A Framing Plan for Farm-co for the Next Decade

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Abstract

This report presents a farming operation plan for Farm-co to maximise their profits for the next decade. A linear programming model is applied to address the resource allocation and the mixed farming planning problem, considering farmland capacity, requirements of feeding livestock, activity choice, and operation costs. The maximum achievable profit would be £211797.5 after the decade.

1 Introduction

Allocating resources efficiently can help optimise various aspects of the business from operating cost to activity planning. To manage mixed agriculture, there are many different purposes for utilising resources with different sets of constraints. This report focuses on the resource allocation and mixed farming plan problem faced by Farm-co, which operates mixed agriculture that combines cattle rearing and crop cultivation with three different crop types including grain, beans and wheat. A linear programming model is proposed to optimise its long-term operation plan for maximum profitability. Determining which block of land would be allotted to what kind of activity and how many calves would be purchased each year, satisfying the feeding requirements and the limitation of the maximal usage of land for any individual activity are the constraints regarding resource allocation. The constraints also include the fixed cost of purchasing feed for cows. This report begins by introducing the problem and defining modelling variables. It then describes the linear programming model to solve the optimisation problem and discusses the numerical results.

2 Problem Description

The farm of Farm-co consists of 10 blocks of land and each block of land would be assigned to a single framing activity every year in the next decade. The activities include growing grain, beans and wheat, and rearing cows. The size of each block and the relevant cost and benefit of framing crops are displayed in the following tables.

Table 1: Size of Block of Land

Block	1	2	3	4	5	6	7	8	9	10
Size (m^2)	100	250	500	125	800	75	900	360	465	90

Table 2: Relevant Cost and Benefit of Framing Crops

	Yield (kg/m^2)	Sewing Cost (£/m^2)	Selling Price (£/kg)
Grain	3.0	15	4.5
Beans	2.0	10	6.0
Wheat	3.5	12	5.0

Aside from growing crops, Farm-co could buy and raise cows on their farm. Each cow requires 5m^2 of space and could be purchased as a calf for £200. However, it is important to note that cows cannot be sold during the middle of the decade; they can only be sold at the end. The table below shows the selling price for cows of different ages.

Table 3: Selling Price for Cows of Different Ages

Age of Cow	1	2	3	4	5	6	7	8	9	10
Selling Price (£/cow)	350	450	550	600	700	950	1100	1250	1400	1700

Additionally, a cow would consume 15kg of grain each year. The grain used for feeding cows could be obtained by growing and harvesting from the previous year, purchasing in the current year, or both methods could be used. The cost of purchasing grain is £50 per kg. Furthermore, to mitigate risk, no single activity should occupy more than half of the total land area.

3 Linear Programming

To represent the problem in a linear programming form, we need to define some variables.

Let

- $A = \{1, 2, 3, 4\}$ be the activity: planting grain, beans and wheat and rearing cows respectively
- $T = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$ be the year
- $B = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$ be the block of land
- $x_{t,b,a}$ be a binary indicator of whether land b is doing activity a in year t or not
- $y_{t,b}$ be the number of cows on land b in the year t
- g_t be the weight of grain through planting for feeding cows in year t
- h_t be the weight of grain through purchasing for feeding cows in year t
- s_b be the size of land b
- w_a be the yield of activity a
- u_a be the sewing cost of activity a
- v_a be the selling price of activity a
- M_b be the sufficient large number of how many cows could be kept in land b
- p be the area of a cow could take up
- q be the weight of grain that a cow would consume for a year
- r be the cost of purchasing a calf
- n be the purchasing price of grain per kg
- d be the maximum percentage of land usage of a single activity
- f_t be the difference of selling price of a cow comparing to a one-year younger cow in an inverse order of year t

In this case,

$$s = [100,\ 250,\ 500,\ 125,\ 800,\ 75,\ 900,\ 360,\ 465,\ 90], \quad w = [3,\ 2,\ 3.5,\ 0], \quad p = 5, \quad r = 200,$$
$$M = [20,\ 50,\ 100,\ 25,\ 160,\ 15,\ 180,\ 72,\ 93,\ 18], \quad u = [15,\ 10,\ 12,\ 0], \quad q = 15, \quad n = 50,$$
$$f = [300,\ 150,\ 150,\ 150,\ 250,\ 100,\ 50,\ 100,\ 100,\ 350], \quad v = [4.5,\ 6,\ 5,\ 0], \quad d = 0.5.$$

Note that M_b is a sufficiently large number that it does not limit the maximal number of how many cows could be kept in land b. The value comes from the land size dividing by the area that a single cow could take.

How to compute f? We first assume that a cow of zero age sells £0. The parameter f is essentially a list contains the difference between Selling Price$_t$ and Selling Price$_{t-1}$ displayed in the following table in an inverse order of t.

Table 4: Parameter f

Age of Cow (t)	0	1	2	3	4	5	6	7	8	9	10
Selling Price (£/cow)	0	350	450	550	600	700	950	1100	1250	1400	1700
Selling Price$_t$ - Selling Price$_{t-1}$	–	350	100	100	50	100	250	150	150	150	300

The linear programming model illustrates the objective function and all the constraints. It aims to maximise the profit of Farm-co.

$$\max \sum_{t\in T}\left(\sum_{a\in A}\left(\sum_{b\in B} x_{t,b,a}s_b\right)(w_a\ v_a - u_a)\right) + \sum_{t\in T}\left(f_t\sum_{b\in B} y_{t,b}\right) - n\sum_{t\in T} h_t - v_1\sum_{t\in T} g_t - r\sum_{b\in B} y_{10,b}$$

subject to

$$\begin{aligned}
\sum_{a\in A} x_{t,b,a} &= 1, && \forall t\in T,\ \forall b\in B\\
\sum_{b\in B}(y_{t,b} - y_{t-1,b}) &\geq 0, && \forall t\in T\backslash\{1\}\\
y_{t,b} &\geq 0.1 - M_b(1 - x_{t,b,4}), && \forall t\in T,\ \forall b\in B\\
y_{t,b} &\leq 0.1 + M_b x_{t,b,4}, && \forall t\in T,\ \forall b\in B\\
q\sum_{b\in B} y_{1,b} &= h_1,\\
q\sum_{b\in B}(y_{t,b}) &= g_t + h_t, && \forall t\in T\backslash\{1\}\\
w_1\sum_{b\in B}(x_{t,b,1}s_b) &\geq g_{t+1}, && \forall t\in T\backslash\{10\}\\
\sum_{b\in B} x_{t,b,a}s_b &\leq d\sum_{b\in B} s_b, && \forall t\in T,\ \forall a\in A\\
x_{t,b,a},\ y_{t,b},\ g_t,\ h_t &\geq 0.
\end{aligned}$$

In the objective function, the first term, $\sum_{t\in T}\left(\sum_{a\in A}\left(\sum_{b\in B} x_{t,b,a}s_b\right)(w_a\ v_a - u_a)\right)$, is the total revenue of selling all the crops that grow in the decade and the second term, $\sum_{t\in T}\left(f_t\sum_{b\in B} y_{t,b}\right)$, is the benefit of selling all the cows at the end of the final year. The cost involves three components: $n\sum_{t\in T} h_t$ is the cost of buying grain for cows; in the first term, we assume all the crops are sold but in fact some grain are left for feeding the cows in the next year so we have to reduce the benefit of selling grain by $v_1\sum_{t\in T} g_t$ term; finally, $r\sum_{b\in B} y_{10,b}$ is the total cost of buying calves.

4 Results

Having solved the problem by linear programming, we now discuss the numerical results we obtained from the model. The ideal farming operation plan achieves a profit of £211797.5 from selling all crops and livestock over the decade.

First, we outline the operating plans. In the first year, mainly grains and wheat are planted, but from the second year onwards, this plan suggests reducing the amount of wheat cultivated and focusing on raising cattle. Although the wheat yield varies yearly, it is cultivated in all years. Beans are planted only in the first and final year. Cultivation of grain, which is also used as feed for livestock, will be the Farm-co's main activity throughout the period except the final year.

Table 5: Land Allocation Plan

	Year 1	Year 2 to 8	Year 9	Year 10
Block 1	wheat	grain	grain	cow (20)
Block 2	grain	grain	grain	cow (50)
Block 3	grain	grain	grain	wheat
Block 4	grain	cow (25)	cow (25)	cow (25)
Block 5	grain	cow (160)	cow (160)	wheat
Block 6	grain	wheat	grain	beans
Block 7	wheat	grain	grain	cow (180)
Block 8	wheat	cow (72)	cow (72)	cow (72)
Block 9	wheat	cow (93)	cow (93)	wheat
Block 10	beans	wheat	wheat	cow (18)
Total Cows	0	350	350	365

Table 6: Yearly Total Land Use and Yield for Individual Activity

	Year 1	Year 2 to 8	Year 9	Year 10
Planting Grain - Total Yield	5250 kg	5250 kg	5475 kg	0 kg
Planting Grain - Land Use	47.75%	47.75%	49.80%	0.00%
Planting Beans - Total Yield	180 kg	0 kg	0 kg	149.99 kg
Planting Beans - Land Use	2.46%	0.00%	0.00%	2.05%
Planting Bheat - Total Yield	6387.5 kg	577.5 kg	315 kg	6177.5 kg
Planting Wheat - Land Use	49.80%	4.50%	2.46%	48.16%
Rearing Cows - Total Cows	0 cows	350 cows	350 cows	365 cows
Rearing Cows - Land Use	0%	47.75%	47.75%	49.80%
Grain Planted in Last Year	0 kg	5250 kg	5250 kg	5475 kg
Grain Purchased in Current year	0 kg	0 kg	0 kg	0 kg

We next explore the details. In the first year, calves are not purchased because the farm does not yet have grain to feed the cows, and it would also not be profitable to purchase grain as feed. However, from the second year onwards, the activity of rearing cattle which is the most profitable would be maximised. Farm-co could purchase 350 calves and start keeping them using 47.75% of the entire farmland. As the cows can only be sold in the final year, the number of cows would not be reduced. The farm would continue to raise 350 cows until year 9, and purchase 15 more calves in year 10, bringing the total number of cows to 375.

When it comes to crop yield, the plan would maximise wheat cultivation in the first year, which is the second most profitable activity after rearing cows, by allocating 49.80% of the entire farmland and harvesting 6387.5 kg. Regarding the grain yield, 5250 kg would be cultivated every year from year 1 to year 8. In year 9, 49.80% of the entire farmland is used to maximise cultivation yielding 5475 kg, but in the final year, no grain would be grown. This is because all cattle would be sold in the final year. Therefore, the farm will return to wheat planting as their main activity in the final year instead of grain cultivation. Harvesting 6177.5 kg using 48.16% of the total land.

Since planting beans is less profitable than the other three activities, it is only cultivated in the first and tenth years, resulting also in very low land use and yield. Finally, as we can see in the table above, purchasing grain

for cattle feed would never be done even once over the decade because it is less cost-effective than producing it by the farm itself.

5 Summary

In a nutshell, choosing this farming plan would result in a profit of £211797.5. In the first year, the farm would produce 5250 kg of grain to feed cows, which would be bought in the second year, as well as 180 kg of beans and 6387.5 kg of wheat. In the second year, the farm would purchase 350 calves. The farming plan would then shift to growing 5250 kg of grain to feed the cows and 577.5 kg of wheat. This pattern would be repeated until year 8, with the yield of grain increasing to 5475 kg and the yield of wheat decreasing to 315 kg in year 9. In the final year, the farm would harvest 6177.5 kg of wheat and 150 kg of beans, along with purchasing 15 more calves and selling all 365 cows at the end of the year.