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In[ ]:= ClearAll["Global`*"]
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(*
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Root System of  $B_2$ 
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$$\Phi = \{\alpha_1, \alpha_1 + \alpha_2, \alpha_1 + 2\alpha_2, \alpha_2, -\alpha_1, -(\alpha_1 + \alpha_2), -(\alpha_1 + 2\alpha_2), -\alpha_2\}$$

```
*)
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In[ ]:=  $\alpha_1 = \{\sqrt{2}, 0\}$ ; T = {{Cos[3  $\pi$  / 4], -Sin[3  $\pi$  / 4]}, {Sin[3  $\pi$  / 4], Cos[3  $\pi$  / 4]}};
```

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 $\alpha_2 = (1/\sqrt{2}) (T.\alpha_1)$ ;
```

```
a1 =  $\alpha_1$ ; a2 =  $\alpha_2$ ;
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```
In[ ]:= (* Plot the Root System of  $B_2$  *)
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```
origin = {0, 0};
```

```
rootOrbit = {{- $\alpha_1$ ,  $\alpha_1$ }, {- $\alpha_2$ ,  $\alpha_2$ }, {-( $\alpha_1 + 2\alpha_2$ ),  $\alpha_1 + 2\alpha_2$ }, {-( $\alpha_1 + \alpha_2$ ),  $\alpha_1 + \alpha_2$ }};
```

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temp = ListLinePlot[rootOrbit, AspectRatio  $\rightarrow$  Automatic,
```

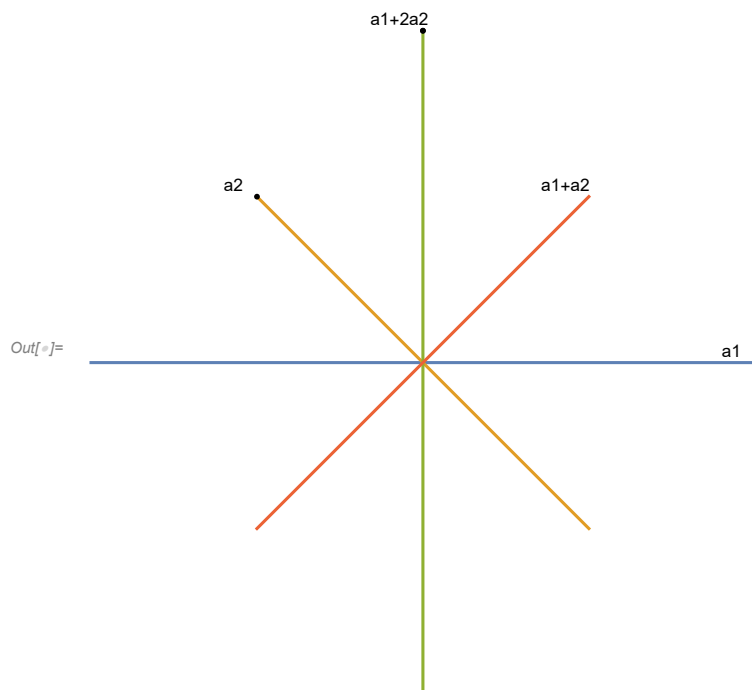
```
Axes  $\rightarrow$  False, PlotLabel  $\rightarrow$  Style["Root System of  $B_2$ ", FontSize  $\rightarrow$  12],
```

```
Epilog  $\rightarrow$  {Point[a1 + 2 a2], Text["a1+2a2", a1 + 2 a2 + {-0.1, 0.05}],
```

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Point[a1 + 2 a2], Text["a1+a2", a1 + a2 + {-0.1, 0.05}], Point[a2],
```

```
Text["a2", a2 + {-0.1, 0.05}], Point[a1], Text["a1", a1 + {-0.1, 0.05}]}
```

Root System of  $B_2$



In[ ]:= (\*

Let  $w_{\alpha_1}, w_{\alpha_2}$  be the simple reflections,  
then the Weyl group 'W' of  $B_2$  could contain such elements:

$W = \{e, w_{\alpha_1}, w_{\alpha_2}w_{\alpha_1}, w_{\alpha_1}w_{\alpha_2}w_{\alpha_1}, w_{\alpha_2}w_{\alpha_1}w_{\alpha_2}w_{\alpha_1}, w_{\alpha_2}w_{\alpha_1}w_{\alpha_2}, w_{\alpha_1}w_{\alpha_2}, w_{\alpha_2}\},$   
and theirs signs are

sign: +   -   +   -   +   -   +   -   ;

The half sum of positives root:  $\delta = \frac{3}{2} \alpha_1 + 2 \alpha_2$  , and the Weyl orbit of  $\delta$  is

$$\left\{ \frac{3}{2} \alpha_1 + 2 \alpha_2, \frac{1}{2} \alpha_1 + 2 \alpha_2, \frac{1}{2} \alpha_1 - \alpha_2, -\frac{3}{2} \alpha_1 - \alpha_2, -\frac{3}{2} \alpha_1 - 2 \alpha_2, -\frac{1}{2} \alpha_1 - 2 \alpha_2, -\frac{1}{2} \alpha_1 + \alpha_2, \frac{3}{2} \alpha_1 + \alpha_2 \right\}$$

\*)

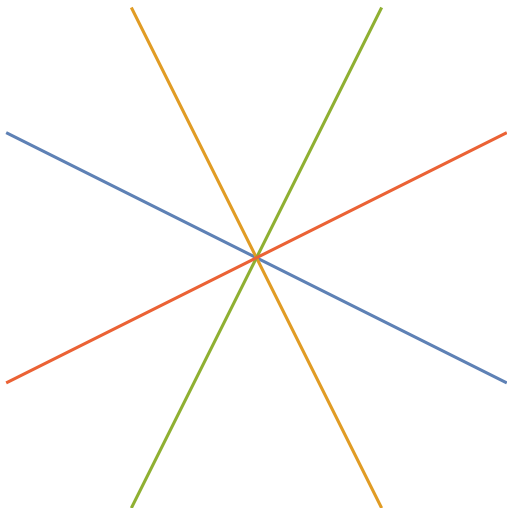
$$\delta = \frac{3}{2} \alpha_1 + 2 \alpha_2;$$

$$\text{deltaOrbitPlot} = \left\{ \left\{ -\frac{1}{2} \alpha_1 + \alpha_2, \frac{1}{2} \alpha_1 - \alpha_2 \right\}, \left\{ -\frac{1}{2} \alpha_1 - 2 \alpha_2, \frac{1}{2} \alpha_1 + 2 \alpha_2 \right\}, \left\{ -\frac{3}{2} \alpha_1 - 2 \alpha_2, \frac{3}{2} \alpha_1 + 2 \alpha_2 \right\}, \left\{ -\frac{3}{2} \alpha_1 - \alpha_2, \frac{3}{2} \alpha_1 + \alpha_2 \right\} \right\};$$

temp = ListLinePlot[deltaOrbitPlot, AspectRatio → Automatic, Axes → False,  
PlotLabel → Style["Weyl Orbit of  $\delta$  of  $B_2$ ", FontSize → 12]]

Weyl Orbit of  $\delta$  of  $B_2$

Out[ ]:=



```

In[ ]:= (* Orbital projection measure of  $\delta$  *)
scale = 100; a1 =  $\alpha_1$ ; a2 =  $\alpha_2$ ;
region = ParametricRegion[{t * a1[[1]] + s * (a1 + a2) [[1]], t * a1[[2]] + s * (a1 + a2) [[2]]},
  {{t, 0, scale}, {s, 0, scale}}];
factor = 1 / Det[Transpose[{a1, a1 + a2}]];
f[x_, y_] := factor * Piecewise[{{1, {x, y}  $\in$  region}}, 0];
g[x_, y_] = Integrate[f[x - t * (a1 + 2 a2) [[1]], y - t * (a1 + 2 a2) [[2]], {t, 0, scale}];
h[x_, y_] = Integrate[g[x - t * a2[[1]], y - t * a2[[2]], {t, 0, scale}];

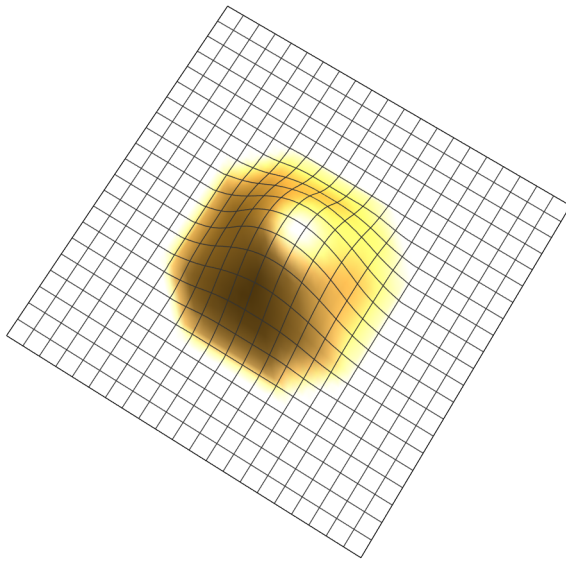
deltaOrbit = { $\frac{3}{2} a_1 + 2 a_2$ ,  $\frac{1}{2} a_1 + 2 a_2$ ,  $\frac{1}{2} a_1 - a_2$ ,
   $-\frac{3}{2} a_1 - a_2$ ,  $-\frac{3}{2} a_1 - 2 a_2$ ,  $-\frac{1}{2} a_1 - 2 a_2$ ,  $-\frac{1}{2} a_1 + a_2$ ,  $\frac{3}{2} a_1 + a_2$ };

 $\mu[x_, y_] := \sum_{i=1}^8 (-1)^{i+1} * h[\text{deltaOrbit}[[i, 1]] - x, \text{deltaOrbit}[[i, 2]] - y];$ 

range = 2.4;
data = Table[{x, y, If[ $\mu[x, y] \leq 0.01$ , 0,  $\mu[x, y]$ ]},
  {x, -range, range, 0.1}, {y, -range, range, 0.1}];
data1 = Flatten[data, 1];
(* temp=ListPointPlot3D[data1]; *)
ListPlot3D[data1, AspectRatio  $\rightarrow$  Automatic,
  Axes  $\rightarrow$  False, Boxed  $\rightarrow$  False, Lighting  $\rightarrow$  "Neutral", PlotRange  $\rightarrow$  All,
  Mesh  $\rightarrow$  {20}, PlotLabel  $\rightarrow$  Style[" $\mu_\delta^P$ ", FontSize  $\rightarrow$  12]]

```

$$\mu_{\delta}^p$$



$$\mu_{\delta}^p$$

