Production planning with contract decisions

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1 Production planning model (second stage decisions)

Sets

M	set of paper mills
PM_m	set of paper machines at mill m
PM	set of paper machines
P	set of paper products
R	set of raw materials
E	set of energy requirements (essentially just variable cost components)
RW_i	set of raw materials needed to produce product i ($RW \subset R$)
E_i	set of energy requirements needed by product i ($E_i \subset E$)
C	set of customers
T	set of time periods

Parameters

D_{itc}	$i \in P, t \in T, c \in C$	demand in time period t for product i from customer c
F_p	$p \in PM$	fixed cost operating paper machine p.
S_p	$p \in PM$	shutdown fixed costs for shutting down paper machine p.
PC_p	$p \in PM$	monthly production capacity of paper machine p.
L_{cm}	$c \in C, m \in M$	logistics costs for transporting products from mill m to customer c
SC_m	$m \in M$	storage costs per ton of paper stored at mill m
PR_{it}	$i \in P, t \in T$	price per ton of product i at time t
WC_{irpt}	$i \in P, r \in R, p \in PM, t \in T$	cost of raw material r at pm p per one ton of product i produced at time t.
EC_{irpt}	$i \in P, e \in E, m \in M$	cost of energy type e at pm p per one ton of product i.

Decision variables

Integer decision variables

y_p	$p \in PM$	paper machine p is running

Cont. decision variables

x_{iptc}	$i \in P, p \in PM, t \in T$	tons of paper product i produced by paper machine p in time t for customer c.
I_{imtc}	$i \in P, m \in M, t \in T$	tons of paper product i in storage at mill m at time t for customer c.

Derived expressions

SALES_{it}	$i \in P, t \in T$	sales (revenue) of product i in time period t
PCOST_{itpc}	$i \in P, t \in T, p \in M$	raw material costs for producing product i in time period t at pm p for customer c.
ICOST_{mt}	$m \in M$	inventory storage costs for mill m in time period t
LCOST_{mt}	$m \in M, t \in T$	logistics costs for mill m in time period t
ECOST_{itpc}	$i \in P, p \in PM, t \in T$	energy costs for producing product i in time period t at pm p for customer c.

Objective function

$$\mathrm{PROFIT} = \sum_{i \in P} \sum_{t \in T} \mathrm{SALES}_{it} - \sum_{c \in C} \sum_{i \in P} \sum_{t \in T} \sum_{p \in PM} \mathrm{PCOST}_{itpc} - \sum_{m \in M} \sum_{t \in T} \mathrm{ICOSTS}_{mt}$$
$$- \sum_{m \in M} \sum_{t \in T} \mathrm{LCOST}_{mt} - \sum_{c \in C} \sum_{i \in P} \sum_{t \in T} \sum_{p \in PM} \mathrm{ECOST}_{itpc} - \sum_{p \in PM} \sum_{t \in T} F_{pt} y_p - \sum_{p \in PM} S_p(1 - y_p)$$

where

$$\mathrm{SALES}_{it} = \sum_{p \in PM} PR_{it} x_{ipt}$$
$$\mathrm{PCOST}_{itpc} = (\sum_{r \in RW_i} WC_{irpt}) x_{iptc}$$
$$\mathrm{ICOST}_{mt} = \sum_{i \in P} \sum_{c \in C} SC_m I_{imtc}$$
$$\mathrm{LCOST}_{mt} = \sum_{p \in PM} \sum_{c \in C} \sum_{i \in P} L_{cm} x_{iptc}$$
$$\mathrm{ECOST}_{itpc} = (\sum_{r \in E_i} EC_{irpt}) x_{iptc}$$

and the last two terms account for fixed monthly operating costs of running machines as well as the single fixed shutdown cost.

Constraints

(i) Demand satisfaction:

$$\sum_{p \in PM} x_{iptc} + I_{im(t-1)c} = D_{itc} + I_{imtc} \quad (\forall t \in T, \forall i \in P, \forall c \in C)$$

(ii) Bounded inventory

$$\sum_{c \in C} \sum_{i \in P} I_{imtc} \leq I_m^U \quad (\forall m \in M, \forall t \in T)$$

(iii) Bounded capacity

$$\sum_{i \in P} x_{ipt} \leq PC_p y_p \quad (\forall t \in T, p \in PM)$$

(iv) Non-negativity

$$x_{ipt} \geq 0 \quad (\forall i \in P, \forall p \in PM, \forall t \in T)$$

$$u_{cmit} \geq 0 \quad (\forall c \in C, \forall m \in M, \forall i \in P, \forall t \in T)$$

$$I_{imt} \geq 0 \quad (\forall i \in P, \forall m \in M, \forall t \in T)$$

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