Statistics 2 Workshop Handout (Distribution Functions) ①
Day 2 (9/23/2025) ; 9:45-10:30 am

examples of PMFs (discrete):

- Bernoulli: $X \sim \text{Bern}(p)$, $p \in [0,1]$
  $p_X(1) = p$; $p_X(0) = 1-p$ } like a flip of a biased coin

- Binomial: $X \sim \text{Binom}(n,p)$
  $p_X(k) = \binom{n}{k} p^k (1-p)^{n-k}$, $k = 0, 1, \dots, n$ } # of 1's in n in n independent flips of biased coin

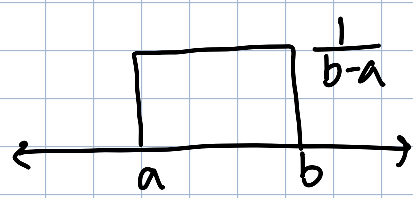
- Geometric: $X \sim \text{Geom}(p)$
  $p_X(k) = (1-p)^{k-1} p$, $k = 1, 2, \dots$ } # of flips until first 1 in flips of biased coin

- Poisson: $X \sim \text{Poisson}(\lambda)$, $\lambda > 0$
  $p_X(k) = e^{-\lambda} \frac{\lambda^k}{k!}$, $k = 0, 1, \dots$ } # of random arrivals in unit time interval, with average number of arrivals per unit time λ

examples of PDFs (continuous):

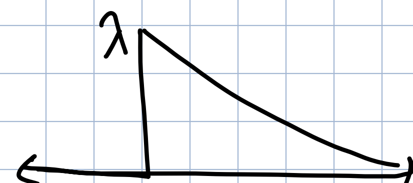
- Uniform: $X \sim \text{unif}[a,b]$

$$f_X(x) = \begin{cases} 0 & , x < a \\ \frac{1}{b-a} & , a \le x \le b \\ 0 & , x \ge b \end{cases}$$



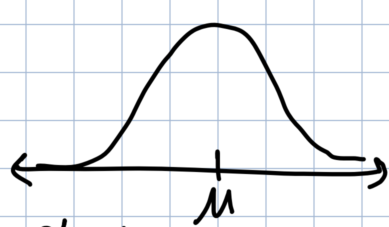
- Exponential: $X \sim \exp(\lambda)$, $\lambda > 0$

$$f_X(x) = \begin{cases} 0 & , x < 0 \\ \lambda e^{-\lambda x} & , x \ge 0 \end{cases}$$



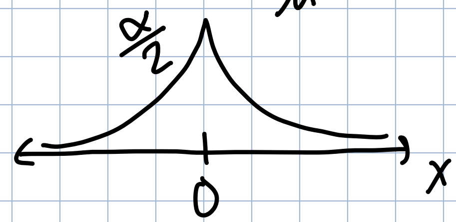
- Gaussian (Normal): $X \sim N(\mu, \sigma^2)$

$$f_X(x) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$



- Laplacian: $X \sim L(\alpha)$, $\alpha > 0$

$$f_X(x) = \frac{\alpha}{2} e^{-\alpha |x|}, \quad -\infty < x < \infty$$



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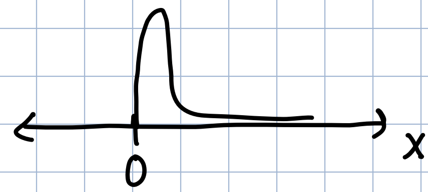
Provided by Luisa Watkins @SIO

- Rayleigh: $X \sim R(\sigma)$, $\sigma > 0$

  $f_X(x) = \frac{x}{\sigma^2} e^{\frac{-x^2}{(2\sigma)^2}}$, $x \geq 0$



examples of E[X]

Discrete

- Bern(p): $E[X] = 1 \cdot p + 0 \cdot (1-p) = p$
- Binom(n,p): $E[X] = \sum_{k=0}^{n} k \binom{n}{k} p^k (1-p)^{n-k} = np$
- Geom(p): $E[X] = \sum_{k=1}^{\infty} kp(1-p)^{k-1} = \frac{p}{(1-(1-p))^2} = \frac{1}{p}$
- Poisson(λ): $E[X] = \sum_{k=0}^{\infty} k \frac{\lambda^k}{k!} e^{-\lambda} = \lambda$

Continuous

- Unif[a,b]: $E[X] = \int_a^b x \left(\frac{1}{b-a}\right) dx = \frac{1}{b-a} \frac{x^2}{2} \Big]_a^b = \frac{b+a}{2}$
- $N(\mu, \sigma^2)$: $E[X] = \int_{-\infty}^{\infty} x \frac{1}{\sqrt{2\pi\sigma^2}} e^{\frac{-(x-\mu)^2}{2\sigma^2}} dx = \mu$ (symmetry!)
- Exp(λ): $E[X] = \int_0^{\infty} x \lambda e^{-\lambda x} dx = \frac{1}{\lambda}$ (integration by parts)

examples of var(X)

Discrete

- Bern(p): $\mathrm{var}(X) = p(1-p)$
- Binom(n,p): $\mathrm{var}(X) = np(1-p)$
- Geom(p): $\mathrm{var}(X) = \frac{1-p}{p^2}$
- Poisson(λ): $\mathrm{var}(X) = \lambda$

Continuous

- Unif[a,b]: $\mathrm{var}(X) = \frac{(b-a)^2}{12}$
- $N(\mu, \sigma^2)$: $\mathrm{var}(X) = \sigma^2$
- Exp(λ): $\mathrm{var}(X) = \frac{1}{\lambda^2}$