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Two Stage Proportional Navigation Guidance Law for Impact Time Control

Abdul Saleem[1] and Ashwini Ratnoo[2]

Abstract— **The problem of controlling the engagement time of an interceptor against a stationary target is considered and a two stage proportional navigation (PN) guidance law is proposed as the solution. The paper first derives the closed form expressions for PN engagement times with navigation constants of $N = 1$ and $N = 2$, respectively. Combining $N = 1$ and $N = 2$ phases, a guidance method is presented which provides a range based switching logic for achieving the desired impact time. The closed-form expression for the switching range is further analyzed for deducing maximum and minimum impact times. The proposed guidance methodology is applied to a salvo attack of multiple missiles. Extensive simulations are carried out to validate the theoretical findings.**

I. Introduction

Modern battleships are equipped with advance self defense mechanism such as close-in-weapon systems (CIWS) which can detect and destroy incoming anti-ship missiles. By presenting multiple simultaneous threats, a salvo attack of missiles can significantly increase the chances against such a defense mechanism. Salvo attack can be achieved if a specific time is set as a common impact time for all missiles and each missile is controlled independently to reach the target point.

The impact time control problem can be solved using finite horizon optimal control which reduces to a two point boundary value (TPBV) problem [1]. However, for nonlinear systems, the solutions are typically obtained using numerical methods. This difficulty was overcome by formulating the problem in a linearized framework [2]. A feed forward term which is proportional to the impact time error is added to the optimal control input and the method gives a near-optimal guidance law for impact time control.

Lyapunov based guidance laws have also been used for impact time control [3], [4]. The method ensures stability of the system in addition to satisfying desired guidance objectives.

Sliding mode control offers another class of solution to the problem. In sliding mode control, a switching surface is considered which assures that the motion of the system on the surface exhibits a desired behavior of output tracking error. Then a discontinuous control function is obtained that brings the system state to the surface in a finite time and thereafter impose the states to remain on this surface [5]. Authors of [6]–[8] uses sliding mode control strategy for achieving the impact time control.

[1] Assistant Professor, Department of Electrical and Electronics Engineering, Government Engineering College, Wayanad-670 644, India abdulsaleempk@gmail.com

[2] Assistant Professor in Autonomous Vehicles Laboratory, Department of Aerospace Engineering, Indian Institute of Science, Bangalore, India ratnoo@aero.iisc.ernet.in

Another approach of guidance law design for impact time control is to consider the guidance command as a polynomial functions of range-to-go [9] or of time-to-go [10]. The unknown coefficients of the polynomial are determined to satisfy zero miss distance and impact time constraints.

Proportional navigation is a commonly used missile guidance law which is simple, easy to implement, and efficient. Implementing proportional navigation essentially requires the line-of-sight rate measurement which can be obtained using an on-board sensor. Further, driving the line-of-sight rate to zero, PN achieves the collision course with the target. In the literature, two approaches present the use of PN guidance for impact time control. In the first approach, the navigation gain is varied as a function of impact time error defined as the difference between desired and estimated impact times [11], [12]. The guidance command of second method consists of a combination of a feedback term to achieve zero miss distance and a feed forward term for controlling the impact time [13]–[18]. In [13]–[15], the feedback term is the conventional PN guidance command and in [16]–[18] it is based on the biased PN command.

Most of the existing approaches require estimation of the engagement time and closed-form solutions for the same. Engagement time expressions given by [4] and [18] can be represented only by using special functions such as incomplete beta function and Gaussian hyper-geometric function

As the main contributions, this paper presents a two-stage PN approach for generating impact time constrained missile trajectories against stationary targets. The control over the engagement time is obtained through a closed-form relationship between the switching-range and the exact total engagement time. Closed-form expressions for minimum and maximum impact times are derived so that a desired and feasible impact time can be chosen from within the permissible set of impact times.

The remainder of this paper is organized as follows: In Section II, the impact time control problem is stated. Section III derives the PN engagement times for navigation gains $N = 1$ and $N = 2$. Section IV analyses the two stage PN guidance framework for impact time control. Simultaneous impact of multiple missiles is discussed in Section V. Simulation studies are presented in Section VI. Section VII summarizes the concluding remarks.

II. Problem Statement

Consider a planar missile engagement against a stationary target as shown in Fig. 1. The missile is assumed to move

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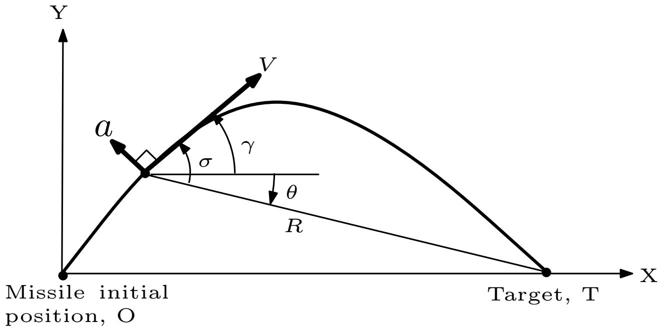


Fig. 1: Engagement geometry

with a constant speed V. Here, R, γ, and θ represents range-to-target, missile heading angle, and line-of-sight (LOS) angle, respectively. The missile heading error is defined as

$$\sigma = \gamma - \theta \tag{1}$$

The relative engagement kinematics is governed by,

$$\dot{R} = -V\cos\sigma \tag{2}$$

$$\dot{\theta} = \frac{-V\sin\sigma}{R} \tag{3}$$

$$\dot{\gamma} = \frac{a}{V} \tag{4}$$

where, a is the lateral acceleration which is applied normal to the velocity. The objective here is to develop a guidance solution governing a which would satisfy a predefined impact time t_f and further extend the method for a simultaneous impact of multiple missiles.

III. Engagement Time

A closed-form expression for engagement time can be obtained if the range or heading error can be expressed as an explicit function of time. Since the solution of differential equations governing range and heading error variations are of non-elementary anti derivatives, an exact closed-form expressions for PN impact time is not possible. However, for a special case of $N = 1$ or $N = 2$, it is possible to solve the governing differential equations in terms of an elementary anti derivative. This section derives engagement times corresponding to the navigation gains $N = 1$ and $N = 2$.

For a PN guidance, the heading angle variation is governed by

$$\dot{\gamma} = N\dot{\theta} \tag{5}$$

Differentiation of Eq. (1), yields

$$\dot{\sigma} = \dot{\gamma} - \dot{\theta} \tag{6}$$

Using Eq. (5) in Eq. (6) results in

$$\dot{\sigma} = (N-1)\dot{\theta} \tag{7}$$

A. Engagement time for N=1

Substituting $N = 1$ in Eq. (7), yields

$$\dot{\sigma} = 0 \tag{8}$$

which implies that heading error is constant for $N = 1$ trajectory. Let σ_0 be the initial heading error. Hence, Eq. (2) can be expressed as

$$\dot{R} = -V\cos\sigma_0 \tag{9}$$

let t_{f_1} be the PN engagement time for $N = 1$. Integrating Eq. (9) from $t = 0$ to $t = t_{f_1}$ leads to

$$t_{f_1} = \frac{R_0 - R_1}{V\cos\sigma_0} \tag{10}$$

where, R_0 is the initial range and R_1 is the range at the instant $t = t_{f_1}$.

B. Engagement time for N=2

Substituting $N = 2$ in Eq. (7) and using Eq. (3), results in

$$\dot{\sigma} = \dot{\theta} = \frac{-V\sin\sigma}{R} \tag{11}$$

Using Eq. (2) and Eq. (11) leads to

$$\frac{\mathrm{d}R}{R} = \frac{\mathrm{d}\sigma}{\tan\sigma} \tag{12}$$

A relation between R and σ can be obtained by integrating Eq. (12) as

$$R = R_0 \frac{\sin\sigma}{\sin\sigma_0} \tag{13}$$

Using Eqs. (3) and (13) in Eq. (11) results in

$$\dot{\sigma} = \frac{-V\sin\sigma_0}{R_0} \tag{14}$$

Integrating Eq. (14), yields

$$t_{f_2} = \frac{R_0}{V}\left(\frac{\sigma_0 - \sigma_2}{\sin\sigma_0}\right) \tag{15}$$

where, t_{f_2} is the PN engagement time for $N = 2$ and σ_2 is the heading error at the instant $t = t_{f_2}$.

IV. Two stage PN guidance Law

If the navigation gain is fixed as $N = 1$ or $N = 2$, then an exact closed-from expression for impact time is obtained. However, with the choice of a fixed navigation gain, the resulting impact time of Eq. (10) and Eq. (15) are constant. A control parameter and thereby a desired impact time can be potentially achieved by considering a two phase PN guidance command with $N = 1$ and $N = 2$ phases. The point at which the switching of the navigation gain from $N = 1$ to $N = 2$ can be controlled to get a desired impact time.

Fig. 2 illustrates the proposed guidance methodology. The missile trajectory from O to S, and from S to T are governed by PN guidance commands with $N = 1$ and $N = 2$, respectively. Here, S is the point at which the navigation gain is switched from $N = 1$ to 2, and R_s is the range-to-target at this switching instant. As implied from Eq. (8), heading error is constant in the first phase. Hence, the initial heading error

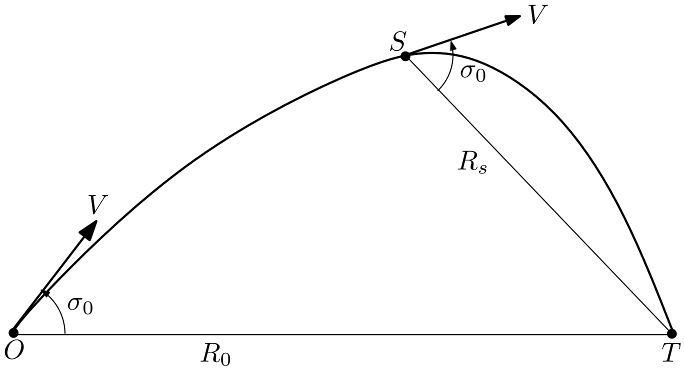


Fig. 2: Two stage engagement.

of second phase is also σ_0. As shown in Fig 2, R_s represents the final range for the first phase and initial range for the second phase. At interception, collision course guarantees $\sigma = 0$. Using that in Eq. (10) and (15) gives the engagement times of the two phases as

$$t_{f_1} = \frac{R_0 - R_s}{V\cos\sigma_0} \tag{16}$$

$$t_{f_2} = \frac{R_s}{V}\left(\frac{\sigma_0}{\sin\sigma_0}\right) \tag{17}$$

The total engagement time is the sum of the engagement times of the two phases and is obtained using Eq. (16) and Eq. (17) as

$$t_f = \frac{R_0 - R_s}{V\cos\sigma_0} + \frac{R_s}{V}\left(\frac{\sigma_0}{\sin\sigma_0}\right) \tag{18}$$

Eq. (18) presents an exact closed-form of expression of the impact time. The control parameter R_s can be obtained from Eq. (18) as

$$R_s = \left(\frac{Vt_f\cos\ \sigma_0 - R_0}{\sigma_0\ \cos\sigma_0 - \sin\sigma_0}\right)\sin\ \sigma_0 \tag{19}$$

Note that the control parameter R_s is a function of the desired impact time and initial conditions. The control strategy can be summarized as follows: (i) Compute the parameter R_s for the desired time t_f using Eq. (19). Apply PN guidance command with $N = 1$ until range becomes R_s and thereafter command PN guidance law with $N = 2$.

A. Minimum and Maximum Impact Times

Differentiating Eq. (18) with respect to R_s leads to

$$\frac{\mathrm{d}t_f}{\mathrm{d}R_s} = \frac{-1}{V\ \cos\ \sigma_0} + \frac{1}{V}\left(\frac{\sigma_0}{\sin\ \sigma_0}\right) \begin{array}{l} < 0 \text{ if } |\sigma_0| < \pi/2 \\ > 0 \text{ if } |\sigma_0| > \pi/2 \end{array} \tag{20}$$

The two conditions of initial heading errors ($|\sigma_0| < 90^\circ$ and $|\sigma_0| > 90^\circ$) are discussed subsequently.

1) $|\sigma_0| < 90^\circ$: Eq. (20) implies that t_f increases with a decrease in R_S . As governed by Eqs. (2), (8), and (11), the range decreases with $N = 1$ as the engagement proceeds. Hence, minimum value of $R_S = 0$ corresponds to the special case with only one phase of $N = 1$. The maximum value of R_S is R_0 and it corresponds to the case where the entire engagement is governed by $N = 2$. Hence, minimum and maximum impact times are obtained when $R = R_S$, and $R = 0$, respectively.

2) $|\sigma_0| > 90^\circ$: Using Eq. (2) and Eqn. (6), the range increases indefinitely for $N = 1$ and $\sigma_0 > \pi/2$. Hence, any impact impact time greater than $\frac{R_0}{V}\frac{1}{\cos\sigma 0}$ can be achieved. Hence the overall minimum and maximum times can be expressed as

$$t_{fmin} = \frac{R_0}{V}\left(\frac{\sigma_0}{\sin\ \sigma_0}\right) \tag{21}$$

$$\begin{aligned} t_{fmax} = &\frac{R_0}{V}\left(\frac{1}{\cos\ \sigma_0}\right) && \text{if } |\sigma_0| < \pi/2 \\ \Rightarrow &\infty && \text{if } |\sigma_0| > \pi/2 \end{aligned} \tag{22}$$

Algorithm 1 summarizes the proposed guidance method for impact time control using the proposed method. Note that

Algorithm 1 Two stage PN guidance law for impact time control

1: Compute t_{fmin} and t_{fmax} using Eqs. (21) and (22), respectively.
2: Determine the permissible values of t_f as $S = [t_{fmin}\ t_{fmax}]$.
3: Select a t_f from S.
4: Find the control parameter R_s from Eq. (19).
5: Apply the guidance command $a = V\dot{\theta}$ until $R = R_s$ and thereafter use the command $a = 2V\dot{\theta}$

computations involved in Algorithm 1 can be done off-line and with an on line sensing, the missile can implement the switching condition.

V. Salvo Attack

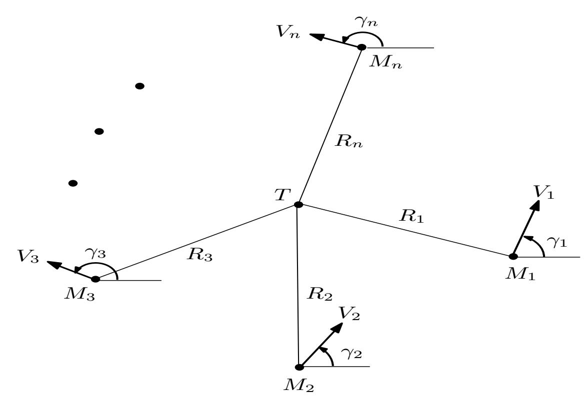


Fig. 3: Salvo attack.

Consider n missiles $M_i,\ i = 1\ldots n$ located at different locations and all are launched at $t = 0$ against a stationary target, T as shown in Fig. 3. Let R_i, γ_i be the initial range to target and initial heading angles of missiles, and let V_i be their speeds. The objective is that all missiles should hit the target simultaneously. This can be achieved if a common desired impact time is chosen for all missiles and all are independently controlled using the Algorithm 1. The

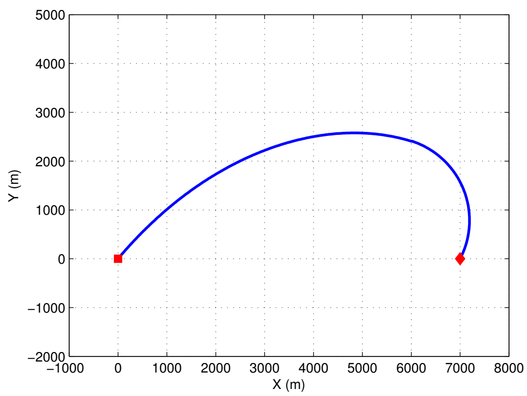


Fig. 4: Trajectory - sample scenario.

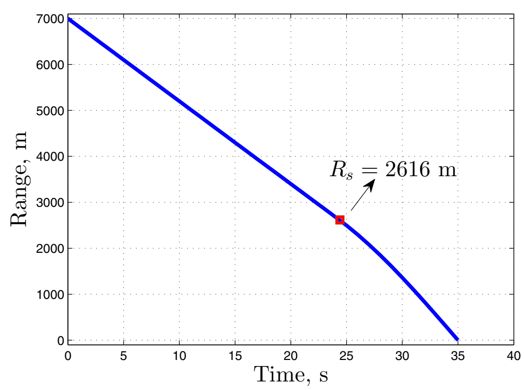


Fig. 5: Range - sample scenario.

common impact time is chosen from the intersection of impact time set of individual missiles.

VI. Simulations

A. Case 1: Sample Scenario

Consider an engagement geometry with $R_0 = 7000$ m, $\theta_0 = 0°$, and $\gamma_0 = 50°$. Let the missile speed is $V = 280$ m/s and its launch point be at $(0,0)$. The minimum and maximum impact times are obtained using Eqs. (21) and (22) as 28.48s and 38.89s, respectively. An impact time $t_f = 35$ s is chosen for the simulation.

Resulting missile trajectory with a successful interception is shown in Fig. 4. The control parameter is obtained from Eq. (19) as $R_s = 2616$ m. PN guidance command with $N = 1$ is applied until $R = R_s = 2616\ m$ and thereafter the navigation gain is switched to $N = 2$. Fig. 5 shows the range variation with an indication of switch point. The corresponding lateral acceleration profile is shown in Fig. 6. The variation of navigation gain and heading error is shown in Fig. 7. The heading error is constant up to $t = 24.5$s, as given by Eq. (8). After the navigation gain is switched to 2, the heading error decreases to zero. The impact time error is less than 10^{-4} s.

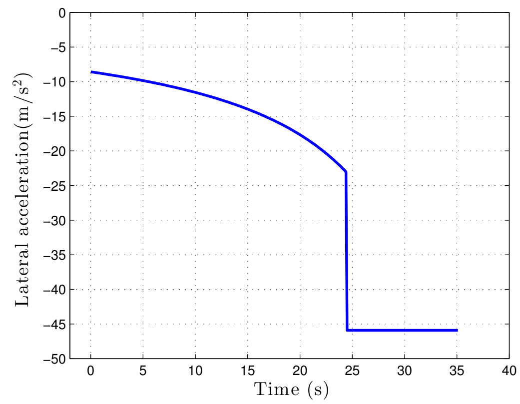


Fig. 6: Lateral acceleration - sample scenario.

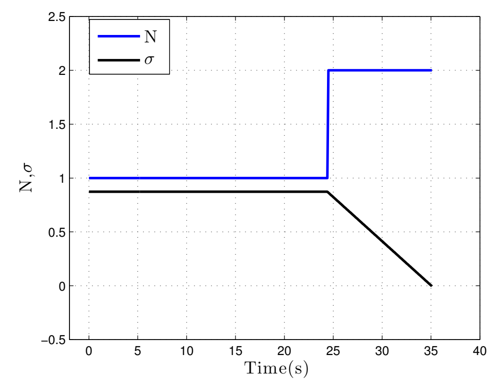


Fig. 7: Navigation gain and heading error variation - sample scenario.

B. Case 2: Salvo Attack

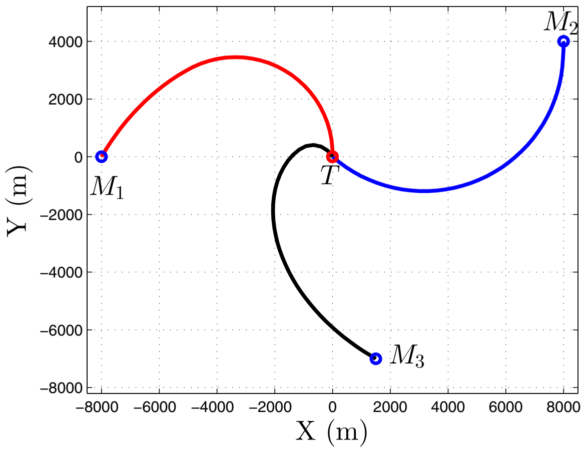
Consider three missiles with initial conditions as shown in Table I, and a stationary target at (0,0). The objective here is that all the missiles should hit the target simultaneously.

TABLE I: Salvo attack - initial conditions and impact time set

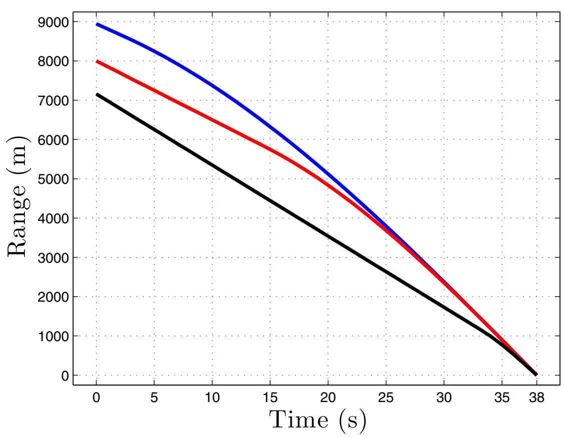
$x_0(m), y_0(m)$	$\gamma_0(deg)$	$V(m/s)$	$S = [t_{fmin}, t_{fmax})$
$(-8000, 0)$	60	300	[32.25,53.33]
$(8000, 4000)$	-90	300	[36.9,66.67]
$(1500, -7000)$	150	270	[29.88,39.55]

The set of achievable impact time for each of the missile is shown in Table I. The intersection set Ω of all impact time sets can be obtained as [36.9 39.55]. An impact time $t_f = 38$ s is chosen from this set. To achieve a simultaneous impact, each missile is independently controlled using the two stage PN guidance algorithm presented in Algorithm 1 and a common impact time $t_f = 38$ s.

The trajectories are shown in Fig. 8a showing an intercept on the target. Fig. 8b shows the range variation of missiles, which illustrates that all missiles hit the target at $t = 38$ s.



(a) Trajectory



(b) Range Variation

Fig. 8: Salvo attack.

VII. Conclusions

A two stage PN guidance law for impact time control of a missile against a stationary target is presented. Closed-form expressions of impact time for PN guidance law with the navigation gains $N = 1$ and $N = 2$ are derived. The range-to-target at the switching from $N = 1$ to $N = 2$ acts as the control parameter, and a closed-form expression for this range is derived as a function of desired impact time. The guidance law is extended for a cooperative attack of multiple missiles for a simultaneous attack against a stationary target. Simulations are carried out illustrating the efficacy of the proposed method.

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