QUESTION 1

For the matrix X with $\mu' = \begin{bmatrix} -2 & 7 & 1 \end{bmatrix}$, $\Sigma = \begin{bmatrix} 3 & 0 & 1 \\ 0 & 3 & 2 \\ 1 & 2 & 1 \end{bmatrix}$ and the matrix $B = \begin{bmatrix} 7 & 2 & 1 \\ 1 & 0 & 2 \\ 0 & 2 & 3 \end{bmatrix}$ find the following:

a) $E(X_2 - X_1)$

b) E(BX)

c) Cov(BX)

(1+2+2 = 5 marks)

QUESTION 2

Let the matrix $X = \begin{bmatrix} 5 & 20 \\ 6 & 17 \\ 7 & 18 \\ 9 & 21 \\ 8 & 19 \end{bmatrix}$

a) Find **S*** and **R** for the matrix **X**

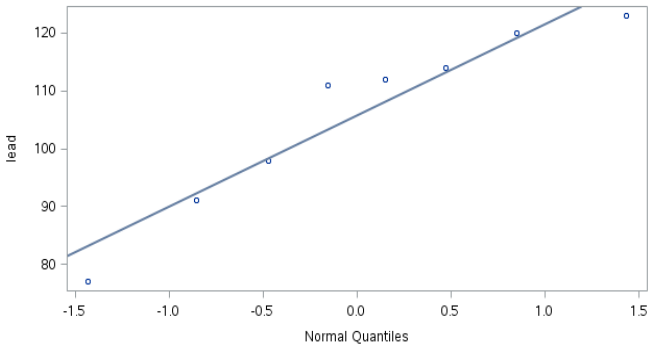
b) Find the eigenvalues for $B = \begin{bmatrix} 0 & 1 \\ -2 & -3 \end{bmatrix}$

(3+3 = 6 marks)

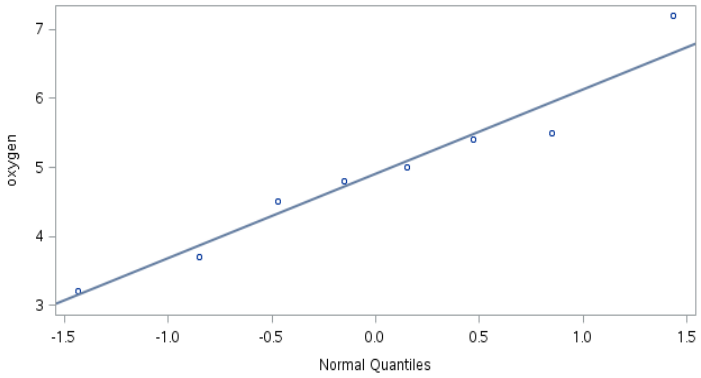
QUESTION 3

The following output from SAS was used to investigate observations of lead contamination and dissolved oxygen in water samples taken from the Yarra River.

Basic Statistical Measures			
Location		Variability	
Mean	105.7500	Std Deviation	15.76388
Median	111.5000	Variance	248.50000
Mode	.	Range	46.00000
		Interquartile Range	22.50000



Basic Statistical Measures			
Location		Variability	
Mean	4.912500	Std Deviation	1.21941
Median	4.900000	Variance	1.48696
Mode	.	Range	4.00000
		Interquartile Range	1.35000



a) Using the output above comment on the distribution of lead

b) Using the sas code and output (continued overleaf) sketch the bivariate contour

(2+4 = 6 marks)

```
data water;
input oxygen lead;
datalines;
4.5 91
3.2 111
3.7 120
5.4 98
4.8 114
7.2 77
5.0 123
5.5 112
;
run;

proc univariate data=water
normal plots;
run;

proc iml;
use water;
read all var _all_ into A;
n=nrow(A);
p=ncol(A);
reset print;
centre=t(mean(A));
cov_A=cov(A);
cor_A=corr(A);
incov=inv(cov_A);
mu0={5,102};
tsq=n*t(centre-
mu0)*incov*(centre-mu0);
fcri=finv(0.95,p,n-p);
lam=eigval(cov_A);
E=eigvec(cov_A);
length=2*sqrt(lam*fcri/n);
mean_A=mean(A);
d2=(A-mean_A)*incov*t(A-
mean_A);
mahala=vecdiag(d2);
ranks=rank(mahala);
f0=(ranks-0.5)/n;
chiq=cinv(f0,p);
sd=vecdiag(cov_x);
ubound1=centre+sqrt(fcri)*sqrt(
sd/n);
lbound1=centre-
sqrt(fcri)*sqrt(sd/n);
```

centre 2 rows 1 col (numeric)

4.9125
105.75

cov_A 2 rows 2 cols (numeric)

1.4869643	-12.21071
-12.21071	248.5

cor_A 2 rows 2 cols (numeric)

1	-0.635225
-0.635225	1

incov 2 rows 2 cols (numeric)

1.127449	0.0554002
0.0554002	0.0067464

mu0 2 rows 1 col (numeric)

5
102

tsq 1 row 1 col (numeric)

0.5371732

fcri 1 row 1 col (numeric)

5.1432528

f0 8 rows 1 col (numeric)

0.6875
0.8125
0.4375
0.0625
0.1875
0.9375
0.5625
0.3125

lam 2 rows 1 col (numeric)

249.10215
0.884814

E 2 rows 2 cols (numeric)

-0.049253	-0.998786
0.9987863	-0.049253

length 2 rows 1 col (numeric)

25.310031
1.5084466

mean_A 1 row 2 cols (numeric)

4.9125	105.75

chiq 8 rows 1 col (numeric)

2.3263016
3.3479529
1.1507283
0.129077
0.4152787
5.5451774
1.6533571
0.7493869

mahala 8 rows 1 col (numeric)

2.3337542
2.4962022
1.1130395
0.254532
0.3706084
4.1889993
2.1833423
1.0595221

ranks 8 rows 1 col (numeric)

6
7
4
1
2
8
5
3

ubound1 2 rows 1 col (numeric)

5.8902419
118.38971

lbound1 2 rows 1 col (numeric)

3.9347581
93.110289

QUESTION 4

For the data matrix $X = \begin{bmatrix} 1 & 3 & -2 \\ 2 & 2 & -4 \\ 3 & 4 & -2 \\ 2 & 3 & -4 \\ 2 & 3 & -3 \end{bmatrix}$ with $S = \begin{bmatrix} 1/2 & 1/4 & 0 \\ 1/4 & 1/2 & 1/2 \\ 0 & 1/2 & 1 \end{bmatrix}$ and using α=0.05

$$S^{-1} = \begin{bmatrix} 4 & -4 & 2 \\ -4 & 8 & -4 \\ 2 & -4 & 3 \end{bmatrix}$$

a) test the hypothesis that $\mu_0 = \begin{bmatrix} 3 \\ 3 \\ -4 \end{bmatrix}$ using the appropriate test statistic and state your conclusion.

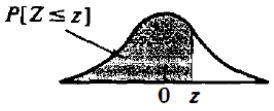
b) Calculate the simultaneous confidence intervals for each of μ_1, μ_2 and μ_3

c) Find the confidence interval for the difference between μ_1 and μ_3

(4+3+1 = 8 marks)

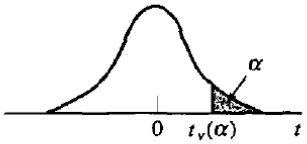
END OF TEST QUESTIONS

TABLE 1 STANDARD NORMAL PROBABILITIES



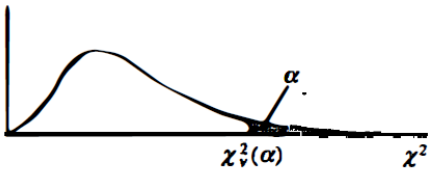
z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
.0	.5000	.5040	.5080	.5120	.5160	.5199	.5239	.5279	.5319	.5359
.1	.5398	.5438	.5478	.5517	.5557	.5596	.5636	.5675	.5714	.5753
.2	.5793	.5832	.5871	.5910	.5948	.5987	.6026	.6064	.6103	.6141
.3	.6179	.6217	.6255	.6293	.6331	.6368	.6406	.6443	.6480	.6517
.4	.6554	.6591	.6628	.6664	.6700	.6736	.6772	.6808	.6844	.6879
.5	.6915	.6950	.6985	.7019	.7054	.7088	.7123	.7157	.7190	.7224
.6	.7257	.7291	.7324	.7357	.7389	.7422	.7454	.7486	.7517	.7549
.7	.7580	.7611	.7642	.7673	.7703	.7734	.7764	.7794	.7823	.7852
.8	.7881	.7910	.7939	.7967	.7995	.8023	.8051	.8078	.8106	.8133
.9	.8159	.8186	.8212	.8238	.8264	.8289	.8315	.8340	.8365	.8389
1.0	.8413	.8438	.8461	.8485	.8508	.8531	.8554	.8577	.8599	.8621
1.1	.8643	.8665	.8686	.8708	.8729	.8749	.8770	.8790	.8810	.8830
1.2	.8849	.8869	.8888	.8907	.8925	.8944	.8962	.8980	.8997	.9015
1.3	.9032	.9049	.9066	.9082	.9099	.9115	.9131	.9147	.9162	.9177
1.4	.9192	.9207	.9222	.9236	.9251	.9265	.9279	.9292	.9306	.9319
1.5	.9332	.9345	.9357	.9370	.9382	.9394	.9406	.9418	.9429	.9441
1.6	.9452	.9463	.9474	.9484	.9495	.9505	.9515	.9525	.9535	.9545
1.7	.9554	.9564	.9573	.9582	.9591	.9599	.9608	.9616	.9625	.9633
1.8	.9641	.9649	.9656	.9664	.9671	.9678	.9686	.9693	.9699	.9706
1.9	.9713	.9719	.9726	.9732	.9738	.9744	.9750	.9756	.9761	.9767
2.0	.9772	.9778	.9783	.9788	.9793	.9798	.9803	.9808	.9812	.9817
2.1	.9821	.9826	.9830	.9834	.9838	.9842	.9846	.9850	.9854	.9857
2.2	.9861	.9864	.9868	.9871	.9875	.9878	.9881	.9884	.9887	.9890
2.3	.9893	.9896	.9898	.9901	.9904	.9906	.9909	.9911	.9913	.9916
2.4	.9918	.9920	.9922	.9925	.9927	.9929	.9931	.9932	.9934	.9936
2.5	.9938	.9940	.9941	.9943	.9945	.9946	.9948	.9949	.9951	.9952
2.6	.9953	.9955	.9956	.9957	.9959	.9960	.9961	.9962	.9963	.9964
2.7	.9965	.9966	.9967	.9968	.9969	.9970	.9971	.9972	.9973	.9974
2.8	.9974	.9975	.9976	.9977	.9977	.9978	.9979	.9979	.9980	.9981
2.9	.9981	.9982	.9982	.9983	.9984	.9984	.9985	.9985	.9986	.9986
3.0	.9987	.9987	.9987	.9988	.9988	.9989	.9989	.9989	.9990	.9990
3.1	.9990	.9991	.9991	.9991	.9992	.9992	.9992	.9992	.9993	.9993
3.2	.9993	.9993	.9994	.9994	.9994	.9994	.9994	.9995	.9995	.9995
3.3	.9995	.9995	.9995	.9996	.9996	.9996	.9996	.9996	.9996	.9997
3.4	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9998
3.5	.9998	.9998	.9998	.9998	.9998	.9998	.9998	.9998	.9998	.9998

TABLE 2 STUDENT'S t-DISTRIBUTION PERCENTAGE POINTS



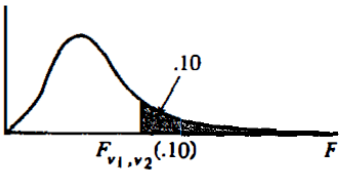
d.f.	α								
ν	.250	.100	.050	.025	.010	.00833	.00625	.005	.0025
1	1.000	3.078	6.314	12.706	31.821	38.190	50.923	63.657	127.321
2	.816	1.886	2.920	4.303	6.965	7.649	8.860	9.925	14.089
3	.765	1.638	2.353	3.182	4.541	4.857	5.392	5.841	7.453
4	.741	1.533	2.132	2.776	3.747	3.961	4.315	4.604	5.598
5	.727	1.476	2.015	2.571	3.365	3.534	3.810	4.032	4.773
6	.718	1.440	1.943	2.447	3.143	3.287	3.521	3.707	4.317
7	.711	1.415	1.895	2.365	2.998	3.128	3.335	3.499	4.029
8	.706	1.397	1.860	2.306	2.896	3.016	3.206	3.355	3.833
9	.703	1.383	1.833	2.262	2.821	2.933	3.111	3.250	3.690
10	.700	1.372	1.812	2.228	2.764	2.870	3.038	3.169	3.581
11	.697	1.363	1.796	2.201	2.718	2.820	2.981	3.106	3.497
12	.695	1.356	1.782	2.179	2.681	2.779	2.934	3.055	3.428
13	.694	1.350	1.771	2.160	2.650	2.746	2.896	3.012	3.372
14	.692	1.345	1.761	2.145	2.624	2.718	2.864	2.977	3.326
15	.691	1.341	1.753	2.131	2.602	2.694	2.837	2.947	3.286
16	.690	1.337	1.746	2.120	2.583	2.673	2.813	2.921	3.252
17	.689	1.333	1.740	2.110	2.567	2.655	2.793	2.898	3.222
18	.688	1.330	1.734	2.101	2.552	2.639	2.775	2.878	3.197
19	.688	1.328	1.729	2.093	2.539	2.625	2.759	2.861	3.174
20	.687	1.325	1.725	2.086	2.528	2.613	2.744	2.845	3.153
21	.686	1.323	1.721	2.080	2.518	2.601	2.732	2.831	3.135
22	.686	1.321	1.717	2.074	2.508	2.591	2.720	2.819	3.119
23	.685	1.319	1.714	2.069	2.500	2.582	2.710	2.807	3.104
24	.685	1.318	1.711	2.064	2.492	2.574	2.700	2.797	3.091
25	.684	1.316	1.708	2.060	2.485	2.566	2.692	2.787	3.078
26	.684	1.315	1.706	2.056	2.479	2.559	2.684	2.779	3.067
27	.684	1.314	1.703	2.052	2.473	2.552	2.676	2.771	3.057
28	.683	1.313	1.701	2.048	2.467	2.546	2.669	2.763	3.047
29	.683	1.311	1.699	2.045	2.462	2.541	2.663	2.756	3.038
30	.683	1.310	1.697	2.042	2.457	2.536	2.657	2.750	3.030
40	.681	1.303	1.684	2.021	2.423	2.499	2.616	2.704	2.971
60	.679	1.296	1.671	2.000	2.390	2.463	2.575	2.660	2.915
120	.677	1.289	1.658	1.980	2.358	2.428	2.536	2.617	2.860
∞	.674	1.282	1.645	1.960	2.326	2.394	2.498	2.576	2.813

TABLE 3 χ^2 DISTRIBUTION PERCENTAGE POINTS



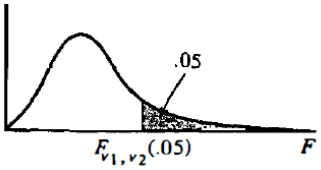
d.f.	α								
ν	.990	.950	.900	.500	.100	.050	.025	.010	.005
1	.0002	.004	.02	.45	2.71	3.84	5.02	6.63	7.88
2	.02	.10	.21	1.39	4.61	5.99	7.38	9.21	10.60
3	.11	.35	.58	2.37	6.25	7.81	9.35	11.34	12.84
4	.30	.71	1.06	3.36	7.78	9.49	11.14	13.28	14.86
5	.55	1.15	1.61	4.35	9.24	11.07	12.83	15.09	16.75
6	.87	1.64	2.20	5.35	10.64	12.59	14.45	16.81	18.55
7	1.24	2.17	2.83	6.35	12.02	14.07	16.01	18.48	20.28
8	1.65	2.73	3.49	7.34	13.36	15.51	17.53	20.09	21.95
9	2.09	3.33	4.17	8.34	14.68	16.92	19.02	21.67	23.59
10	2.56	3.94	4.87	9.34	15.99	18.31	20.48	23.21	25.19
11	3.05	4.57	5.58	10.34	17.28	19.68	21.92	24.72	26.76
12	3.57	5.23	6.30	11.34	18.55	21.03	23.34	26.22	28.30
13	4.11	5.89	7.04	12.34	19.81	22.36	24.74	27.69	29.82
14	4.66	6.57	7.79	13.34	21.06	23.68	26.12	29.14	31.32
15	5.23	7.26	8.55	14.34	22.31	25.00	27.49	30.58	32.80
16	5.81	7.96	9.31	15.34	23.54	26.30	28.85	32.00	34.27
17	6.41	8.67	10.09	16.34	24.77	27.59	30.19	33.41	35.72
18	7.01	9.39	10.86	17.34	25.99	28.87	31.53	34.81	37.16
19	7.63	10.12	11.65	18.34	27.20	30.14	32.85	36.19	38.58
20	8.26	10.85	12.44	19.34	28.41	31.41	34.17	37.57	40.00
21	8.90	11.59	13.24	20.34	29.62	32.67	35.48	38.93	41.40
22	9.54	12.34	14.04	21.34	30.81	33.92	36.78	40.29	42.80
23	10.20	13.09	14.85	22.34	32.01	35.17	38.08	41.64	44.18
24	10.86	13.85	15.66	23.34	33.20	36.42	39.36	42.98	45.56
25	11.52	14.61	16.47	24.34	34.38	37.65	40.65	44.31	46.93
26	12.20	15.38	17.29	25.34	35.56	38.89	41.92	45.64	48.29
27	12.88	16.15	18.11	26.34	36.74	40.11	43.19	46.96	49.64
28	13.56	16.93	18.94	27.34	37.92	41.34	44.46	48.28	50.99
29	14.26	17.71	19.77	28.34	39.09	42.56	45.72	49.59	52.34
30	14.95	18.49	20.60	29.34	40.26	43.77	46.98	50.89	53.67
40	22.16	26.51	29.05	39.34	51.81	55.76	59.34	63.69	66.77
50	29.71	34.76	37.69	49.33	63.17	67.50	71.42	76.15	79.49
60	37.48	43.19	46.46	59.33	74.40	79.08	83.30	88.38	91.95
70	45.44	51.74	55.33	69.33	85.53	90.53	95.02	100.43	104.21
80	53.54	60.39	64.28	79.33	96.58	101.88	106.63	112.33	116.32
90	61.75	69.13	73.29	89.33	107.57	113.15	118.14	124.12	128.30
100	70.06	77.93	82.36	99.33	118.50	124.34	129.56	135.81	140.17

TABLE 4 *F*-DISTRIBUTION PERCENTAGE POINTS ($\alpha = .10$)



ν_2 \ ν_1	1	2	3	4	5	6	7	8	9	10	12	15	20	25	30	40	60
	39.86	49.50	53.59	55.83	57.24	58.20	58.91	59.44	59.86	60.19	60.71	61.22	61.74	62.05	62.26	62.53	62.79
2	8.53	9.00	9.16	9.24	9.29	9.33	9.35	9.37	9.38	9.39	9.41	9.42	9.44	9.45	9.46	9.47	9.47
3	5.54	5.46	5.39	5.34	5.31	5.28	5.27	5.25	5.24	5.23	5.22	5.20	5.18	5.17	5.17	5.16	5.15
4	4.54	4.32	4.19	4.11	4.05	4.01	3.98	3.95	3.94	3.92	3.90	3.87	3.84	3.83	3.82	3.80	3.79
5	4.06	3.78	3.62	3.52	3.45	3.40	3.37	3.34	3.32	3.30	3.27	3.24	3.21	3.19	3.17	3.16	3.14
6	3.78	3.46	3.29	3.18	3.11	3.05	3.01	2.98	2.96	2.94	2.90	2.87	2.84	2.81	2.80	2.78	2.76
7	3.59	3.26	3.07	2.96	2.88	2.83	2.78	2.75	2.72	2.70	2.67	2.63	2.59	2.57	2.56	2.54	2.51
8	3.46	3.11	2.92	2.81	2.73	2.67	2.62	2.59	2.56	2.54	2.50	2.46	2.42	2.40	2.38	2.36	2.34
9	3.36	3.01	2.81	2.69	2.61	2.55	2.51	2.47	2.44	2.42	2.38	2.34	2.30	2.27	2.25	2.23	2.21
10	3.29	2.92	2.73	2.61	2.52	2.46	2.41	2.38	2.35	2.32	2.28	2.24	2.20	2.17	2.16	2.13	2.11
11	3.23	2.86	2.66	2.54	2.45	2.39	2.34	2.30	2.27	2.25	2.21	2.17	2.12	2.10	2.08	2.05	2.03
12	3.18	2.81	2.61	2.48	2.39	2.33	2.28	2.24	2.21	2.19	2.15	2.10	2.06	2.03	2.01	1.99	1.96
13	3.14	2.76	2.56	2.43	2.35	2.28	2.23	2.20	2.16	2.14	2.10	2.05	2.01	1.98	1.96	1.93	1.90
14	3.10	2.73	2.52	2.39	2.31	2.24	2.19	2.15	2.12	2.10	2.05	2.01	1.96	1.93	1.91	1.89	1.86
15	3.07	2.70	2.49	2.36	2.27	2.21	2.16	2.12	2.09	2.06	2.02	1.97	1.92	1.89	1.87	1.85	1.82
16	3.05	2.67	2.46	2.33	2.24	2.18	2.13	2.09	2.06	2.03	1.99	1.94	1.89	1.86	1.84	1.81	1.78
17	3.03	2.64	2.44	2.31	2.22	2.15	2.10	2.06	2.03	2.00	1.96	1.91	1.86	1.83	1.81	1.78	1.75
18	3.01	2.62	2.42	2.29	2.20	2.13	2.08	2.04	2.00	1.98	1.93	1.89	1.84	1.80	1.78	1.75	1.72
19	2.99	2.61	2.40	2.27	2.18	2.11	2.06	2.02	1.98	1.96	1.91	1.86	1.81	1.78	1.76	1.73	1.70
20	2.97	2.59	2.38	2.25	2.16	2.09	2.04	2.00	1.96	1.94	1.89	1.84	1.79	1.76	1.74	1.71	1.68
21	2.96	2.57	2.36	2.23	2.14	2.08	2.02	1.98	1.95	1.92	1.87	1.83	1.78	1.74	1.72	1.69	1.66
22	2.95	2.56	2.35	2.22	2.13	2.06	2.01	1.97	1.93	1.90	1.86	1.81	1.76	1.73	1.70	1.67	1.64
23	2.94	2.55	2.34	2.21	2.11	2.05	1.99	1.95	1.92	1.89	1.84	1.80	1.74	1.71	1.69	1.66	1.62
24	2.93	2.54	2.33	2.19	2.10	2.04	1.98	1.94	1.91	1.88	1.83	1.78	1.73	1.70	1.67	1.64	1.61
25	2.92	2.53	2.32	2.18	2.09	2.02	1.97	1.93	1.89	1.87	1.82	1.77	1.72	1.68	1.66	1.63	1.59
26	2.91	2.52	2.31	2.17	2.08	2.01	1.96	1.92	1.88	1.86	1.81	1.76	1.71	1.67	1.65	1.61	1.58
27	2.90	2.51	2.30	2.17	2.07	2.00	1.95	1.91	1.87	1.85	1.80	1.75	1.70	1.66	1.64	1.60	1.57
28	2.89	2.50	2.29	2.16	2.06	2.00	1.94	1.90	1.87	1.84	1.79	1.74	1.69	1.65	1.63	1.59	1.56
29	2.89	2.50	2.28	2.15	2.06	1.99	1.93	1.89	1.86	1.83	1.78	1.73	1.68	1.64	1.62	1.58	1.55
30	2.88	2.49	2.28	2.14	2.05	1.98	1.93	1.88	1.85	1.82	1.77	1.72	1.67	1.63	1.61	1.57	1.54
40	2.84	2.44	2.23	2.09	2.00	1.93	1.87	1.83	1.79	1.76	1.71	1.66	1.61	1.57	1.54	1.51	1.47
60	2.79	2.39	2.18	2.04	1.95	1.87	1.82	1.77	1.74	1.71	1.66	1.60	1.54	1.50	1.48	1.44	1.40
120	2.75	2.35	2.13	1.99	1.90	1.82	1.77	1.72	1.68	1.65	1.60	1.55	1.48	1.45	1.41	1.37	1.32
∞	2.71	2.30	2.08	1.94	1.85	1.77	1.72	1.67	1.63	1.60	1.55	1.49	1.42	1.38	1.34	1.30	1.24

TABLE 5 *F*-DISTRIBUTION PERCENTAGE POINTS ($\alpha = .05$)



ν_2 \ ν_1	1	2	3	4	5	6	7	8	9	10	12	15	20	25	30	40	60
1	161.5	199.5	215.7	224.6	230.2	234.0	236.8	238.9	240.5	241.9	243.9	246.0	248.0	249.3	250.1	251.1	252.2
2	18.51	19.00	19.16	19.25	19.30	19.33	19.35	19.37	19.38	19.40	19.41	19.43	19.45	19.46	19.46	19.47	19.48
3	10.13	9.55	9.28	9.12	9.01	8.94	8.89	8.85	8.81	8.79	8.74	8.70	8.66	8.63	8.62	8.59	8.57
4	7.71	6.94	6.59	6.39	6.26	6.16	6.09	6.04	6.00	5.96	5.91	5.86	5.80	5.77	5.75	5.72	5.69
5	6.61	5.79	5.41	5.19	5.05	4.95	4.88	4.82	4.77	4.74	4.68	4.62	4.56	4.52	4.50	4.46	4.43
6	5.99	5.14	4.76	4.53	4.39	4.28	4.21	4.15	4.10	4.06	4.00	3.94	3.87	3.83	3.81	3.77	3.74
7	5.59	4.74	4.35	4.12	3.97	3.87	3.79	3.73	3.68	3.64	3.57	3.51	3.44	3.40	3.38	3.34	3.30
8	5.32	4.46	4.07	3.84	3.69	3.58	3.50	3.44	3.39	3.35	3.28	3.22	3.15	3.11	3.08	3.04	3.01
9	5.12	4.26	3.86	3.63	3.48	3.37	3.29	3.23	3.18	3.14	3.07	3.01	2.94	2.89	2.86	2.83	2.79
10	4.96	4.10	3.71	3.48	3.33	3.22	3.14	3.07	3.02	2.98	2.91	2.85	2.77	2.73	2.70	2.66	2.62
11	4.84	3.98	3.59	3.36	3.20	3.09	3.01	2.95	2.90	2.85	2.79	2.72	2.65	2.60	2.57	2.53	2.49
12	4.75	3.89	3.49	3.26	3.11	3.00	2.91	2.85	2.80	2.75	2.69	2.62	2.54	2.50	2.47	2.43	2.38
13	4.67	3.81	3.41	3.18	3.03	2.92	2.83	2.77	2.71	2.67	2.60	2.53	2.46	2.41	2.38	2.34	2.30
14	4.60	3.74	3.34	3.11	2.96	2.85	2.76	2.70	2.65	2.60	2.53	2.46	2.39	2.34	2.31	2.27	2.22
15	4.54	3.68	3.29	3.06	2.90	2.79	2.71	2.64	2.59	2.54	2.48	2.40	2.33	2.28	2.25	2.20	2.16
16	4.49	3.63	3.24	3.01	2.85	2.74	2.66	2.59	2.54	2.49	2.42	2.35	2.28	2.23	2.19	2.15	2.11
17	4.45	3.59	3.20	2.96	2.81	2.70	2.61	2.55	2.49	2.45	2.38	2.31	2.23	2.18	2.15	2.10	2.06
18	4.41	3.55	3.16	2.93	2.77	2.66	2.58	2.51	2.46	2.41	2.34	2.27	2.19	2.14	2.11	2.06	2.02
19	4.38	3.52	3.13	2.90	2.74	2.63	2.54	2.48	2.42	2.38	2.31	2.23	2.16	2.11	2.07	2.03	1.98
20	4.35	3.49	3.10	2.87	2.71	2.60	2.51	2.45	2.39	2.35	2.28	2.20	2.12	2.07	2.04	1.99	1.95
21	4.32	3.47	3.07	2.84	2.68	2.57	2.49	2.42	2.37	2.32	2.25	2.18	2.10	2.05	2.01	1.96	1.92
22	4.30	3.44	3.05	2.82	2.66	2.55	2.46	2.40	2.34	2.30	2.23	2.15	2.07	2.02	1.98	1.94	1.89
23	4.28	3.42	3.03	2.80	2.64	2.53	2.44	2.37	2.32	2.27	2.20	2.13	2.05	2.00	1.96	1.91	1.86
24	4.26	3.40	3.01	2.78	2.62	2.51	2.42	2.36	2.30	2.25	2.18	2.11	2.03	1.97	1.94	1.89	1.84
25	4.24	3.39	2.99	2.76	2.60	2.49	2.40	2.34	2.28	2.24	2.16	2.09	2.01	1.96	1.92	1.87	1.82
26	4.23	3.37	2.98	2.74	2.59	2.47	2.39	2.32	2.27	2.22	2.15	2.07	1.99	1.94	1.90	1.85	1.80
27	4.21	3.35	2.96	2.73	2.57	2.46	2.37	2.31	2.25	2.20	2.13	2.06	1.97	1.92	1.88	1.84	1.79
28	4.20	3.34	2.95	2.71	2.56	2.45	2.36	2.29	2.24	2.19	2.12	2.04	1.96	1.91	1.87	1.82	1.77
29	4.18	3.33	2.93	2.70	2.55	2.43	2.35	2.28	2.22	2.18	2.10	2.03	1.94	1.89	1.85	1.81	1.75
30	4.17	3.32	2.92	2.69	2.53	2.42	2.33	2.27	2.21	2.16	2.09	2.01	1.93	1.88	1.84	1.79	1.74
40	4.08	3.23	2.84	2.61	2.45	2.34	2.25	2.18	2.12	2.08	2.00	1.92	1.84	1.78	1.74	1.69	1.64
60	4.00	3.15	2.76	2.53	2.37	2.25	2.17	2.10	2.04	1.99	1.92	1.84	1.75	1.69	1.65	1.59	1.53
120	3.92	3.07	2.68	2.45	2.29	2.18	2.09	2.02	1.96	1.91	1.83	1.75	1.66	1.60	1.55	1.50	1.43
∞	3.84	3.00	2.61	2.37	2.21	2.10	2.01	1.94	1.88	1.83	1.75	1.67	1.57	1.51	1.46	1.39	1.32