Producti-Co's Weekly Production Plan

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Abstract

This report presents worker schedules and production plans for Producti-Co to maximise profits. Our solution uses a linear programming model to address the scheduling and production planning problem, considering worker schedules, union requirements, factory capacity, and production cost. The maximum achievable profit is £3508090. If a new machine claims to decrease the production time of product 7 to 5.5 hours, Producti-Co should not pay any money for the machine because it is not profitable.

1 Introduction

Proper scheduling can help optimise various aspects of business from employment cost to production plans. To produce products, there are many different purposes for utilising labour with different sets of constraints. This report focuses on the scheduling and production plan problem faced by Producti-Co, which produces seven different products and aims to optimise its weekly production plan for maximum profitability. Determining how many workers would be hired each day, satisfying the union requirements and limitation of the maximal number of workers on the production line are the constraints regarding scheduling. Also, production plan constraints include the extra cost of producing a certain product and additional man-hours under some conditions. This report begins by introducing the problem and defining modelling variables. It then describes the linear programming model to solve the optimisation problem and discusses the numerical results.

2 Problem Description and Modelling

2.1 Scheduling and Production Plan Problem

Producti-Co could produce seven different products, labelling one to seven, and the corresponding sale price and production time are displayed in the following table.

Table 1: Product with Its Sale Price and Production Time

Product	1	2	3	4	5	6	7
Sale Price (£)	100	420	350	490	550	100	1115
Production time (hours)	1.0	2.0	2.7	2.4	4.5	0.7	9.5

There is an additional £2000 cost if any unit of product 7 is produced. Regarding product 2, an extra one man-hour would be consumed when the production exceeds 100 units. In terms of product 3 and product 4, if these two products are produced on the same day, it requires 75 labour hours to set up the production line.

Besides, there are two types of working type: full-time and part-time. It costs £80 and £35 per hour to hire full-time and part-time workers. A part-time employee is expected to work for six hours per day while a full-time worker would work eight hours per day. Both of these two kinds of employee work for five constitutive days followed by two days for rest. The union has set a maximum limit of 25% of the total labour hours to be contributed by part-time workers. Moreover, the factory can only accommodate a maximum of 500 workers.

2.2 Linear Programming

To represent the problem in a linear programming form, we need to define some variables.

Let

- $T = \{1, 2, 3, 4, 5, 6, 7\}$ be Monday, Tuesday, Wednesday and so on
- $P = \{1, 2, 3, 4, 5, 6, 7\}$ be the product label
- $W = \{1, 2\}$ be the working type, 1 for full-time and 2 for part-time
- $x_{t,p}$ be the production of product p on day t
- $y_{t,w}$ be the number of worker starting working on day t with working type w
- s_p be the sale price of product p
- k_p be the production time of product p
- c_b be the cost of worker with working type w
- h_b be the working hour of working type w
- $\max_{\text{workers}}$ be the maximal number of workers on duty per day
- $\max_{\text{parttime}}$ be the maximal part-time working hour rate
- g_p be the fixed cost for producing product p
- e_p be the extra production time threshold of product p
- f_p be the man-hour increment of product p after producing over its extra production time threshold
- d be the extra labour hours while producing both product 3 and product 4
- $z_{t,p}$ be the binary indicator of whether product p is produced on day t or not
- l_t be the binary indicator of whether product 3 and product 4 are producing on the same day t or not
- $u_{t,p}$ be the production of product p below its extra production time threshold
- $v_{t,p}$ be the production of product p above its extra production time threshold
- M_p be the sufficient larger number than the largest possible production for product p.

In this case,

$$
\begin{aligned}
c &= [80, 35]\,, & s &= [100, 420, 350, 490, 550, 100, 1115]\,, & g &= [0, 0, 0, 0, 0, 0, 2000]\,, & \max_{\text{workers}} &= 500, \\
h &= [8, 6]\,, & k &= [1.0, 2.0, 2.7, 2.4, 4.5, 0.7, 0.95]\,, & e &= [0, 100, 0, 0, 0, 0, 0]\,, & \max_{\text{parttime}} &= 0.25, \\
d &= 75, & M &= [4000, 2000, 1482, 1667, 889, 5715, 422]\,, & f &= [0, 1, 0, 0, 0, 0, 0]\,.
\end{aligned}
$$

It is worthy to discuss the M which is sufficiently large that it does not limit the production of products. The number comes form the maximal production of single product under the condition of 500 full-time workers all devoting on producing that product.

The linear programming model illustrates the objective function and all the constraints. It aims to maximise the profit of Producti-Co.

$$
\max \sum_{t \in T} \sum_{p \in P} (x_{t,p} s_p - g_p z_{t,p}) - \sum_{t \in T} \sum_{w \in W} \sum_{i=0}^{4} y_{(6-t+i) \bmod 7+1,w} c_w h_w
$$

subject to

$$
\sum_{t \in T} \sum_{i=0}^{4} y_{(6+t-i) \bmod 7+1,2} h_2 \le \max_{\text{parttime}} \sum_{t \in T} \sum_{w \in W} \sum_{i=0}^{4} y_{(6+t-i) \bmod 7+1,w} h_w,
$$

$$\sum_{w \in W} \sum_{i=0}^{4} y_{(6+t-i)\text{mod}7+1,w} \leq \text{max}_{\text{workers}}, \qquad \forall t \in T$$

$$\sum_{p \in P} (x_{t,p} k_p + f_p v_{t,p}) + dl_t \leq \sum_{w \in W} \sum_{i=0}^{4} y_{(6+t-i)\text{mod}7+1,w} h_w, \qquad \forall t \in T$$

$$1 - x_{t,p} \leq M_p \left(1 - z_{t,p}\right), \qquad \forall p \in P, \ \forall t \in T$$

$$x_{t,p} \leq M_p z_{t,p}, \qquad \forall p \in P, \ \forall t \in T$$

$$l_t \leq z_{t,3}, \ l_t \leq z_{t,4}, \ l_t \geq z_{t,3} + z_{t,4} - 1, \qquad \forall t \in T$$

$$u_{t,p} + v_{t,p} = x_{t,p}, \qquad \forall p \in P, \ \forall t \in T$$

$$u_{t,p} \leq e_p \qquad \forall p \in P, \ \forall t \in T$$

Regarding $\sum_{i=0}^{4} y_{(6-t+i)\text{mod}7+1}$, this presents the total number of workers starting working form different days. For example, while $t = 1$ it is $y_1 + y_4 + y_5 + y_6 + y_7$.

3 Results

Having solved the problem by linear programming, we now discuss the numerical results we obtained from the model. The ideal weekly production includes 697 units of product 2 and 10190 units of product 4. In total, 485 full-time workers and 215 part-time workers are hired. The total profit from selling all of these products is £3508090.

Table 2: Production Plan and Schedule

	Mon	Tue	Wed	Thr	Fri	Sat	Sun	Total
Product 2 (units)	100	97	100	100	100	100	100	697
Product 4 (units)	1485	1415	1415	1410	1485	1490	1490	10190
FT Workers Start Working	92	5	98	92	95	98	5	485
PT Workers Start Working	8	95	2	8	5	2	95	215
Workers on Duty	500	500	500	500	500	500	500	–

As we can see form the table, the production plan suggests only producing product 2 and product 4. The reason is that these two products have the first and second highest sale price-production time ratio (SP ratio). Regrading product 2, the benefit gained is £210 for every hour of production cost. However, while the production for product 2 exceeds 100 units, the ratio becomes 140 £/hour. In terms of product 4, the ratio is 204.17 £/hour. On the other hand, the other products have the ratios somewhere around 100 to 142.86 £/hour.

Table 3: Sale Price/Production time Ratio (£/hour)

Product	1	2	3	4	5	6	7
SP ratio	100	210 (140)	129.63	204.17	122.22	142.86	117.37

4 Discussion

4.1 Discussion of What-If Analysis

For further discussion, we aim to provide Producti-Co with nine conceivable cases to improve their business.

First, we will analyse the given scenario of purchasing a new machine to produce product 7 in 5.5 hours. The following tables present other business cases for weekly production plans with the different combinations of sale prices and production times. Here is a scenario for product 7 and it only shows the changing conditions from the original scenario in Case 1 while fixed all other factors.

Table 4: Scenarios for Cases 1 to 5

	P7 Production Time (hour)	P7 Sale Price (£)
Case 1	9.50	1,115
Case 2	5.50	1,115
Case 3	5.44	1,115
Case 4	5.50	1,126
Case 5	5.50	1,127

Table 5: Results of Cases 1 to 5 - Production Plans

	Profit (£)	Growth (%)	P1	P2	P3	P4	P5	P6	P7	FT Workers	PT Workers
Case 1	3,508,090	0.00	0	697	0	10190	0	0	0	485	215
Case 2	3,508,090	0.00	0	697	0	10190	0	0	0	485	215
Case 3	3,513,570	0.16	0	699	0	11	0	0	4490	485	215
Case 4	3,508,232	0.00	0	700	0	7135	0	0	1332	485	215
Case 5	3,512,255	0.12	0	700	0	1	0	0	4445	485	215

In Case 2, the production time of product 7 decreases by 5.5 hours, but the result does not show any improvement since it is still not profitable to produce this product. Even though the SP ratio for product 7 increases to 202.7 £/hour, it would not be a sensible option yet to produce because of the additional fixed cost of £2,000.

To make product 7 profitable, Producti-Co would need to negotiate with the manufacturer to develop a higher efficient machine with a production time of 5.44 hours as shown in Case 3. If this is not feasible, and they can only obtain the 5.5-hour machine, we recommend that they consider raising the sale price of product 7 to at least £1,126 to make it profitable like in Case 4, instead of the scenario in Case 3. However, this may not generate enough profit to pay back for purchasing a new machine, unless the machine has a considerably lower price. Therefore, we have a suggestion here in Case 5. If the sale price is further raised to £1.127, Case 5's increase in profit over Case 4 might allow sufficient margin to purchase the new machine. This would make Case 5 comparable enough to Case 3 as the table below.

Table 6: Estimated Purchase Price of the New Machine

	Profit(£)	Growth(%)	Increment(£)	26 weeks	52 weeks	156 weeks
Case 1	3,508,090	-	-	-	-	-
Case 3	3,513,570	0.16	5,480	142,480	284,960	854,880
Case 4	3,508,232	0.00	142	3,692	7,384	22,152
Case 5	3,512,255	0.12	4,165	108,290	216,580	649,740

Additionally, we suggest that Producti-Co selects new machines taking into account the general market price for comparable equipment, as well as their preferred payback period. For their reference, we have provided three examples under the assumption that the payback period is between half a year and three years. In these cases, the reasonable price range for the new machine would be between £108,290 - £854,880.

Secondly, we would like to discuss four possible scenarios with other products to bring Producti-Co more profits. As shown in Table 3, the product 6 has the third highest sale price-production time ratio.

Table 7: Scenarios for Cases 6 to 9

	P5 Production Time (hour)	P6 Production Time (hour)	P6 Sale Price (£)
Case 6	4.5	0.70	143
Case 7	4.5	0.70	148
Case 8	4.5	0.45	100
Case 9	2.6	0.70	100

Table 8: Results of Cases 6 to 9 - Production Plans

	Profit(£)	Growth(%)	P1	P2	P3	P4	P5	P6	P7	FT Workers	PT Workers
Case 6	3,511,015	0.1	0	700	0	1	0	34925	0	485	215
Case 7	3,687,650	5.1	0	10	0	0	0	36900	0	485	215
Case 8	3,981,980	13.5	0	4	0	0	0	62160	0	698	2
Case 9	3,690,510	5.2	0	3	0	0	9940	0	0	485	215

What kind of situations on product 6 can make any improvement in their profits? We could first consider the case of a price up. Since product 2 and product 4 have higher sale price-production time ratios, those two are produced generally. However, if Product-Co is able to raise the price of product 6 to about £143 in Case 6, it would be the great timing of switching the main production from product 4 to product 6 while product 2 remains with the similar production. Likewise, if they decide to sell product 6 at £148, we would highly recommend that they switch almost the entire production to focus on product 6 only, enabling Producti-Co to acquire more than 5% in growth shown in Case 7.

Furthermore, they will potentially earn the highest profit at £3,981,980 which would be a growth rate of 13.5% if they ever find any way of reducing their production time of product 6 from 0.7 hours to around 0.45 hours in Case 8. The rest of products, such as product 5 (Case 9) and product 1, are not beneficial enough to produce unless they could almost double those prices or reduce half their production time which are most likely infeasible in the actual business.

4.2 Proposal for Producti-Co

To conclude our discussion, we would like to propose that Producti-Co find and purchases the higher quality machine to produce product 7 in 5.44 hours as 5.5 hours is not cost-effective enough to be profitable. In case that any machine with 5.44 hours is unavailable but the one with 5.5 hours exist, it is worthwhile that they consider a price up to £1,127 additionally. Otherwise, they will not make any profits by purchasing the new machine. Finally, if Producti-Co would like to switch their main production from product 2, 4, or 7 for some reasons such as the change of their circumstances and the public trends change, we would recommend product 6 rather than product 5 or product 1.

5 Summary

To summarise, the optimal production plan for Producti-Co involves producing 697 units of product 2 and 10,190 units of product 4, while employing 485 full-time workers and 215 part-time workers, resulting in the highest profit of £3,508,090. If a new machine is purchased to reduce the production time of product 7 to 5.5 hours, it would not change the profit because the production plan remains the same. Therefore, Producti-Co should not invest a new machine in this case.