Lyon 1

Mesh and Computational Geometry

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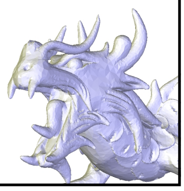

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Mesh Simplification

- Reducing the complexity of a mesh
- Elimination of redundancy
- Reduction in the number of vertices, edges and faces while preserving a specific property that may depend on the application

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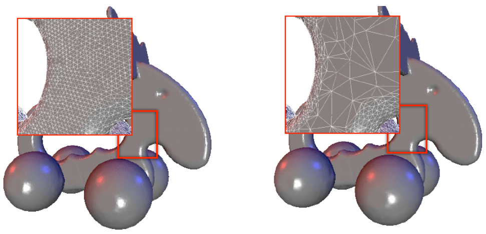
Redundancy reduction

- Useful for meshes obtained from depth images (oversampling in flat areas)



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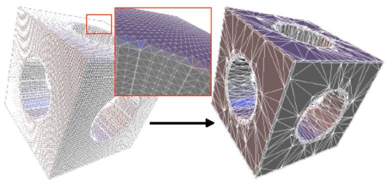
Suppression of grid effect

- Useful for implicit surface meshes obtained by a classic *marching-cube*



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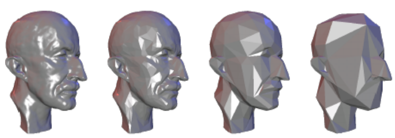
Simplification as a tool for multi-resolution

- Case of a simplification in several steps
- Definition of a mesh hierarchy
- Allows to introduce the notion of scale in geometry processing



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- Possibility of progressive mesh transmission -> transmission of operations allowing to transform one mesh into another (from the most rudimentary to the most refined)
  - Transmission over the network
  - Adaptation to material capacities (trade-off size / quality)
  - Visualization
  - Adaptation to the perceptual properties of the human eye

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2 ways to see the problem

- Given a mesh M={S,F}
- Finding M' ={S',F' } such that
  1. |S'|=n < |S| minimizing the distance (M,M')
  2. distance(M,M') < ε and |S'|minimum
- Difficult problem
  - Suboptimal solution

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Hausdorff Distance

- Given 2 objects A and B

$$H(A,B) = max_{P\in A}(d(P,B))$$

Non-symmetrical distance!
Efficient but expensive

Metro : Approximation of the Hausdorff distance between triangular meshes (points to surface computations)

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- In some cases S' may be a subset of S (sub-sampling)
- Possibility of adding additional quality constraints:
  - Triangle shapes
  - Control on the deviation of the normal vector
  - Respect of attributes such as color

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2 main classes of methods

- Fusion of vertices in 1 step
  - generally fast, but limited quality of the result
  - Algorithm in O(S) (S : number of vertices)
- Iterative decimation
  - Quality result
  - Algorithm in O(Slg(S)) on average (requires a priority-queue)

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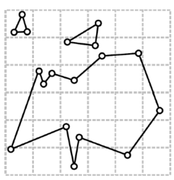
Vertex fusion

- Fusion parameterized by an approximation tolerance ε
- Partition of the space including the mesh using cells of diameter < ε



ε

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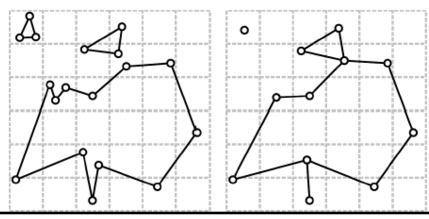
Vertex fusion

- Merging of vertices present in the same cell (*cluster*)
- Choice of one representative per *cluster*
- Can be used to produce a mesh hierarchy
  - *cluster* hierarchy



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Vertex fusion

- Some input faces become degenerate
- Transfer of connectivity to representatives
  - 2 representatives are connected by an edge, if two vertices of their respective *clusters* are connected
  - 3 representatives are connected by one triangle if 3 vertices of their respective *clusters* are connected by one triangle

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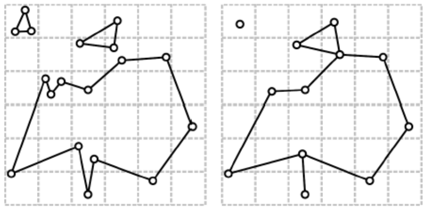
Vertex fusion



Possibility of topological genus changes if several pieces of the surface pass through the same cell (non *manifold* mesh)

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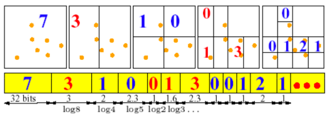
Vertex fusion

- Mainly meaningful for dense point sets (or dense sets of vertices)
- Can be extended for the compression of point sets (Gandoin et al)



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Vertex fusion

- The possibility of changing the topology can also be an advantage:
  - Removal of holes or handles smaller than ε
- The resulting mesh is generally not optimal
  - Control on the position of the vertices but not on the position of the simplified faces and edges
  - Possibility of carrying out *clusters* on the basis of other criteria

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Influence of the choice of representatives

1. Average position of *cluster points*
2. Median position of *cluster points*
3. Position optimizing a representativeness criterion
   - Minimization of a mean square error

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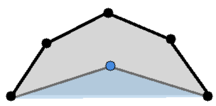
Choosing an average position



The representative is not a vertex of the initial mesh

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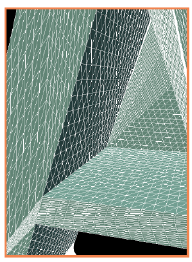

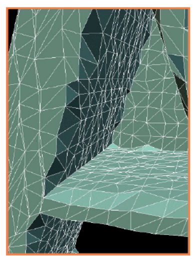

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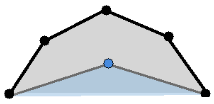
Choosing an average position

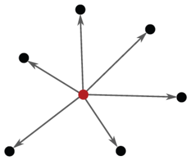


$$\hat{m} = argmin_m \sum_i d^2(x_i, m)$$

Direct construction

$$\sum_i \mathbf{X_i\hat{M}} = \vec{0}$$

$$\hat{m} = \frac{1}{n}\sum x_i$$



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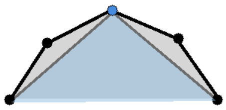
Choice of a median position



Subsampling

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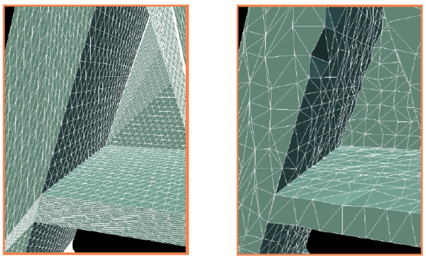

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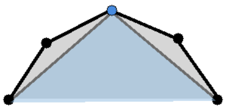
Choice of a median position



Median

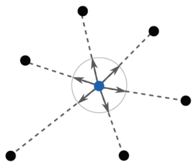
$$\tilde{m} = argmin_m \sum_i d(x_i, m)$$

Non-direct construction

$$\sum_i \frac{\mathbf{X_i\tilde{M}}}{\|\mathbf{X_i\tilde{M}}\|} = \vec{0}$$

By iteration of the Weiszfeld algorithm

$$\tilde{m}_k = \sum \frac{x_i}{\|x_i - m_{k-1}\|} / \sum \frac{1}{\|x_i - m_{k-1}\|}$$



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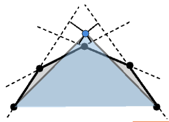
Minimization of square error



Point that best respects the connectivity in the surroundings

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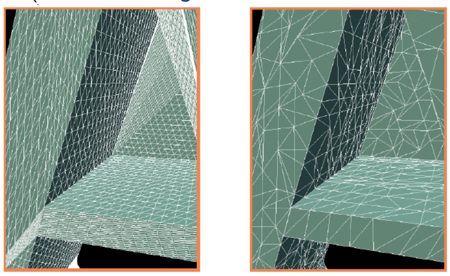

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Quadratic error

1. Find the point that minimizes the sum of (distances)^2 to the facets of the *cluster*

Other possibility :

1. Find the point minimizing the sum of (distances)^2 to the tangent planes to the vertices of the *cluster*

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Quadratic error

- Distance from a point Q=(x,y,z) to a plane P_i of normal n_i passing through a point R_i

$$d(Q, P_i)^2 = ((Q - R_i).n_i)^2$$
$$= (Q.n_i + d_i)^2$$
$$\text{où } d_i = -R_i.n_i$$

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Quadratic error

- Sum of the $(\text{distances})^2$ from a point $Q=(x,y,z)$ to a set of planes P_i of normal n_i passing through points R_i

$$E(Q) = \sum_i (Q.n_i + d_i)^2$$

$$\text{où } d_i = -R_i.n_i$$

•Search for the minimizing point E

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Quadratic error

Gradient is vanishing at the minimum

$$\frac{\delta E(Q)}{\delta Q} = \sum_i \frac{\delta (Q.n_i + d_i)^2}{\delta Q}$$

$$= \sum_i \frac{\delta (n_i^T Q + d_i)^2}{\delta Q}$$

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$$\frac{\delta E(Q)}{\delta Q} = \sum_i n_i n_i^T Q + n_i d_i$$

•So E minimal for Q verifying

$$(\sum_i n_i n_i^T) Q = -\sum_i n_i d_i$$

•Resolution of a linear system with 3 equations and 3 unknowns

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- The iso-values of E are ellipsoids, hence the name *quadric error metric* (*QEM*)

$$E(Q) = \sum_i (Q.n_i + d_i)^2$$

$$\text{où } d_i = -R_i.n_i$$

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Iterative decimation

- Deleting one vertex at a time
- Deleting a vertex affects a "region".
- Ranking of the possibility of deleting a vertex
  - Boolean
  - Or a more nuanced value that can be used as a priority
- Side effect of a vertex decimation : evolution of the ranking of the neighboring vertices
  - update cost

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- The deletion of a vertex is interpreted as the remeshing of a small surface patch
- Cases where the possible remeshing of a patch is evaluated in a Boolean manner:
  - As long as there are still "patches" to be simplified
    - Simplification of one of these patches
    - Updating of simplificability criteria for close patches

  Until there are no more reducible patches

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- Cases where the possible reduction of a patch is ranked by a priority
  - Creating a queue of patches
  - As long as there are still patches to be reduced
    - Reduction of the highest priority patch
    - Update of reducibility criteria and queuing

  Until there are no more reducible patches

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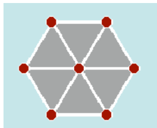
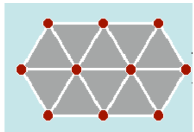
Simplification operations

- Depending on the algorithms, the patch may be the incident faces at a vertex or at an edge





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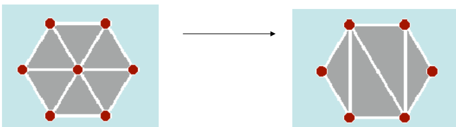
1. Removing vertices
   (region = incident faces at the vertex)
- Subsampling
- The evaluation of the removal should take into account the different ways of re-triangulating the region



- Z-triangulation?
- Triangulation minimizing the area of the patch, $O(b^3)$?
  (Dynamic programming, b vertices on the boundary)

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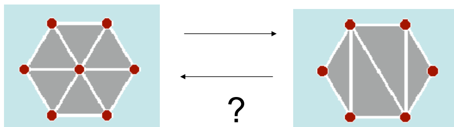
1. Removing vertices
   (region = incident faces at the vertex)
- Which code to be transmitted for the reversion of the simplication?



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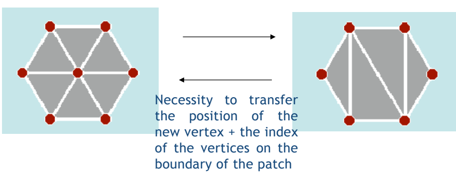
1. Removing vertices
   (region = incident faces at the vertex)
- Which code to be transmitted for a reversion of the simplification?



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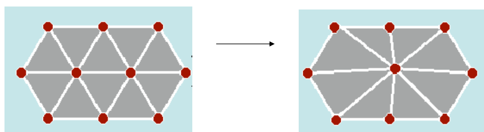
2. Edge collapse
   (region = faces incident to one edge)
- Resampling
- The edge collapse evaluation must take into account the position of the resulting vertex



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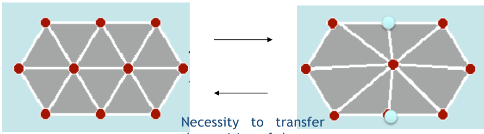
Edge collapse / Vertex split

(region = faces incident to one edge)

- Reverse operation = vertex fission



Necessity to transfer the position of the two created vertices + the index of the 2 vertices whose degree gets increased

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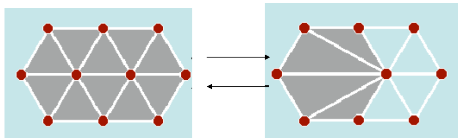
2. Oriented edge collapse

(region = faces incident to one edge)

- Subsampling
  (contraction towards the target vertex)
- Operation not symmetrical with respect to edge orientation



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- Attention:
  - The vertex resulting from edge contraction must have an admissible position
- Problem with updating the priority queue
  - ~~Deletion and~~ reinsertion in 0(lgn) of the reevaluated regions (i. e. vertex or edges)

Too costly!!!

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Local error metric

- To evaluate the deletion of a vertex
  - Calculation of the average plane corresponding to the adjacent vertices
  - Error = distance to this plane (Shroeder et al 92)
  - Retriangulation in Z
- To evaluate the contraction of an edge
  - Error=distance between the two extremities of the edge
  - Or any other distance better respecting local differential geometry

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Use of the quadric error metric

- Garland Heckbert 97
  - Each vertex P_i is associated with the function $Q_i(Q)$ which gives the quadric distance from any point Q to the planes/faces surrounding P_i (the minimum is 0 for Q= P_i at the start)
  - Cost of contracting an edge P_iP_j = minimum value of Q_i+Q_j(Q) which implies to find the point Q where this value is reached

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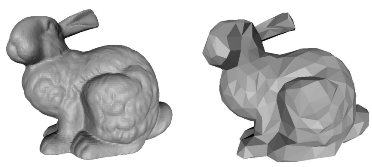
Use of quadric error metric

- Garland Heckbert 97



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Total error metric

1. The region after simplification must remain in an offset of the original surface (Cohen et al 96)
2. Preservation of the Hausdorf distance (symmetrical or not)
   - For example, the one measuring the distance from the vertices of one mesh to the faces of the other mesh

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Other possible criteria

- Approximation error
- Control of dihedral angles
- Control the valence of the vertices
- Control the shape of the triangles
- Respect for colour variations
- Limit the deviation of the normal vector

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