Practice Problems 7

$$S = \begin{bmatrix} 4 & 1 & 1 \\ 1 & 3 & 2 \\ 1 & 2 & 2 \end{bmatrix}$$

from SAS
$\lambda_1 = 5.714$
$\lambda_2 = 2.856$
$\lambda_3 = 0.429$

90% CI for λ_i

$$\left(\frac{\hat{\lambda}_i}{1+z_{\alpha/2}\sqrt{2/n}} , \frac{\hat{\lambda}_i}{1-z_{\alpha/2}\sqrt{2/n}} \right)$$

90% CI $\alpha = 0.1$
$z_{0.1/2} = 1.645$

for λ_1 $$\left(\frac{5.714}{1+1.645\times\sqrt{2/14}} , \frac{5.714}{1-1.645\times\sqrt{2/14}} \right) = (3.523, 15.106)$$

λ_2 $$\left(\frac{2.856}{1+1.645\times\sqrt{2/14}} , \frac{2.856}{1-1.645\times\sqrt{2/14}} \right) = (1.761, 7.551)$$

λ_3 $$\left(\frac{0.429}{1+1.645\times\sqrt{2/14}} , \frac{0.429}{1-1.645\times\sqrt{2/14}} \right) = (0.265, 1.134)$$

2. hypothesis test for Intraclass Covariance Pattern

$H_0: \Sigma = \Sigma^*$ or $H_0: \lambda_2 = \lambda_3 = \lambda_4 = \lambda_5$

$H_a: \Sigma \neq \Sigma^*$ $H_a: \{\lambda_2, \lambda_3, \lambda_4, \lambda_5\} \neq \lambda$

Criteria $\alpha = 0.05$ $n = 14$

$$df = k = 0.5p(p-1) - 1$$
$$= 0.5\times 5(4) - 1$$
$$= 9$$

$\chi^2(0.05), 9 = 16.919$

must assume multivariate normal

Sample evidence

$$Q = (p-1)(n-1)\ln(\bar{\lambda}_1) - (n-1)\sum_2^p \ln(\hat{\lambda}_i)$$

$$\bar{\lambda}_1 = \frac{1}{p-1}\sum_2^p \hat{\lambda}_i = \frac{1}{5-1}(1.7867 + 0.3892 + 0.2300 + 0.0143)$$
$$= 0.60505$$

$$Q = (5-1)(14-1)\ln(0.60505) - (14-1)(\ln(1.7867) + \ln(0.3892)$$
$$+ \ln(0.2300) + \ln(0.0143))$$
$$= 52.919$$

Conclusion

With $Q = 52.919$ and $\chi^2_{0.05,9} = 16.919$ we reject the null hypothesis. The sample provides statistically significant evidence that the population covariance matrix does not show an Intraclass Covariance Pattern.

3.

PC from sample covariance matrix

$$Y_1 = 0.0168\,B_1 - 0.0478\,B_2 + 0.8171\,B_3 + 0.5743\,B_4$$

$$Y_2 = 0.4955\,B_1 + 0.8224\,B_2 + 0.1851\,B_3 - 0.2094\,B_4$$

PC from sample correlation matrix

$$U_1 = 0.5437\,B_1 + 0.5979\,B_2 - 0.2910\,B_3 - 0.5121\,B_4$$

$$U_2 = 0.4347 B_1 + 0.3517 B_2 + 0.6661 B_3 + 0.4936 B_4$$