Demonstration of Law of Large Numbers and Central Limit Theorem using exponential distribution.

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Overview

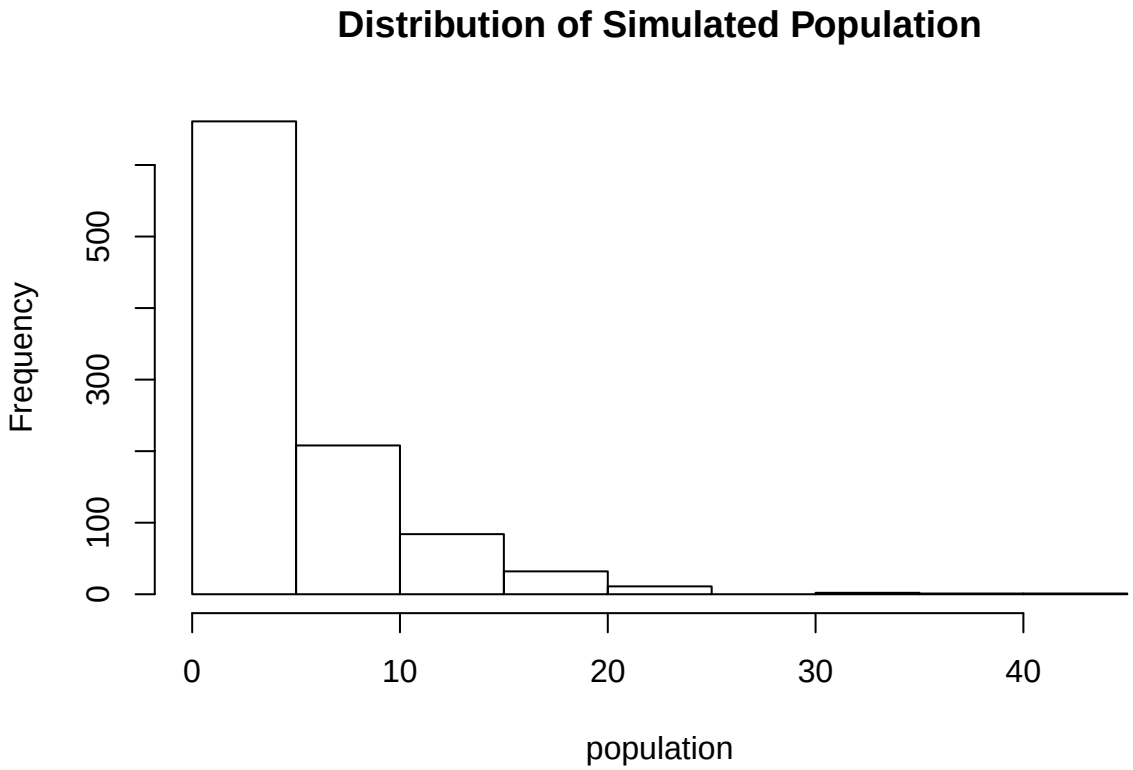
Law of Large Numbers tells us the mean of a sample of size n converges to mu, the population mean, as n becomes increasingly large with a probability of 100%.

The Central Limit Theorem tells us that the mean of a large number of iterations of independent random variables, each with a well defined mean and variance, will be approximately normally distributed, regardless of the underlying distribution.

The report will demonstrate the Central Limit Theorem using the exponential distribution. We will use a lambda parameter = 0.2 for the simulation. Therefore, the theoritical mean and standard deviation is 1/0.2 = 5.

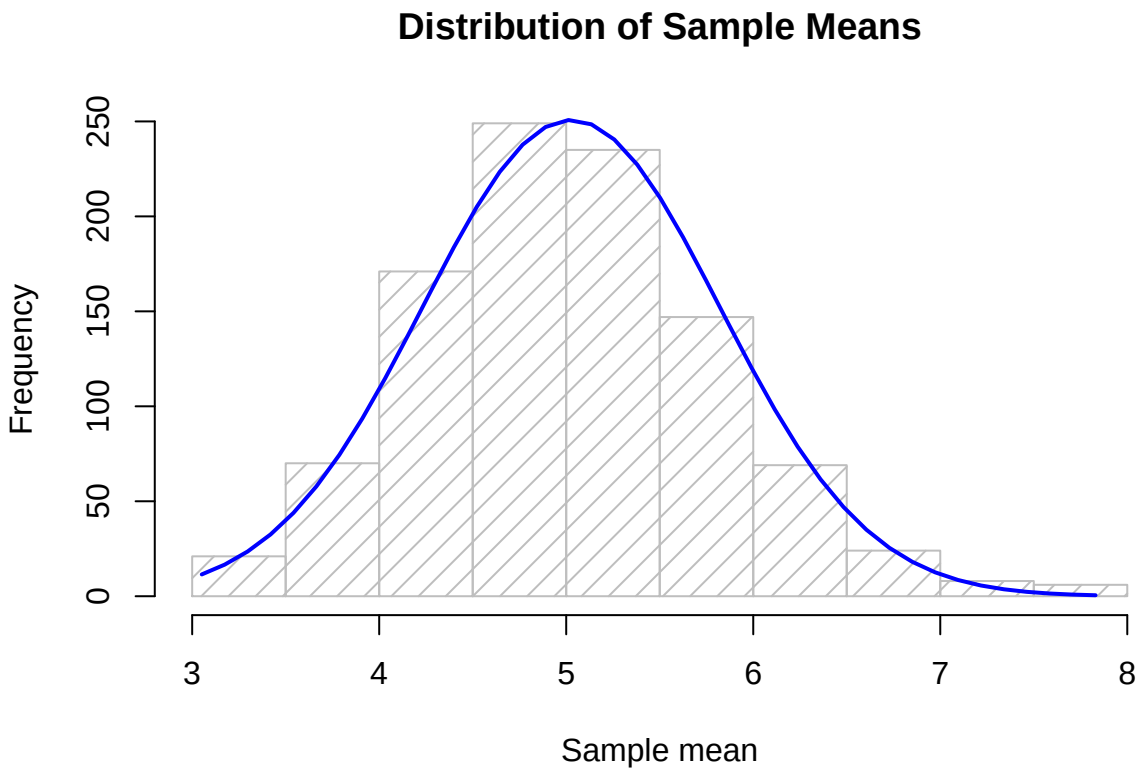
Simulations

```
lambda <- 0.2
N <- 1000
n <- 40
population <- rexp(N, lambda)
hist(population, main = "Distribution of Simulated Population")
```



The mean of the simulated population is 4.7604466 and the standard deviation is 5.0034578 compared to the theoritical values of 5. The simulated population is a good representation of the theoritical population, and is clearly not normally distributed.

```
samples <- matrix(rexp(N*n,lambda), N)
sample_vars <- apply(samples, 1, var)
sample_means <- apply(samples, 1, mean)
h<-hist(sample_means, breaks=10, density=10, col="gray", xlab="Sample mean", main="Distribution of Samp
xfit<-seq(min(sample_means),max(sample_means),length=n)
yfit<-dnorm(xfit,mean=mean(sample_means),sd=sd(sample_means))
yfit <- yfit*diff(h$mids[1:2])*length(sample_means)
lines(xfit, yfit, col="blue", lwd=2)
```



The mean of the sample means is 5.0267111 compared with the theorical population mean of 5, and the predicted standard error of the mean is 0.7905694 compared with the calculated value of 0.7954171. The 95% confidence interval for the mean of the sample means distribution is 4.928112, 5.1253102, which contains the theoritical population mean of 5.

The distribution of sample means is shown to be normally distributed with sample mean equal to the population mean, and sample standard distribution equal to the population standard error for a sample size of 40.