

MATH1309 – Practice Problems 6

Let \mathbf{X} be a random vector with mean $(2, 5)^T$

1. Find the eigenvalue and eigenvectors for the following covariance matrix (either by hand or using PROC IML)

$$\Sigma = \begin{pmatrix} 1 & 4 \\ 4 & 100 \end{pmatrix}$$

2. State the Principal Components
3. Find the correlation of each Principal Component with each variable

4. Find the eigenvalue and eigenvectors for the corresponding correlation matrix (either by hand or using PROC IML)

$$\rho = \begin{pmatrix} 1 & 0.4 \\ 0.4 & 1 \end{pmatrix}$$

5. State the Principal Components
6. Find the correlation of each Principal Component with each variable

Let $\mathbf{X}^T = (X_1, X_2, X_3, X_4)$ be a random vector and

$$\mathbf{Cov}(\mathbf{X}) = \mathbf{\Sigma} = \begin{pmatrix} 9 & 1 & 2 & 3 \\ 1 & 9 & 3 & 2 \\ 2 & 3 & 9 & 1 \\ 3 & 2 & 1 & 9 \end{pmatrix}.$$

- (a) Prove that $(15, -0.5(1, 1, 1, 1)^T, (9, 0.5(1, -1, -1, 1)^T, (7, 0.5(1, -1, 1, -1)^T$ and $(5, 0.5(-1, -1, 1, 1)^T$ are the eigenvalue-vector pairs of $\mathbf{\Sigma}$.
- (b) Obtain the principal components and their variances.
- (c) Discuss the significance of these pc's.