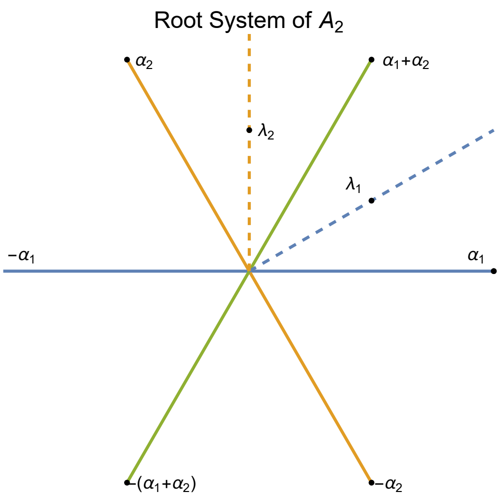
In[∘]:= 
```
ClearAll["Global`*"]
(*
   The orbital projection measure μ_λ:

      μ_λ^p = Σ_{w∈W} sgn(w) e_{wλ} * P,      P = ∏_{α∈Φ⁺} * F_α

   My appoarch is:

      μ_λ^p = lim_{n → ∞} Σ_{w∈W} sgn(w) e_{wλ} * P_n        where P_n = ∏_{α∈Φ⁺} *F_{nα}
*)
```

In[∘]:= 
```
(* Definie the basic infomation including root system of A_2 *)
α1 = {√2, 0}; T = {{Cos[2 π / 3], -Sin[2 π / 3]}, {Sin[2 π / 3], Cos[2 π / 3]}};
δ = α1 + T.α1; λ1 = (1 / 3) * (2 α1 + T.α1); λ2 = (1 / 3) * (α1 + 2 T.α1);  a1 = α1;  a2 = T.α1;
origin = {0, 0};
roots = {δ, -δ, α1, -α1, T.α1, -T.α1};
```

In[∘]:= 
```
(* Plot the Root System of A_2 *)
p1 = ListLinePlot[{{-α1, α1}, {-T.α1, T.α1}, {-(α1 + T.α1), α1 + T.α1}},
   AspectRatio → Automatic, Axes → False,
   PlotLabel → Style["Root System of A_2", FontSize → 12], Epilog →
    {Point[α1], Text["α1", α1 + {-0.1, 0.1}], Point[T.α1], Text["α2", T.α1 + {0.1, 0}],
     Point[α1 + T.α1], Text["α1+α2", α1 + T.α1 + {0.2, 0}], Point[α1],
     Text["-α1", -α1 + {0.1, 0.1}], Point[-T.α1], Text["-α2", -T.α1 + {0.1, 0}],
     Point[-α1 - T.α1], Text["-(α1+α2)", -α1 - T.α1 + {0.2, 0}], Point[λ1],
     Text["λ1", λ1 + {-0.1, 0.1}], Point[λ2], Text["λ2", λ2 + {0.1, 0}]}];
p2 = ListLinePlot[{{{0, 0}, 2 λ1}, {{0, 0}, 2 λ2}}, AspectRatio → Automatic, Axes → False,
   Epilog → {Point[λ1], Text["λ1", λ1 + {0, 0.1}], Point[λ2], Text["λ2", λ2 + {0.1, 0}]},
   PlotStyle → Dashed];
Show[
 p1,
 p2]
```

Out[∘]=

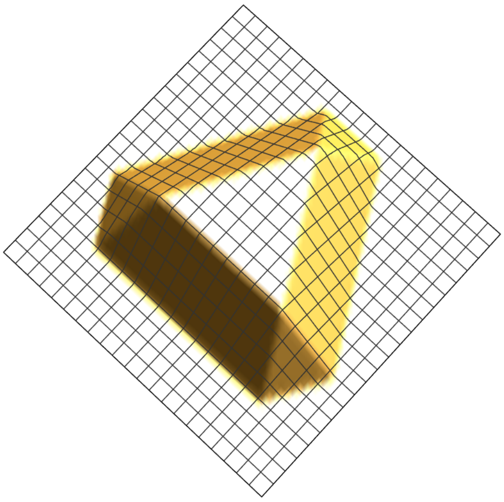


In[ ]:=
```
(* Define the Weyl group of A₂ *)
Wa1[λ_] := λ - 2 * (λ.α₁) / (α₁.α₁) α₁;
Wa2[λ_] := λ - 2 * (λ.(T.α₁)) / ((T.α₁).(T.α₁)) (T.α₁);

W1[λ_] := Wa1[Wa1[λ]]; (* + *)
W2[λ_] := Wa1[λ]; (* - *)
W3[λ_] := Wa1[Wa2[λ]]; (* + *)
W4[λ_] := Wa2[Wa1[Wa2[λ]]]; (* - *)
W5[λ_] := Wa2[Wa1[λ]]; (* + *)
W6[λ_] := Wa2[λ]; (* - *)

W = {W1, W2, W3, W4, W5, W6};
roots = {W1[δ], W2[δ], W3[δ], W4[δ], W5[δ], W6[δ]};
```

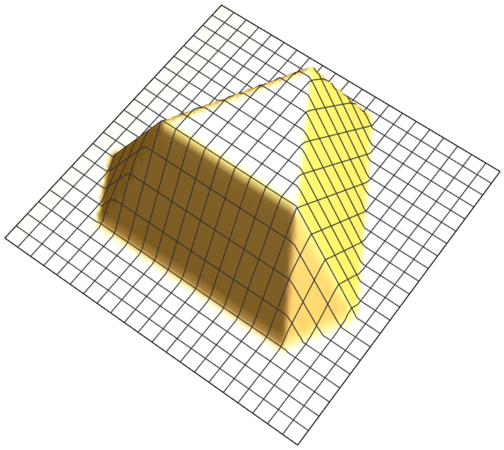
In[ ]:=
```
(* Orbital projection measure of regular orbit λ *)
λ = λ1 + 3 λ2;
orbit = {W1[λ], W2[λ], W3[λ], W4[λ], W5[λ], W6[λ]};
scale = 100;
region = ParametricRegion[
   {t * a1[[1]] + s * a2[[1]], t * a1[[2]] + s * a2[[2]]}, {{t, 0, scale}, {s, 0, scale}}];
factor = 1 / Det[Transpose[{a1, a2}]];
f[x_, y_] := factor * Piecewise[{{1, {x, y} ∈ region}}, 0];
g[x_, y_] = Integrate[f[x - t * δ[[1]], y - t * δ[[2]]], { t, 0, scale}];
```

In[ ]:=
```
(* Orbital projection measure formula *)
μ[x_, y_] := Sum[(-1)^(i+1) * g[orbit[[i, 1]] - x, orbit[[i, 2]] - y], {i, 1, 6}];
range = 3.3;
data2 = Table[{x, y, If[μ[x, y] ≤ 0.01, 0, μ[x, y]]},
   {x, -range, range, 0.1}, {y, -range, range, 0.1}];
data1 = Flatten[data2, 1];
temp = ListPointPlot3D[data1];
ListPlot3D[data1, AspectRatio → Automatic,
 PlotLabel → Style["μ_λ^p , λ= λ₁ + 3 λ₂", FontSize → 12], Axes → False,
 Boxed → False, Lighting → "Neutral", PlotRange → All, Mesh → {20}, PlotRange → All]
```

$\mu_\lambda{}^p$ , $\lambda$= $\lambda_1$ + 3 $\lambda_2$



$\mu_\lambda{}^p$ , $\lambda$= $\lambda_1$ + 3 $\lambda_2$



In[ ]:= **(* Convolution of coadjoint orbits: $\mu_\delta$ * $\mu_\lambda$ *)**

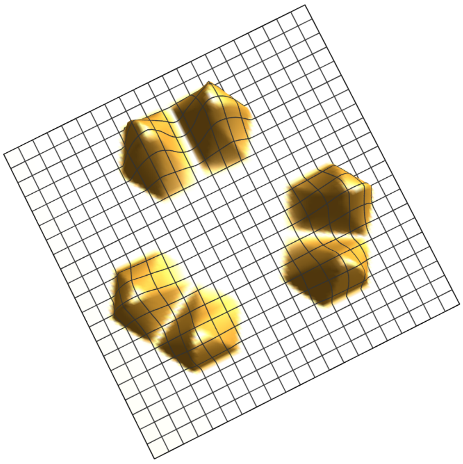
$$\mu2[x\_,\ y\_] := \sum_{i=1}^{6} (-1)^{i+1} * g[roots[\![i,\ 1]\!] - x,\ roots[\![i,\ 2]\!] - y];$$

$$h2[x\_,\ y\_] := \left(\sum_{i=1}^{6} (-1)^{i+1} * \mu2[(x - orbit[\![i,\ 1]\!]),\ (y - orbit[\![i,\ 2]\!])]\right);$$

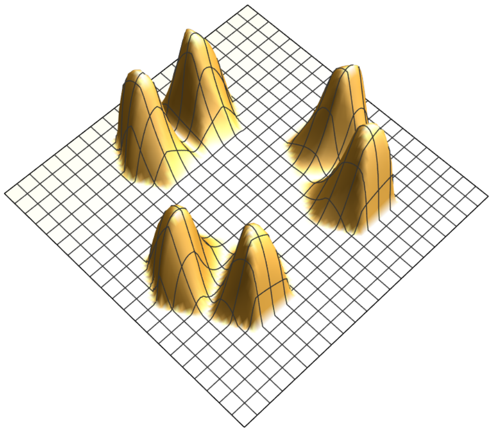
$$h2[x\_,\ y\_] := \left(\sum_{i=1}^{6} (-1)^{i+1} * \mu2[(x - orbit[\![i,\ 1]\!]),\ (y - orbit[\![i,\ 2]\!])]\right) * \frac{(\{x,\ y\}.a1) * (\{x,\ y\}.a2) * (\{x,\ y\}.\delta)}{(\delta.a1) * (\delta.a2) * (\delta.\delta)};$$

```
range2 = 5;
data = Table[{x, y, h2[x, y]}, {x, -range2, range2, 0.1}, {y, -range2, range2, 0.1}];
data1 = Flatten[data, 1];

ListPlot3D[data1, AspectRatio → Automatic,
 PlotLabel → Style["μδ * μλ |t* , λ= 2 λ1 + 3 λ2", FontSize → 12], Axes → False,
 Boxed → False, Lighting → "Neutral", PlotRange → All, Mesh → {20}, PlotRange → All]
```

$\mu_\delta * \mu_\lambda \mid_{t^*}$ , $\lambda$= 2 $\lambda_1$ + 3 $\lambda_2$

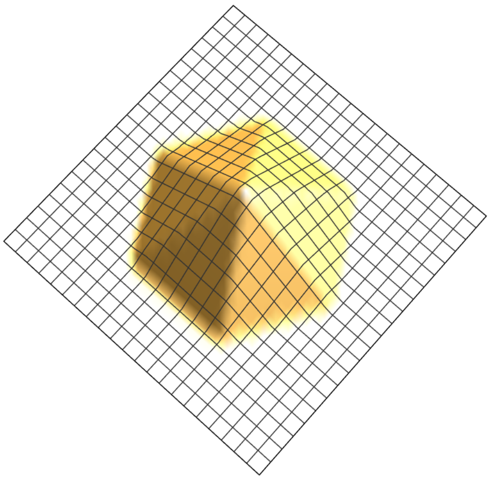


$\mu_\delta * \mu_\lambda \mid_{t^*}$ , $\lambda$= 2 $\lambda_1$ + 3 $\lambda_2$

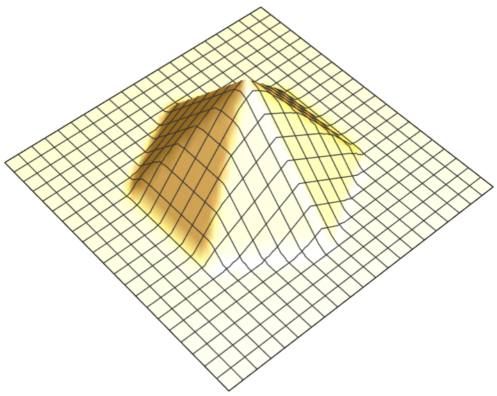


In[ ]:=
```
(* Orbital projection measure of δ *)
μθ[x_, y_] := Sum_{i=1}^{6} (-1)^(i+1) * g[roots⟦i, 1⟧ - x, roots⟦i, 2⟧ - y];
range3 = 2;
data = Table[{x, y, If[μθ[x, y] ≤ 0.01, 0, μθ[x, y]]},
    {x, -range3, range3, 0.1}, {y, -range3, range3, 0.1}];
data1 = Flatten[data, 1];
temp = ListPointPlot3D[data1];
ListPlot3D[data1, AspectRatio → Automatic, Axes → False,
 Boxed → False, Lighting → "Neutral", PlotRange → All, Mesh → {20},
 PlotRange → All, PlotLabel → Style["μδ^p , δ = λ1 + λ2", FontSize → 12]]
```

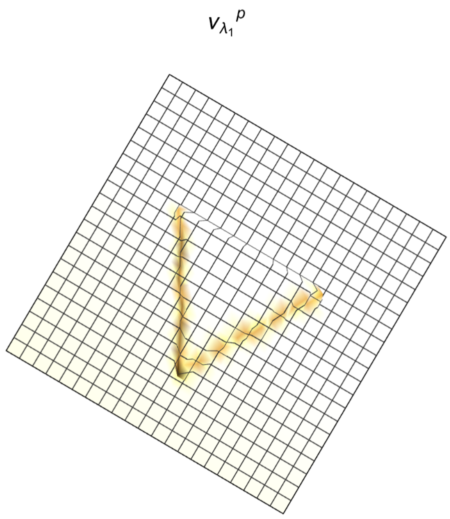
$\mu_\delta{}^p$ , $\delta = \lambda_1 + \lambda_2$

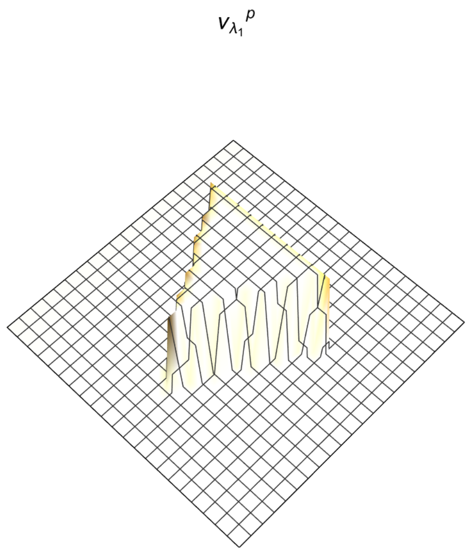


$\mu_\delta{}^p$ , $\delta = \lambda_1 + \lambda_2$



In[∘]:=
```
(* Orbital projection measure of singular orbit: fundamental weight λ₁ *)
orbitl1 = {λ1, λ1 - α₁, λ1 - δ};
factor2 = ((δ.α₁) (δ.(T.α₁)) (δ.δ)) / (((T.α₁).((1 / 2) T.α₁)) (λ1.α₁) (λ1.δ));
region1 = ParametricRegion[
   {t * a1⟦1⟧ + s * δ⟦1⟧, t * a1⟦2⟧ + s * δ⟦2⟧}, {{t, 0, scale}, {s, 0, scale}}];
region2 = ParametricRegion[{t * a1⟦1⟧ + s * a2⟦1⟧, t * a1⟦2⟧ + s * a2⟦2⟧},
   {{t, 0, scale}, {s, 0, scale}}];
region3 = ParametricRegion[{t * a2⟦1⟧ + s * δ⟦1⟧, t * a2⟦2⟧ + s * δ⟦2⟧},
   {{t, 0, scale}, {s, 0, scale}}];

f1[x_, y_] := factor2 * Piecewise[{{1, {x, y} ∈ region1}}, 0];
f2[x_, y_] := factor2 * Piecewise[{{1, {x, y} ∈ region2}}, 0];
f3[x_, y_] := factor2 * Piecewise[{{1, {x, y} ∈ region3}}, 0];

ν₁[x_, y_] := f1[orbitl1⟦1, 1⟧ - x, orbitl1⟦1, 2⟧ - y] -
   f2[orbitl1⟦2, 1⟧ - x, orbitl1⟦2, 2⟧ - y] + f3[orbitl1⟦3, 1⟧ - x, orbitl1⟦3, 2⟧ - y];

data = Table[{x, y, If[ν₁[x, y] ≤ 0.01, 0, ν₁[x, y]]},
   {x, -1.5, 1.5, 0.1}, {y, -1.5, 1.5, 0.1}];
data1 = Flatten[data, 1];
ListPlot3D[data1, AspectRatio → Automatic,
 Axes → False, Boxed → False, Lighting → "Neutral", PlotRange → All,
 Mesh → {20}, PlotLabel → Style["ν_λ₁^p", FontSize → 12]]
```

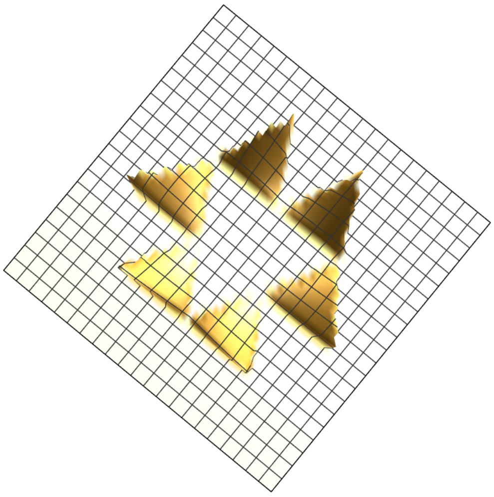


In[•]:=

```
(* Convolution of coadjoint orbits: μδ * νλ1 *)
h3[x_, y_] := Sum[(-1)^(i+1) *
    ν1[x - roots[[i, 1]], y - roots[[i, 2]]] * (({x, y}.a1) * ({x, y}.a2) * ({x, y}.δ)) / ((δ.a1) * (δ.a2) * (δ.δ)), {i, 1, 6}];
data = Table[{x, y, h3[x, y]}, {x, -3, 3, 0.1}, {y, -3, 3, 0.1}];
data1 = Flatten[data, 1];
ListPlot3D[data1, AspectRatio → Automatic, Axes → False,
 Boxed → False, Lighting → "Neutral", PlotRange → All, Mesh → {20},
 PlotLabel → Style["μδ * νλ1 |t* , λ= 2 λ1 + 3 λ2", FontSize → 12]]
```

$\mu_\delta * \nu_{\lambda_1}\ |_{t^*}$ , $\lambda = 2\,\lambda_1 + 3\,\lambda_2$



$\mu_\delta * \nu_{\lambda_1}\ |_{t^*}$ , $\lambda = 2\,\lambda_1 + 3\,\lambda_2$