

chapter 5 Performance Analysis

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Regret Analysis

- A player chooses an action $\theta^{(t)} \in K$ every t period, where K is a feasible set of actions.
- The cost function $f^{(t)}$ determines the cost $f^{(t)}(\theta^{(t)})$ for action $\theta^{(t)}$.
- The player decides his action based on the strategy.

Regret Analysis

- How does the player choose an action which minimizes a total cost $\sum f^{(t)}(\theta^{(t)})$?
- Can the cost function be minimized even if it is not unknown?
- We introduce a regret about the strategy.

definition (Regret)

The difference between the total cost of an action based on a strategy A and the total cost of the optimal strategy θ^ is defined as the regret $\text{Regret}(A)$ of strategy A .*

$$\text{Regret}(A) = \sum_{t=1}^T f^{(t)}(\theta^{(t)}) - \sum_{t=1}^T f^{(t)}(\theta^*)$$

Regret Analysis

Regret analysis in online learning

- Let action be the parameter of the online learner $\theta^{(t)} \in \mathbb{R}^m$ given the training data $(\mathbf{x}^{(t)}, y^{(t)})$.
- Let the cost function be a loss function $f^{(t)} = (\mathbf{x}^{(t)}, y^{(t)}, \theta)$.
- In this case, the optimal strategy is the strategy that chooses the action that minimizes the cost function for all training data.

Follow the Leader

- At first, We consider strategy for choosing action that minimizes the total cost to date.

$$\boldsymbol{\theta}^{(t)} = \arg \min_{\boldsymbol{\theta} \in K} \sum_{i=1}^{t-1} f^{(i)}(\boldsymbol{\theta})$$

- This strategy is called Follow the Leader (FTL).