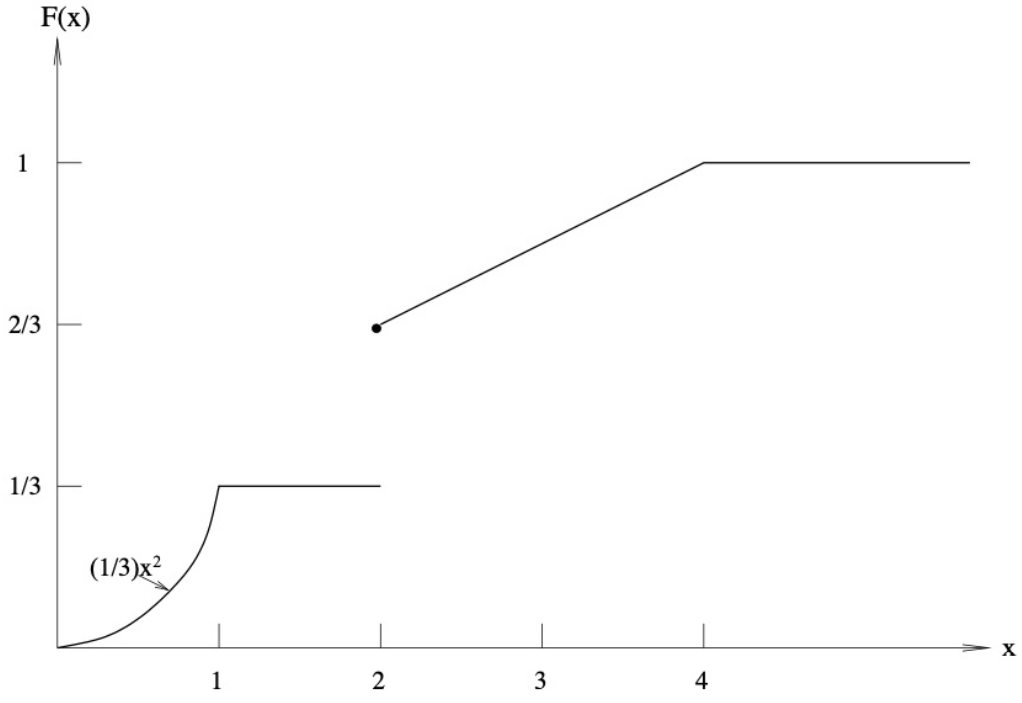
Statistics 2 Workshop Exercises (Distribution Functions)

Day 2 (9/23/2025); 9:45-10:30 am

① *Probabilities from a cdf.* Let X be a random variable with the cdf shown below.



Find the probabilities of the following events.

(a) $\{X = 2\}$.

(b) $\{X < 2\}$.

(c) $\{X = 2\} \cup \{0.25 \leq X \leq 1.5\}$.

(d) $\{X = 2\} \cup \{0.25 \leq X \leq 3\}$.

② Let X be a random variable with PDF given by

$$f_X(x) = \begin{cases} cx^2 & ; \ |x| \leq 1 \\ 0 & ; \ \text{otherwise} \end{cases}$$

a) Find the constant c.

b) Find E[X] and var(X)

c) Find $P(X \geq 1/2)$

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Solutions:

① (a) There is a jump at $X = 2$, so we have

$$\begin{aligned}\mathsf{P}\{X = 2\} &= \mathsf{P}\{X \le 2\} - \mathsf{P}\{X < 2\} \\ &= F(2) - F(2^-) \\ &= \frac{2}{3} - \frac{1}{3} \\ &= \frac{1}{3}.\end{aligned}$$

(b) $\mathsf{P}\{X < 2\} = F(2^-) = \frac{1}{3}$.

(c) since $\{X = 2\}$ and $\{0.25 \le X \le 1.5\}$ are two disjoint events,

$$\begin{aligned}\mathsf{P}(\{X = 2\} \cup \{0.25 \le X \le 1.5\}) &= \mathsf{P}\{X = 2\} + \mathsf{P}\{0.25 \le X \le 1.5\} \\ &= \frac{1}{3} + F(1.5) - F(0.25^-) \\ &= \frac{1}{3} + \frac{1}{3} - \frac{1}{3} \times 0.25^2 \\ &= \frac{31}{48}.\end{aligned}$$

(d) We have

$$\begin{aligned}\mathsf{P}(\{X = 2\} \cup \{0.25 \le X \le 3\}) &= \mathsf{P}\{0.25 \le X \le 3\} \\ &= F(3) - F(0.5^-) \\ &= \frac{5}{6} - \frac{1}{3} \times 0.25^2 \\ &= \frac{39}{48}.\end{aligned}$$

② a) To find c, we can use $\int_{-\infty}^{\infty} f_X(u)du = 1$:

$$\int_{-\infty}^{\infty} f_X(x)dx = 1$$

↳ $\int_{-1}^{1} cx^2 dx = 1$

↳ $\frac{2}{3}c = 1$

↳ $c = 3/2$

b) To find $E[X]$, we can write

$$E[X] = \int_{-1}^{1} x f_X(x)dx$$

$$= \frac{3}{2}\int_{-1}^{1} x^3 dx = 0$$

To find $\text{var}(X)$,

$$\text{var}(X) = E[X^2] - (E[X])^2 = E[X^2]$$

$$= \int_{-1}^{1} x^2 f_X(x)dx$$

$$= \frac{3}{2}\int_{-1}^{1} x^4 dx$$

$$= 3/5$$

c) To find $P(X \geq 1/2)$,

$$P(X \geq 1/2) = \int_{1/2}^{1} f_X(x)dx$$

$$= \frac{3}{2}\int_{1/2}^{1} x^2 dx$$

$$= 7/16$$