Assignment-2
Optimization Methods (Spring 2022)
Submission Deadline: 26th March 2022 (11.59pm)
Total Marks: 75

Instructions

1. Attempting all questions is mandatory.
2. Marks for each of the question are mentioned at the question itself.
3. You are expected to solve all the questions using python programming language.
4. Use of any in-built libraries to solve the problem directly is not allowed.
5. Submission Format: Submit a zip file with the name rollno_Assignment2 containing the files 1.py, 2.py, 3.py, 4.py corresponding to codes for Question 1, 2, 3, 4 respectively.
6. Plagiarism is a strict No. We will pass all codes through the plagiarism checking tool to verify if the code is copied from somewhere. In that case, you get **F** grade in the course.
7. If any two students codes are found exactly same (if they copy from each other), both will get **F** grade.

1 Question 1 - Simplex Algorithm (25 marks)

Implement simplex algorithm to solve given optimization problem. Make sure you handle the cases of unbounded solutions, no solutions, degenerate solutions. Use two phase method for initialization of simplex algorithm.

$$\begin{aligned} \min \quad & \mathbf{c}^T\mathbf{x} \\ \text{subject to} \quad & A\mathbf{x} \leq \mathbf{b} \\ & \mathbf{x} \geq \mathbf{0} \end{aligned}$$

1.1 Input Format

A file named: input.txt (note the extension) would be given to you which will have the following structure for each test case.

< start A>
<row 1 of matrix>
<row 2 of matrix>
...
<row n of matrix>
<end A>
<start b>
<vector b>
<end b>
<start c>
<vector c>
<end c>

1.2 Output Format

- **If the optimal solution exists:** Then output the optimal value. Also, output the vector of optimal values of the variables.
- **If there is unbounded solution:** print "Unbounded".
- **If infeasible:** Print "Infeasible".

2 Question 2 - Branch and Bound (15 marks)

Given a set of villages and distance between every pair of villages, find a tour (a cycle that visits all the nodes) of minimum cost. Formulate it as integer linear program. You can use cut set formulation of travelling salesman problem that was discussed in class. Use branch and bound algorithm to find a tour with minimum cost. To solve a LP relaxation problem, use the Simplex algorithm routing developed in Question 1.

2.1 Input Format

A file named: input.txt (note the extension) would be given to you which will have the following structure for each test case.

< start A>
<row 1 of matrix>
<row 2 of matrix>
...
<row n of matrix>
<end A>

2.2 Output Format

- **If a tour exists:** (a) Output the cost of the minimum distance tour. (b) Output a Boolean vector of dimension $|E|$, where i^{th} value denotes whether i^{th} edge is included in the tour or not. (c) Output the number of nodes explored (or output the number of LP relaxations solved).
- If there does not exist any tour, then output "Infeasible Problem". Also, Output the number of nodes explored (or output the number of LP relaxations solved).

3 Question 3 - Ellipsoid Method (20 marks)

Implement Ellipsoid method to solve given optimization problem. Make sure you handle the cases of no solutions. Use sliding objective method discussed in the class to find the solution.

$$\begin{aligned} \min \quad & \mathbf{c}^T\mathbf{x} \\ \text{subject to} \quad & A\mathbf{x} \leq \mathbf{b} \\ & \mathbf{x} \geq \mathbf{0} \end{aligned}$$

3.1 Input Format

A file named: input.txt (note the extension) would be given to you which will have the following structure for each test case.

< start A>
<row 1 of matrix>
<row 2 of matrix>
...
<row n of matrix>
<end A>
<start b>
<vector b>

<end b>
<start c>
<vector c>
<end c>

3.2 Output Format

- **If the optimal solution exists:** Then output the optimal value. Also, output the vector of optimal values of the variables.
- **If infeasible:** Print "Infeasible".

4 Question 4 - Cutting Plane Method (15 marks)

Implement Gomory's cutting plane method discussed in the class to solve given integer programming problem. Make sure you handle the cases of no solutions.

$$\begin{aligned} \max \quad & \mathbf{c}^T\mathbf{x} \\ \text{subject to} \quad & A\mathbf{x} \leq \mathbf{b} \\ & \mathbf{x} \geq \mathbf{0} \end{aligned}$$

4.1 Input Format

A file named: input.txt (note the extension) would be given to you which will have the following structure for each test case. To solve a LP relaxation problem, use the Simplex Algorithm routine developed in Question 1.

< start A>
<row 1 of matrix>
<row 2 of matrix>
...
<row n of matrix>
<end A>
<start b>
<vector b>
<end b>
<start c>
<vector c>
<end c>

4.2 Output Format

- (a) Output the optimal value (up to 6 decimals). (b) Output an N-dimensional integer vector, denoting optimal solution. (c) Output the number of cutting planes generated (or the number of LP relaxations solved).

- In case of unbounded solution, print "Unbounded". Also, output the number of cutting planes generated (or the number of LP relaxations solved).
- In case of no solution, print "Infeasible Solution". Also, output the number of cutting planes generated (or the number of LP relaxations solved).