



Echo State Networks for Time Series Data

Open Data Science Conference, Boston

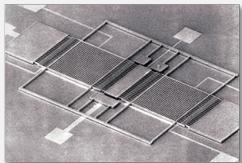
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About Draper



Apollo Guidance Computer



Microelectromechanical System (MEMS)



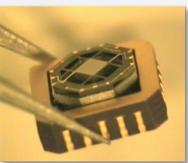
First Algebraic Compiler



Precision Guided Munitions



Tissue Engineering



Precision Timing



Ballistic Missile Guidance



Digital Fly-by-Wire



Shuttle Docking



SPIRE Jr.



Mark 14 Gunsight



Precision Airdrop



Engineering the World Around You

Independent not-for-profit research and development company

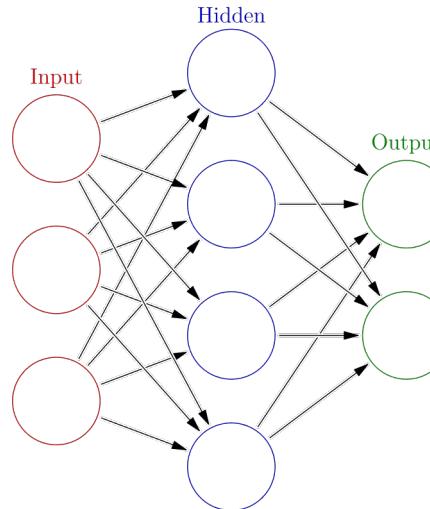
- Headquartered in Cambridge, MA
- 1500+ employees, \$520 million revenue
- Serving the nation since 1932
- Independent organization since 1973

Outline

- Neural Networks in general
- Echo State Networks in detail
- Demo: https://github.com/tguidici/ODSC_2020_ESN
- Example: ESN for finance

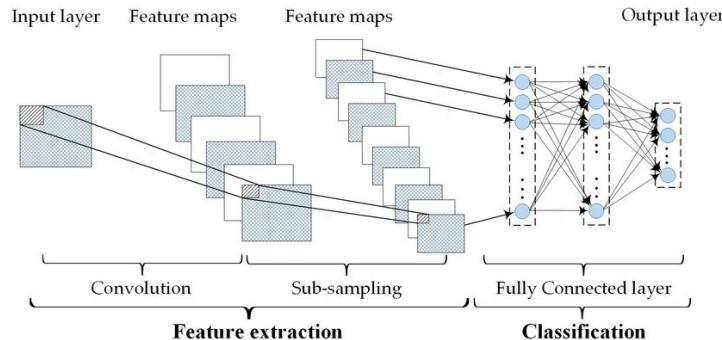
Neural Networks (NN)

- Loosely (!) modeled on human/animal brains
 - Many simple processing nodes
 - Densely connected (usually)
 - “Learn” to perform a task by analyzing training examples
- First proposed in 1944; on a boom-bust cycle of interest
 - Dropped out of favor ~ 1969
 - Resurgence in 1980’s!
 - Disinterest in 2000’s!
 - ALL THE INTEREST IN 2010’s!!



- Circular nodes represent artificial neurons
- Arrows illustrate connections between output of one neuron to input of another

Convolutional vs Recurrent Neural Nets

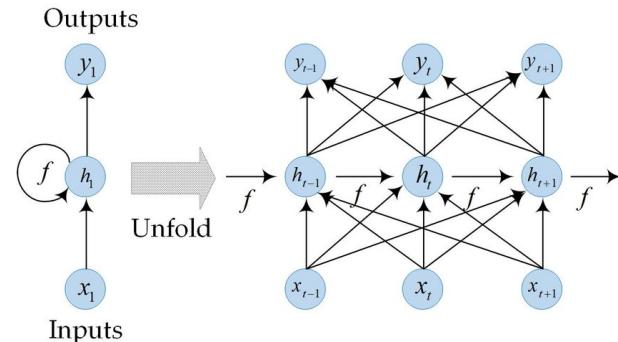


Convolutional Neural Net (CNN)



Spatial structure matters
eg: image classification

Am I a CAT or a COWPOKE?

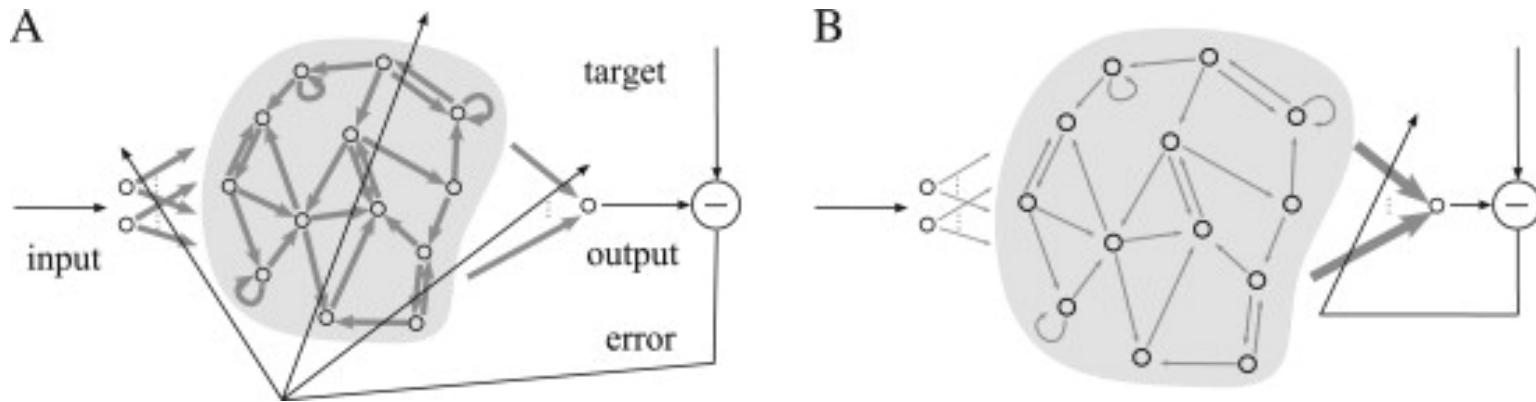


Recurrent Neural Net (RNN)

Sequential structure matters
eg: weather prediction

It rained the last two days, will it rain today?

Reservoir Computing – a better RNN

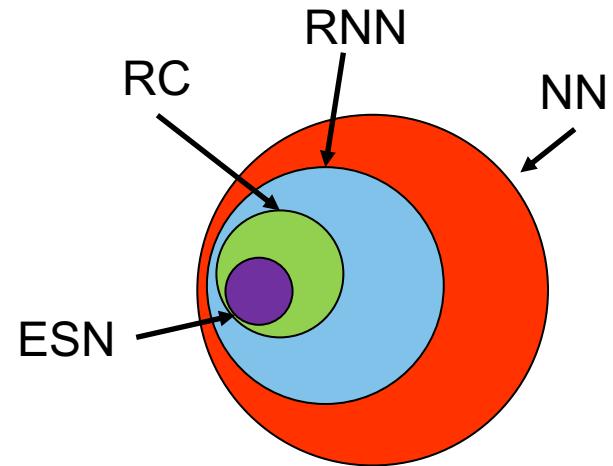


- A. Traditional RNN: all connection weights adapted
 - Computationally intensive, hard to tune
- B. Reservoir Computing (RC): only output weights are adapted
 - Fast! Accurate! Still challenging to tune

Bold arrows
indicate trained
weights

Echo State Networks (ESN) – introduction!

- ESNs are a recurrent neural network
 - Captures **dynamic behavior** of a time series or sequential data.
 - Internal state (**memory**) to process sequences of inputs and remember context
- ESNs are also type of reservoir computing
 - **Faster training** by only learning the output weights vs. back propagation approaches
 - Captures more complex interactions by manipulating topologies and neurons within network
 - Non-linear transformation of inputs into higher dimension of reservoir
 - Output weights are trained ~ “what linear combination of reservoir elements best approximates my target?”



ESNs are a type of Reservoir Computing, which is a subset of RNNs, which are a type of Neural Network

ESN: Overview

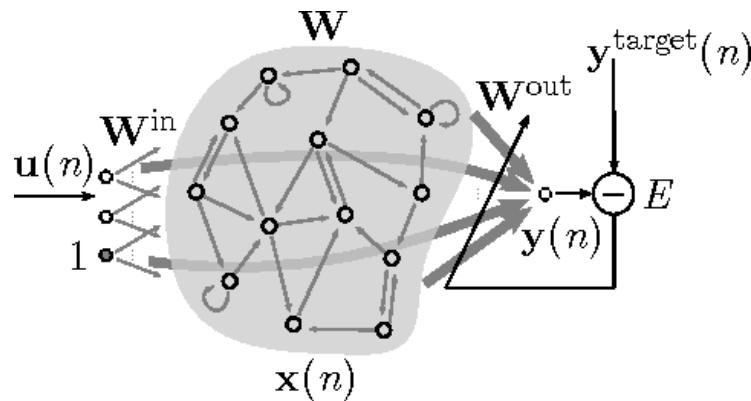


Fig. 1: An echo state network.

For each time step n

- $u(n)$: input
- $x(n)$: reservoir activations
- $y(n)$: network output
- $y^{\text{target}}(n)$: desired output

Randomly Initialized & fixed

- W^{in} : input weights
 - Linearly map input into reservoir
- W : recurrent weights
 - How are the reservoir neurons connected to each other?

Learned parameters

- W^{out} : output weights
 - Linearly map $(u(n), x(n))$ to $y(n)$
 - Minimize Error($y^{\text{target}}(n)$, $y(n)$)

ESN: Overview

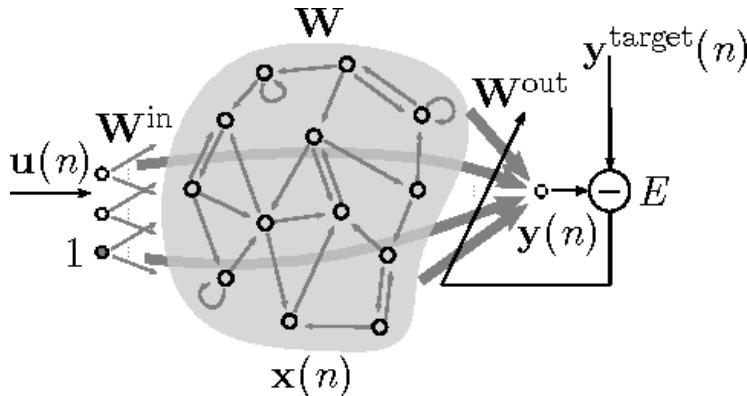


Fig. 1: An echo state network.

How does it work?

1. **Setup the network!** : Generate a large random reservoir RNN with
 - Input weights W^{in}
 - Recurrent weights W
 - Leak rate α
2. **Map to higher dimension!**: Run input $u(n)$ through network and collect activation states $x(n)$
 - Non-linear high dimensional transform!
3. **Optimize!**: Compute W^{out} such that W^{out} minimizes Error($y(n)$, $y^{target}(n)$)
 - The only training we need!
 - Error minimized across all time points
4. **Predict something new!**: Trained network can be used to compute predicted $y(n)$ from new $u(n)$

Definitions and update equations

$W^{in} \in \mathbb{R}^{N_x \times (1+N_u)}$: input weights

$W \in \mathbb{R}^{N_x \times N_x}$: recurrent weights

$\alpha \in (0,1]$: leak rate

$x(n) \in \mathbb{R}^{N_x}$: neuron activations

$\tilde{x}(n) \in \mathbb{R}^{N_x}$: neuron updates

$$x(n) = (1 - \alpha)x(n - 1) + \alpha\tilde{x}(n)$$

$$\tilde{x}(n) = \tanh(W^{in}[1; u(n)] + Wx(n - 1))$$

Key Reservoir Parameters: Input Weights

$$W^{in} \in \mathbb{R}^{N_x \times (1+N_u)}$$

$$W_{ij}^{in} \sim Unif(-a, a)$$

Tune/scale input weights by varying a

- Generally drawn from a Uniform distribution, but not required!
- Input scaling determines how nonlinear reservoir responses are
 - $a < 1$ (generally)
- Scaling input weights helps remove the influence of outliers.
- Different sets of inputs can be scaled differently if the corresponding portions of $u(n)$ contribute differently to task

Key Reservoir Parameters: Spectral Radius

$$W \in \mathbb{R}^{N_x \times N_x}$$

$$\rho(W) = \text{spectral radius of } W$$

- Spectral radius = eigenvalue of W with greatest magnitude
- W defines which reservoir neurons are connected and how
- $\rho(W) < 1$ ensures echo state property. But not always necessary.
- $y(n)$ requires extensive historical knowledge? Large $\rho(W)$
- $y(n)$ depends on more recent history? Small $\rho(W)$
- Practical approach: Select $\rho(W)$ that maximizes accuracy, using $\rho(W)=1$ as initial starting value

Key Reservoir Parameters: Leak Rate

$\alpha \in (0, 1]$: leak rate

$x(n) = (1 - \alpha)x(n - 1) + \alpha\tilde{x}(n)$: neuron activations

$\tilde{x}(n) = \tanh(W^{in}[1; u(n)] + Wx(n - 1))$: neuron updates

- Captures temporal dynamics of system
- "speed of reservoir update dynamics discretized in time"
- Goal: set α to match the speed of the dynamics of $u(n)$ and/or $y^{\text{target}}(n)$
- Different components of $u(n)$ have different speed dynamics? No problem!

$$\alpha \in \mathbb{R}^{N_x}$$

Other reservoir parameters

Can generally be set to default/“reasonable” values

- Sparsity of the reservoir (connectivity)
 - Reservoirs should be very sparsely connected, regardless of size.
 - Potentially each neuron only connected to 10 others!
 - This sparsity is what makes it fast! fast! fast!
- Size of the reservoir (number of neurons)
 - Pick as many as you think you'll need computationally
 - But you probably need fewer than you think (10's, not 100's)
- Distribution used to generate weights for W
 - Uniform distribution is probably good enough.
 - Can also be used for generating weights for W^{in}

More choices – Which error?

Output weights, W^{out} , computed to minimize error....but which error?

$$W^{out} = \underset{W^{out}}{\operatorname{argmin}} E(y, y^{target})$$

- Simplest: Mean Squared Error
- Potential for overfitting or feedback instability? Ridge Regression
- No overfitting and lots of compute power? Pseudoinverse
- Some time points more (less) important than others? Regression Weighting
 - Weight more (less) heavily error for important timepoints

D R A P E R®

ESN DEMO

DEMO

DEMO can be found at: https://github.com/tguidici/ODSC_2020_ESN

D R A P E R®

ESN for Finance

Echo State Networks for Market Modeling

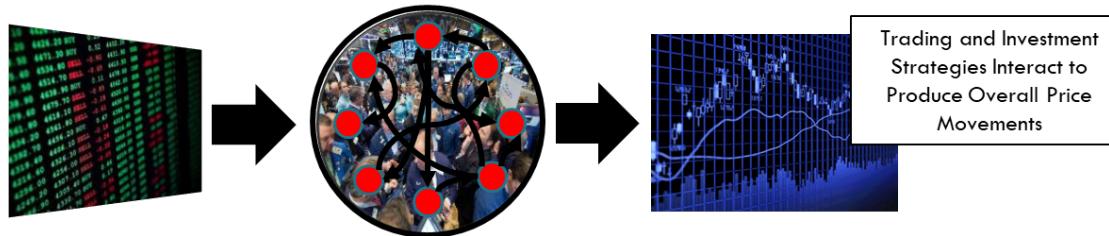
1. The financial market is a complex system
 - Large system of non linearly interacting information and behaviors.
 - Temporal/sequential information crucial (“memory”)
2. Echo State Networks are uniquely capable of modeling non-linear, interacting dynamics with temporal dependencies.

Theory : The Stock Market is a Network of Interacting Strategies

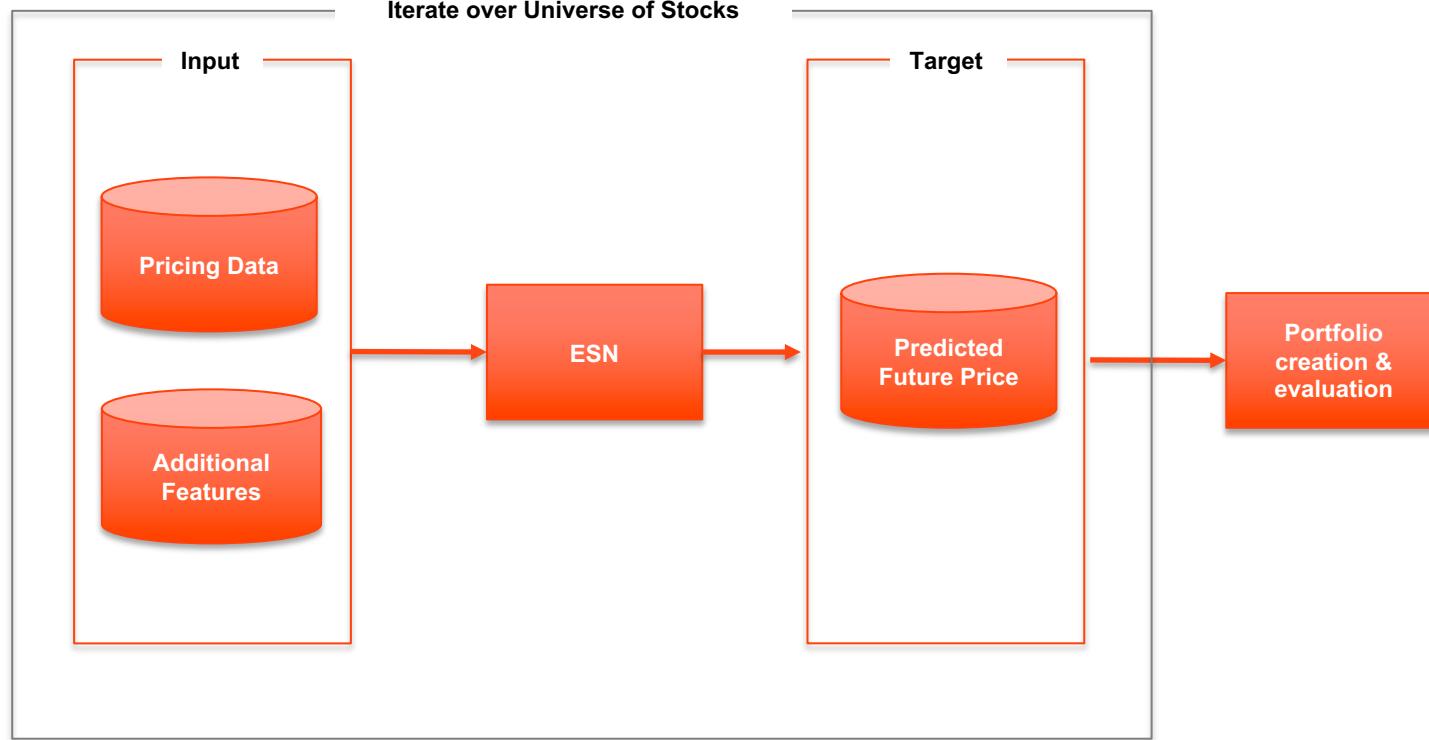
Input – Market, company news, external news, world events, opinions

Computation – Complex system dynamics of strategies

Output – Predicted stock price movement and corresponding volume



ESN for Market Modeling pipeline



Test Setup: parameter tuning & data leakage

Data Leakage: when your model knows more than it should

	2004	2005	2006	2007	2008	2009	2010	2011	2012	2013
Development Test										
Train										
Validate										
Out of Sample Test										
Final Test #1										
Train										
Validate										
Out of Sample Test										

- All development tests in the pre-2010 timeframe
 - True out-of-sample tests in the post-2010 timeframe
 - Final tests were executed a single time using parameters determined during development
-

ESN for Market Modeling: data leakage

Data Leakage: when your model knows more than it should

What we did well:

- Held out “final” datasets for test of the entire pipeline
- Separate training and validation datasets for hyperparameter tuning
- Temporal buffer to separate validation and test datasets

What we didn’t do well:

- Optional PCA featurization used data before splitting into train/validate/test
 - features in training data “knew” something about variance of test data
(Caught this error before final tests!)

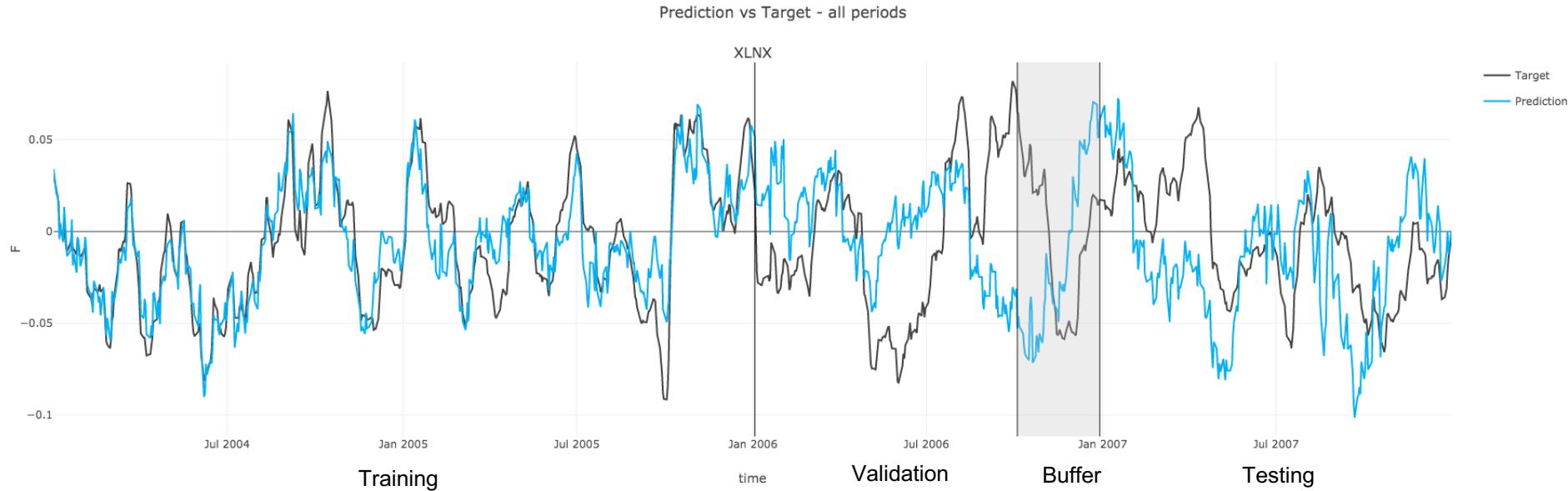
Automated hyperparameter tuning

ESNs have many hyperparameters – how to pick good values?

We automated the pipeline to iterate over a "reasonable" range for select parameters, pick values with lowest error, on average.

- Training:
 - Spectral radius: tune over reservoir
 - Input scaling: tune over inputs
- Validation:
 - Leak rate: tune over temporal dynamics of input

ESN for Finance – Results



Portfolios constructed based on ESN predictions were robust and performed well

Where to next?

- Ensemble or on-line training
 - Data generating process not stationary? Model needs to adapt
- Output feedback
 - Trained readouts fed back into reservoir to change dynamics
- Semi-structured network topologies
 - Groups of neurons linked to specific inputs
- Attention based ESN
 - Original input modulated by attention response to capture differences in importance
- Different error metrics
 - Some types of error matter more than others (eg: when predicting counts, output should not be negative....)