1. $m(a+bX) = a + b \cdot m(X)$

$$\frac{1}{N}\sum_{i=1}^{N}(a+bX_i) = a + b\,m(X)$$

$$\frac{1}{N}\sum_{i=1}^{N} a + \frac{1}{N}\sum_{i=1}^{N} bX_i$$

$$\frac{1}{N}aN + \frac{b}{N}\sum_{i=1}^{N} X = a + bm(X)$$

$$a + bm(X) = a + bmX$$

2. $\mathrm{cov}(X, a+bY)$

$$= \frac{1}{N}\sum_{i=1}^{N}\left(X_i - m(X)\right)\left(a+bY_i - m(a+bY)\right)$$

$$= \frac{1}{N}\sum_{i=1}^{N}\left(X_i - m(X)\right)\left(a+bY_i - a - bm(Y)\right)$$

$$= \frac{1}{N}\sum_{i=1}^{N}\left(X_i - m(X)\right)b\left(Y_i - m(Y)\right)$$

$$= \frac{b}{N}\sum_{i=1}^{N}\left(X_i - m(X)\right)\left(Y_i - m(Y)\right)$$

$$= b\,\mathrm{cov}(X, Y)$$

3. $cov(a+bx, a+bx)$

$$= \frac{1}{N}\sum_{i=1}^{N}\left(a+bx_i - m(a+bx)\right)\left(a+bx_i - m(a+bx)\right)$$

$$= \frac{1}{N}\sum_{i=1}^{N}\left(a+bX_i - a - bm(X)\right)\left(a+bx_i - a - bm(X)\right)$$

$$= \frac{1}{N}\sum_{i=1}^{N} b^2\left(X_i - m(x)\right)\left(X_i - m(x)\right)$$

$$= \frac{b^2}{N}\sum_{i=1}^{N}\left(X_i - m(X)\right)\left(X_i - m(X)\right) = \frac{b^2}{N}\sum_{i=1}^{N}\left(X_i - m(X)\right)^2 = b^2 s^2$$

$$= b^2\, cov(X, X)$$

if $b = 1$:

$cov(X, X) = s^2$

4. $x = (1, 2, 3)$ Median $(x) = [1]$ $3 - 1 = 2$

$g(x) = 2 + 5x = (7, 12, 17)$ Median $(g) = [1] = 2 + 5\,\text{median}(x)$ $17 - 7 = 10$

$h(x) = \text{arsinh}(x) = (.88, 1.44, 1.82)$ Median$(h) = [1] = \text{arcsinh}(\text{median}(x))$ $1.82 - .88 = .94$

Any nondecreasing transformation doesn't affect the order so median will be the same. Same applies to any quartile but not IQR or range since distance will be affected.

5. $m(g(x)) = \frac{7+12+17}{3} = 12$ ✓

$g(m(x)) = 2 + 5(2) = 12$

$m(h(x)) = \frac{.88 + 1.44 + 1.82}{3} = 1.38$ ✗

$h(m(x)) = \text{arcsinh}(2) = 1.44$

Not true that $m(g(x)) = g(m(x))$, but can be true sometimes.