Production planning with contract decisions

Tom Holt

July 28, 2022

1 Contract decisions (first stage decisions)

2 Production planning model (second stage decisions)

Sets

M	set of paper mills
PM	set of paper machines
P	set of paper products
R	set of raw materials
E	set of energy requirements (essentially just variable cost components)
E_i	set of energy requirements needed by product i ($E_i \subset E$)
C	set of customers
T	set of time periods

Parameters

P_{rt}	$t \in T, r \in R$	price of raw material r purchased at time t.
α_{ir}	$i \in P, r \in R$	conversion factor to convert 1 ton of product i into required raw material r.
D_{itc}	$i \in P, t \in T, c \in C$	demand in time period t for product i from customer c
F_p	$p \in PM$	fixed cost operating paper machine p.
S_p	$p \in PM$	shutdown fixed costs for shutting down paper machine p.
PC_p	$p \in PM$	monthly production capacity of paper machine p.
L_{cp}	$c \in C, m \in M$	logistics costs for transporting products from pm p to customer c
SC_m	$m \in M$	storage costs per ton of paper stored at mill m
PR_{it}	$i \in P, t \in T$	price per ton of product i at time t
EC_{iept}	$i \in P, e \in E, m \in M$	cost of energy type e at pm p per one ton of product i.

Decision variables

Integer decision variables

y_p	$p \in PM$	paper machine p is running

Cont. decision variables

x_{iptc}	$i \in P, p \in PM, t \in T$	tons of paper product i produced by paper machine p in time t for customer c.
I_{imtc}	$i \in P, m \in M, t \in T$	tons of paper product i in storage at mill m at time t for customer c.
R_{tr}	$t \in T, r \in R$	tons of raw material r purchased at time t.
RI_{trm}	$t \in T, r \in R, m \in M,$	raw material inventory of material r at time t at mill m.

Objective function

$$\text{PROFIT} = \text{SALES} - \text{ICOSTS} - \text{RCOST} - \text{LCOST} - \text{ECOST} \\ - \sum_{p \in PM} \sum_{t \in T} F_{pt} y_p - \sum_{p \in PM} S_p (1 - y_p)$$

where

$$\begin{aligned}
\text{SALES} &= \sum_{i \in P} \sum_{t \in T} \sum_{p \in PM} PR_{it} x_{ipt} \\
\text{RCOST} &= \sum_{r \in R} \sum_{t \in T} P_{rt} R_{rt} \\
\text{ICOST} &= \sum_{m \in M} \sum_{t \in T} SC_m (\sum_{i \in P} \sum_{c \in C} I_{imtc} + \sum_{r \in R} RI_{trm}) \\
\text{LCOST} &= \sum_{t \in T} \sum_{p \in PM} \sum_{c \in C} \sum_{i \in P} L_{cp} x_{iptc} \\
\text{ECOST} &= \sum_{c \in C} \sum_{i \in P} \sum_{t \in T} \sum_{p \in PM} (\sum_{e \in E_i} EC_{iept}) x_{iptc}
\end{aligned}$$

and the last two terms account for fixed monthly operating costs of running machines as well as the single fixed shutdown cost.

Constraints

(i) Demand satisfaction:

$$\sum_{p \in PM} x_{iptc} + \sum_{m \in M} I_{im(t-1)c} = D_{itc} + \sum_{m \in M} I_{imtc} \quad (\forall t \in T, \forall i \in P, \forall c \in C)$$

(ii) Raw material balance:

$$R_{tr} + \sum_{m \in M} RI_{(t-1)rm} = \sum_{c \in C} \sum_{p \in PM} \alpha_{ir} x_{iptc} + \sum_{m \in M} RI_{trm} \quad (\forall t \in T, \forall i \in P, \forall r \in R)$$

(iii) Bounded inventory

$$\sum_{i \in P} \sum_{c \in C} I_{imtc} + \sum_{r \in R} RI_{trm} \leq I_m^U \quad (\forall m \in M, \forall t \in T)$$

(iv) Bounded capacity

$$\sum_{i \in P} x_{ipt} \leq PC_p y_p \quad (\forall t \in T, p \in PM)$$

(v) Non-negativity: all continuous decision variables are non-negative.