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Echo State Networks for Time Series Data

Open Data Science Conference, Boston

Teal Guidici, PhD

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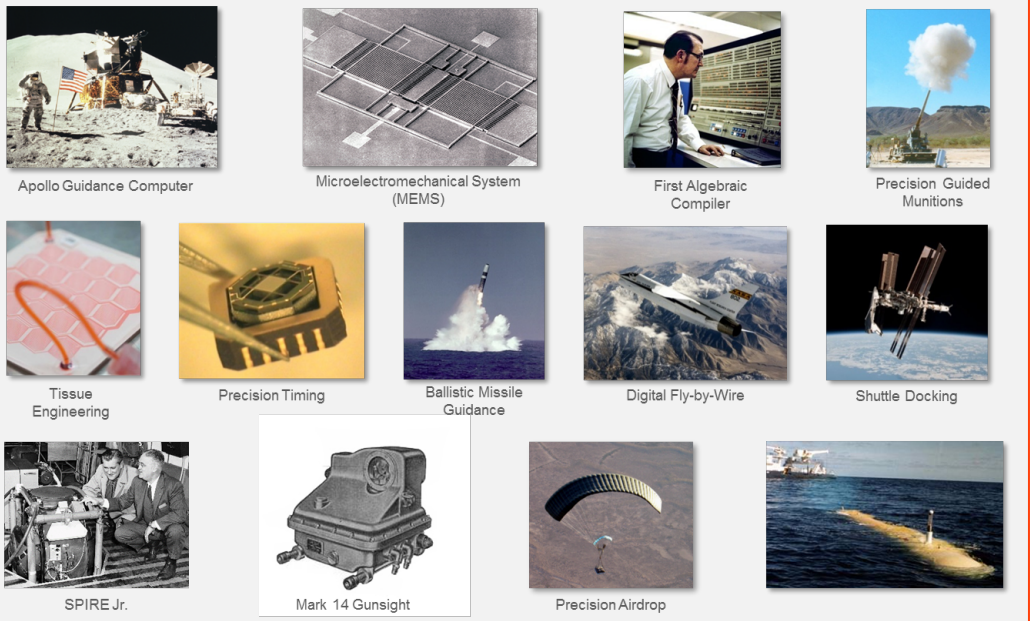
The Charles Stark Draper Laboratory, Inc.
555 Technology Square, Cambridge Mass. 02139-3563

About Draper



Independent not-for-profit research and development company

- Headquartered in Cambridge, MA
- 1500+ employees, $520 million revenue
- Serving the nation since 1932
- Independent organization since 1973

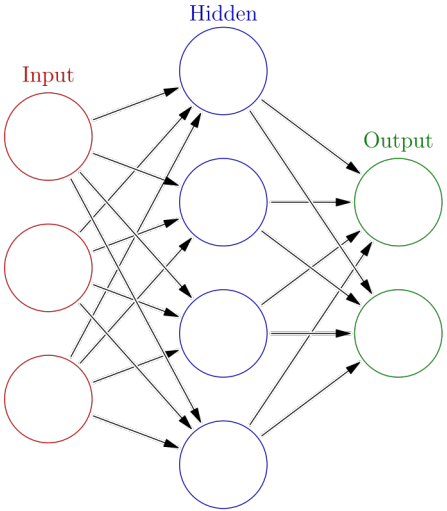
Outline

- Neural Networks in general
- Echo State Networks in detail
- Demo: https://github.com/tguidici/ODSC_2020_ESN
- Example: ESN for finance

Neural Networks (NN)

- Loosely (!) modeled on human/animal brains
  - Many simple processing nodes
  - Densely connected (usually)
  - “Learn” to perform a task by analyzing training examples

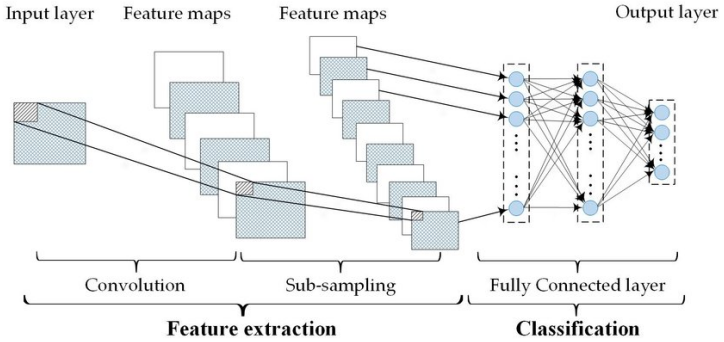
- First proposed in 1944; on a boom-bust cycle of interest
  - Dropped out of favor ~ 1969
  - Resurgence in 1980’s!
  - Disinterest in 2000’s!
  - ALL THE INTEREST IN 2010’s!!



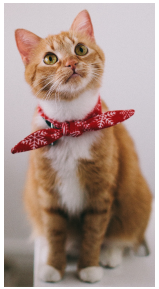
- Circular nodes represent artificial neurons
- Arrows illustrate connections between output of one neuron to input of another

Image credit: By Glosser.ca - Own work, Derivative of File:Artificial neural network.svg, CC BY-SA 3.0, https://commons.wikimedia.org/w/index.php?curid=24913461

Convolutional vs Recurrent Neural Nets



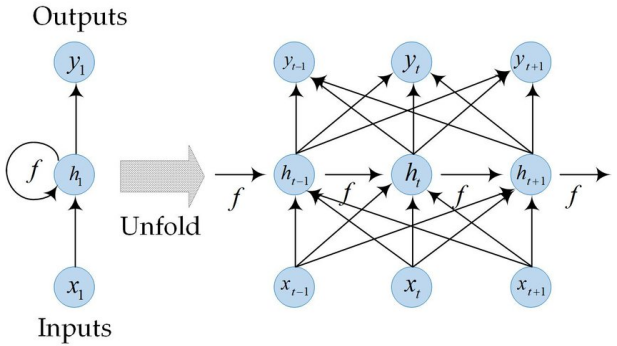
Convolutional Neural Net



Spatial structure matters

eg: image classification

Am I a CAT or a COWPOKE?



Recurrent Neural Net (RNN)

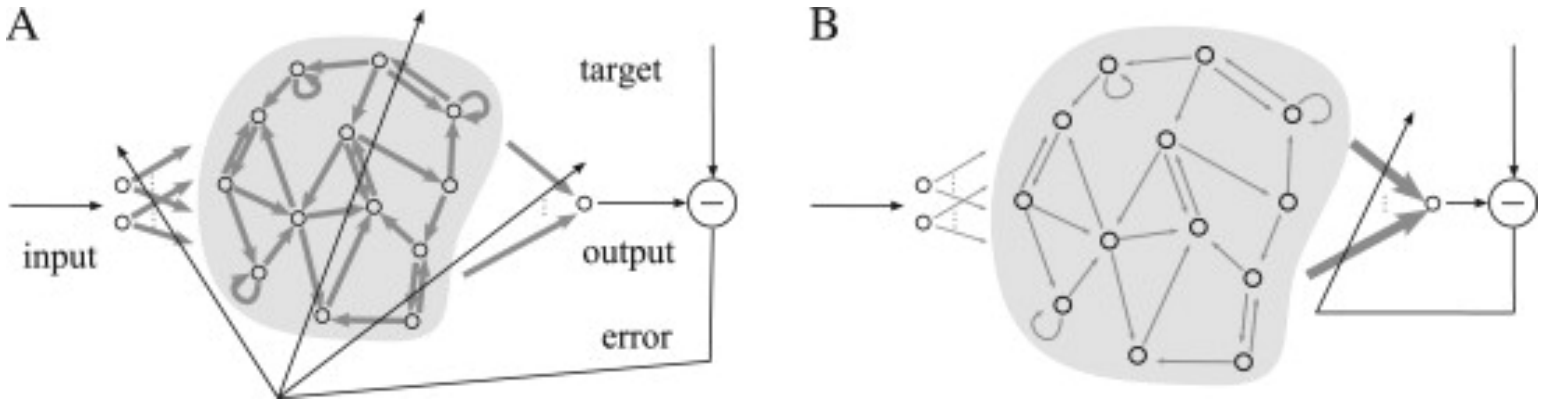
Sequential structure matters

eg: weather prediction

It rained the last two days, will it rain today?

Neural Net Images From: Zhang, Sushen & Hosseini Bamakan, Seyed Mojtaba & Qu, Qiang & Li, Sha. (2018). Learning for Personalized Medicine: A Comprehensive Review From a Deep Learning Perspective. IEEE Reviews in Biomedical Engineering. PP. 1-1. 10.1109/RBME.2018.2864254.

Reservoir Computing – a better RNN



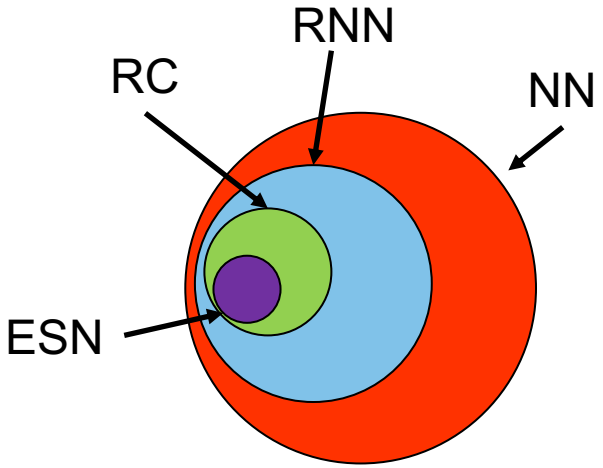
Bold arrows indicate trained weights

A. Traditional RNN: all connection weights adapted
   - Computationally intensive, hard to tune
B. Reservoir Computing (RC): only output weights are adapted
   - Fast! Accurate! Still challenging to tune

Image and caption from Lukoševičius, Mantas & Jaeger, Herbert. (2009). Jaeger, H.: Reservoir computing approaches to recurrent neural network training. Computer Science Review 3, 127-149. Computer Science Review. 3. 127-149. 10.1016/j.cosrev.2009.03.005.

Echo State Networks (ESN) – introduction!

- ESNs are a recurrent neural network
  - Captures **dynamic behavior** of a time series or sequential data.
  - Internal state (**memory**) to process sequences of inputs and remember context
- ESNs are also type of reservoir computing
  - **Faster training** by only learning the output weights vs. back propagation approaches
  - Captures more complex interactions by manipulating topologies and neurons within network
  - Non-linear transformation of inputs into higher dimension of reservoir
  - Output weights are trained ~ “what linear combination of reservoir elements best approximates my target?”



ESNs are a type of Reservoir Computing, which is a subset of RNNs, which are a type of Neural Network

ESN: Overview

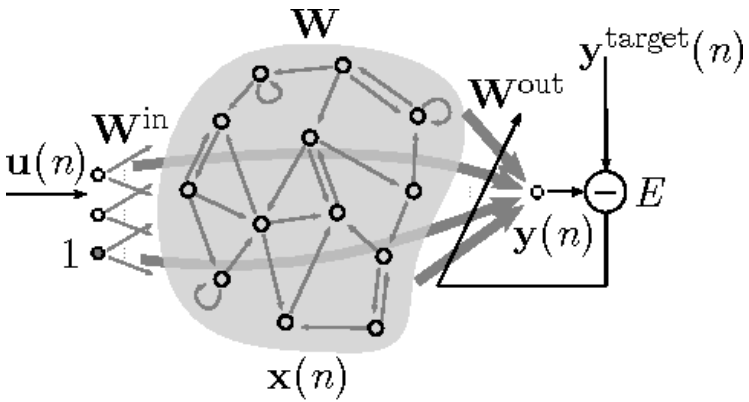


Fig. 1: An echo state network.

For each time step n

- u(n): input
- x(n): reservoir activations
- y(n): network output
- $y^{target}(n)$: desired output

Randomly Initialized & fixed

- W^{in}: input weights
  - Linearly map input into reservoir
- W: recurrent weights
  - How are the reservoir neurons connected to each other?

Learned parameters

- W^{out}: output weights
  - Linearly map (u(n), x(n)) to y(n)
  - Minimize Error($y^{target}(n)$, y(n))

Image From: M. Lukoševičius (2012): **A Practical Guide to Applying Echo State Networks.** In: G. Montavon, G. B. Orr, and K.-R. Müller (eds.) Neural Networks: Tricks of the Trade, 2nd ed. Springer LNCS 7700, pp 659-686

ESN: Overview

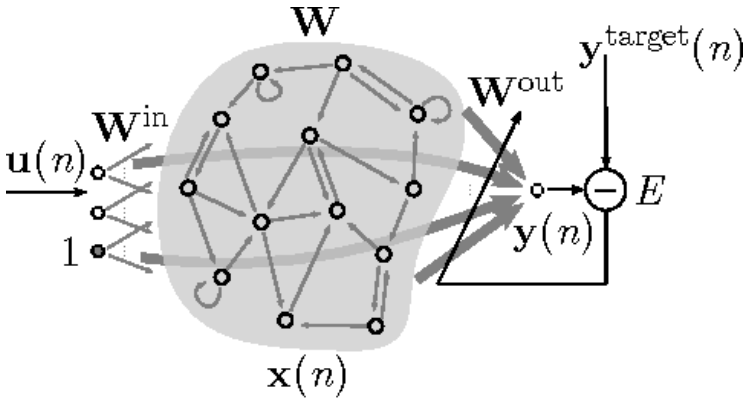


Fig. 1: An echo state network.

How does it work?

1. **Setup the network!** : Generate a large random reservoir RNN with
   - Input weights W^{in}
   - Recurrent weights W
   - Leak rate α
2. **Map to higher dimension!**: Run input u(n) through network and collect activation states x(n)
   - Non-linear high dimensional transform!
3. **Optimize!**: Compute W^{out} such that W^{out} minimizes Error(y(n), y^{target}(n))
   - The only training we need!
   - Error minimized across all time points
4. **Predict something new!**: Trained network can be used to compute predicted y(n) from new u(n)

Definitions and update equations

$W^{in} \in \mathbb{R}^{N_x \times (1+N_u)}$: input weights

$W \in \mathbb{R}^{N_x \times N_x}$: recurrent weights

$\alpha \in (0,1]$: leak rate

$x(n) \in \mathbb{R}^{N_x}$: neuron activations

$\tilde{x}(n) \in \mathbb{R}^{N_x}$: neuron updates

$$x(n) = (1-\alpha)x(n-1) + \alpha\tilde{x}(n)$$

$$\tilde{x}(n) = tanh(W^{in}[1; u(n)] + Wx(n-1))$$

Key Reservoir Parameters: Input Weights

$$W^{in} \in \mathbb{R}^{N_x \times (1+N_u)}$$
$$W^{in}_{ij} \sim Unif(-a, a)$$
Tune/scale input weights by varying a

- Generally drawn from a Uniform distribution, but not required!
- Input scaling determines how nonlinear reservoir responses are
  - a <1 (generally)
- Scaling input weights helps remove the influence of outliers.
- Different sets of inputs can be scaled differently if the corresponding portions of u(n) contribute differently to task

Key Reservoir Parameters: Spectral Radius

$$\boldsymbol{W} \in \mathbb{R}^{\boldsymbol{N_x} \times \boldsymbol{N_x}}$$

$$\boldsymbol{\rho(W)} = \textbf{ spectral radius of W}$$

- Spectral radius = eigenvalue of W with greatest magnitude
- W defines which reservoir neurons are connected and how
- $\rho(W) < 1$ ensures echo state property. But not always necessary.
- y(n) requires extensive historical knowledge? Large $\rho(W)$
- y(n) depends on more recent history? Small $\rho(W)$
- Practical approach: Select $\rho(W)$ that maximizes accuracy, using $\rho(W)=1$ as initial starting value

Key Reservoir Parameters: Leak Rate

$$\alpha \in (0, 1]\text{: leak rate}$$
$$x(n) = (1 - \alpha)x(n-1) + \alpha\tilde{x}(n)\text{: neuron activations}$$
$$\tilde{x}(n) = tanh(W^{in}[1; u(n)] + Wx(n-1))\text{: neuron updates}$$

- Captures temporal dynamics of system
- ”speed of reservoir update dynamics discretized in time”
- Goal: set α to match the speed of the dynamics of u(n) and/or $y^{target}(n)$
- Different components of u(n) have different speed dynamics? No problem!

$$\alpha \in \mathbb{R}^{N_x}$$

Other reservoir parameters

Can generally be set to default/“reasonable” values

- Sparsity of the reservoir (connectivity)
  - Reservoirs should be very sparsely connected, regardless of size.
    - Potentially each neuron only connected to 10 others!
  - This sparsity is what makes it fast! fast! fast!
- Size of the reservoir (number of neurons)
  - Pick as many as you think you’ll need computationally
  - But you probably need fewer than you think (10’s, not 100’s)
- Distribution used to generate weights for W
  - Uniform distribution is probably good enough.
  - Can also be used for generating weights for W^{in}

More choices – Which error?!

Output weights, W^{out}, computed to minimize error….but which error?

$$W^{out} = \underset{W^{out}}{\operatorname{argmin}} E(y, y^{target})$$

- Simplest: Mean Squared Error
- Potential for overfitting or feedback instability? Ridge Regression
- No overfitting and lots of compute power? Pseudoinverse
- Some time points more (less) important than others? Regression Weighting
  - Weight more (less) heavily error for important timepoints

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ESN DEMO

DEMO

DEMO can be found at: https://github.com/tguidici/ODSC_2020_ESN

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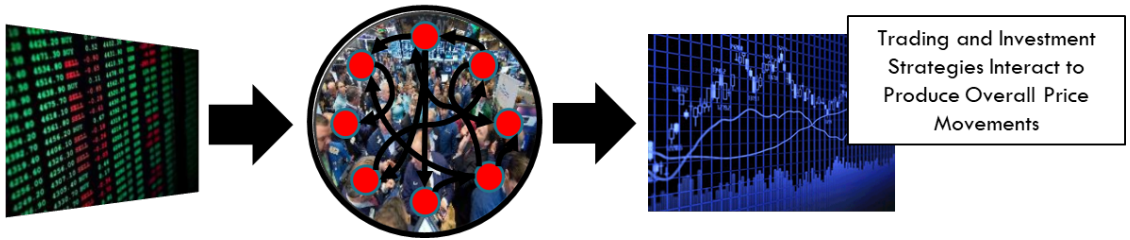
ESN for Finance

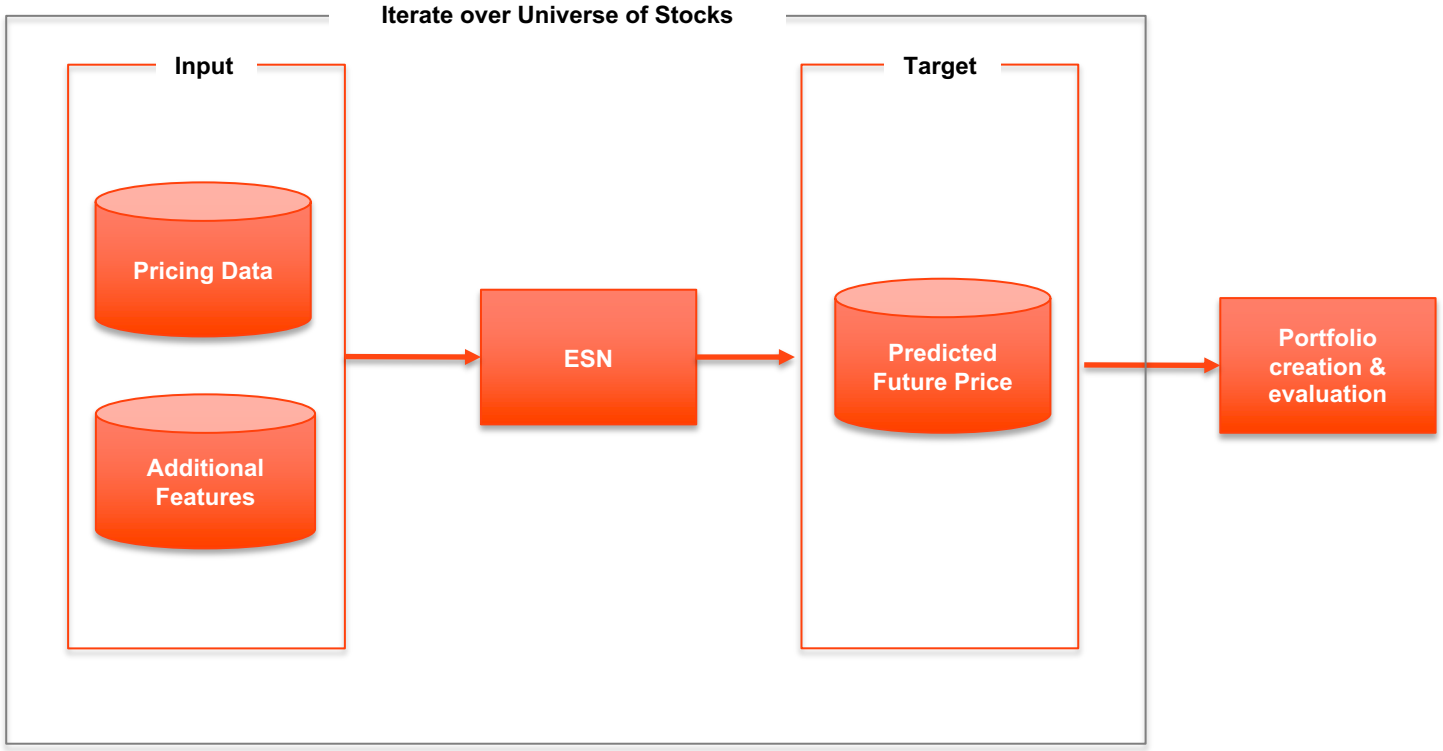
Echo State Networks for Market Modeling

1. The market is a complex system
    - Large system of non linearly interacting information and behaviors.
    - Temporal/sequential information crucial (“memory”)
2. Echo State Networks are uniquely capable of modeling non-linear, interacting dynamics with temporal dependencies.

Theory : The Stock Market is a Network of Interacting Strategies
Input – Market, company news, external news, world events, opinions
Computation – Complex system dynamics of strategies
Output – Predicted stock price movement and corresponding volume

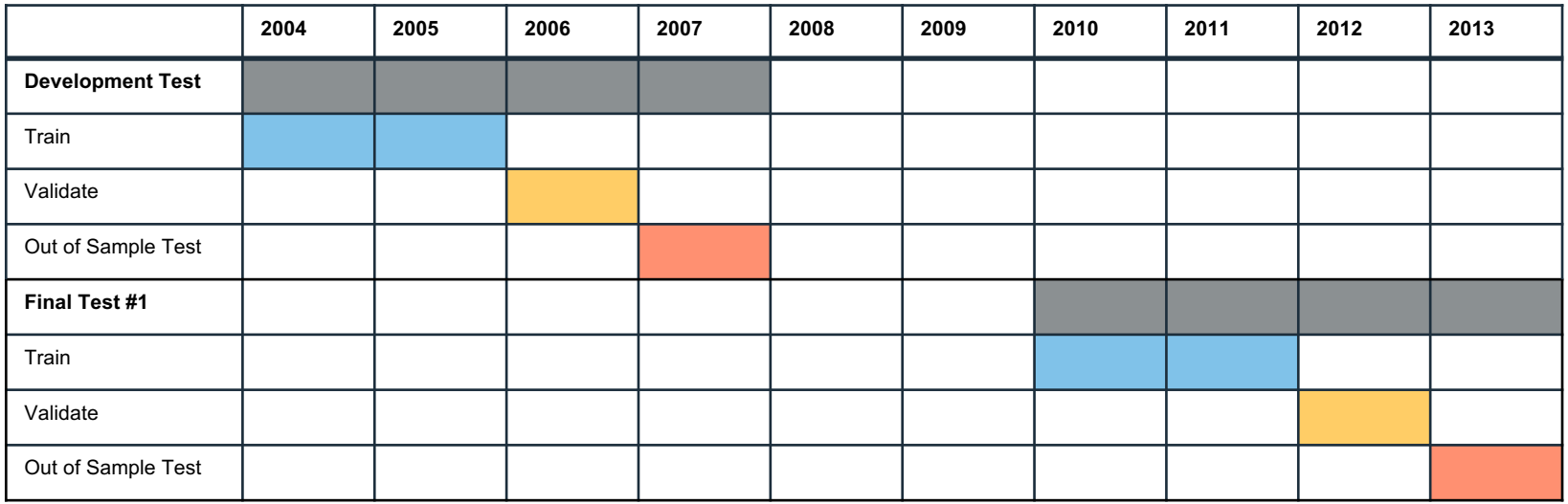


ESN for Market Modeling pipeline



Test Setup: parameter tuning & data leakage

Data Leakage: when your model knows more than it should



- All development tests in the pre-2010 timeframe
- True out-of-sample tests in the post-2010 timeframe
- Final tests were executed a single time using parameters determined during development

ESN for Market Modeling: data leakage

Data Leakage: when your model knows more than it should

What we did well:

- Held out “final” datasets for test of the entire pipeline
- Separate training and validation datasets for hyperparameter tuning
- Temporal buffer to separate validation and test datasets

What we didn’t do well:

- Optional PCA featurization used data before splitting into train/validate/test
  - features in training data “knew” something about variance of test data (Caught this error before final tests!)

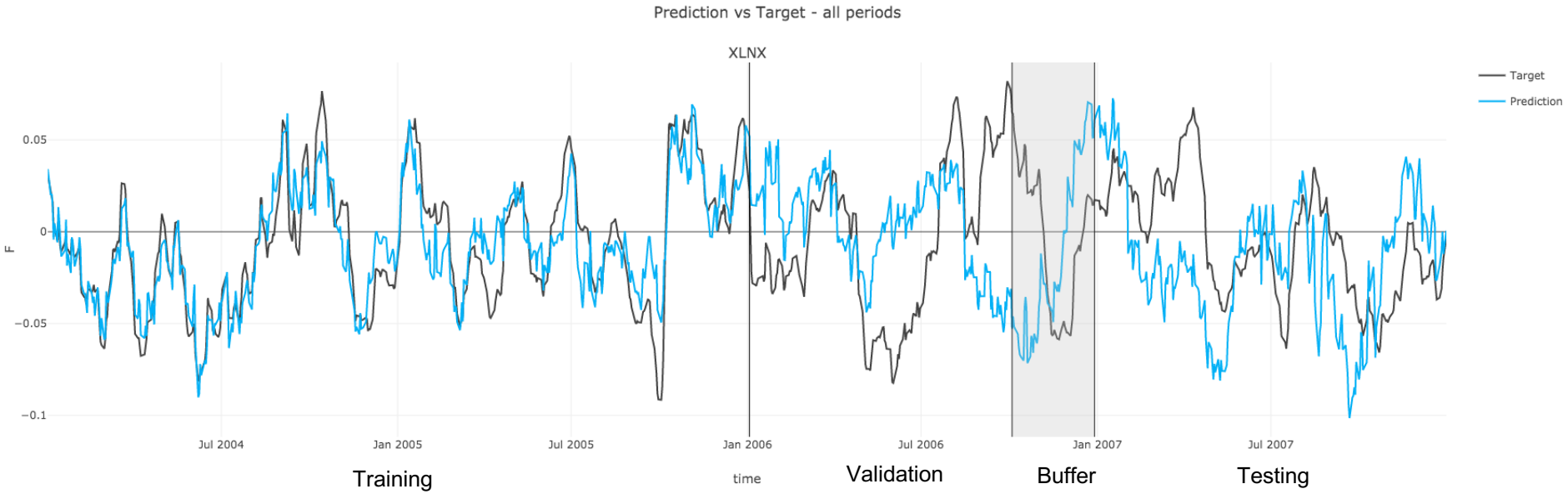
Highly recommend ODSC talk on data leakage from Yuriy Guts – Friday!
https://github.com/YuriyGuts/odsc-target-leakage-workshop

Automated hyperparameter tuning

ESNs have many hyperparameters – how to pick good values?
We automated the pipeline to iterate over a ”reasonable” range for select parameters, pick values with lowest error, on average.

- Training:
  - Spectral radius: tune over reservoir
  - Input scaling: tune over inputs
- Validation:
  - Leak rate: tune over temporal dynamics of input

ESN for Finance – Results



Portfolios constructed based on ESN predictions were robust and performed well

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Where to next?

- Ensemble or on-line training
  - Data generating process not stationary? Model needs to adapt
- Output feedback
  - Trained readouts fed back into reservoir to change dynamics
- Semi-structured network topologies
  - Groups of neurons linked to specific inputs
- Different error metrics
  - Some types of error matter more than others (eg: when predicting counts, output should not be negative….)