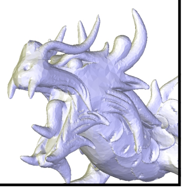
Lyon 1

Mesh and Computational Geometry

Raphaëlle Chaine
Université Claude Bernard Lyon 1

M2 ID3D
Image, Développement
et Technologie 3D
et 3A Centrale

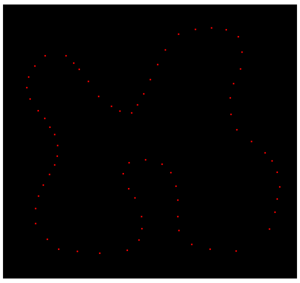
2024-25



1

Use of Voronoi Center for line reconstruction from a 2D point set (Crust)

- Let consider a set of points sampled on a line
- The points are not ordered
- How to approximate the input line with a polygonal line?

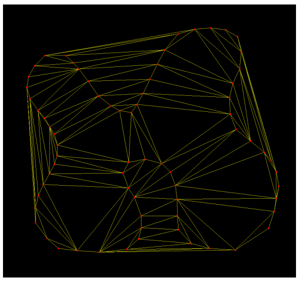


214

214

Use of Voronoi Center for line reconstruction from a point set (Crust)

- If the sampling is dense enough, Delaunay encloses a good candidate
- How to remove the edges crossing the shape?

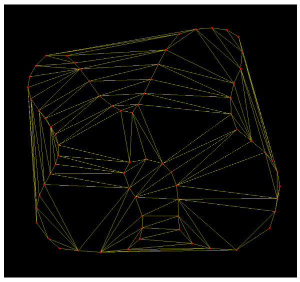


215

215

Use of Voronoi Center for line reconstruction from a point set (Crust)

- Add points as far as possible from the input line
  - Points located on the internal and external skeleton of the shape
  - Centers of maximal empty circles

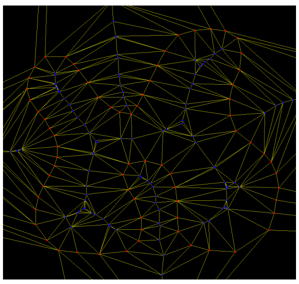


216

216

Use of Voronoi Center for line reconstruction from a point set (Crust)

- The Voronoi centers are close to the shape skeleton
  - Let's add them to break edges that cross the shape while preserving the boundaries

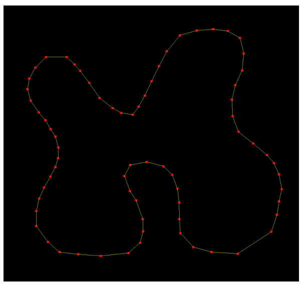


217

217

Use of Voronoi Center for line reconstruction from a point set (Crust)

- Keep the edges that join initial input points only
- Correctness of the algorithm if the input point set is locally denser than a given proportion ε of the distance to the skeleton (**ε-sampling**)

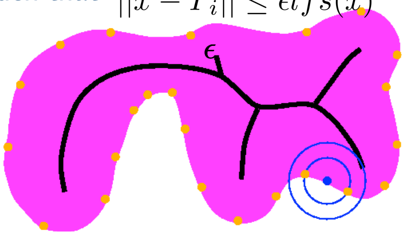


218

218

Local Feature Size (lfs) and ε-sampling

- Given ε, an **ε-sampling** of a shape is a set of samples P_i such that for each x there is a i such that $||x - P_i|| \leq \epsilon lfs(x)$



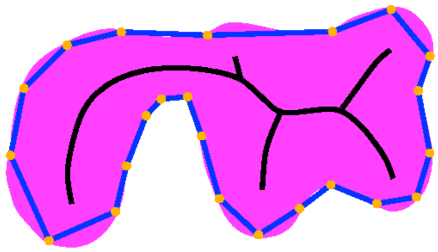
Sampling density locally proportional to 1/lfs
Measure of thickness and curvature

219

219

Reconstruction

- Crust provides guarantees
  - Homoeomorphism of the curve and its approximation in the case of a 0.25-sampling

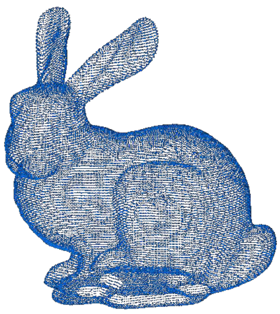


220

220

Crust in 3D?

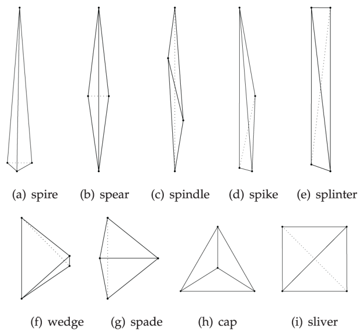
In 3D the Delaunay triangulation provides a tetrahedrisation of the shape



221

Crust in 3D?

- Zoology of Delaunay tetrahedra based on point samples from an object's surface

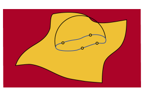


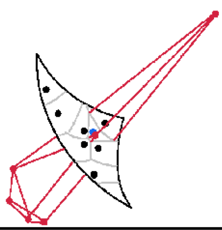
222

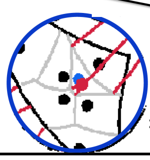
222

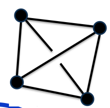
Crust in 3D

- Some Voronoi centers can be far from the skeleton and close to the surface...
  - No control over the position of the centers of flat tetrahedrons: 4 neighboring points almost cocyclic may have their center near the surface (sliver)









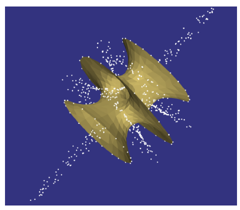
Amenta et al.

223

223

3D Crust

- Need to filter Voronoi centers



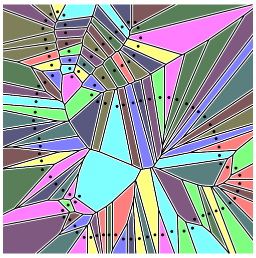
- Consider only the poles!
  - ie. Voronoi vertices certified to be far from the surface by one of the point samples

224

224

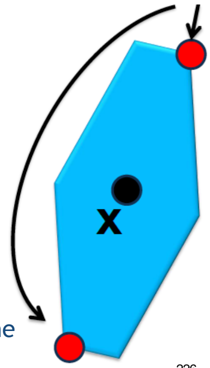
Poles

- In presence of a dense and non noisy sampling
  - long and thin Voronoi cells,
  - direction similar to the normal to the surface.



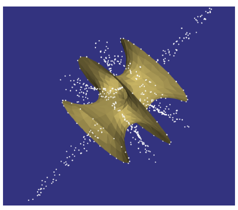
Poles

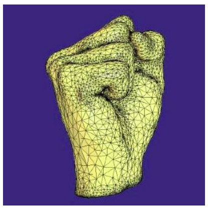
- Let V_x be the Voronoi's cell of a point x
- Positive pole p^+ : Voronoi Vertex of V_x further away from x
- Vector pole xp^+ : approximation of the normal direction at x
- The negative pole p^- : farthest vertex of V_x in the opposite direction to the vector xp^+



Crust in 3D

- Adding poles in the 3D triangulation



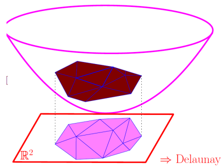


Amenta et al 98

- Reconstructed surface composed of Delaunay faces relying on 3 input point samples

Interpretation of 2D Voronoi in the space of the spheres

- By lifting the points in a higher dimensional space, there is another geometric interpretation of Voronoi
- Parabolic lift?
  - Of what?
  - For the interpretation of Delaunay we lifted the points, and we used the fact that the lift of the points of a circle were coplanar



Space of spheres

- Voronoi cell of one site P : locus of the center of the empty circles passing through P
- How to represent 2D circles by 3D points?
  - The circle C of center c and radius R will be represented by the point $(c,c^2\text{-}R^2)$
    - Lift the center point $c(x_c, y_c)$ to the altitude $c^2\text{-}R^2$ ie. at the coordinate point $(x_c, y_c, x_c^2+y_c^2\text{-}R^2)$
  - Note that 2D points alone correspond to circles of radius 0
    - Where are they located in the space of spheres?

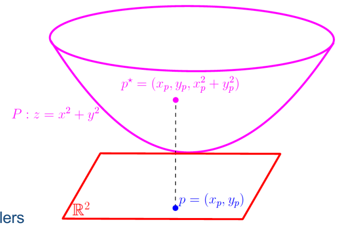
Demonstration

- 2D points correspond to circles of radius 0
  - Where are they located in the space of the spheres?
  - On the paraboloid $z = x^2 + y^2$

Demonstration

- 2D points correspond to circles of radius 0
  - Where are they located in the space of spheres?
  - On the paraboloid

$$z = x^2 + y^2$$



Images by O. Devillers

231

231

Nice interpretation of Voronoi in the space of spheres

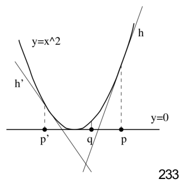
- Representation of a circle of center c and radius R by the point (c,c^2-R^2)
- What is the lift of all the circles passing through a point P?

232

232

Nice interpretation of Voronoi in the space of the spheres

- Representation of a circle of center c and radius R by the point (c,c^2-R^2)
- All circles passing through a point P : hyperplane tangent in Φ(P) to the paraboloid (Φ(P) lift of P on the paraboloid)

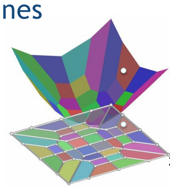


233

233

Nice interpretation of Voronoi in the space of the spheres

- We consider the lifting Φ(P_i) of all the input points P_i on the paraboloid
- Correspondence between Voronoi and the intersection of the half spaces located above the previous hyperplanes



234

234