MATH1309 – Practice Problems 3

By hand

1. Let **X** be $N_3(\boldsymbol{\mu}, \boldsymbol{\Sigma})$ with $\boldsymbol{\mu}' = [-3,1,4]$ and $\boldsymbol{\Sigma} = \begin{bmatrix} 1 & -2 & 0 \\ -2 & 5 & 0 \\ 0 & 0 & 2 \end{bmatrix}$ Which of the following random variables are independent? Explain.
   a. X_1 and X_2
   b. X_2 and X_3
   c. (X_1,X_2) and X_3
   d. $\frac{X_1+X_2}{2}$ and X_3
   e. X_2 and $X_2 - \frac{5}{2}X_1 - X_3$

SOLUTIONS

a) No as the covariance is not equal to 0
b) Yes the covariance is equal to 0
c) Yes the covariance is equal to 0 $\sigma_{13} = \sigma_{23} = 0$
d) Yes they are jointly normal $\frac{1}{2}\sigma_{13} + \frac{1}{2}\sigma_{23} = 0$
e) No the covariance not equal to 0

$$\sigma_{22} - \frac{5}{2}\sigma_{12} - \sigma_{23} = 5 - -\frac{5}{2}(-2) - (0) = 10$$