

```
In[164]:= ClearAll["Global`*"]
```

```
(* Basic information of  $A_2$  *)
```

```
 $\alpha_1 = \{\sqrt{2}, 0\}$ ;  $T = \{\{\cos[2\pi/3], -\sin[2\pi/3]\}, \{\sin[2\pi/3], \cos[2\pi/3]\}\}$ ;
```

```
 $\delta = \alpha_1 + T.\alpha_1$ ;  $\lambda_1 = (1/3) * (2\alpha_1 + T.\alpha_1)$ ;  $\lambda_2 = (1/3) * (\alpha_1 + 2T.\alpha_1)$ ;  $a_1 = \alpha_1$ ;  $a_2 = T.\alpha_1$ ;
```

```
In[167]:=
```

```
(* Weyl group of  $A_2$  *)
```

```
In[168]:= (* Liouville measure of  $I_\delta$  *)
```

```
scale = 10000;
```

```
In[169]:= region1 = ParametricRegion[{t * (- $\delta$ ) [[1]] + s * (- $\delta$  -  $a_1$ ) [[1]], t * (- $\delta$ ) [[2]] + s * (- $\delta$  -  $a_1$ ) [[2]]},  
{{t, 0, scale}, {s, 0, scale}}];
```

```
factor1 = 1;
```

```
f1[x_, y_] := factor1 * Piecewise[{{1, {x, y} ∈ region1}}, 0];
```

```
h1[x_, y_] = Integrate[f1[x - t * (- $\delta$  -  $a_2$ ) [[1]], y - t * (- $\delta$  -  $a_2$ ) [[2]], {t, 0, scale}];
```

```
In[173]:= region2 = ParametricRegion[{t * ( $a_2$  -  $a_1$ ) [[1]] + s * (- $a_1$ ) [[1]], t * ( $a_2$  -  $a_1$ ) [[2]] + s * (- $a_1$ ) [[2]]},  
{{t, 0, scale}, {s, 0, scale}}];
```

```
factor2 = 1;
```

```
f2[x_, y_] := factor2 * Piecewise[{{1, {x, y} ∈ region2}}, 0];
```

```
h2[x_, y_] = Integrate[f2[x - t * (- $\delta$  -  $a_1$ ) [[1]], y - t * (- $\delta$  -  $a_1$ ) [[2]], {t, 0, scale}];
```

```
In[177]:= region3 = ParametricRegion[{t * ( $a_1$  -  $a_2$ ) [[1]] + s * (- $a_2$ ) [[1]], t * ( $a_1$  -  $a_2$ ) [[2]] + s * (- $a_2$ ) [[2]]},  
{{t, 0, scale}, {s, 0, scale}}];
```

```
factor3 = 1;
```

```
f3[x_, y_] := factor3 * Piecewise[{{1, {x, y} ∈ region3}}, 0];
```

```
h3[x_, y_] = Integrate[f3[x - t * (- $\delta$  -  $a_2$ ) [[1]], y - t * (- $\delta$  -  $a_2$ ) [[2]], {t, 0, scale}];
```

```
In[181]:= region4 = ParametricRegion[
```

```
{t * ( $\delta$ ) [[1]] + s * ( $a_1$ ) [[1]], t * ( $\delta$ ) [[2]] + s * ( $a_1$ ) [[2]]}, {{t, 0, scale}, {s, 0, scale}}];
```

```
factor4 = 1;
```

```
f4[x_, y_] := factor4 * Piecewise[{{1, {x, y} ∈ region4}}, 0];
```

```
h4[x_, y_] = Integrate[f4[x - t * ( $a_2$ ) [[1]], y - t * ( $a_2$ ) [[2]], {t, 0, scale}];
```

```
In[185]:= region5 = ParametricRegion[{t * ( $\delta$  +  $a_2$ ) [[1]] + s * ( $\delta$ ) [[1]], t * ( $\delta$  +  $a_2$ ) [[2]] + s * ( $\delta$ ) [[2]]},  
{{t, 0, scale}, {s, 0, scale}}];
```

```
factor5 = 1;
```

```
f5[x_, y_] := factor5 * Piecewise[{{1, {x, y} ∈ region5}}, 0];
```

```
h5[x_, y_] = Integrate[f5[x - t * ( $\delta$  +  $a_1$ ) [[1]], y - t * ( $\delta$  +  $a_1$ ) [[2]], {t, 0, scale}];
```

```
In[189]:= region6 = ParametricRegion[{t * ( $\delta$  +  $a_2$ ) [[1]] + s * ( $a_2$ ) [[1]], t * ( $\delta$  +  $a_2$ ) [[2]] + s * ( $a_2$ ) [[2]]},  
{{t, 0, scale}, {s, 0, scale}}];
```

```
factor6 = 1;
```

```
f6[x_, y_] := factor6 * Piecewise[{{1, {x, y} ∈ region6}}, 0];
```

```
h6[x_, y_] = Integrate[f6[x - t * (- $a_1$  +  $a_2$ ) [[1]], y - t * (- $a_1$  +  $a_2$ ) [[2]], {t, 0, scale}];
```

```
region7 = ParametricRegion[{t * ( $\delta$  +  $a_1$ ) [[1]] + s * ( $a_1$ ) [[1]], t * ( $\delta$  +  $a_1$ ) [[2]] + s * ( $a_1$ ) [[2]]},  
{{t, 0, scale}, {s, 0, scale}}];
```

```
factor7 = 1;
```

```
f7[x_, y_] := factor7 * Piecewise[{{1, {x, y} ∈ region7}}, 0];
```

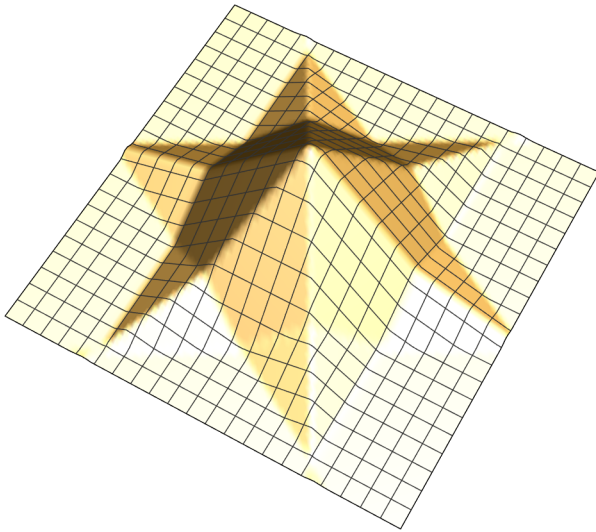
```
h7[x_, y_] = Integrate[f7[x - t * (- $a_2$  +  $a_1$ ) [[1]], y - t * (- $a_2$  +  $a_1$ ) [[2]], {t, 0, scale}];
```

```
In[197]:= h[x_, y_] := h1[x - δ[[1]], y - δ[[2]]] + h2[x - a1[[1]], y - a1[[2]]] + h3[x - a2[[1]], y - a2[[2]]] +
h5[x + δ[[1]], y + δ[[2]]] + h6[x + a2[[1]], y + a2[[2]]] + h7[x + a1[[1]], y + a1[[2]]];
```

```

In[203]:= range = 1.305;
data2 = Table[{x, y, h[x, y]}, {x, -range, range, 0.03}, {y, -range, range, 0.03}];
data1 = Flatten[data2, 1];
temp = ListPointPlot3D[data1];
ListPlot3D[data1, AspectRatio → Automatic,
  PlotLabel → Style[" $v_{I_\lambda}$ , Moment Set  $I_\delta$ ", FontSize → 12] PlotRange → All,
  Mesh → {20}, Axes → False, Boxed → False, Lighting → "Accent"]
PlotRange  $v_{I_\lambda}$ , Moment Set  $I_\delta \rightarrow$  All

```



PlotRange v_{I_λ} , Moment Set $I_\delta \rightarrow$ All

