FINAL EXAM

ISyE6420
Spring 2022

Released April 28, 12:00 am – due May 1, 11:59 pm. This exam is not proctored and not time limited except the due date. Late submissions will not be accepted.

Use of all available electronic and printed resources is allowed except direct communication that violates Georgia Tech Academic Integrity Rules. However, internet search on course-related topics is not allowed during the exam period. Furthermore, public Ed Discussion posting about the exam questions is not permitted. If you need any clarification on the questions, please use private posting that are visible only to the instructors.

Name _____

Problem	1	2	3	Total
Score	/45	/30	/25	/100

1. Orthodontic Distance. A longitudinal study was conducted to understand the effect of age and sex on the orthodontic distance (y). Measurements on 27 children are given in the file `ortho.csv`. There are a total of 16 boys and 11 girls, which are identified in the dataset using the column `Subject`. Consider the following random effects model:

$$\begin{aligned} y_{ij}|\beta_0,\beta_1,\beta_2,u_i,\sigma^2_\epsilon &\sim^{ind.} N(\beta_0+\beta_1 age_{ij}+\beta_2 sex_i+u_i,\sigma^2_\epsilon),\\ u_i|\sigma^2_u &\sim^{iid} N(0,\sigma^2_u), \end{aligned}$$

for $i = 1,\cdots,27$ and $j = 1,\cdots,4$. Here u_i represents the random effect of the ith subject. The sex variable should be coded as -1 for female and 1 for male. Assume the following prior distributions:

$$\begin{aligned} \beta_k &\sim^{iid} N(0,\sigma^2=10^8),\ k=0,1,2\\ \tau_\epsilon &\sim Gamma(.01,.01)\\ \tau_u &\sim Gamma(.01,.01). \end{aligned}$$

where $\tau = \frac{1}{\sigma^2}$

1. Fit the random effects model and plot the posterior densities of the five parameters $\beta_0,\beta_1,\beta_2,\sigma^2_\epsilon$, and σ^2_u. (use 100,000 samples with 10,000 burn-in.)

2. The intraclass correlation coefficient is defined as

$$\rho = \frac{\sigma^2_u}{\sigma^2_u+\sigma^2_e}.$$

   Plot the posterior density of ρ. Does it appear to be significantly different from 0?

3. Fit the model ignoring the random effects (that is, set all the u_i's to be 0) and plot the posterior densities of the four parameters β_0,β_1,β_2, and σ^2_ϵ. What differences do you see from the previous analysis using random effects (compare the posterior means and credible intervals of the four parameters)?

2. Nanowire density. Consider the problem of predicting the density of nanowires (y) with respect to the thickness of polymer films (x) in a solution-based growth process (see Figure 1). Eight experiments were conducted with two replicates (except for one run). The data are in the file `nanowire.csv`. The density of nanowires is assumed to follow a Poisson distribution with mean:

$$\mu(x) = \theta_1\exp(-\theta_2 x^2)+\theta_3\{1-\exp(-\theta_2 x^2)\}\Phi(-x/\theta_4),$$

where $\Phi(\cdot)$ is the standard normal CDF - note that there is a `phi()` function in BUGS for this, and in pymc you may use the `invprobit()` function. Assume the following prior distribution for the parameters:

$$\begin{aligned} \log\theta_1,\log\theta_3,\log\theta_4 &\sim^{iid} N(0,\sigma^2=10)\\ \theta_2 &\sim U(0,1). \end{aligned}$$

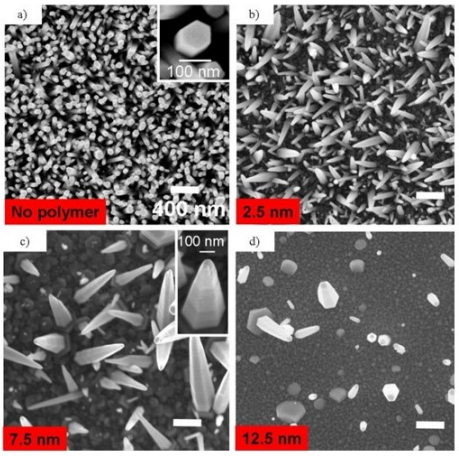

Figure 1: Scanning Electron Microscopy images showing nanowire density at different levels of polymer thickness.

If using pymc: when transforming your priors using another function, you must wrap the result in pm.Deterministic to properly use the transformed variable.

1. Obtain the posterior samples of the four parameters $(\theta_1, \ldots, \theta_4)$ using MCMC. Provide their mean and 95% credible intervals (use 100,000 samples with 10,000 burn-in).

2. Find the predictive distribution of the density of nanowires when the thickness is 2.0 nm.

3. Color Attraction for Oulema melanopus. Some colors are more attractive to insects than others. Wilson and Shade (1967)[1] conducted an experiment aimed at determining the best color for attracting cereal leaf beetles (*Oulema melanopus*). Six boards in each of four selected colors (lemon yellow, white, green, and blue) were placed in a field of oats during summer time. The following table (modified from Wilson and Shade, 1967) gives data on the number of cereal leaf beetles trapped:

Board color	Insects trapped					
Lemon yellow	45	59	48	46	38	47
White	21	12	14	17	13	17
Green	16	11	20	21	14	7
Blue	37	32	15	25	39	41

(a) Use MCMC software to conduct ANOVA analysis of the color "treatments." Use STZ constraints.

(b) Based on MCMC software output, state your conclusions about the attractiveness of these colors to the beetles.

[1]C. M. Wilson and R. E. Shade (1967). Relative Attractiveness of Various Luminescent Colors to the Cereal Leaf Beetle and the Meadow Spittlebug. *Journal of Economic Entomology*, 60, 578–580.