SIO Math Workshop – Algebra Worksheet

CHOOSE 1-2 PRACTICE PROBLEMS TO SOLVE INDEPENDENTLY. WE'LL WORK PROBLEM #5 TOGETHER, LATER!

For the following practice problems let $x, a \in \mathbb{R}$ ('x and a are real numbers'). [Partial solutions are in red for selected practice problems.]

Solving and simplifying algebraic expressions.

1. Reduce the following to simplest terms. $[5x^2 + 32x + 12]$

$$\frac{5x^2 - 18x - 8}{\frac{x-4}{x+6}}$$

2. Reduce the following to simplest terms. $\left[\frac{x^2-4}{x^3+3x^2+3x+1}\right]$

$$\frac{1}{x+1} - \frac{2}{(x+1)^2} - \frac{3}{(x+1)^3}$$

3. Solve for x. $[x = \frac{1}{3}e^{10a}]$

$$\frac{6a(3+2)}{3} - 5 = \ln(3x) - 5$$

4. Solve for x by factoring. Do not evaluate complex terms. Hint: Reduce to quadratic form via the substitution $y = x^4$ $[x = \sqrt[4]{-2}]$

$$x^8 + 4x^4 + 4 = 0$$

5. An experiment requires 25% saline solution. The lab has 14% and 60% solutions. What volume of each should we mix to obtain 100 L of the 25% solution?

[Mix 76.09 L of the 14% solution and 23.91 L of the 60% solution.]

6. We begin with 100 g of some radioactive isotope. After 50 years, we know that 80 g of the isotope will still remain. Formulate an equation to describe the exponential decay of the substance. Determine the substance's half-life.

Hint: Try an equation of the form $X(t) = X_0 e^{kt}$, where X(t) is the mass of isotope remaining, X_0 is some constant, k is a decay rate, and t is time

[Decay equation is $X(t) = 100 \cdot (\frac{4}{5})^{t/50}$. Half-life of isotope is 155.3 years.]

Graphing.

Sketch the following curves.

7.
$$y = 3|x|$$

8.
$$f(x) = \begin{cases} x^2, & \text{if } x \le 0\\ 2x, & \text{if } 0 < x \end{cases}$$