & \sqrt{2} x_3 - \mathbf{i} \sqrt{2} y_3 & -x_1 + \mathbf{i} y_1 & -\mathbf{i} h_1 \end{pmatrix}$$

(\*

**G - induced differential operators:**

$$X_1 : 2y_1 dh_1 + (2h_2 - 2h_1) dy_1 + x_2 dx_3 + y_2 dy_3 - x_3 dx_2 - y_3 dy_2 ;$$

$$Y_1 : -2x_1 dh_1 - (2h_2 - 2h_1) dx_1 - y_2 dx_3 + x_2 dy_3 - y_3 dx_2 + x_3 dy_2 ;$$

$$X_2 : 2y_2 dh_2 + (h_1 - 2h_2) dy_2 + 2x_3 dx_1 + 2y_3 dy_1 + (-x_1 - \sqrt{2} x_4) dx_3 + (-y_1 - \sqrt{2} y_4) dy_3 - \sqrt{2} x_3 dx_4 - \sqrt{2} y_3 dy_4 ;$$

$$Y_2 : -2x_2 dh_2 + (-h_1 + 2h_2) dx_2 + 2y_3 dx_1 - 2x_3 dy_1 + (y_1 - \sqrt{2} y_4) dx_3 + (-x_1 + \sqrt{2} x_4) dy_3 + \sqrt{2} y_3 dx_4 - \sqrt{2} x_3 dy_4 ;$$

$$Y_{\alpha_1}^G = -\frac{1}{2} (X_1^G + \mathbf{i} Y_1^G),$$

$$\text{we know } Y_{\alpha_1} \cdot \pi_{1,1} = \pi_{1,2} \text{ and } -Y_{\alpha_1}^G \cdot \pi_{1,1}(\exp) = \pi_{1,2}(\exp);$$

This holds true for all  $\lambda = \sum_i c_i \lambda_i$

\*)

$$\begin{pmatrix} 0 & 2x_3 + 2\mathbf{i}y_3 & -\sqrt{2}x_1 - 2x_4 - \mathbf{i}\sqrt{2}y_1 - 2\mathbf{i}y_4 & -2x_3 - 2 \\ -2x_3 + 2\mathbf{i}y_3 & 4\mathbf{i}y_2 & \mathbf{i}\sqrt{2}h_1 - 2\mathbf{i}\sqrt{2}h_2 & 0 \\ \sqrt{2}x_1 + 2x_4 - \mathbf{i}\sqrt{2}y_1 - 2\mathbf{i}y_4 & \mathbf{i}\sqrt{2}h_1 - 2\mathbf{i}\sqrt{2}h_2 & 0 & \mathbf{i}\sqrt{2}h_1 - 2 \\ 2x_3 - 2\mathbf{i}y_3 & 0 & \mathbf{i}\sqrt{2}h_1 - 2\mathbf{i}\sqrt{2}h_2 & -4\mathbf{i}y_2 \\ 0 & 2x_3 - 2\mathbf{i}y_3 & -\sqrt{2}x_1 - 2x_4 + \mathbf{i}\sqrt{2}y_1 + 2\mathbf{i}y_4 & -2x_3 + 2 \end{pmatrix}$$

$$\begin{pmatrix} \mathbf{i} h_1 & x_1 + \mathbf{i} y_1 & \sqrt{2} x_3 + \mathbf{i} \sqrt{2} y_3 & -\sqrt{2} x_4 - \mathbf{i} \sqrt{2} y_4 & 0 \\ -x_1 + \mathbf{i} y_1 & -\mathbf{i} h_1 + 2 \mathbf{i} h_2 & \sqrt{2} x_2 + \mathbf{i} \sqrt{2} y_2 & 0 & -\sqrt{2} x_4 - \mathbf{i} \sqrt{2} y_4 \\ -\sqrt{2} x_3 + \mathbf{i} \sqrt{2} y_3 & -\sqrt{2} x_2 + \mathbf{i} \sqrt{2} y_2 & 0 & \sqrt{2} x_2 + \mathbf{i} \sqrt{2} y_2 & -\sqrt{2} x_3 - \mathbf{i} \sqrt{2} y_3 \\ \sqrt{2} x_4 - \mathbf{i} \sqrt{2} y_4 & 0 & -\sqrt{2} x_2 + \mathbf{i} \sqrt{2} y_2 & \mathbf{i} h_1 - 2 \mathbf{i} h_2 & x_1 + \mathbf{i} y_1 \\ 0 & \sqrt{2} x_4 - \mathbf{i} \sqrt{2} y_4 & \sqrt{2} x_3 - \mathbf{i} \sqrt{2} y_3 & -x_1 + \mathbf{i} y_1 & -\mathbf{i} h_1 \end{pmatrix}$$

$$\begin{pmatrix} 0 & -2i x_3 + 2 y_3 & -i \sqrt{2} x_1 + 2i x_4 + \sqrt{2} y_1 - 2 y_4 & -2i x_3 + 2 y_3 \\ -2i x_3 - 2 y_3 & -4i x_2 & -\sqrt{2} h_1 + 2 \sqrt{2} h_2 & 0 \\ -i \sqrt{2} x_1 + 2i x_4 - \sqrt{2} y_1 + 2 y_4 & \sqrt{2} h_1 - 2 \sqrt{2} h_2 & 0 & -\sqrt{2} h_1 + 2 \sqrt{2} h_2 \\ -2i x_3 - 2 y_3 & 0 & \sqrt{2} h_1 - 2 \sqrt{2} h_2 & 4i x_2 \\ 0 & -2i x_3 - 2 y_3 & i \sqrt{2} x_1 - 2i x_4 + \sqrt{2} y_1 - 2 y_4 & -2i x_3 - 2 y_3 \end{pmatrix}$$

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G - induced differential operators:

$$X1 : 2y1 \, dh1 + (2h2-2h1) \, dy1 + x2 \, dx3 + y2 \, dy3 - x3 \, dx2 - y3 \, dy2$$

$$Y1: -2x1 \, dh1 + (2h1-2h1) \, dx1 - y2 \, dx3 + x2 \, dy3 - y3 \, dx2 - x3 \, dy2$$

\*)

In[ ]:= Tr[M1.M1] // Simplify

$$\text{Out[ ]} = -4 \left( h1^2 + x1^2 + y1^2 \right)$$

(\* B<sub>2</sub> : λ<sub>1</sub> measure \*)

$$a1 = \{2, 0\};$$

$$a2 = \{0, 2\}; \lambda1 = a1 + a2; H1 = \left(1, \frac{-1}{2}\right);$$

$$H2 = (-1, 1); (* H1 = \left(1, \frac{-1}{2}\right), H2 = (-1, 1) *)$$

$$a1 = \{1, 1\};$$

$$a2 = \{0, -1\}; \lambda1 = a1 + a2; (* H1 = (1, 1),$$

$$H2 = (0, -2) *) (* I need a new basis *)$$

$$\text{In[ ]} = a1 = \{2, 0\}; a2 = \{0, 2\}; \lambda1 = a1 + a2; H1 = \left\{1, \frac{-1}{2}\right\}; H2 = \{-1, 1\};$$

$$\text{In[ ]} = a2.H2$$

$$\text{Out[ ]} = 2$$

$$\text{In[ ]} =$$

$$f[x_, y_] := \frac{e^{i(\lambda1) \cdot \{x, y\}}}{(i a1 \cdot \{x, y\}) (i (a1 + a2) \cdot \{x, y\}) (i (a1 + 2 a2) \cdot \{x, y\})} - \frac{e^{i(\lambda1 - a1) \cdot \{x, y\}}}{(i a1 \cdot \{x, y\}) (i a2 \cdot \{x, y\}) (i (a1 + 2 a2) \cdot \{x, y\})} + \frac{e^{i(\lambda1 - a1 - 2 a2) \cdot \{x, y\}}}{(i a1 \cdot \{x, y\}) (i a2 \cdot \{x, y\}) (i (a1 + 2 a2) \cdot \{x, y\})} - \frac{e^{i(\lambda1 - 2 a1 - 2 a2) \cdot \{x, y\}}}{(i a1 \cdot \{x, y\}) (i (a1 + a2) \cdot \{x, y\}) (i (a1 + 2 a2) \cdot \{x, y\})};$$



$$\text{In}[*]:= \text{fff}[h1\_ , h2\_ ] := \frac{e^{i h1}}{-i (2 h1 - 2 h2) (h1) (2 h2)} - \frac{e^{i (-h1)}}{-i (2 h1 - 2 h2) (-h1 + 2 h2) (2 h2)} + \frac{e^{-i (-h1)}}{-i (2 h1 - 2 h2) (-h1 + 2 h2) (2 h2)} - \frac{e^{-i h1}}{-i (2 h1 - 2 h2) (h1) (2 h2)} ;$$

**In[\*]:= f[h1 - 2 h2, h1] // FullSimplify**

$$\text{Out}[*]= \frac{-h1 \sin[h1] + 2 h2 \sin[h1] + h1 \sin[h1 - 2 h2]}{2 h1 h2 (h1^2 - 3 h1 h2 + 2 h2^2)}$$

**In[\*]:= fff[h1, h2] // FullSimplify**

$$\text{Out}[*]= \frac{\sin[h1]}{h1^3 - 3 h1^2 h2 + 2 h1 h2^2}$$

**In[\*]:= (2 h1 - 2 h2) (-h1 + 2 h2) (2 h2) // Expand**

$$\text{Out}[*]= -4 h1^2 h2 + 12 h1 h2^2 - 8 h2^3$$

**In[\*]:=  $\frac{e^{i h1}}{-i (2 h1 - 2 h2) (h1) (2 h2)} - \frac{e^{i (-h1)}}{-i (2 h1 - 2 h2) (-h1 + 2 h2) (2 h2)}$  // Simplify**

$$\text{Out}[*]= \frac{i e^{-i h1} (h1 + e^{2 i h1} h1 - 2 e^{2 i h1} h2)}{4 h1 (h1 - 2 h2) (h1 - h2) h2}$$

**In[\*]:=  $-\frac{e^{i (-h1+2 h2)}}{-i (2 h1 - 2 h2) (-h1 + 2 h2) (2 h2)} + \frac{e^{-i (-h1+2 h2)}}{-i (2 h1 - 2 h2) (-h1 + 2 h2) (2 h2)}$  // FullSimplify**

$$\text{Out}[*]= \frac{\sin[h1 - 2 h2]}{2 h1^2 h2 - 6 h1 h2^2 + 4 h2^3}$$

$$\frac{-\sin[2 h2]}{4 h2^3}$$

**In[\*]:= f[h1, h1 - 2 h2] // FullSimplify**

$$\text{Out}[*]= \frac{-h1 \sin[h1] + 2 h2 \sin[h1] + h1 \sin[h1 - 2 h2]}{2 h1 h2 (h1^2 - 3 h1 h2 + 2 h2^2)}$$

**In[\*]:= D[-h1 Sin[h1] + h1 Sin[h1 - 2 h2], h2] // FullSimplify**

$$\text{Out}[*]= -2 h1 \cos[h1 - 2 h2]$$

**In[\*]:= D[2 h1 h2 (h1^2 - 3 h1 h2 + 2 h2^2), h2] // FullSimplify**

$$\text{Out}[*]= 2 h1 (h1^2 - 6 h1 h2 + 6 h2^2)$$

**In[\*]:=  $\frac{-2 h1 \cos[h1 - 2 h2] + 2 \sin[h1]}{2 h1 (h1^2 - 6 h1 h2 + 6 h2^2)}$  // Simplify**

$$\text{In}[*]:= \text{g}[h1\_ , h2\_ ] := \frac{-h1 \sin[h1] + h1 \sin[h1 - 2 h2]}{2 h1 h2 (h1^2 - 3 h1 h2 + 2 h2^2)} ;$$

In[\*]:= **g[1, h2]**

$$\text{Out[*]} = \frac{-\sin[1] + \sin[1 - 2 h2]}{2 h2 (1 - 3 h2 + 2 h2^2)}$$

(\* Use L'Hopital rule to approximate  $\lambda_1$  measure restriction to  $h_1$  \*)

$$\text{In[*]} = \text{measure0}[h2\_ , x2\_ , y2\_ ] := - \frac{\cos\left[\sqrt{h2^2 + x2^2 + y2^2}\right] \sin\left[\sqrt{h2^2 + x2^2 + y2^2}\right]}{\left(2 \left(\sqrt{h2^2 + x2^2 + y2^2}\right)^3\right)};$$

**h2 D[measure0[h2, x2, y2], h2] +  
x2 D[measure0[h2, x2, y2], x2] + y2 D[measure0[h2, x2, y2], y2] // Simplify**

$$\text{Out[*]} = - \frac{\cos\left[2 \sqrt{h2^2 + x2^2 + y2^2}\right]}{2 (h2^2 + x2^2 + y2^2)} + \frac{3 \sin\left[2 \sqrt{h2^2 + x2^2 + y2^2}\right]}{4 (h2^2 + x2^2 + y2^2)^{3/2}}$$

$$\text{In[*]} = \text{measure1}[h2\_ , x2\_ , y2\_ ] := - \frac{\cos\left[2 \sqrt{h2^2 + x2^2 + y2^2}\right]}{2 (h2^2 + x2^2 + y2^2)} + \frac{3 \sin\left[2 \sqrt{h2^2 + x2^2 + y2^2}\right]}{4 (h2^2 + x2^2 + y2^2)^{3/2}};$$

**h2 D[measure1[h2, x2, y2], h2] +  
x2 D[measure1[h2, x2, y2], x2] + y2 D[measure1[h2, x2, y2], y2] // Simplify**

$$\text{Out[*]} = \frac{5 \cos\left[2 \sqrt{h2^2 + x2^2 + y2^2}\right]}{2 (h2^2 + x2^2 + y2^2)} + \frac{(-9 + 4 h2^2 + 4 x2^2 + 4 y2^2) \sin\left[2 \sqrt{h2^2 + x2^2 + y2^2}\right]}{4 (h2^2 + x2^2 + y2^2)^{3/2}}$$

**measure2[h2\_, x2\_, y2\_] :=**

$$\frac{5 \cos\left[2 \sqrt{h2^2 + x2^2 + y2^2}\right]}{2 (h2^2 + x2^2 + y2^2)} + \frac{(-9 + 4 h2^2 + 4 x2^2 + 4 y2^2) \sin\left[2 \sqrt{h2^2 + x2^2 + y2^2}\right]}{4 (h2^2 + x2^2 + y2^2)^{3/2}};$$

**h2 D[measure2[h2, x2, y2], h2] +  
x2 D[measure2[h2, x2, y2], x2] + y2 D[measure2[h2, x2, y2], y2] // Simplify**

$$\text{Out[*]} = \frac{(-19 + 4 h2^2 + 4 x2^2 + 4 y2^2) \cos\left[2 \sqrt{h2^2 + x2^2 + y2^2}\right]}{2 (h2^2 + x2^2 + y2^2)} - \frac{3 \times (-9 + 8 h2^2 + 8 x2^2 + 8 y2^2) \sin\left[2 \sqrt{h2^2 + x2^2 + y2^2}\right]}{4 (h2^2 + x2^2 + y2^2)^{3/2}}$$

$$\text{In[*]} = \text{measure3}[h2\_ , x2\_ , y2\_ ] := \frac{(-19 + 4 h2^2 + 4 x2^2 + 4 y2^2) \cos\left[2 \sqrt{h2^2 + x2^2 + y2^2}\right]}{2 (h2^2 + x2^2 + y2^2)} -$$

$$\frac{3 \times (-9 + 8 h2^2 + 8 x2^2 + 8 y2^2) \sin\left[2 \sqrt{h2^2 + x2^2 + y2^2}\right]}{4 (h2^2 + x2^2 + y2^2)^{3/2}};$$

```
In[ ]:= h2 D[measure3[h2, x2, y2], h2] +
        x2 D[measure3[h2, x2, y2], x2] + y2 D[measure3[h2, x2, y2], y2] // Simplify
```

$$\text{Out[ ]} = -\frac{(-65 + 24 h2^2 + 24 x2^2 + 24 y2^2) \cos\left[2 \sqrt{h2^2 + x2^2 + y2^2}\right]}{2 (h2^2 + x2^2 + y2^2)} - \frac{1}{4 (h2^2 + x2^2 + y2^2)^{3/2}}$$

$$(81 + 16 h2^4 + 16 x2^4 - 100 y2^2 + 16 y2^4 + 4 x2^2 (-25 + 8 y2^2) + 4 h2^2 (-25 + 8 x2^2 + 8 y2^2)) \sin\left[2 \sqrt{h2^2 + x2^2 + y2^2}\right]$$

```
In[ ]:= measure4[h2_, x2_, y2_] :=
```

$$-\frac{(-65 + 24 h2^2 + 24 x2^2 + 24 y2^2) \cos\left[2 \sqrt{h2^2 + x2^2 + y2^2}\right]}{2 (h2^2 + x2^2 + y2^2)} - \frac{1}{4 (h2^2 + x2^2 + y2^2)^{3/2}}$$

$$(81 + 16 h2^4 + 16 x2^4 - 100 y2^2 + 16 y2^4 + 4 x2^2 (-25 + 8 y2^2) + 4 h2^2 (-25 + 8 x2^2 + 8 y2^2)) \sin\left[2 \sqrt{h2^2 + x2^2 + y2^2}\right];$$

```
In[ ]:= A // Expand
```

```
In[ ]:= A[[2, 2]] // Expand
```

```
Out[ ]:= 24 + 50 dγ + 35 dγ^2 + 10 dγ^3 + dγ^4 + 48 i h2 + 52 i dγ h2 + 18 i dγ^2 h2 +
        2 i dγ^3 h2 + 24 x2^2 + 14 dγ x2^2 + 2 dγ^2 x2^2 + 24 y2^2 + 14 dγ y2^2 + 2 dγ^2 y2^2
```

```
In[ ]:= 24 measure0[h2, x2, y2] + 50 measure1[h2, x2, y2] + 35 measure2[h2, x2, y2] +
        10 measure3[h2, x2, y2] + measure4[h2, x2, y2] + 48 i h2 measure0[h2, x2, y2] +
        52 i h2 measure1[h2, x2, y2] + 18 i h2 measure2[h2, x2, y2] +
        2 i h2 measure3[h2, x2, y2] + 24 x2^2 measure0[h2, x2, y2] +
        14 x2^2 measure1[h2, x2, y2] + 2 x2^2 measure2[h2, x2, y2] + 24 y2^2 measure0[h2, x2, y2] +
        14 y2^2 measure1[h2, x2, y2] + 2 y2^2 measure2[h2, x2, y2] // FullSimplify
```

```
In[ ]:= \frac{1}{h2^2 + x2^2 + y2^2} 2 \times (3 (x2^2 + y2^2) + 2 i h2 (h2 (-2 i + h2) + x2^2 + y2^2))
```

$$\left(\cos\left[\sqrt{h2^2 + x2^2 + y2^2}\right]^2 - \sin\left[\sqrt{h2^2 + x2^2 + y2^2}\right]^2\right) -$$

$$\frac{1}{\sqrt{h2^2 + x2^2 + y2^2}} 2 (h2 (-3 i + 2 h2) + x2^2 + y2^2)$$

$$\left(\sin\left[\sqrt{h2^2 + x2^2 + y2^2}\right] \cos\left[\sqrt{h2^2 + x2^2 + y2^2}\right] - \cos\left[\sqrt{h2^2 + x2^2 + y2^2}\right] \sin\left[\sqrt{h2^2 + x2^2 + y2^2}\right]\right)$$

```
Out[ ]:= \frac{1}{h2^2 + x2^2 + y2^2} 2 \times (3 (x2^2 + y2^2) + 2 i h2 (h2 (-2 i + h2) + x2^2 + y2^2))
```

$$\left(\cos\left[\sqrt{h2^2 + x2^2 + y2^2}\right]^2 - \sin\left[\sqrt{h2^2 + x2^2 + y2^2}\right]^2\right)$$

$$\text{In}[*]:= \text{t1}[\text{h2\_}, \text{x2\_}, \text{y2\_}] := \frac{1}{\text{h2}^2 + \text{x2}^2 + \text{y2}^2} 2 \times \left( 3 (\text{x2}^2 + \text{y2}^2) + 2 \, \text{i} \, \text{h2} (\text{h2} (-2 \, \text{i} + \text{h2}) + \text{x2}^2 + \text{y2}^2) \right) \\ \left( \text{Cos} \left[ \sqrt{\text{h2}^2 + \text{x2}^2 + \text{y2}^2} \right]^2 - \text{Sin} \left[ \sqrt{\text{h2}^2 + \text{x2}^2 + \text{y2}^2} \right]^2 \right);$$

$$\text{t2}[\text{h2\_}, \text{x2\_}, \text{y2\_}] := \left( \text{Cos} \left[ \sqrt{\text{h2}^2 + \text{x2}^2 + \text{y2}^2} \right] + \text{i} \, \text{h2} \frac{\text{Sin} \left[ \sqrt{\text{h2}^2 + \text{x2}^2 + \text{y2}^2} \right]}{\sqrt{\text{h2}^2 + \text{x2}^2 + \text{y2}^2}} \right)^2;$$

$$\text{In}[*]:= \text{t1}[1, 1, 1] // \text{N}$$

$$\text{Out}[*]= -6.32295 - 3.79377 \, \text{i}$$

$$\text{In}[*]:= \text{t2}[1, 1, 1] // \text{N}$$

$$\text{Out}[*]= -0.298962 - 0.18299 \, \text{i}$$

$$\text{In}[*]:= \text{temp3}[\text{h2\_}, \text{x2\_}, \text{y2\_}] := \frac{1}{\text{h2}^2 + \text{x2}^2 + \text{y2}^2} \\ 2 \left( \text{x2}^2 + \text{y2}^2 + (3 (\text{x2}^2 + \text{y2}^2) + 2 \, \text{i} \, \text{h2} (\text{h2} (-2 \, \text{i} + \text{h2}) + \text{x2}^2 + \text{y2}^2)) \text{Cos} \left[ 2 \sqrt{\text{h2}^2 + \text{x2}^2 + \text{y2}^2} \right] - \right. \\ \left. \sqrt{\text{h2}^2 + \text{x2}^2 + \text{y2}^2} (\text{h2} (-3 \, \text{i} + 2 \, \text{h2}) + \text{x2}^2 + \text{y2}^2) \text{Sin} \left[ 2 \sqrt{\text{h2}^2 + \text{x2}^2 + \text{y2}^2} \right] \right);$$

$$\text{In}[*]:= \text{temp3}[\text{h2}, 0, 0] // \text{Simplify}$$

$$\text{Out}[*]= (8 + 4 \, \text{i} \, \text{h2}) \text{Cos} \left[ 2 \sqrt{\text{h2}^2} \right] + \frac{2 \times (3 \, \text{i} - 2 \, \text{h2}) \sqrt{\text{h2}^2} \text{Sin} \left[ 2 \sqrt{\text{h2}^2} \right]}{\text{h2}}$$

$$\text{In}[*]:= \text{A}[[1, 1]] // \text{Expand}$$

$$\text{Out}[*]= 24 + 50 \, \text{d}\gamma + 35 \, \text{d}\gamma^2 + 10 \, \text{d}\gamma^3 + \text{d}\gamma^4 + 24 \, \text{i} \, \text{h1} + 26 \, \text{i} \, \text{d}\gamma \, \text{h1} + 9 \, \text{i} \, \text{d}\gamma^2 \, \text{h1} + \text{i} \, \text{d}\gamma^3 \, \text{h1} + \\ 12 \, \text{h1}^2 + 7 \, \text{d}\gamma \, \text{h1}^2 + \text{d}\gamma^2 \, \text{h1}^2 + 4 \, \text{i} \, \text{h1}^3 + \text{i} \, \text{d}\gamma \, \text{h1}^3 + 12 \, \text{x1}^2 + 7 \, \text{d}\gamma \, \text{x1}^2 + \text{d}\gamma^2 \, \text{x1}^2 + \\ 4 \, \text{i} \, \text{h1} \, \text{x1}^2 + \text{i} \, \text{d}\gamma \, \text{h1} \, \text{x1}^2 + 12 \, \text{y1}^2 + 7 \, \text{d}\gamma \, \text{y1}^2 + \text{d}\gamma^2 \, \text{y1}^2 + 4 \, \text{i} \, \text{h1} \, \text{y1}^2 + \text{i} \, \text{d}\gamma \, \text{h1} \, \text{y1}^2$$

$$\text{In}[*]:= \text{measure00}[\text{h1\_}, \text{x1\_}, \text{y1\_}] := \frac{\text{Sin} \left[ \sqrt{\text{h1}^2 + \text{x1}^2 + \text{y1}^2} \right]}{\left( \sqrt{\text{h1}^2 + \text{x1}^2 + \text{y1}^2} \right)^3};$$

$$\text{In}[*]:= \text{h1 D}[\text{measure00}[\text{h1}, \text{x1}, \text{y1}], \text{h1}] +$$

$$\text{x1 D}[\text{measure00}[\text{h1}, \text{x1}, \text{y1}], \text{x1}] + \text{y1 D}[\text{measure00}[\text{h1}, \text{x1}, \text{y1}], \text{y1}] // \text{Simplify}$$

$$\text{Out}[*]= \frac{\text{Cos} \left[ \sqrt{\text{h1}^2 + \text{x1}^2 + \text{y1}^2} \right]}{\text{h1}^2 + \text{x1}^2 + \text{y1}^2} - \frac{3 \text{Sin} \left[ \sqrt{\text{h1}^2 + \text{x1}^2 + \text{y1}^2} \right]}{(\text{h1}^2 + \text{x1}^2 + \text{y1}^2)^{3/2}}$$

$$\text{In}[*]:= \text{measure11}[\text{h1\_}, \text{x1\_}, \text{y1\_}] := \frac{\text{Cos} \left[ \sqrt{\text{h1}^2 + \text{x1}^2 + \text{y1}^2} \right]}{\text{h1}^2 + \text{x1}^2 + \text{y1}^2} - \frac{3 \text{Sin} \left[ \sqrt{\text{h1}^2 + \text{x1}^2 + \text{y1}^2} \right]}{(\text{h1}^2 + \text{x1}^2 + \text{y1}^2)^{3/2}};$$

In[\*]:= h1 D[measure11[h1, x1, y1], h1] +  
 x1 D[measure11[h1, x1, y1], x1] + y1 D[measure11[h1, x1, y1], y1] // Simplify

$$\text{Out[*]} = -\frac{5 \cos\left[\sqrt{h_1^2 + x_1^2 + y_1^2}\right]}{h_1^2 + x_1^2 + y_1^2} - \frac{(-9 + h_1^2 + x_1^2 + y_1^2) \sin\left[\sqrt{h_1^2 + x_1^2 + y_1^2}\right]}{(h_1^2 + x_1^2 + y_1^2)^{3/2}}$$

In[\*]:= measure22[h1\_, x1\_, y1\_] :=

$$-\frac{5 \cos\left[\sqrt{h_1^2 + x_1^2 + y_1^2}\right]}{h_1^2 + x_1^2 + y_1^2} - \frac{(-9 + h_1^2 + x_1^2 + y_1^2) \sin\left[\sqrt{h_1^2 + x_1^2 + y_1^2}\right]}{(h_1^2 + x_1^2 + y_1^2)^{3/2}};$$

In[\*]:= h1 D[measure22[h1, x1, y1], h1] +  
 x1 D[measure22[h1, x1, y1], x1] + y1 D[measure22[h1, x1, y1], y1] // Simplify

$$\text{Out[*]} = -\frac{(-19 + h_1^2 + x_1^2 + y_1^2) \cos\left[\sqrt{h_1^2 + x_1^2 + y_1^2}\right]}{h_1^2 + x_1^2 + y_1^2} +$$

$$\frac{3 \times (-9 + 2 h_1^2 + 2 x_1^2 + 2 y_1^2) \sin\left[\sqrt{h_1^2 + x_1^2 + y_1^2}\right]}{(h_1^2 + x_1^2 + y_1^2)^{3/2}}$$

In[\*]:= measure33[h1\_, x1\_, y1\_] := -\frac{(-19 + h\_1^2 + x\_1^2 + y\_1^2) \cos\left[\sqrt{h\_1^2 + x\_1^2 + y\_1^2}\right]}{h\_1^2 + x\_1^2 + y\_1^2} +

$$\frac{3 \times (-9 + 2 h_1^2 + 2 x_1^2 + 2 y_1^2) \sin\left[\sqrt{h_1^2 + x_1^2 + y_1^2}\right]}{(h_1^2 + x_1^2 + y_1^2)^{3/2}};$$

In[\*]:= h1 D[measure33[h1, x1, y1], h1] +  
 x1 D[measure33[h1, x1, y1], x1] + y1 D[measure33[h1, x1, y1], y1] // Simplify

$$\text{Out[*]} = \frac{(-65 + 6 h_1^2 + 6 x_1^2 + 6 y_1^2) \cos\left[\sqrt{h_1^2 + x_1^2 + y_1^2}\right]}{h_1^2 + x_1^2 + y_1^2} +$$

$$\frac{(81 + h_1^4 + x_1^4 - 25 y_1^2 + y_1^4 + x_1^2 (-25 + 2 y_1^2) + h_1^2 (-25 + 2 x_1^2 + 2 y_1^2)) \sin\left[\sqrt{h_1^2 + x_1^2 + y_1^2}\right]}{(h_1^2 + x_1^2 + y_1^2)^{3/2}}$$

In[\*]:= measure44[h1\_, x1\_, y1\_] := \frac{(-65 + 6 h\_1^2 + 6 x\_1^2 + 6 y\_1^2) \cos\left[\sqrt{h\_1^2 + x\_1^2 + y\_1^2}\right]}{h\_1^2 + x\_1^2 + y\_1^2} +

$$\frac{1}{(h_1^2 + x_1^2 + y_1^2)^{3/2}} (81 + h_1^4 + x_1^4 - 25 y_1^2 + y_1^4 + x_1^2 (-25 + 2 y_1^2) + h_1^2 (-25 + 2 x_1^2 + 2 y_1^2))$$

$$\sin\left[\sqrt{h_1^2 + x_1^2 + y_1^2}\right];$$

In[ ]:= M1 // MatrixForm

Out[ ]:= MatrixForm=

$$\begin{pmatrix} i h1 & x1 + i y1 & 0 & 0 & 0 \\ -x1 + i y1 & -i h1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & i h1 & x1 + i y1 \\ 0 & 0 & 0 & -x1 + i y1 & -i h1 \end{pmatrix}$$

In[ ]:= A[[1, 1]] // Expand

Out[ ]:=  $24 + 50 d_\gamma + 35 d_\gamma^2 + 10 d_\gamma^3 + d_\gamma^4 + 24 i h1 + 26 i d_\gamma h1 + 9 i d_\gamma^2 h1 + i d_\gamma^3 h1 + 12 h1^2 + 7 d_\gamma h1^2 + d_\gamma^2 h1^2 + 4 i h1^3 + i d_\gamma h1^3 + 12 x1^2 + 7 d_\gamma x1^2 + d_\gamma^2 x1^2 + 4 i h1 x1^2 + i d_\gamma h1 x1^2 + 12 y1^2 + 7 d_\gamma y1^2 + d_\gamma^2 y1^2 + 4 i h1 y1^2 + i d_\gamma h1 y1^2$

In[ ]:=  $24 \text{measure00}[h1, x1, y1] + 50 \text{measure11}[h1, x1, y1] + 35 \text{measure22}[h1, x1, y1] + 10 \text{measure33}[h1, x1, y1] + \text{measure44}[h1, x1, y1] + 24 i h1 \text{measure00}[h1, x1, y1] + 26 i h1 \text{measure11}[h1, x1, y1] + 9 i h1 \text{measure22}[h1, x1, y1] + i h1 \text{measure33}[h1, x1, y1] + 12 h1^2 \text{measure00}[h1, x1, y1] + 7 h1^2 \text{measure11}[h1, x1, y1] + h1^2 \text{measure22}[h1, x1, y1] + 4 i h1^3 \text{measure00}[h1, x1, y1] + i h1^3 \text{measure11}[h1, x1, y1] + 12 x1^2 \text{measure00}[h1, x1, y1] + 7 x1^2 \text{measure11}[h1, x1, y1] + x1^2 \text{measure22}[h1, x1, y1] + 4 i h1 x1^2 \text{measure00}[h1, x1, y1] + i h1 x1^2 \text{measure11}[h1, x1, y1] + 12 y1^2 \text{measure00}[h1, x1, y1] + 7 y1^2 \text{measure11}[h1, x1, y1] + y1^2 \text{measure22}[h1, x1, y1] + 4 i h1 y1^2 \text{measure00}[h1, x1, y1] + i h1 y1^2 \text{measure11}[h1, x1, y1]$  // Simplify

Out[ ]:=  $-2 \cos \left[ \sqrt{h1^2 + x1^2 + y1^2} \right] - \frac{2 i h1 \sin \left[ \sqrt{h1^2 + x1^2 + y1^2} \right]}{\sqrt{h1^2 + x1^2 + y1^2}}$

In[ ]:= A[[1, 2]] // Expand

Out[ ]:=  $24 x1 + 26 d_\gamma x1 + 9 d_\gamma^2 x1 + d_\gamma^3 x1 + 4 h1^2 x1 + d_\gamma h1^2 x1 + 4 x1^3 + d_\gamma x1^3 + 24 i y1 + 26 i d_\gamma y1 + 9 i d_\gamma^2 y1 + i d_\gamma^3 y1 + 4 i h1^2 y1 + i d_\gamma h1^2 y1 + 4 i x1^2 y1 + i d_\gamma x1^2 y1 + 4 x1 y1^2 + d_\gamma x1 y1^2 + 4 i y1^3 + i d_\gamma y1^3$

In[ ]:=  $24 x1 \text{measure00}[h1, x1, y1] + 26 x1 \text{measure11}[h1, x1, y1] + 9 x1 \text{measure22}[h1, x1, y1] + x1 \text{measure33}[h1, x1, y1] + 4 h1^2 x1 \text{measure00}[h1, x1, y1] + h1^2 x1 \text{measure11}[h1, x1, y1] + 4 x1^3 \text{measure00}[h1, x1, y1] + x1^3 \text{measure11}[h1, x1, y1] + 24 i y1 \text{measure00}[h1, x1, y1] + 26 i y1 \text{measure11}[h1, x1, y1] + 9 i y1 \text{measure22}[h1, x1, y1] + i y1 \text{measure33}[h1, x1, y1] + 4 i h1^2 y1 \text{measure00}[h1, x1, y1] + i h1^2 y1 \text{measure11}[h1, x1, y1] + 4 i x1^2 y1 \text{measure00}[h1, x1, y1] + i x1^2 y1 \text{measure11}[h1, x1, y1] + 4 x1 y1^2 \text{measure00}[h1, x1, y1] + x1 y1^2 \text{measure11}[h1, x1, y1] + 4 i y1^3 \text{measure00}[h1, x1, y1] + i y1^3 \text{measure11}[h1, x1, y1]$  // Simplify

Out[ ]:=  $-\frac{2 (x1 + i y1) \sin \left[ \sqrt{h1^2 + x1^2 + y1^2} \right]}{\sqrt{h1^2 + x1^2 + y1^2}}$

```
In[ ]:= adH1 = {{0, 0, 0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0, 0, 0},
               {0, 0, 2, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, -1, 0, 0, 0, 0, 0, 0},
               {0, 0, 0, 0, 1, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0, 0, 0},
               {0, 0, 0, 0, 0, 0, -2, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 1, 0, 0},
               {0, 0, 0, 0, 0, 0, 0, 0, -1, 0}, {0, 0, 0, 0, 0, 0, 0, 0, 0, 0}};
```

```
adX1 = {{0, 0, 0, 0, 0, 0, 1, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0, 0, 0},
        {-2, 2, 0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0, 0, 0},
        {0, 0, 0, 1, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0, 0, 0},
        {0, 0, 0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0, -1, 0},
        {0, 0, 0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0, 0, 0}};
```

```
adY1 = {{0, 0, -1, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0, 0, 0},
        {0, 0, 0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 1, 0, 0, 0, 0, 0},
        {0, 0, 0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0, 0, 0},
        {2, -1, 0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0, 0, 0},
        {0, 0, 0, 0, 0, 0, -1, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0, 0, 0}};
```

```
In[ ]:= adH1 // MatrixForm
```

```
Out[ ]//MatrixForm=
```

$$\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

```
In[ ]:= adX1 // MatrixForm
```

```
Out[ ]//MatrixForm=
```

$$\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -2 & 2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

```
In[ ]:= adY1 // MatrixForm
```

```
Out[ ]//MatrixForm=
```

$$\begin{pmatrix} 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 2 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

```
In[ ]:= T = h1 i adH1 + x1 (adX1 - adY1) + i y1 (adX1 + adY1);
```

```
In[ ]:= Tr[T.T] // Simplify
```

```
Out[ ]:= -12 (h1^2 + x1^2 + y1^2)
```