SmeltCo Ltd's Monthly Production Plan

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Abstract

In this report, a production plan is provided for SmeltCo Ltd. to maximise the profit. We proposed a solution using a linear programming model to address the blending problem, mixing two raw materials at a specific ratio. The production capacity limits, raw material proportion requirements, and cost of raw materials were taken into account in the model. The maximum profit achievable is £975555.56. However, if negotiations with the supplier are successful and additional aluminium and copper can be obtained, a different production plan could increase the profit to £1193333.32.

Introduction

Metallurgical companies' profits are directly impacted by the cost of raw materials and consumption rates. As a result, it is critical to find the ratio of differnet raw materials and production strategies to maximise the revenues. Such a challenge can broadly be defined as a blending problem concerning mixing the ingredients to obtain a product with a certain proportion. Linear programming is a powerful tool in this scenario, providing an efficient approach. This report focuses on the blending problem faced by SmeltCo Ltd., producing brass, bronze and duralumin, and aims to optimise its monthly production plan for maximum profitability. Constraints such as production capacity, raw material quantities and proportion requirements add an extra layer of complexity to modelling. This report begins by introducing the problem and defining modelling variables. It then describes the linear programming model to solve the optimisation problem, followed by a discussion of the numerical results and sensitivity analysis.

Problem Description and Modelling

Blending Problem

SmeltCo. Ltd has specific requirements for the raw materials used in producing brass, bronze, and duralumin. For brass, a blend of copper and zinc must contain at least 20% copper and no more than 50%. Bronze must have a copper proportion of 60 to 80% combined with tin. Duralumin requires a minimum of 45% aluminium and at least 10% copper but no more than 50%. We refer to copper as "Material 1" for every product and zinc, tin, and aluminium as "Material 2" for brass, bronze, and duralumin respectively.

It is worth mentioning that although there is a restriction on the proportion of aluminium when producing duralumin, this requirement will be satisfied as long as we fulfil the condition of the proportion of copper. The reason is that if duralumin contains 10 to 50% copper, it is equivalent to containing 50 to 90% aluminium. Thus, we do not include the aluminium ratio constraint while modelling.

While determining production, it is important to consider the amount and cost of raw materials, production capacity and profits. SmeltCo. Ltd purchases 50,000 kg of aluminium, 80,000 kg of copper, 75,000 kg of tin, and 100,000 kg of zinc monthly, priced at £8, £12, £2, and £5 per kg respectively. Because of production limitations, the factory can only produce up to 70,000 kg of brass, 100,000 kg of bronze, and 90,000 kg of duralumin and during the blending process, there is a 10% loss of raw materials. In terms of sell prices, the company can sell brass for £8 per kg, bronze for £16 per kg and £14 per kg and expect to sell everything they produce.

Linear Programming

To represent the problem in a linear programming form, we need to define some variables.

Let

- x_1 be the amount of copper used in producing brass
- x_2 be the amount of copper used in producing bronze
- x_3 be the amount of copper used in producing duralumin
- y_1 be the amount of zinc used in producing brass
- y_2 be the amount of tin used in producing bronze
- y_3 be the amount of aluminium used in producing duralumin.

First, we consider the ratio requirement. The proportion of copper can be expressed in the following form

$$\frac{x_i}{x_i + y_i}$$

where $i = 1, 2, 3$ present brass, bronze and duralumin respectively and we assume the production of each product is greater than zero kg.

Thus, for example the minimal ratio constraint of copper in brass can be write as

$$\frac{x_1}{x_1 + y_1} \geq 0.2$$

and the maximal ratio constraint of copper in brass is

$$\frac{x_1}{x_1 + y_1} \leq 0.5.$$

Similar arguments can be applied to copper proportion limitations in the other two products.

Second, the constraint of maximal quantity of copper we can use is

$$x_1 + x_2 + x_3 \leq 80000.$$

We are also restricted by the maximum amount of zinc, tin and aluminium.

$$y_1 \leq 100000, \quad y_2 \leq 75000, \quad y_3 \leq 50000.$$

Third, regarding to production capacity, we multiply 0.9 which presents 10% loss by sum of two raw material uses and this should not exceed the maximum production of the product. Taking brass as an example, we have the inequality

$$0.9(x_1 + y_1) \leq 70000.$$

Finally, our objective function would be

$$0.9 \times \text{sum of sell price times production for three products} - \text{cost of raw materials for three products}$$

where 0.9 again presents 10% loss of raw materials.

The linear programming model illustrates the objective function and all the constraints. It aims to maximise the profit of SmeltCo Ltd.

$$\max \quad 0.9(8(x_1 + y_1) + 16(x_2 + y_2) + 14(x_3 + y_3)) - (12(x_1 + x_2 + x_3) + 5y_1 + 2y_2 + 8y_3)$$

subject to

$$
\begin{aligned}
x_1 + x_2 + x_3 &\le 80000, & & \\
0.9(x_1 + y_1) &\le 70000, & y_1 &\le 100000, \\
0.9(x_2 + y_2) &\le 100000, & y_2 &\le 75000, \\
0.9(x_3 + y_3) &\le 90000, & y_3 &\le 50000, \\
x_1/(x_1 + y_1) &\le 0.5, & x_1/(x_1 + y_1) &\ge 0.2, \\
x_2/(x_2 + y_2) &\le 0.8, & x_2/(x_2 + y_2) &\ge 0.6, \\
x_3/(x_3 + y_3) &\le 0.5, & x_3/(x_3 + y_3) &\ge 0.1, \\
x_1,\ x_2,\ x_3,\ y_1,\ y_2,\ y_3 &\ge 0 & &
\end{aligned}
$$

Results

Having solved the blending problem by linear programming, we now discuss the numerical results we obtained from the model. The ideal production includes 38888.89 kg of brass, 111111.1 kg of bronze and 55555.56 kg of duralumin. However, due to a 10% loss of raw materials, the actual production becomes 35000 kg of brass, 100000 kg of bronze, and 50000 kg of duralumin. The total profit from selling all of these products is £975555.56. Note that we rounded up all float values to two decimal places in the report.

Next, we explore the composition of each product. To produce 35000 kg of brass, we will need 7777.78 kg of copper and 31111.11 kg of zinc. The ratio of copper in this product is 20%. For 100000 kg of bronze, which consists of 60% copper, it will consume 66666.67 kg of copper and 44444.44 kg of tin. Lastly, blending 5555.56 kg of copper and 50000 kg of aluminium will obtain 50000 kg of duralumin after considering the loss and the product comprises 10% of copper.

Table 1: Case 1. Production Plan for SmeltCo. Ltd (unit: kg)

Product	Actual Production	Copper Uses	Material 2	Material 2 Uses	Ratio of Copper
Brass	35000.00	7777.78	Zinc	31111.11	20%
Bronze	100000.00	66666.67	Tin	44444.44	60%
Duralumin	50000.00	5555.56	Aluminium	50000.00	10%

Finally, we discuss the sensitivity analysis of the raw materials. If the shadow price of a material is greater than zero, it means that this material is fully used. The following table displays the shadow prices and dual range of each raw material. The interpretation of these numbers and dual range is as follows: for instance, increasing the maximum quantity of copper by one more kg could lead to a total profit increase of £4. However, if the maximum quantity of copper exceeds 87777.78 kg, it will no longer contribute to the profit. The same applies to aluminium. For zinc and tin, a one-unit increase of them will not change the optimal profit because they are not fully used. On the other hand, since the shadow prices of copper and aluminium are greater than zero, if we have the opportunity to negotiate with the raw material supplier, we would request more copper and aluminium.

Table 2: Shadow Price for Case 1

Material	Shadow Price (£)	Dual Range	Estimated New Total Profit (£)
Zinc	0.00	–	–
Tin	0.00	–	–
Aluminium	4.22	0 - 90000.00	1144444.44
Copper	4.00	72222.22 - 87777.78	1006666.67

Discussion

Discussion of Sensitivity Analysis

For further discussion, we aim to provide SemltCo. Ltd two more different cases based on the results of the sensitivity analysis we acquired in the Case 1 as we have non-zero values in the shadow price for aluminium and copper. This means that we could consider using the upper bound of the dual range for each material to calculate a new optimal value, i.e. a new total profit. To do so, we could set that value as a new maximum quantity for the corresponding material. For example, if SmeltCo ltd would raise the maximum quantity for aluminium up to 90000 kg and fixed others, the profit would increase to £1144444.44. Likewise, increasing copper up to 87777.78 kg enables the company to make total profit of £1006666.67.

Since the new estimated profit by increasing the maximum quantity of aluminium is larger than increasing copper, we employ the upper bound of the dual range for aluminium, namely, 90000 kg, and fixed others to compute the new production plan referred to Case 2, which obtains the profit of £1144444.44. The result of Case 2 as well as its sensitivity analysis can be found in the table below.

Table 3: Case 2. Production Plan for SmeltCo. Ltd (unit: kg)

Product	Actual Production	Copper Uses	Material 2	Material 2 Uses	Ratio of Copper
Brass	15000.00	3333.33	Zinc	13333.33	20%
Bronze	100000.00	66666.67	Tin	44444.44	60%
Duralumin	90000.00	10000.00	Aluminium	90000.00	10%

Table 4: Shadow Price for Case 2

Material	Shadow Price (£)	Dual Range	Estimated New Total Profit (£)
Zinc	0.00	–	–
Tin	0.00	–	–
Aluminium	4.22	0 - 90000.00	1144444.44
Copper	4.00	76666.67 - 92222.22	1193333.33

In the same fashion, we continue another calculation for Case 3 using the upper bound of the dual range for cooper, specifically 92222.22 kg, based on the sensitivity analysis of Case 2. The profit of Case 3 achievable is £1193333.33. The production plan for Case 3 is displayed as below.

Table 5: Case 3. Production Plan for SmeltCo. Ltd (unit: kg)

Product	Actual Production	Copper Uses	Material 2	Material 2 Uses	Ratio of Copper
Brass	70000.00	15555.56	Zinc	62222.22	20%
Bronze	100000.00	66666.67	Tin	44444.44	60%
Duralumin	90000.00	10000.00	Aluminium	90000.00	10%

Table 6: Shadow Price for Case 3

Material	Shadow Price (£)	Dual Range	Estimated New Total Profit (£)
Zinc	0.00	–	–
Tin	0.00	–	–
Aluminium	4.00	89999.99 - 90000.00	1193333.33

Material	Shadow Price (£)	Dual Range	Estimated New Total Profit (£)
Copper	0.00	–	–

As we can see from the new production plan for the Case 3, the new profit increases to £1193333.33. Now, non-zero value in shadow price can be seen only for aluminium. However, the upper bound of the dual range for aluminium is as the same value as the amount we set in the maximum quantity in the Case 1. It means that this is the end of the sensitivity analysis for this blending problem and there is no more improvement in the maximum quantity for any material nor the total profit. Therefore, the total profit at £1193333.33 is the final optimal value for this blending problem when SmeltCo. Ltd considers additional procurement.

Supplementary Explanation

As a supplementary explanation before ending up the discussion, we would like to mention about the maximum quantity limit for zinc and tin. The limits were given initially at 100000 kg and 75000 kg respectively. We assumed that SmeltCo ltd pays the cost to supplier after they actually used for the amount of consumption.

Even when we have the maximised total profit as the final optimal value by increasing the maximum quantities for aluminium and copper, the amount of actual consumption for zinc is only 62222.22 kg, using 62.22% of the initial quantity. Likewise, one for tin is only 44444.44 kg, namely 59.26% of maximum quantity.

According to the results of our supplemental computation for zinc and tin by using the same model, even if we let the limit for zin and tin decreased until the same rate as the actual consumption for those two materials, the profit did not show any improvements. Thus, we would recommend that SmeltCo. Ltd purchases less amount of zinc and tin as much as the estimated amount of actual consumption not for the maximum quantity, only in case that they ever must pay for the cost in advance, before their actual production process.

Proposal for SmeltCo. Ltd

To conclude this discussion, we would like to propose that SmeltCo ltd procures alminium and cooper more and negotiate with their supplier to increase the maximum quantity for these two materials up to 90000 kg and 92222.23 kg respectively. In case that it is not feasible for them to afford the additional cost for all the increase, we still encourage them to purchase these two materials even with the slight increase in amount. In this case, we would also highlight for them to put more priority on aluminium rather than copper, not the other way around.

Summary

To sum up, SmeltCo Ltd. can achieve the highest profit of £975555.56 by producing 35000 kg of brass, 100000 kg of bronze, and 50000 kg of duralumin, with copper proportions of 20%, 60%, and 10% respectively. If negotiations with the supplier are successful, increasing the maximum quantities of copper and aluminium to 92222.22 and 90000 kg would result in a higher profit of £1193333.33.