MATH1309 – Practice Problems 8

This week we will explore the solutions to Examples 3 to 7 in the lecture notes using the procedure PROC FACTOR

The first two examples below will use the PCA method method=principal, the last two examples will use maximum likelihood method=m.

Note:

When working with summary information only, you need to add an additional argument into the first line of your data step to define the type of information you have ie:

(type=corr)

(type=cov)

Example 3: The sample mean $\overline{\boldsymbol{X}}$ and sample covariance matrix $\boldsymbol{S}$ for a multivariate population with unknown mean $\boldsymbol{\mu}$ and covariance matrix Σ is obtained using a random sample of size 25. They are:

$$\overline{\boldsymbol{X}} = \begin{pmatrix} 5 \\ 10 \\ 8 \end{pmatrix} \quad \boldsymbol{S} = \begin{pmatrix} 19 & 30 & 2 \\ 30 & 57 & 5 \\ 2 & 5 & 38 \end{pmatrix}.$$

Estimate a suitable orthogonal factor model.

Example 4: A random sample of size 25 was obtained form multivariate population. The computed values for the sample mean and the sample covariance matrix are given below:

$$\overline{\boldsymbol{X}} = \begin{pmatrix} 5 \\ 2 \\ 8 \\ 8 \end{pmatrix} \qquad \boldsymbol{S} = \begin{pmatrix} 18.87 & 26.86 & 7.55 & -5.15 \\ 26.86 & 47.24 & 5.10 & -15.23 \\ 7.55 & 5.10 & 92.19 & 58.89 \\ -5.15 & -15.23 & 58.89 & 48.52 \end{pmatrix}.$$

Estimate a suitable orthogonal factor model.

Example 5: The sample correlation matrix, $\boldsymbol{R}$ given of below is computed for seven descriptive characteristics observed on 30 residential properties. The characteristics are as follows:

X_1 = area in square feet,

X_2 = total number of rooms,

X_3 = number of bedrooms,

X_4 = number of bathrooms,

X_5 = age of the property,

X_6 = attached garage or car-port

(0=no garage or car-port, 1=single, 2= double or higher) and

X_7 = view(1=good, 0=poor).

Estimate a suitable factor model using the maximum likelihood procedure.

$$\boldsymbol{R} = \begin{pmatrix} 1.00 & 0.55 & 0.40 & 0.53 & 0.45 & 0.47 & 0.28 \\ 0.55 & 1.00 & 0.43 & 0.75 & 0.35 & 0.42 & 0.18 \\ 0.40 & 0.43 & 1.00 & 0.31 & 0.45 & 0.21 & 0.15 \\ 0.53 & 0.75 & 0.31 & 1.00 & 0.64 & 0.40 & 0.44 \\ 0.45 & 0.35 & 0.45 & 0.64 & 1.00 & 0.48 & 0.25 \\ 0.47 & 0.42 & 0.21 & 0.40 & 0.48 & 1.00 & 0.01 \\ 0.28 & 0.18 & 0.15 & 0.44 & 0.25 & 0.01 & 1.00 \end{pmatrix}.$$

Example 7: Let X_1, X_2, X_3, X_4, X_5 denote the weekly rate of return for five selected stocks listed in the Melbourne stock exchange. The correlation matrix between the stocks based on 50 observations is given below:

$$\boldsymbol{R} = \begin{pmatrix} 1.00 & 0.79 & 0.42 & 0.71 & 0.50 \\ 0.79 & 1.00 & 0.01 & 0.85 & 0.11 \\ 0.42 & 0.01 & 1.00 & 0.02 & 0.96 \\ 0.71 & 0.85 & 0.02 & 1.00 & 0.13 \\ 0.50 & 0.11 & 0.96 & 0.13 & 1.00 \end{pmatrix}.$$

Obtain two factor orthogonal model using maximum likelihood method. Apply varimax rotation to the results and explain the model.