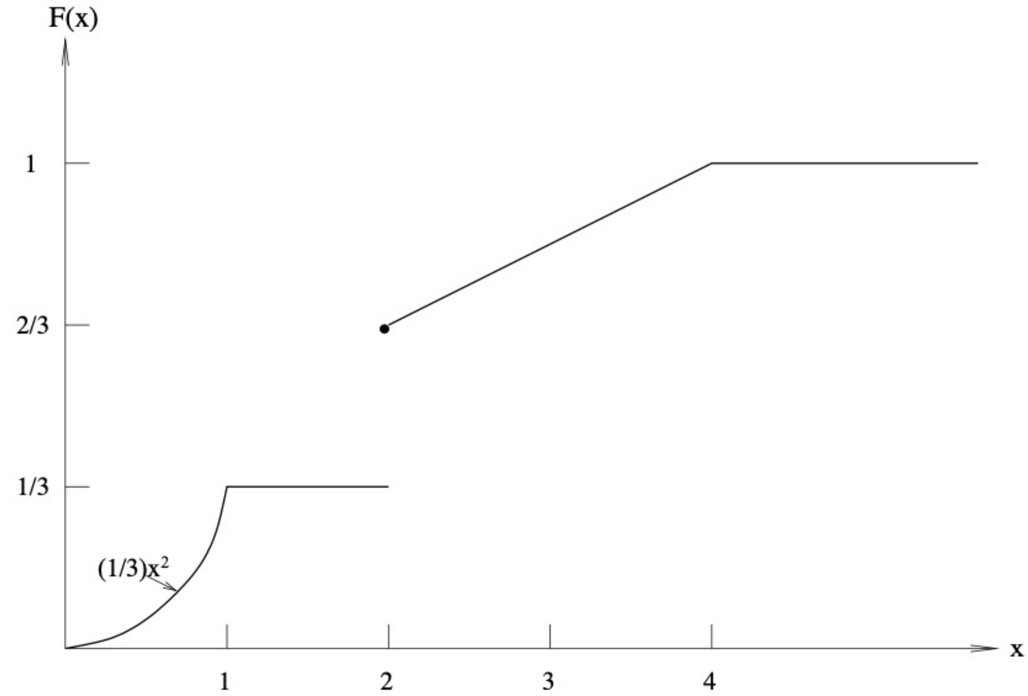
Statistics 2 Workshop Exercises (Distribution Functions)
Day 2 (9/23/2025); 9:45-10:30 am

1. *Probabilities from a cdf.* Let X be a random variable with the cdf shown below.



Find the probabilities of the following events.

(a) $\{X = 2\}$.
(b) $\{X < 2\}$.
(c) $\{X = 2\} \cup \{0.25 \leq X \leq 1.5\}$.
(d) $\{X = 2\} \cup \{0.25 \leq X \leq 3\}$.

2. Let X be a random variable with PDF given by

$$f_X(x) = \begin{cases} cx^2 & ; \ |x| \leq 1 \\ 0 & ; \ \text{otherwise} \end{cases}$$

a) Find the constant c.

b) Find $E[X]$ and $\mathrm{var}(X)$

c) Find $P(X \geq 1/2)$

Provided by Luisa Watkins @ SIO

l1watkins@ucsd.edu

Solutions:

1

(a) There is a jump at $X = 2$, so we have

$$\begin{aligned} \mathsf{P}\{X = 2\} &= \mathsf{P}\{X \le 2\} - \mathsf{P}\{X < 2\} \\ &= F(2) - F(2^-) \\ &= \frac{2}{3} - \frac{1}{3} \\ &= \frac{1}{3}. \end{aligned}$$

(b) $\mathsf{P}\{X < 2\} = F(2^-) = \frac{1}{3}$.

(c) since $\{X = 2\}$ and $\{0.25 \le X \le 1.5\}$ are two disjoint events,

$$\begin{aligned} \mathsf{P}(\{X = 2\} \cup \{0.25 \le X \le 1.5\}) &= \mathsf{P}\{X = 2\} + \mathsf{P}\{0.25 \le X \le 1.5\} \\ &= \frac{1}{3} + F(1.5) - F(0.25^-) \\ &= \frac{1}{3} + \frac{1}{3} - \frac{1}{3} \times 0.25^2 \\ &= \frac{31}{48}. \end{aligned}$$

(d) We have

$$\begin{aligned} \mathsf{P}(\{X = 2\} \cup \{0.25 \le X \le 3\}) &= \mathsf{P}\{0.25 \le X \le 3\} \\ &= F(3) - F(0.5^-) \\ &= \frac{5}{6} - \frac{1}{3} \times 0.25^2 \\ &= \frac{39}{48}. \end{aligned}$$

2 a) To find c, we can use $\int_{-\infty}^{\infty} f_X(u)du = 1$:

$$\int_{-\infty}^{\infty} f_X(x)dx = 1$$

$$\hookrightarrow \int_{-1}^{1} cx^2 d = 1$$

$$\hookrightarrow \frac{2}{3}cx = 1$$

$$\hookrightarrow c = 3/2$$

b) To find E[X], we can write

$$E[X] = \int_{-1}^{1} x f_X(x) dx$$

$$= \frac{3}{2}\int_{-1}^{1} x^3 dx = 0$$

To find var(X),

$$var(X) = E[X^2] - (E[X])^2 = E[X^2]$$

$$= \int_{-1}^{1} x^2 f_X(x)dx$$

$$= \frac{3}{2}\int_{-1}^{1} x^4 dx$$

$$= 3/5$$

c) To find $P(X \geq 1/2)$,

$$P(X \geq 1/2) = \int_{1/2}^{1} f_X(x)dx$$

$$= \frac{3}{2}\int_{1/2}^{1} x^2 dx$$

$$= 7/16$$