ASSIGNMENT – 8

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Starting year: 1993

IMPLEMENTATION METHODOLOGY:

- Below datasets have been used for the project:
  - CRSP daily returns (DSF)
  - Fama-French 3 factor (daily)

- The assignment implementation is divided into 4 parts as below:
  - **8.1:** Computation of semi-betas (monthly) and comparison with beta (monthly).
  - **8.2:** Computation of daily VaR (Value-at-Risk) and daily ES (Expected Shortfall).
  - **8.3:** Volatility Modelling using JP Morgan's Risk Metrics Dynamic Variance model.
  - **8.4:** Volatility Modelling using GARCH (1,1) model and comparison of volatility obtained using 8.3 for a random sample of 5 firms amongst the randomly sampled 250 firms.

- **8.1. Beta estimation**:

  - Betas for all the randomly sampled 250 firms have been computed using the past 1-month data as a monthly estimate. All the 4 semi-betas have been computed on similar lines per the below formulas, as provided:

$$\beta_{t,i}^{\mathcal{N}} \equiv \frac{\sum_{k=1}^{m} r_{t,k,i}^{-} f_{t,k}^{-}}{\sum_{k=1}^{m} f_{t,k}^{2}}$$

$$\beta_{t,i}^{\mathcal{P}} \equiv \frac{\sum_{k=1}^{m} r_{t,k,i}^{+} f_{t,k}^{+}}{\sum_{k=1}^{m} f_{t,k}^{2}}$$

$$\beta_{t,i}^{\mathcal{M}-} \equiv -\frac{\sum_{k=1}^{m} r_{t,k,i}^{+} f_{t,k}^{-}}{\sum_{k=1}^{m} f_{t,k}^{2}}$$

$$\beta_{t,i}^{\mathcal{M}+} \equiv -\frac{\sum_{k=1}^{m} r_{t,k,i}^{-} f_{t,k}^{+}}{\sum_{k=1}^{m} f_{t,k}^{2}}$$

  - The time-period length has been chosen as 1 month as instructed. The function- ***beta_comp_func*** contains the implementation for the same.
  - All these beta estimates including Upbeta, Downbeta have been stored in a single dataset – ***Monthly_Beta_estimates.***

$$\beta^{-}_{t,i} \equiv \frac{\sum_{k=1}^{m} r_{t,k,i} f^{-}_{t,k}}{\sum_{k=1}^{m} (f^{-}_{t,k})^2}$$

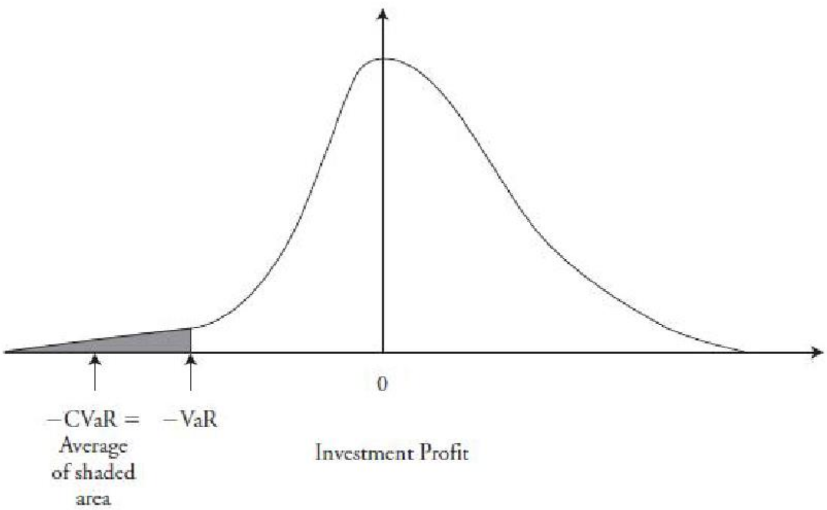
$$\beta^{+}_{t,i} \equiv \frac{\sum_{k=1}^{m} r_{t,k,i} f^{+}_{t,k}}{\sum_{k=1}^{m} (f^{+}_{t,k})^2}$$

- The 4 semi-betas should add up to the monthly beta estimate for each firm which have been captured as a separate column – ***Beta_diff***.
- The Co-skewness, Co-kurtosis measures have been calculated per the given formulas and stored in the dataset – ***Moment_estimates.***

$$\widehat{\text{coskew}}_{it} = \frac{\frac{1}{m}\sum r_{t,k,i} f^2_{t,k}}{\sqrt{\frac{1}{m}\sum r^2_{t,k,i}}\left(\frac{1}{m}\sum f^2_{t,k}\right)}$$

$$\widehat{\text{cokurt}}_{it} = \frac{\frac{1}{m}\sum r_{t,k,i} f^3_{t,k}}{\sqrt{\frac{1}{m}\sum r^2_{t,k,i}}\left(\frac{1}{m}\sum f^2_{t,k}\right)^{\frac{3}{2}}}$$

- **<u>8.2. Var and ES Calculation:</u>**
  - An equal-weighted portfolio has been constructed for Var, ES computation with an initial capital of $1mn. And daily Var, $Var, ES, ES_Value has been calculated for each day for the period of 10 years using the past 1 year (252 days) (Historical Var approach) as the estimation period using a sliding window approach at a 95% confidence level.
  - As ExpectedShortfall is defined as the conditional Var or tail Var, the dollar loss for Expected Shortfall has been computed as an equal-probability weighted average (mean) of the portfolio losses in the tail of the return distribution (as per below).



8.3. Volatility Modelling:

- Below is the Risk Metric for dynamic variance calculation at a daily frequency used for each firm for the period of 10 years.

$$\sigma_{t+1}^2 = \lambda\sigma_t^2 + (1-\lambda)r_t^2$$

  - The initial estimate for the recursive variance estimation has been selected as the daily variance estimate for each firm over a period of 1 year (i.e., 1993).
  - These daily variance estimates *sigma_t+1* are then plugged into the equation as *sigma_t* to get the subsequent forecast for variance.

8.4. GARCH Modelling:

- For modelling variance using the GARCH (1,1) model, the parameters *omega, alpha, beta* have been estimated using the package – **rugarch** in R with the following initial parameters for each firm in the sample. ***The mentioned package has been used since the parameters cannot be estimated using simple regression as the dependent variable (forecasted variance) and current variance are not independent variables. These variables are auto correlated.***
  - Omega = 0.0
  - Alpha = .06
  - Beta = 0.94

$$\sigma_{t+1}^2 = \omega + \beta\sigma_t^2 + \alpha r_t^2.$$

- The function ***GARCH_parameters_func*** contains the implementation for estimation of these parameters.
- The parameters estimated for each stock have been taken as constant throughout the 10 years' period for the project.
- The parameters obtained via regression have then been plugged into the GARCH (1,1) equation to obtain dynamic variance estimates.
- The knitted pdf contains the plots for comparison of these variance estimates for randomly sampled 5 firms.

ANALYSIS:

Beta Computation:

1. The semibetas computed in 8.1. can be described as a 4-way decomposition of the traditional market beta i.e., the CAPM beta depending on the covariation between asset returns and market returns as per Bollerslev's 2021 paper titled – Realized semibetas: Disentangling "good" and "bad" downside risks.
2. As evident, the decomposed semibetas should add up to the CAPM beta for each stock, which is evident from the results in the csv attached – Monthly_Beta_estimates.

Monthly_Beta_estimates.csv

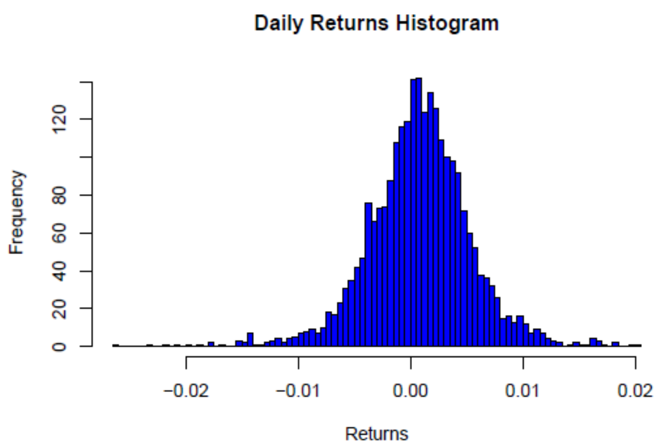
3. The difference between the sum of the decomposed semibetas and actual CAPM beta tends to 0 for almost all the randomly sampled firms and hence, the relationship is consistent.
4. These semibetas represent all possible combinations of co-variation between the market and asset returns, and hence, the summation of all these betas should add up to the CAPM beta.
5. The 3rd (co-skewness) and 4th (co-kurtosis) co-moment estimates have been captured in the below.

Moment_estimates.csv

6. An interesting observation for co-skewness is the wide range of values each stock assumes over the period of 10 years (range: 4, mean: -0.03) and a similar observation for co-kurtosis (range: 22, mean: 0.44).

Historical Var, ES Computation:

1. Var, as per definition measures the potential loss on a portfolio/security from extreme negative returns.
2. The Expected Shortfall, an in-industry practice measure for risk monitoring and management, is an improved metric since it captures the expected loss on a portfolio conditional on a firm's losses being greater than the worst **alpha%** losses, where alpha is described as the level of significance and 1-alpha as the degree of confidence.
3. It is thus better than Var, in the sense that Var does not capture the worst losses, or the magnitude of the losses should there be a loss on the portfolio greater than the worst alpha% return on the portfolio.
4. The below distribution of daily returns depicts a ballpark figure for the worst 5% losses on the portfolio at current time.
5. NOTE: The actual VaR computation for each day for the portfolio have been captured in the attached excel files - Var, ES estimates 8.2.1, Var, ES estimates 8.2.2.



6. As expected, the ES value (ranges from a daily predicted loss of $7.2k to $10k) is always greater than or equal to the VaR predicted losses (ranges from $4.3k to $6.6k) for a portfolio of a $1mn initial capital.
7. In terms of percentage the Var varies from 0.35% to 0.65% of the total invested value.

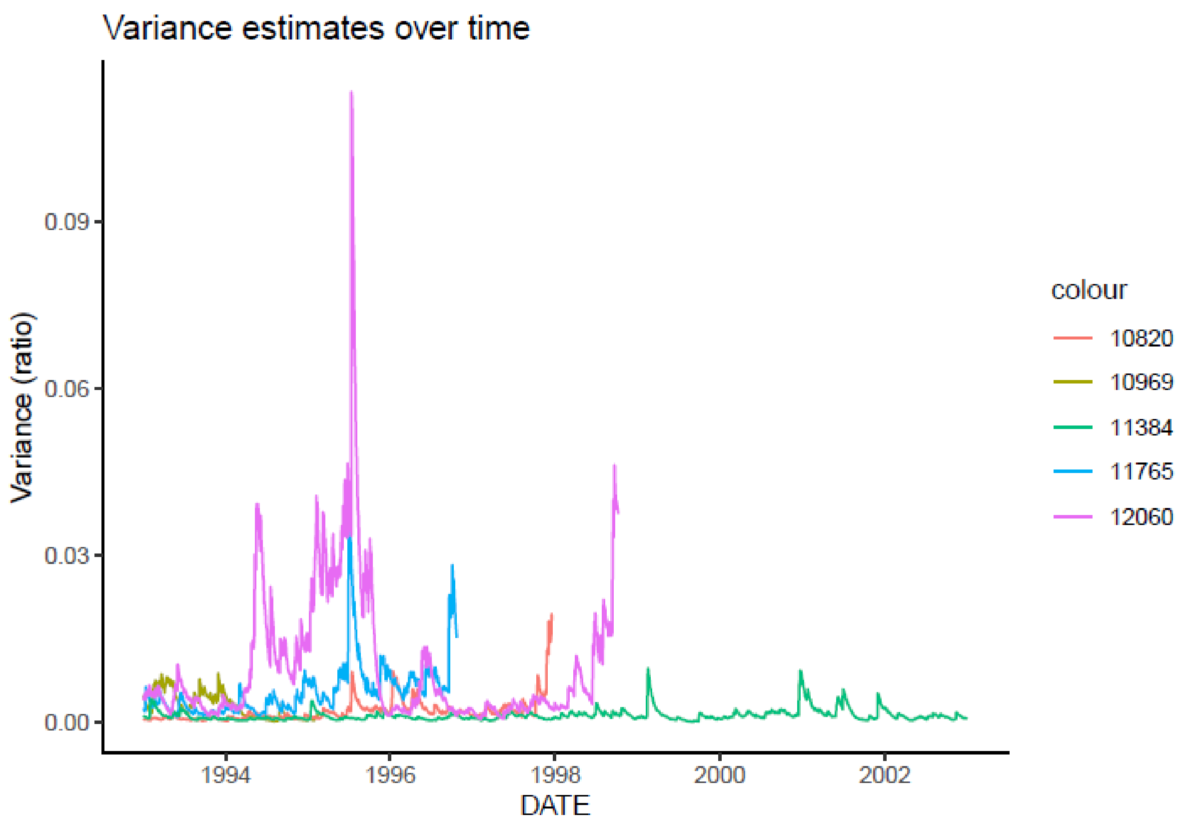
Portfolio Comparison:

8. As compared to the portfolio constructed in 8.2.1, the portfolio constructed in 8.2.2 better captures the Var, ES estimates for the starting year 1993 since the past 1-year returns are unavailable for Historical Var, ES computation for portfolio 1.

Variance Computation

1. JP Morgan's Risk Metrics Variance Model:

a. The daily variance computed using this model varies between 0.1% to more than 1% for the pool for firms over a period of 1 year, with a mean of 0.3%.
b. The model assumes a constant coefficient of 0.94 for current variance value and the remaining weight to squared daily returns, which is nothing but a simpler case of the more general GARCH (p,q) model implemented in 8.4.
c. Below is the variation of daily variances of 5 randomly selected firms from the sample.

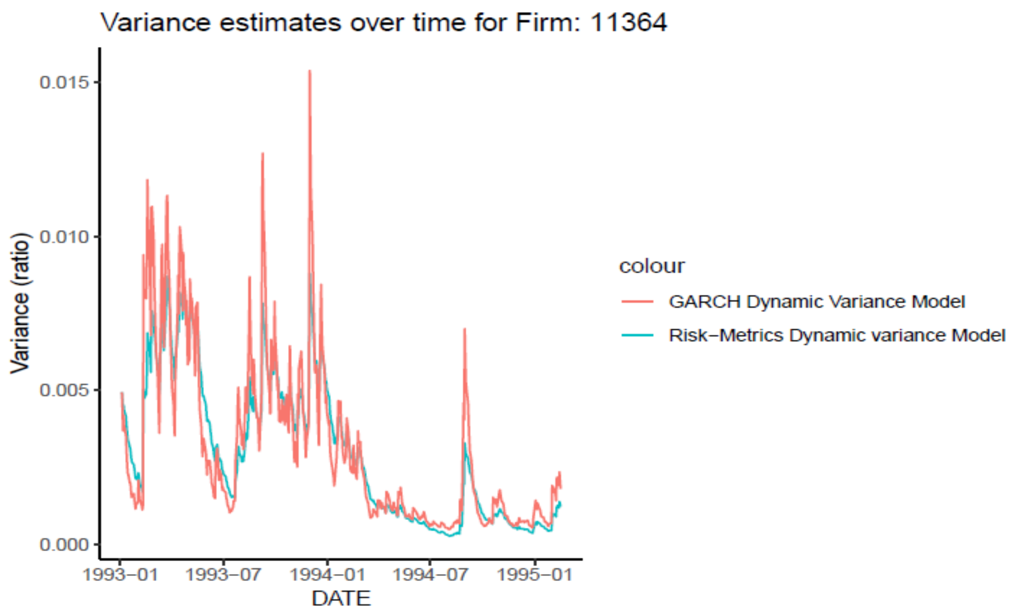


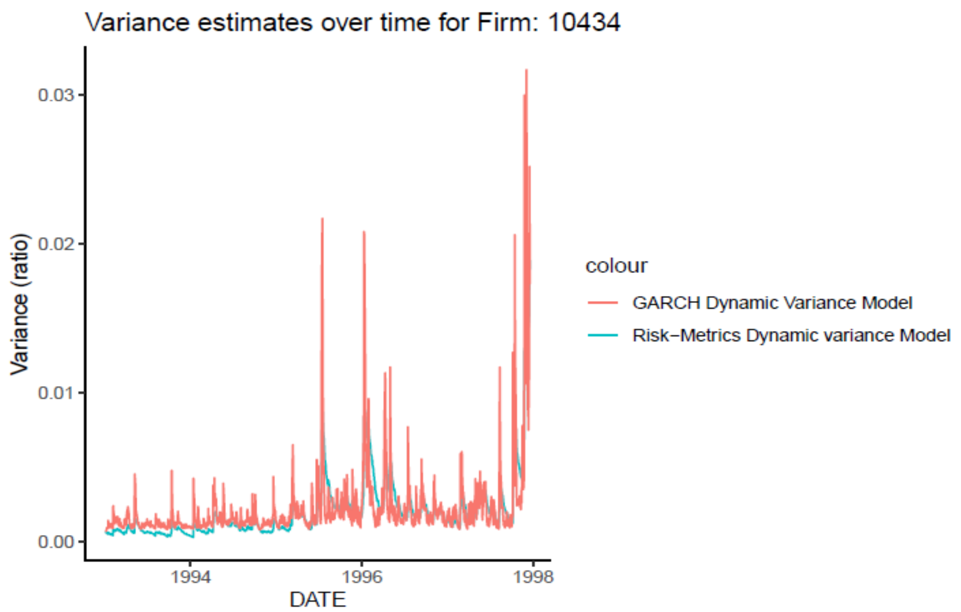
2. GARCH (1,1) variance computation:

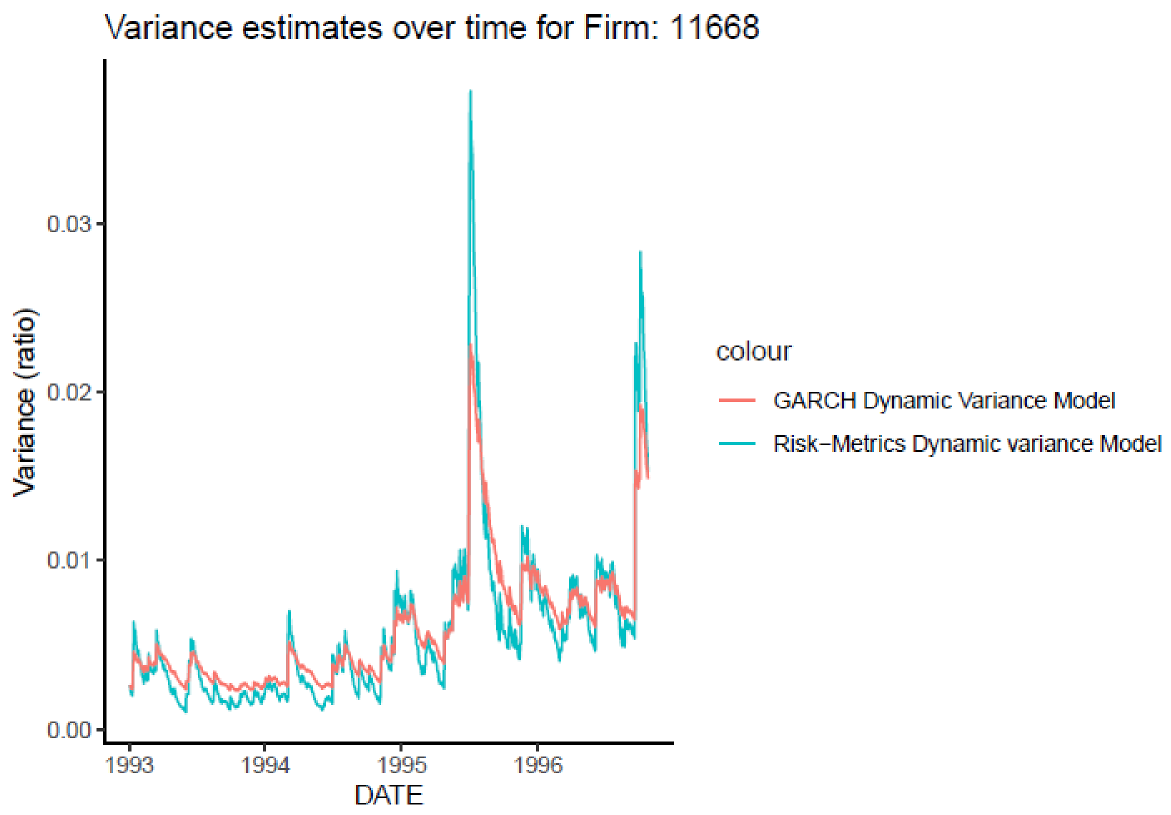
a. The GARCH (1,1) variance estimation seems to be quite close to the variance calculated using the Risk-Metrics dynamic variance model with a mean of 0.33%, however, captures the slightest variations and fluctuations in volatility for every day.

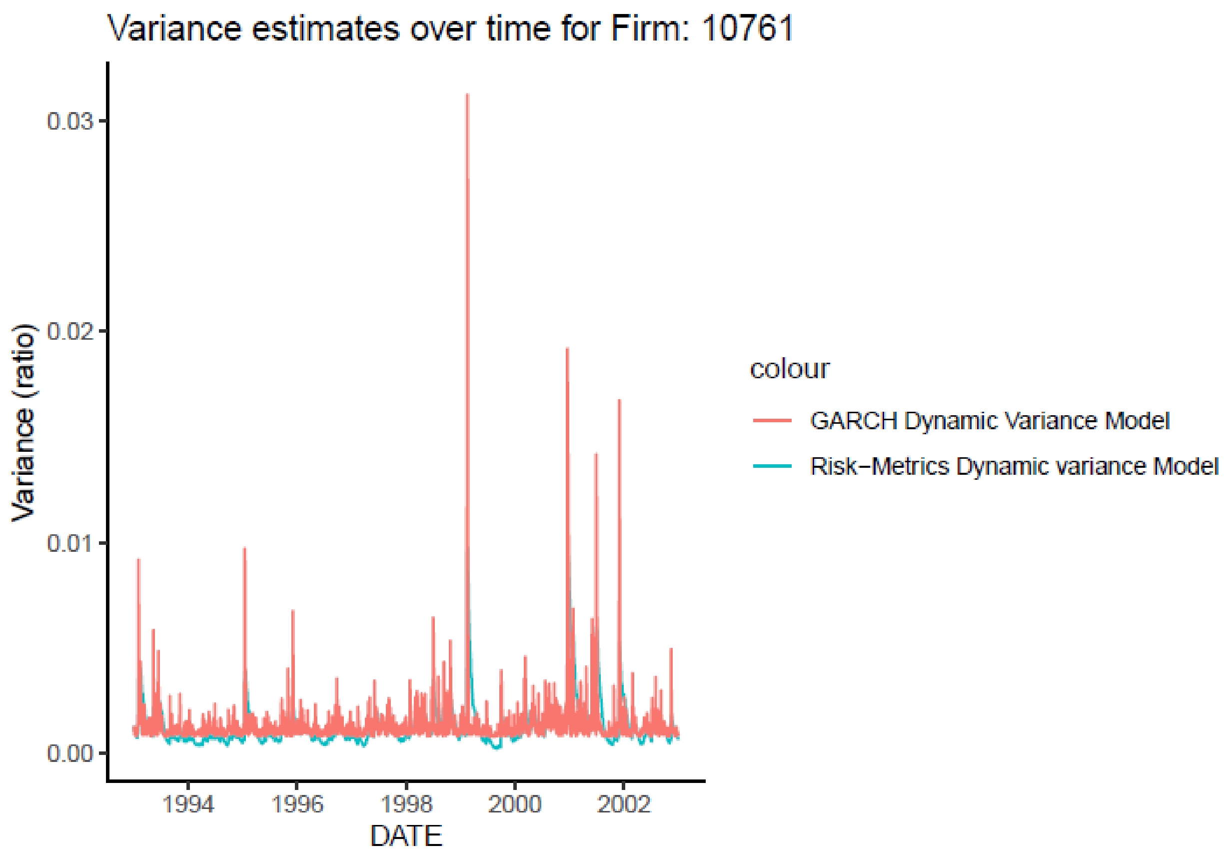
b. The parameters – omega, alpha, beta obtained for each firm, as expected revolve around the 0.00, 0.94, 0.06 as expected given the variances estimates are close to that calculated using the Risk-Metrics dynamic variance model.

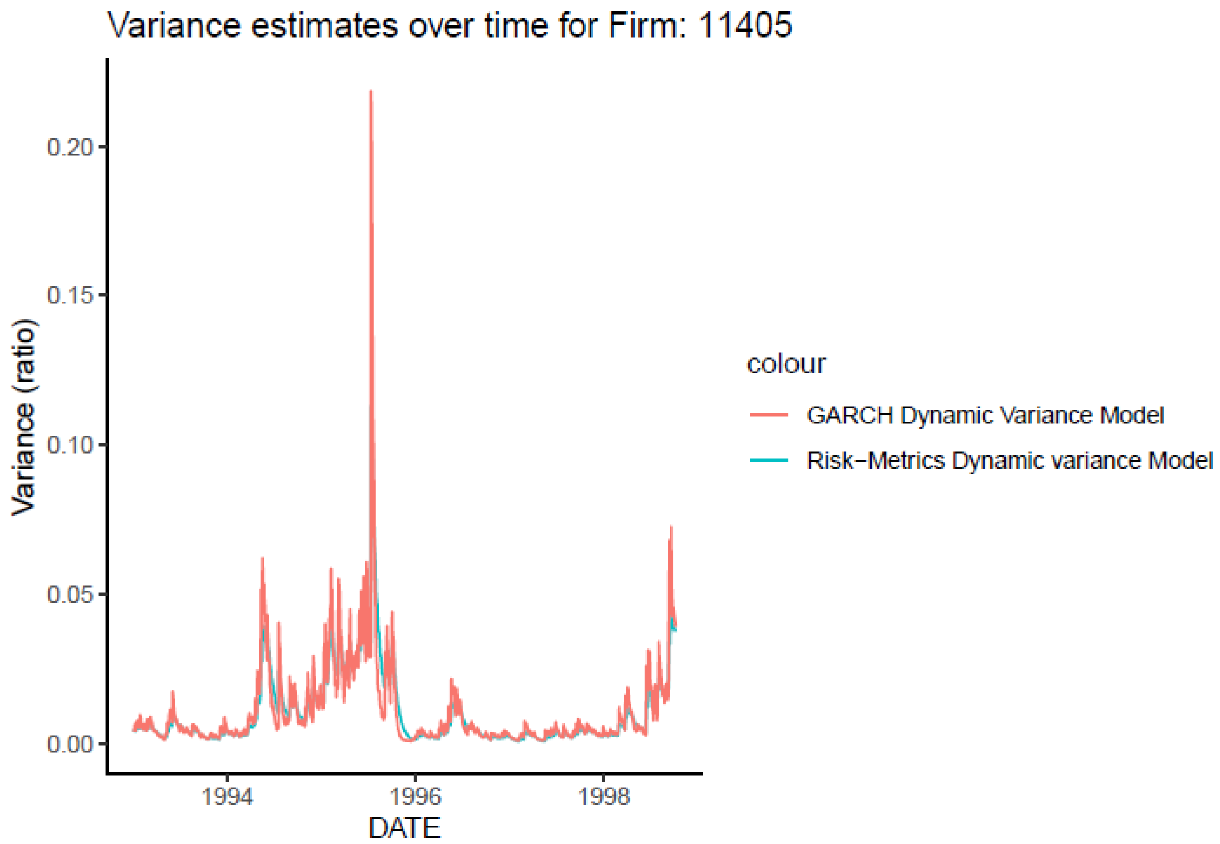
Below is the volatility comparison of these 5 firms, used in 8.3, over time.






Variance estimates over time for Firm: 11668
Variance (ratio)
0.03
0.02
0.01
0.00
1993
1994
1995
1996
DATE
colour
GARCH Dynamic Variance Model
Risk−Metrics Dynamic variance Model


Variance estimates over time for Firm: 10761
Variance (ratio)
0.03
0.02
0.01
0.00
1994
1996
1998
2000
2002
DATE
colour
GARCH Dynamic Variance Model
Risk−Metrics Dynamic variance Model



Variance estimates over time for Firm: 11405
Variance (ratio)
0.20
0.15
0.10
0.05
0.00
1994
1996
1998
DATE
colour
GARCH Dynamic Variance Model
Risk−Metrics Dynamic variance Model