

## chapter 5 Performance Analysis

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# Regret Analysis

- A player chooses an action  $\theta^{(t)} \in K$  every  $t$  period, where  $K$  is a feasible set of actions.
- The cost function  $f^{(t)}$  determines the cost  $f^{(t)}(\theta^{(t)})$  for action  $\theta^{(t)}$ .
- The player decides his action based on the strategy.

# Regret Analysis

- How does the player choose an action which minimizes a total cost  $\sum f^{(t)}(\theta^{(t)})$ ?
- Can the cost function be minimized even if it is not unknown?
- We introduce a regret about the strategy.

## definition (Regret)

*The difference between the total cost of an action based on a strategy  $A$  and the total cost of the optimal strategy  $\theta^*$  is defined as the regret  $\text{Regret}(A)$  of strategy  $A$ .*

$$\text{Regret}(A) = \sum_{t=1}^T f^{(t)}(\theta^{(t)}) - \sum_{t=1}^T f^{(t)}(\theta^*)$$

# Regret Analysis

## Regret analysis in online learning

- Let action be the parameter of the online learner  $\theta^{(t)} \in \mathbb{R}^m$  given the training data  $(\mathbf{x}^{(t)}, y^{(t)})$ .
- Let the cost function be a loss function  $f^{(t)} = (\mathbf{x}^{(t)}, y^{(t)}, \theta)$ .
- In this case, the optimal strategy is the strategy that chooses the action that minimizes the cost function for all training data.