SIO Math Workshop – Mixing Problem Solution

Summary of Known Quantities.

Sample	AABW	NADW
$T = 1.97\ ^\circ\text{C}$	$T_1 = -0.5\ ^\circ\text{C}$	$T_2 = 3\ ^\circ\text{C}$
$S = 34.83$ psu	$S_1 = 34.67$ psu	$S_2 = 34.90$ psu
$V = 1\ \text{dm}^3$	$V_1 = ?$	$V_2 = ?$

Relevant Equations.

$$V_1T_1 + V_2T_2 = VT \quad \text{Conservation temperature.} \tag{4}$$
$$V_1S_1 + V_2S_2 = VS \quad \text{Conservation salt.} \tag{5}$$
$$V_1 + V_2 = V \tag{6}$$

Solution.

Note: We have two unknowns (V_1, V_2) but three equations–our system is over-constrained! Proceed by choosing any two of our equations to solve for the unknowns. All choices should yield identical results, barring errors in our 'Known Quantities'.

Proceed with (5) and (6). Solve (6) for V_1.

$$V_1 + V_2 = V$$
$$V_1 = V - V_2$$

Substitute result into (5).

$$(V - V_2)S_1 + V_2S_2 = VS$$
$$V_2 = \frac{V(S - S_1)}{S_2 - S_1}$$
$$\boxed{V_2 \approx 0.696\ \text{dm}^3}$$ Substitution for V, S, S_1, S_2.

Use solution for V_2 to find V_1.

$$V_1 + V_2 = 1$$
$$V_1 = V - V_2$$
$$\boxed{V_1 \approx 0.304\ \text{dm}^3}$$ Substituting for V, V_2.

Note: Choosing equations (4) and (6) yields $V_1 = 0.294\ dm^3$ and $V_2 = 0.706\ dm^3$. Choosing equations (4) and (5) yields $V_1 = 0.294\ dm^3$ and $V_2 = 0.706\ dm^3$, also.

Scientifically, this discrepancy could arise because: (1) the reported temperature and salinity values contained some sampling error, or (2) the characteristic temperature and salinity values for the example water masses (AABW and NADW) are actually ranges.

In reality, this discrepancy occurred because...I chose bad numbers!

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