LEAST SQUARES AND SMOLYAK'S ALGORITHM

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ABSTRACT. We present novel, large-scale experiments for polynomial interpolation in high dimensional settings using some of the most popular algorithms available. We compare Smolyak's Algorithm (SA) on sparse grids and Least Squares (LS) using random data. We empirically confirm that interpolation using LS performs equally as good on smooth and better on non-smooth functions if SA is given n and LS is given $2 \cdot n$ points.

1. INTRODUCTION

Smolyak's algorithm on Sparse Grids has been of high theoretical and practical interest for a long time [2, 5, 13, 20]. In [15], it was recently found that Smolyak's algorithm is *not* optimal in the sampling numbers

$$e_n(H) := \inf_{\substack{\mathbf{x}_1,\ldots,\mathbf{x}_n \in D \\ \varphi_1,\ldots,\varphi_n \in L_2}} \sup_{f \in B(H)} \left\| f - \sum_{i=1}^{n} f(\mathbf{x}_i)\varphi_i \right\|_{L_2}$$

disproving conjecture 5.26 in [7]. We extend upon these theoretical findings with an empirical study comparing the performance of Smolyak's algorithm and Least Squares on simple function recovery problems on the unit cube. Our implementation is available at `https://github.com/th3lias/NumericalExperiments`.

2. NOTATION

We denote the indexset containing all indices $i \in \mathbb{Z}$ where $m \leq i \leq n$ for $m \leq n$ with $[m : n]$. For the set of polynomials p from $D \subseteq \mathbb{C}^K$, $K \in \mathbb{N}$ to the field $\mathbb{F}$ of F maximal degree N we use the notation $p \in \mathcal{P}^N(V, \mathbb{F})$. We use $f \lesssim g$ to denote $f \leq cg$ for functions f, g. With $B(X)$ we denote the closed unit ball of a normed space X. With $X^\star$ we denote the topological dual space of X.

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3. Interpolation

Given $i \in \mathbb{N}$ and distinct data points $\mathbf{X}_i := \{x_{i,0}, \dots, x_{i,n}\} \subseteq D_i \subseteq \mathbb{R}$, function values $\mathbf{Y}_i := \{y_{i,0}, \dots, y_{i,n}\} = \{f(x_{i,0}), \dots, f(x_{i,n})\} \subseteq \mathbb{R}$ for $f : D_i \to \mathbb{R}$, it is a well known fact that there exists a polynomial $\mathcal{I}(\mathbf{Z}_i)$ of degree n such that

$$\mathcal{I}(\mathbf{Z}_i) = y_{i,j}, \quad j \in [0 : n].$$

for $\mathbf{Z}_i := (\mathbf{X}_i, \mathbf{Y}_i)$ We then call $\mathcal{I}(\mathbf{Z}_i)$ an *interpolating* polynomial. It can be written down explicitly as

$$\mathcal{I}(\mathbf{Z}_i) = \sum_{j=0}^{n} y_{i,j} \ell_{i,j} \tag{3.1}$$

for basis functions $\ell_{i,j} \in \mathcal{P}^n(\mathbb{R}, \mathbb{R})$. Moreover, in a single dimension, the formula for this basis is

$$\ell_{i,j}(w) = \prod_{m \in [0:n] \setminus \{j\}} \frac{w - x_{i,m}}{x_{i,j} - x_{i,m}},$$

see [17, 29]. We call $\mathcal{I}$ the *Lagrange interpolator*.

Suppose now we have d point sets $\mathbf{X}_i$. By taking the full cartesian product of point sets

$$\mathbf{X} := \bigtimes_{i=1}^{d} \mathbf{X}_i,$$

we may identify our grid by $\mathbf{X} = \{\mathbf{x_j}\}_{\mathbf{j}}$ with $\mathbf{j} \in \{0, 1, \dots, n\}^d$ and suppose we have according function evaluations $\mathbf{Y} := \{y_{\mathbf{j}}\}_{\mathbf{j}} = \{f(\mathbf{x_j})\}_{\mathbf{j}}$ for $f : D \to \mathbb{R}$, $D \subseteq \mathbb{R}^d$. Let $\mathbf{Z} := (\mathbf{X}, \mathbf{Y})$. Then, we may construct an interpolating polynomial as in (3.1). Indeed, the general formula stays the same but we take tensor product of basis functions, i.e.

$$\ell_{\mathbf{j}}(\mathbf{w}) = \bigotimes_{i=1}^{d} \ell_{j_i}(w_i) = \prod_{i=1}^{d} \ell_{j_i}(w_i) = \prod_{i=1}^{d} \prod_{\substack{k=0 \\ k \neq j_i}}^{n} \frac{w_i - x_{i,k}}{x_{i,j_i} - x_{i,k}} \tag{3.2}$$

and the sum now ranges over all multiindices $\mathbf{j} = (j_1, \dots, j_d) \in [0 : n]^d$. We denote

$$\mathcal{I}(\mathbf{Z}) := \sum_{j_1=0}^{n} \cdots \sum_{j_d=0}^{n} y_{j_1,\dots,j_d} \ell_{j_1,\dots,j_d} \tag{3.3}$$

for the interpolant on $\mathbb{R}^d$. Formula 3.2 is not a great choice for performing calculations on hardware. Leaving the storage of $(n+1)^d$ grid points aside, for computing one evaluation of the map $\mathbf{w} \mapsto \mathcal{I}(\mathbf{Z})(\mathbf{w})$, one must compute the sum of $(n+1)^d$ terms, where in each summand, the evaluation of d basis functions is computed, each of which is a product of n terms. This evaluation is thus as costly as $\mathcal{O}((n+1)^d dn)$. One can counter the costly computation of the interpolating polynomial at a given point by reformulating the above (numerically instable) Lagrange interpolator in it's barycentric form

$$\mathcal{I}(\mathbf{Z})(\mathbf{w}) = \frac{\sum_{j_1=0}^{n} \ell_{j_1}^{(1)}(w_1) \cdots \sum_{j_d=0}^{n} \ell_{j_d}^{(d)}(w_d) y_{\mathbf{j}}}{\sum_{j_1=0}^{n} \ell_{j_1}^{(1)}(w_1) \cdots \sum_{j_d=0}^{n} \ell_{j_d}^{(d)}(w_d)} \tag{3.4}$$

where

$$\ell_{j_k}^{(k)}(w) := \frac{m_{j_k}}{w - x_{j_k}}, \qquad k \in [1:d]. \tag{3.5}$$

with x_{j_k} being the j_k–th entry of $\mathbf{x}$. m_{j_k} are the *barycentric* weights which can be computed in $\mathcal{O}(n^2)$ time for general point distributions but admit $\mathcal{O}(1)$ computation for well–known and widely used point sets [14]. For example, due to [3], for the *Chebyshev points of the second kind*

$$x_l = x_{l,n} = \cos\frac{l\pi}{n},\ l \in [0:n] \tag{3.6}$$

one gets the closed form

$$m_l = (-1)^l \cdot \begin{cases} 1/2 & l \in \{0, n\} \\ 1 & \text{else.} \end{cases} \tag{3.7}$$

In this case, one achieves an evaluation time of $\mathcal{O}((n+1)^d)$. Hence, in this construction, one cannot get around this bottleneck.

4. Smolyak's Algorithm & Sparse Grids

The following description follows [2]. The question arises, how one may reduce the complexity of exact interpolation on grids. The central idea of sparse grids is to *not* take the full tensor product of one dimensional point sets but restrict the number of simultaneously large directions. To that end, take a variable number of points in each direction, i.e. let $N(j)$ specify how many points are used for the index j and let $\mathbf{X}_j := \left\{x_{1,j}, \ldots, x_{N(j),j}\right\}$ and $\mathcal{I}_j$ be the corresponding one–dimensional interpolator with $\mathcal{I}_0 := 0$, $\Delta_j := \mathcal{I}_j - \mathcal{I}_{j-1}$. Then Smolyak's algorithm is given in a simple recursive manner

$$\mathcal{A}(q,d) := \sum_{\|\mathbf{j}\|_1 \le q} \bigotimes_{k=1}^{d} \Delta_{j_k} = \mathcal{A}(q-1,d) + \sum_{\|\mathbf{j}\|_1 = q} \bigotimes_{k=1}^{d} \Delta_{j_k} \tag{4.1}$$

with $\mathcal{A}(q,d) = 0$, $q < d$ and $\mathbf{j}$, $\mathcal{I}$ as before. Evidently, only a relatively small number of knots, through the restriction $\|\mathbf{j}\|_1 \le q$, is needed. To be precise, $\mathcal{O}(n(\log n)^{d-1})$ points are needed. Hence q can be thought of as a resolution parameter. In the implementation, we will refer to $q-d$ as the scale parameter. By this form, one only needs to assess the function values at the *sparse grid*

$$H(q,d) := \bigcup_{q-d+1\leq \|\mathbf{j}\|_1 \leq q} \mathbf{X}_{j_1} \times \cdots \times \mathbf{X}_{j_d}$$

where the number of nodes in a given direction can never be *large* for all directions simultaneously. Hence, given that $\mathbf{X}_j \subset \mathbf{X}_{j+1}$, one may write the interpolator given by Smolyak's construction as

$$\mathcal{A}(q,d)(f) = \sum_{\|\mathbf{j}\|_1 \leq q} f(\mathbf{x_j})\ell_{\mathbf{j}},$$

which is familiar from before. The number of knots $N(j)$ used for each one–dimensional interpolation rule $\mathcal{I}_j$ remains to be specified. In order to obtain nested points, i.e. $\mathbf{X}_j \subset \mathbf{X}_{j+1}$ and thus $H(q,d) \subset H(q+1,d)$ together with collocation rules such as (3.6) it is usual to choose a doubling rule, i.e.

$$N(1) = 1, \quad N(j) = 2^{j-1} + 1,\ j > 1.$$

4.1. **Polynomial Exactness.** Without loss of generality, we restrict ourselves to the symmetric cube $[-1,1]^d$ ***We have*** $[0,1]^d$***. It appears multiple times, therefore I didn't want to change it everywhere now. But we need to change that!*** for interpolation of unknowns instead of a general domain D. In this case, Smolyak's algorithm is well known to exactly reproduce functions on certain polynomial spaces given that the rules $\mathcal{I}_j$ are exact on nested spaces V_j.

Lemma 4.1 ([6, 19]). *Assume $\mathcal{I}_j$ is exact on the vector space $V_j \subseteq C([-1,1])$ and assume*

$$V_1 \subset V_2 \subset V_3 \subset \dots,$$

then $\mathcal{A}(q,d)$ is exact on

$$\sum_{\|j\|_1 = q} V_{j_1} \otimes \cdots \otimes V_{j_d}$$

Moreover, we can exactly specify the spaces on which Smolyak's algorithm is exact.

Lemma 4.2 ([2]). *$\mathcal{A}(q,d)$ is exact on*

$$E(q,d) := \sum_{\|\mathbf{j}\|_1 = q} \mathcal{P}^{m_{j_1}-1}(\mathbb{R},\mathbb{R}) \otimes \cdots \otimes \mathcal{P}^{m_{j_d}-1}(\mathbb{R},\mathbb{R})$$

and $\mathcal{A}(d+k,d)$ is exact for all polynomials of degree k.

Indeed, the following also follows from [2]. Note that we now abuse the notation from 3.1 by writing $\mathcal{I}(\mathbf{Z}) =: \mathcal{I}(f, \mathbf{X}) =: \mathcal{I}(f)$.

Lemma 4.3. *Assume $\mathbf{X}_1 \subset \mathbf{X}_2 \subset \dots$ and $\mathcal{I}_j(f)(x) = f(x)$ for every $f \in C([-1,1])$ and every $x \in \mathbf{X}_j$. Then*

$$\mathcal{A}(q,d)(f)(x) = f(x)$$

for every $f \in C([-1,1]^d)$ and $x \in H(q,d)$.

4.2. **Error Bounds.** Since the interpolation operator $\mathcal{I}_j$ as defined before is exact on $\mathcal{P}^{N(j)-1}(\mathbb{R},\mathbb{R})$ one concludes

$$\|f - \mathcal{I}_j(f)\|_\infty \leq \mathrm{err}_{N(j)-1}(f)(1 + \Lambda_{N(j)})$$

where err_n is the error of the best approximation by $p \in \mathcal{P}^n(\mathbb{R},\mathbb{R})$, Λ_n is the Lebesgue constant for the point set in (3.6) and $n \geq 2$, in which case it is known that

$$\Lambda_n \leq \frac{2}{\pi}\log(n-1) + 1,$$

see for example [9] and [8]. The following bounds can be found in [2] and is well known since [20, 26, 30].

Lemma 4.4. *For the space*

$$F_d^k := \left\{ f : [-1,1]^d \to \mathbb{R} \mid D^\alpha f \text{ continuous if } \alpha_i \leq k \text{ for all } i \right\}$$

the error of $\mathcal{A}(q,d)$ can be bounded as

$$\|I_d - \mathcal{A}(q,d)\|_{\mathrm{op}} \leq c_{d,k} n^{-k} (\log n)^{(k+2)(d-1)+1}$$

where I_d is the embedding $F_d^k \hookrightarrow C([-1,1]^d)$ and $c_{d,k}$ is a positive constant only dependent on d and k.

Moreover, for the Sobolev–Hilbert space H_w^k with

$$H_w^k := \left\{ f \in L_w^2 : \|f\|_k^2 = \sum_{\ell \in \mathbb{N}_0} (1+\ell^2)^k \langle f, b_\ell \rangle^2 < \infty \right\} \tag{4.2}$$

with $\langle f, b_\ell \rangle$ being the ℓ–th Fourier coefficient, one obtains

$$\|f - \mathcal{A}(q,d)(f)\|_0 \leq c_{d,k} n^{-k} (\log n)^{(k+1)(d-1)} \|f\|_k .$$

Here, L_w^2 is the L^2 weighted by the Chebyshev weight

$$\left(1 - x^2\right)^{-1/2} . \tag{4.3}$$

In that case, it is well known that the Chebyshev polynomials

$$T_n(x) := \cos(n \arccos(x)). \tag{4.4}$$

form an orthonormal basis.

5. Least Squares

Contrary to the construction of exactly interpolating approximants in the case of Smolyak's algorithm, Least Squares is a conceptually simpler algorithm. We are given the overdetermined system

$$A\mathbf{z} = \mathbf{y}$$

where $A \in \mathbb{R}^{n\times m}$ with $n > m$. It is well–known that this system may be inconsistent and no exact solution exists. However, one may always pose the optimisation problem solving for $\mathbf{z} \in \mathbb{R}^m$ with the smallest error

$$\inf_{\mathbf{z}\in\mathbb{R}^m} \|A\mathbf{z} - \mathbf{y}\| . \tag{5.1}$$

It is further known that, in case of a full–rank matrix V, the unique solution to (5.1) is given by

$$\mathbf{z}_* = \left(A^\top A\right)^{-1} A^\top \mathbf{y} \in \mathbb{R}^m. \tag{5.2}$$

In our specific case of polynomial interpolation, A is the Vandermonde matrix, consisting of basis polynomials $b_1, b_2, \ldots, b_m$ evaluated at the n different sampled points $x_1, x_2, \ldots, x_n$ in $D \subseteq \mathbb{R}^d$ and $\mathbf{y}$ is the vector of function values sampled from the unknown function $f\colon D \to \mathbb{R}$, i.e. $\mathbf{y} = (f(\mathbf{x}_j))_{j=1}^n$. As for such points, the Vandermonde matrix is never singular, the solution to the approximation problem

$$\inf_p \|f - p\|$$

can analytically be expressed as

$$p_*\colon D \to \mathbb{R}, \quad t \mapsto \sum_{j=1}^m z_{*_j} b_j(t)$$

where $\mathbf{z}_* = \left(z_{*_j}\right)_{j=1}^m$ is given by (5.2). For the later implementation, we choose the basis polynomials b_j as the j–th weighted Chebyshev polynomial (4.4).

5.1. Error Bounds for L^2 recovery.

The notation in the following is borrowed from [27]. In this section we introduce a formal setting to the former considerations. That is, we consider a Hilbert space H of real-valued functions on a set D such that point evaluation $\delta_x : f \mapsto \int_D f \, \mathrm{d}\delta_x = f(x)$ is a bounded, linear functional on H. The general formulation of Least Squares allows for a broad class of recovery problems. In our specific case, the function recovery of real–valued functions on a d–dimensional (compact) subset D using basis functions of a k–dimensional subspace $V_k := \operatorname{span}\{b_1, \dots, b_k\}$, we consider the specific form of Least Squares, given by

$$A_{n,k}(f) := \operatorname*{argmin}_{g \in V_k} \sum_{i=1}^{n} \frac{|g(x_i) - f(x_i)|^2}{\varrho_k(x_i)}$$

where

$$\varrho_k(x) = \frac{1}{2}\left(\frac{1}{k}\sum_{j<k} b_{j+1}(x)^2 + \frac{1}{\sum_{j\geq k} a_j^2}\sum_{j\geq k} a_j^2 b_{j+1}(x)^2\right)$$

and $x_1, \dots, x_n \in D$. Whenever $f \in V_k$, then, of course, $f = A_{n,k}(f)$. With

$$e\left(A_{n,k}, H\right) := \sup_{f \in B(H)} \|f - A_{n,k}(f)\|_{L_2}, \tag{5.3}$$

we denote the worst case error of $A_{n,k}$, where we measure the error of the reconstruction in the space $L_2 := L_2(D, \Sigma, \mu)$ of square integrable functions on D with respect to the measure μ, such that H is embedded into L_2. In light of this, the n–th minimal error (also called n–th sampling number) is denoted by

$$e_n(H) := \inf_{\substack{x_1,\dots,x_n \in D \\ \varphi_1,\dots,\varphi_n \in L_2}} \sup_{f \in B(H)} \left\| f - \sum_{i=1}^{n} f(x_i)\varphi_i \right\|_{L_2}$$

and can be understood as the worst case error of the optimal algorithm using n function values. We get the clear inequality $e_n(H) \leq e(A_{n,k}, H)$ for any point set $\{x_1, \dots, x_n\}$. With

$$a_n(H) := \inf_{\substack{h_1^\star,\dots,h_n^\star \in H^\star \\ \varphi_1,\dots,\varphi_n \in L_2}} \sup_{f \in B(H)} \left\| f - \sum_{i=1}^{n} h_i^\star(f)\varphi_i \right\|_{L_2}$$

we denote the n-th approximation number, which is the worst-case error of an optimal algorithm that uses the n best arbitrary linear and bounded functionals as information about the unknown. This quantity is equal to the n-th singular value of the embedding $\mathrm{id}\colon H \to L_2$.

The following is known since [15].

Theorem 5.1 (Krieg–Ullrich)**.** *There exist constants $C, c > 0$ and a sequence of natural numbers (k_n) with each $k_n \geq cn/\log(n+1)$ and for any $n \in \mathbb{N}$ and measure space (D, Σ, μ), and any RKHS H of real-valued functions on D embedded into $L_2(D, \Sigma, \mu)$, we have*

$$e_n(H) \leq \sqrt{\frac{C}{k_n}\sum_{j\geq k_n} a_j(H)^2}.$$

In particular, for

$$a_n(H) \lesssim n^{-s} \log^{\alpha+s}(n) \tag{5.4}$$

with $s > 1/2, \alpha \in \mathbb{R}$, this implies

$$e_n(H) \lesssim n^{-s} \log^{\alpha+s}(n).$$

The following follows from [27].

Theorem 5.2 (Ullrich). *Given $n \geq 2$ and $c > 0$, let*

$$k_n := \left\lfloor \frac{n}{2^8(2+c)\log n} \right\rfloor,$$

then, for any measure space (D, Σ, μ) and any RKHS H of real-valued functions on D, embedded into $L_2(D, \Sigma, \mu)$, it holds that

$$e_n\left(A_{n,2k_n}, H\right) \leq \sqrt{\frac{2}{k_n} \sum_{j>k_n} a_j(H)^2}$$

with probability at least $1 - 8n^{-c}$.

Examples. In particular, (5.4) is satisfied for the approximation numbers on the Sobolev space of dominating mixed smoothness,

$$\begin{aligned} H &:= H^s_{\mathrm{mix}}\left(\mathbb{T}^d\right) \\ &:= \left\{ f \in L_2\left(\mathbb{T}^d\right) : \|f\|_H^2 := \sum_{m \in \mathbb{N}_0^d} \prod_{j=1}^d (1 + |m_j|^{2s}) \langle f, b_m \rangle_{L_2}^2 < \infty \right\} \end{aligned}$$

with $\mathbb{T}^d \cong [0,1)^d$ where $b_m := \otimes_{j=1}^d b^{(1)}_{m_j}$ and $m = (m_1, \ldots, m_d)$ with

$$\begin{aligned} b^{(1)}_{2m} &:= \sqrt{2}\cos(2\pi m x) \\ b^{(1)}_{2m-1} &:= \sqrt{2}\sin(2\pi m x) \end{aligned}$$

and $b^{(1)}_0 := 1$. This satisfies the assumption for $s > 1/2$. In particular, we can say

$$e_n\left(H^s_{\mathrm{mix}}\left(\mathbb{T}^d\right)\right) \lesssim n^{-s} \log^{sd}(n) \tag{5.5}$$

whenever $s > 1/2$. This disproves a previously posted conjecture (Conjecture 5.26) in [7] and shows that Smolyak's algorithm is not optimal in this case. The surprising fact is that, despite an optimal, deterministic construction of the point sets used for reconstruction being unknown, random i.i.d. points suffice for a reconstruction error that is on the order of optimal points, with probability tending to 1. We verify this by our experimental findings, presented in Section 6 with a much better relative number of points used for LS function recovery vs. SA recovery than guaranteed in this section, i.e. better constants than explicitly known before. It remains an open problem to rigorously improve upon the constants in (5.5).

5.2. **Error Bounds for L^∞ recovery.** Perhaps unexpectedly, least squares using a mean error criterion also performs well in the uniform norm. To be precise, in the following we consider the space of complex, bounded functions G, i.e. $G := \{f : D \to \mathbb{C} \mid \sup_{x \in D} |f(x)| < \infty\}$ $V_n \subset G$ as before. In this case, we *unfix* the point set in question.

Lemma 5.3 ([16])**.** *There is $C > 0$ universal such that for $V_k \subset G$, there exists a point set of cardinality at most $2k$ with the property that for all $f \in G$*

$$\|f - A_{2k,k}(f)\|_\infty \leq C\sqrt{k} \inf_{g \in V_k} \|f - g\|_\infty .$$

Moreover, we can deduce the following by assigning a probability measure to D.

Lemma 5.4 ([16])**.** *Given (D, μ) a probability space and V_k as before, there are points $x_1, \ldots, x_{4k} \in D$ such that for $f \in G_\mu$ with the space of bounded, measurable functions G_μ, then for all $1 \leq p \leq \infty$ we have*

$$\|f - A_{4k,k}(f)\|_{L^p(\mu)} \lesssim k^{(1/2-1/p)_+} \inf_{g \in V_k} \|f - g\|_\infty .$$

Above, the notation $a_+ := \max(0, a)$, $a \in \mathbb{R}$ as usual.

Remark 5.5. As noted earlier, it may be surprising that the same least squares algorithm, namely optimized in the usual L^2 norm, yields good results in different L^p norms as well.

6. Experimental Findings

For assessing the performance of the Least Squares algorithms in comparison to the Sparse Grid alternative, we use the following 12 families of test functions from [25], each defined of the d-dimensional unit-cube $[0,1]^d$. ***boldface here in the formulas for c, w, x?***

1. Continuous: $f_1(x) = \exp\left(-\sum_{i=1}^d c_i \left|x_i - w_i\right|\right)$
2. Corner Peak: $f_2(x) = \left(1 + \sum_{i=1}^d c_i x_i\right)^{-(d+1)}$
3. Discontinuous: $f_3(x) = \begin{cases} 0, & x_1 > w_1 \vee x_2 > w_2, \\ \exp\left(\sum_{i=1}^d c_i x_i\right), & \text{else} \end{cases}$
4. Gaussian: $f_4(x) = \exp\left(-\sum_{i=1}^d c_i^2 (x_i - w_i)^2\right)$
5. Oscillatory: $f_5(x) = \cos\left(2\pi w_1 + \sum_{i=1}^d c_i x_i\right)$
6. Product Peak: $f_6(x) = \prod_{i=1}^d \left(c_i^{-2} + (x_i - w_i)^2\right)^{-1}$
7. G-Function: $f_7(x) = \prod_{i=1}^d \frac{|4x_i - 2 - w_i| + c_i}{1 + c_i}$
8. Morokoff & Calfisch 1: $f_8(x) = (1 + 1/d)^d \prod_{i=1}^d \left(c_i x_i + w_i\right)^{1/d}$
9. Morokoff & Calfisch 2: $f_9(x) = \frac{1}{(d-0.5)^d} \prod_{i=1}^d \left(d - c_i x_i + w_i\right)$
10. Roos & Arnold: $f_{10}(x) = \prod_{i=1}^d \left|4c_i x_i - 2 - w_i\right|$
11. Bratley: $f_{11}(x) = \sum_{d=1}^d (-1)^i \prod_{j=1}^d \left(c_i x_i - w_i\right)$
12. Zhou: $f_{12}(x) = \frac{10^d}{2} \left[\varphi\left(x - \frac{1}{3}\right) + \varphi\left(x - \frac{2}{3}\right)\right]$

    with $\varphi(x) = \frac{10}{(2\pi)^{d/2}} \exp\left(-\frac{1}{2}\left\|c\left(x - w\right)\right\|_2^2\right)$

Note that the first 6 function classes are also known as the *Genz Integrand Families* and were introduced by Genz in [10, 11]. Unlike the original definition in [25], we extend some function classes by introducing the parameters $\mathbf{c}$ and $\mathbf{w}$ which make something like an affine-linear transformation of the input $\mathbf{x}$. This allows for testing multiple realizations of these classes.

6.1. **Implementation Details.** Generating a function from a specific family is done by sampling the random vectors $\mathbf{c}, \mathbf{w} \in \mathbb{R}^d$. In our experiments, we sample each entry of $\mathbf{c}$ and $\mathbf{w}$ from a uniform distribution $\mathcal{U}(0,1)$ and rescale $\mathbf{c}$ afterwards such that $\|\mathbf{c}\|_1 = d$.

Remark 6.1. In [2], experiments were performed for the Genz families, defined on $[-1,1]^d$. We use $[0,1]^d$ as this ensures that also the other families are well-defined for any sampled (and possibly rescaled) $\mathbf{c}, \mathbf{w} \in \mathcal{U}(0,1)^d$ that .

In the following experiments, we compare Smolyak's algorithm with two realizations of the *weighted* Least Squares algorithm. In the first realization we use random points that are uniformly distributed in $[0,1]^d$, and we don't reweigh those points. In the second realization we sample the points from $w(x) = (1-x^2)^{1/2}$, as in 4.3 and we use the value of this density at each point as the basis for the weight calculation. We call the first realization *Least Squares Uniform* (LS-Uniform) and the second one *Least Squares-Chebyshev* (LS-Chebyshev). To ensure reproducibility, we initialized our random number generation with a fixed seed. For actually finding the least–squares solution we utilize Scipy [28] with their lstsq method which uses the gelsy driver from the standard linear algebra package Lapack [1] in the backend. Other drivers or for example the standard *solve*-method from Numpy ([12]) are a possible choice but did not meet our performance– and numerical precision requirements. All 3 algorithms use the same basis functions (4.4) which, as mentioned, form an ONB for L^2_w. The implementation thereof is based on the implementation in [13]. For the implementation of Smolyak's algorithm we employ the standard library Tasmanian with it's Python frontend [18, 21–24]. All algorithms are tested for all families of functions with multiple (≥ 10) realizations and for all dimensions $d \in [2:10]$. In each dimension, the resolution $q \in \mathbb{N}$ was varied. In our experiments, the resolution parameter is usually denoted by *scale*, which is just $q - d$, since $q > d$. Depending on d smaller or larger values for the scale were possible because of computational bottlenecks based on the exponential increase in the used points, see also [4] for an overview on the number of points in a sparse grid. Both Least Squares algorithms enjoyed twice the amount of points, compared to Smolyak's algorithm, sampled in their respective distributions.

For assessing the quality of the interpolants, we again generate n random points $\mathbf{x}_1, \ldots, \mathbf{x}_n$ distributed uniformly in $[0,1]^d$, where n is the number of points in the Sparse Grid at the specific scale which serve as our testing points. For each function from our function classes for a fixed dimension and at each scale, we calculate the *uniform error* and the *mean squared error* by comparing the true function values and the values from our interpolant at those specific points.

The distribution, emerged by collecting all function realizations for each function class, is now depicted in Figures 1 to 26 for specific function classes and dimensions. For more detailed results, we want to refer to Tables 1 to 9, showcasing the exact errors for all three algorithms and different scales for each dimension.

Should we also show something for fixed scale and varying dimension? $\rightarrow$ If yes, add pictures and also mention them here before.

Tables 10 to 21 showcase the interpolation capabilities of those 3 algorithms for each function class and various scales and dimensions.

In most of the experiments, at least one of the Least Squares methods performs beneficial compared to Smolyak's Algorithm. ***Add few more sentences, however full conclusion on the next section.***

Decide whether to use boxplots or max error distribution Decide which plots we want to keep Decide whether we want to have fixed scale plots as well For those plots that we to keep, add more text in the captions

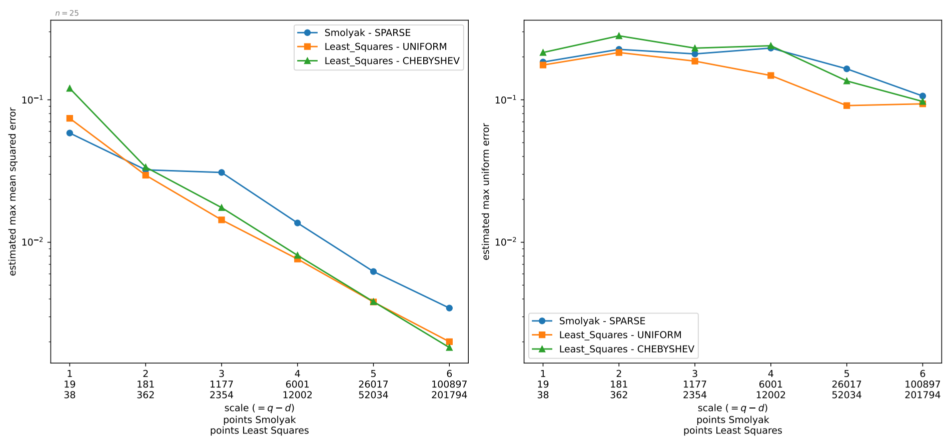

FIGURE 1. Visualization of the results for dim = 9 and various scales tested with $n = 25$ realizations for function class *Continuous*. Left plot shows the estimated max mean squared error and the right one shows the estimated max uniform error.

Add more detailed description mentioning whether Smolyak or Least Squares is preferable.

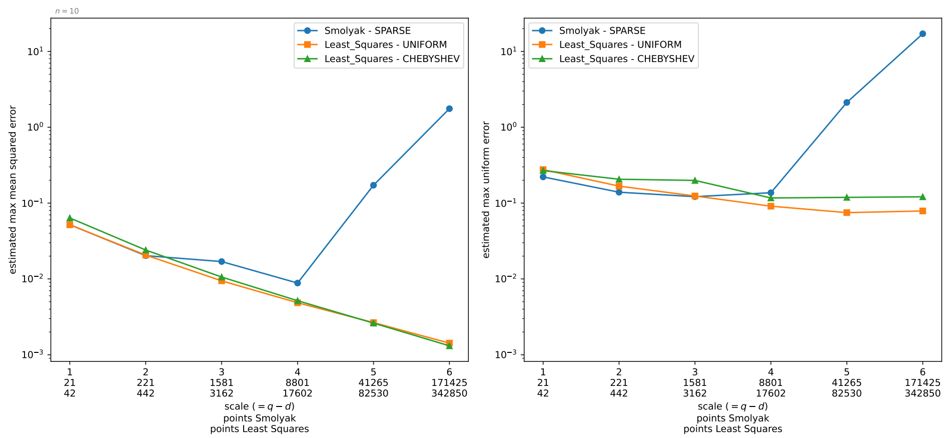

FIGURE 2. Visualization of the results for dim = 10 and various scales tested with $n = 10$ realizations for function class *Continuous*. Left plot shows the estimated max mean squared error and the right one shows the estimated max uniform error.

		Scale1 max		Scale2 max		Scale3 max		Scale4 max		Scale5 max		Scale6 max		Scale7 max		Scale8 max		Scale9 max	
		ℓ_2	ℓ_∞	ℓ_2	ℓ_∞	ℓ_2	ℓ_∞	ℓ_2	ℓ_∞	ℓ_2	ℓ_∞	ℓ_2	ℓ_∞	ℓ_2	ℓ_∞	ℓ_2	ℓ_∞	ℓ_2	ℓ_∞
Bratley $n = 25$	Smolyak	**7.53e-02**	**9.57e-02**	**3.25e-16**	**6.66e-16**	**6.27e-16**	**1.33e-15**	**7.09e-16**	**2.22e-15**	**1.16e-15**	**4.88e-15**	**1.61e-15**	**6.22e-15**	**2.44e-15**	**1.15e-14**	**3.38e-15**	**1.55e-14**	5.34e-15	**2.66e-14**
	LS-Uniform	8.32e-02	1.05e-01	7.53e-16	1.33e-15	1.11e-15	3.55e-15	1.79e-14	1.19e-13	4.14e-15	2.71e-14	2.62e-15	2.80e-14	5.71e-15	7.59e-14	2.04e-14	7.78e-13	3.48e-11	1.97e-09
	LS-Chebyshev	1.92e-01	2.99e-01	3.05e-15	6.11e-15	2.45e-15	7.11e-15	4.02e-15	1.55e-14	2.29e-15	9.33e-15	2.58e-15	8.88e-15	4.24e-15	2.27e-14	3.67e-15	3.57e-14	**2.99e-15**	2.82e-14
Continuous $n = 25$	Smolyak	1.55e-01	2.28e-01	1.01e-01	1.97e-01	2.50e-02	9.26e-02	**7.88e-03**	3.37e-02	**2.72e-03**	**1.92e-02**	**1.35e-03**	1.35e-02	**4.62e-04**	**5.29e-03**	**1.97e-04**	**2.42e-03**	**7.01e-05**	**1.22e-03**
	LS-Uniform	**9.13e-02**	**1.35e-01**	**4.27e-02**	**1.09e-01**	7.79e-02	2.84e-01	6.99e-02	4.70e-01	2.56e-02	2.24e-01	5.99e-03	7.54e-02	5.35e-03	8.21e-02	1.44e-02	5.47e-01	7.67e+00	4.39e+02
	LS-Chebyshev	2.16e-01	3.24e-01	5.49e-02	1.20e-01	**1.90e-02**	**6.58e-02**	9.06e-03	**3.05e-02**	3.58e-03	2.46e-02	1.92e-03	**1.30e-02**	6.80e-04	6.77e-03	2.50e-04	3.94e-03	1.06e-04	2.05e-03
Corner Peak $n = 25$	Smolyak	7.91e-02	1.17e-01	1.67e-02	3.26e-02	4.52e-03	8.68e-03	5.81e-04	**1.48e-03**	3.31e-05	1.04e-04	2.55e-07	**8.90e-07**	1.75e-09	6.08e-09	7.94e-14	**2.56e-13**	**8.06e-16**	**6.22e-15**
	LS-Uniform	**3.50e-02**	**4.91e-02**	**6.20e-03**	**1.21e-02**	3.23e-03	1.54e-02	5.19e-04	3.31e-03	5.36e-05	4.50e-04	4.37e-07	4.13e-06	1.07e-08	2.01e-07	8.45e-13	2.33e-11	1.38e-11	7.92e-10
	LS-Chebyshev	1.47e-01	2.28e-01	3.10e-02	5.44e-02	**2.62e-03**	**8.10e-03**	**3.80e-04**	1.94e-03	**1.72e-05**	**5.50e-05**	**1.86e-07**	8.95e-07	**1.10e-09**	**4.69e-09**	**6.70e-14**	2.87e-13	1.89e-15	2.48e-14
Discontinuous $n = 25$	Smolyak	**1.41e+00**	**2.44e+00**	2.23e+00	5.65e+00	1.12e+00	**2.83e+00**	8.57e-01	3.83e+00	6.10e-01	4.35e+00	3.93e-01	4.18e+00	**2.89e-01**	3.33e+00	2.68e-01	4.30e+00	1.67e-01	3.31e+00
	LS-Uniform	2.32e+00	3.51e+00	**1.27e+00**	**3.36e+00**	2.65e+00	1.07e+01	1.60e+01	1.06e+02	5.31e+00	4.74e+01	2.15e+00	3.57e+01	2.82e+00	5.96e+01	9.45e+00	3.56e+02	7.90e+03	4.49e+05
	LS-Chebyshev	1.55e+00	2.98e+00	1.80e+00	3.45e+00	**1.06e+00**	4.16e+00	**8.25e-01**	**3.66e+00**	**5.28e-01**	**3.07e+00**	**3.67e-01**	**2.95e+00**	2.91e-01	**3.31e+00**	**2.00e-01**	**3.70e+00**	**1.54e-01**	**2.96e+00**
Gaussian $n = 25$	Smolyak	1.01e-01	1.20e-01	8.34e-03	1.29e-02	5.85e-04	8.95e-04	1.95e-05	**3.77e-05**	3.01e-07	7.14e-07	1.23e-10	**3.57e-10**	1.59e-14	4.77e-14	**2.21e-15**	**8.10e-15**	3.76e-15	**2.07e-14**
	LS-Uniform	**7.49e-02**	**1.15e-01**	**4.30e-03**	**8.14e-03**	8.71e-04	3.82e-03	1.43e-05	8.85e-05	4.97e-07	3.64e-06	1.91e-10	1.58e-09	1.15e-13	2.26e-12	1.46e-14	4.63e-13	5.66e-11	3.25e-09
	LS-Chebyshev	1.53e-01	2.34e-01	1.22e-02	2.79e-02	**2.86e-04**	**7.18e-04**	**1.37e-05**	6.92e-05	**1.68e-07**	**4.77e-07**	**8.37e-11**	3.88e-10	**9.38e-15**	**3.20e-14**	2.38e-15	1.70e-14	**2.14e-15**	2.94e-14
G-Function $n = 25$	Smolyak	3.93e-01	**4.99e-01**	1.99e-01	3.67e-01	8.16e-02	2.13e-01	**3.26e-02**	**1.10e-01**	**1.11e-02**	7.23e-02	**4.40e-03**	**3.70e-02**	**1.80e-03**	**2.05e-02**	**8.20e-04**	**1.25e-02**	**2.77e-04**	**6.82e-03**
	LS-Uniform	**3.41e-01**	6.53e-01	**1.19e-01**	**2.95e-01**	3.64e-01	1.48e+00	3.63e-01	2.42e+00	1.11e-01	9.17e-01	2.37e-02	2.70e-01	2.67e-02	5.62e-01	3.48e-02	1.19e+00	3.54e+01	2.01e+03
	LS-Chebyshev	7.29e-01	1.01e+00	1.42e-01	4.01e-01	**6.08e-02**	**1.44e-01**	3.41e-02	1.25e-01	1.32e-02	**6.48e-02**	5.15e-03	3.77e-02	2.08e-03	2.43e-02	9.49e-04	1.81e-02	3.63e-04	9.36e-03
Morokoff Calfisch 1 $n = 25$	Smolyak	**4.43e-02**	**6.43e-02**	1.31e-02	2.59e-02	1.66e-03	3.21e-03	**1.02e-04**	**3.05e-04**	**3.02e-06**	**9.59e-06**	**7.42e-08**	**2.66e-07**	**1.15e-09**	**4.95e-09**	**2.37e-12**	**1.10e-11**	**1.21e-14**	**6.17e-14**
	LS-Uniform	6.26e-02	7.99e-02	**2.14e-03**	**3.95e-03**	**4.82e-04**	**1.93e-03**	3.77e-04	2.33e-03	7.05e-06	5.76e-05	9.23e-08	7.85e-07	6.04e-09	1.41e-07	1.92e-10	7.21e-09	1.09e-09	6.19e-08
	LS-Chebyshev	1.45e-01	2.06e-01	1.06e-02	2.72e-02	1.82e-03	3.73e-03	2.03e-04	5.55e-04	3.40e-06	1.16e-05	9.34e-08	4.62e-07	2.03e-09	9.98e-09	3.13e-11	1.45e-10	4.23e-13	1.88e-12
Morokoff Calfisch 2 $n = 25$	Smolyak	**3.35e-02**	**4.25e-02**	**7.84e-16**	**1.78e-15**	**7.91e-16**	**1.78e-15**	**1.35e-15**	**3.55e-15**	**2.06e-15**	**5.77e-15**	**2.69e-15**	**9.33e-15**	**4.36e-15**	**1.60e-14**	**6.31e-15**	**2.09e-14**	1.04e-14	**5.28e-14**
	LS-Uniform	3.70e-02	4.65e-02	1.59e-15	3.11e-15	1.93e-15	7.11e-15	3.78e-14	2.50e-13	7.75e-15	6.08e-14	6.06e-15	5.55e-14	8.93e-15	1.74e-13	3.18e-14	1.07e-12	9.76e-11	5.62e-09
	LS-Chebyshev	8.56e-02	1.33e-01	5.59e-15	1.07e-14	5.53e-15	1.47e-14	7.19e-15	2.02e-14	4.98e-15	1.69e-14	2.94e-15	1.29e-14	9.84e-15	5.73e-14	8.39e-15	8.28e-14	**6.89e-15**	9.59e-14
Oscillatory $n = 25$	Smolyak	**7.28e-02**	**9.21e-02**	3.22e-03	5.02e-03	4.66e-05	**7.06e-05**	2.02e-07	**4.00e-07**	3.76e-10	8.91e-10	**1.24e-15**	**4.11e-15**	**1.62e-15**	**7.44e-15**	**2.33e-15**	**8.33e-15**	3.89e-15	**1.98e-14**
	LS-Uniform	8.09e-02	1.01e-01	**9.64e-04**	**2.00e-03**	5.94e-05	2.74e-04	2.32e-07	1.52e-06	5.88e-10	4.00e-09	2.19e-15	2.13e-14	3.93e-15	7.05e-14	1.38e-14	5.34e-13	3.69e-11	2.13e-09
	LS-Chebyshev	1.84e-01	2.88e-01	4.61e-03	1.07e-02	**2.43e-05**	7.37e-05	**1.43e-07**	6.98e-07	**2.14e-10**	**6.16e-10**	1.48e-15	7.11e-15	3.68e-15	1.91e-14	2.42e-15	1.63e-14	**1.64e-15**	2.38e-14
Product Peak $n = 25$	Smolyak	**6.06e-02**	**9.32e-02**	**4.82e-16**	**1.11e-15**	**4.52e-16**	**1.33e-15**	**8.43e-16**	**2.44e-15**	**1.14e-15**	**3.77e-15**	**1.71e-15**	**6.22e-15**	**2.50e-15**	**1.33e-14**	**3.56e-15**	**1.55e-14**	5.65e-15	**2.62e-14**
	LS-Uniform	7.11e-02	9.82e-02	2.44e-15	4.66e-15	2.17e-15	7.55e-15	4.26e-14	2.82e-13	5.88e-15	4.12e-14	4.23e-15	3.73e-14	7.79e-15	1.42e-13	2.34e-14	8.82e-13	5.96e-11	3.43e-09
	LS-Chebyshev	1.10e-01	1.90e-01	7.19e-15	1.44e-14	8.31e-15	2.15e-14	1.22e-14	3.43e-14	5.17e-15	2.04e-14	3.52e-15	1.60e-14	7.11e-15	4.31e-14	6.33e-15	6.73e-14	**4.77e-15**	5.48e-14
Roos Arnold $n = 25$	Smolyak	1.41e+00	2.15e+00	9.44e-01	1.95e+00	2.99e-01	7.45e-01	**1.10e-01**	**4.09e-01**	4.15e-02	2.38e-01	**1.53e-02**	**1.04e-01**	**5.97e-03**	**5.52e-02**	**2.10e-03**	**3.87e-02**	**8.52e-04**	**1.80e-02**
	LS-Uniform	**1.40e+00**	**1.90e+00**	**4.45e-01**	**9.78e-01**	1.23e+00	5.07e+00	1.15e+00	7.74e+00	3.65e-01	3.51e+00	6.03e-02	7.97e-01	1.32e-01	2.77e+00	9.68e-02	3.67e+00	1.33e+02	7.64e+03
	LS-Chebyshev	2.48e+00	3.77e+00	8.71e-01	2.04e+00	**2.41e-01**	**6.78e-01**	1.24e-01	5.05e-01	**4.00e-02**	**1.98e-01**	2.08e-02	1.66e-01	7.15e-03	7.05e-02	2.74e-03	4.05e-02	1.07e-03	2.87e-02
Zhou $n = 25$	Smolyak	2.44e-01	2.85e-01	2.03e-02	2.93e-02	9.62e-04	1.46e-03	1.74e-05	**3.47e-05**	1.51e-07	3.60e-07	1.43e-11	4.15e-11	**2.49e-14**	**1.14e-13**	**3.56e-14**	**1.37e-13**	6.11e-14	**3.13e-13**
	LS-Uniform	**2.04e-01**	**2.50e-01**	**6.21e-03**	**1.13e-02**	9.46e-04	4.19e-03	1.81e-05	1.19e-04	2.42e-07	1.68e-06	2.45e-11	2.47e-10	5.85e-14	1.24e-12	1.69e-13	6.49e-12	6.28e-10	3.62e-08
	LS-Chebyshev	4.56e-01	7.18e-01	2.94e-02	7.10e-02	**4.76e-04**	**1.20e-03**	**1.24e-05**	6.05e-05	**8.58e-08**	**2.46e-07**	**9.42e-12**	**4.02e-11**	4.92e-14	2.74e-13	4.09e-14	3.55e-13	**3.66e-14**	6.13e-13

TABLE 1. Visualization of the results for dim = 2 and various scales tested with $n = 25$ realizations for each function class. Best algorithm per function class, dimension and scale is depicted bold.

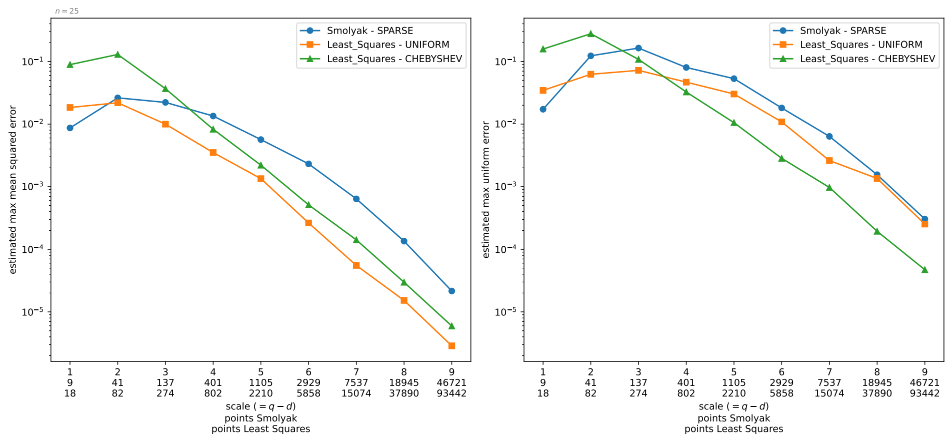

FIGURE 3. Visualization of the results for dim = 4 and various scales tested with $n = 25$ realizations for function class *Corner Peak*. Left plot shows the estimated max mean squared error and the right one shows the estimated max uniform error.

		Scale1 max		Scale2 max		Scale3 max		Scale4 max		Scale5 max		Scale6 max		Scale7 max		Scale8 max		Scale9 max	
		ℓ_2	ℓ_∞	ℓ_2	ℓ_∞	ℓ_2	ℓ_∞	ℓ_2	ℓ_∞	ℓ_2	ℓ_∞	ℓ_2	ℓ_∞	ℓ_2	ℓ_∞	ℓ_2	ℓ_∞	ℓ_2	ℓ_∞
Bratley $n = 25$	Smolyak	**1.57e-01**	**2.57e-01**	**1.69e-02**	**4.33e-02**	**7.02e-16**	**2.22e-15**	**1.07e-15**	**3.11e-15**	**1.76e-15**	**9.33e-15**	2.37e-15	**1.11e-14**	3.26e-15	**1.38e-14**	**4.44e-15**	**2.66e-14**	**6.74e-15**	**4.71e-14**
	LS-Uniform	1.88e-01	3.37e-01	2.27e-02	6.09e-02	1.97e-15	7.33e-15	2.00e-15	1.02e-14	2.68e-15	1.83e-14	**1.91e-15**	2.96e-14	**2.01e-15**	5.76e-14	4.50e-15	1.84e-13	2.81e-14	1.68e-12
	LS-Chebyshev	4.85e-01	7.74e-01	2.86e-02	5.51e-02	1.84e-15	5.55e-15	5.32e-15	1.80e-14	8.33e-15	3.15e-14	7.42e-15	5.87e-14	8.79e-15	5.16e-14	8.00e-15	8.22e-14	1.19e-14	1.33e-13
Continuous $n = 25$	Smolyak	1.45e-01	2.48e-01	5.89e-02	1.90e-01	2.87e-02	**8.67e-02**	1.19e-02	5.77e-02	4.54e-03	**2.78e-02**	1.80e-03	**1.70e-02**	**6.65e-04**	1.07e-02	**2.69e-04**	**4.95e-03**	**1.04e-04**	**3.40e-03**
	LS-Uniform	**9.91e-02**	**1.51e-01**	1.67e-01	4.57e-01	5.12e-02	2.39e-01	1.48e-02	1.07e-01	7.14e-03	6.28e-02	6.91e-03	1.87e-01	2.41e-03	7.40e-02	1.31e-03	7.25e-02	9.36e-03	8.41e-01
	LS-Chebyshev	1.54e-01	2.93e-01	**5.14e-02**	**1.56e-01**	**1.86e-02**	9.44e-02	**8.80e-03**	**4.80e-02**	**3.55e-03**	2.93e-02	**1.63e-03**	1.72e-02	6.98e-04	**9.45e-03**	3.01e-04	7.23e-03	1.24e-04	3.79e-03
Corner Peak $n = 25$	Smolyak	**4.34e-02**	**6.73e-02**	3.96e-02	1.57e-01	1.36e-02	5.59e-02	5.80e-03	3.01e-02	1.43e-03	7.14e-03	2.46e-04	1.34e-03	2.77e-05	1.84e-04	1.88e-06	1.24e-05	4.08e-08	3.12e-07
	LS-Uniform	5.36e-02	9.50e-02	**1.69e-02**	**4.45e-02**	1.20e-02	5.84e-02	**1.65e-03**	**9.08e-03**	**3.77e-04**	7.16e-03	**6.96e-05**	1.03e-03	1.36e-05	5.16e-04	2.32e-06	8.73e-05	2.51e-07	1.69e-05
	LS-Chebyshev	2.22e-01	3.36e-01	4.79e-02	1.26e-01	**1.12e-02**	**3.55e-02**	3.33e-03	1.16e-02	5.53e-04	**2.93e-03**	8.54e-05	**4.00e-04**	**1.15e-05**	**6.56e-05**	**9.61e-07**	**4.63e-06**	**2.12e-08**	**1.34e-07**
Discontinuous $n = 25$	Smolyak	2.70e+00	4.58e+00	3.31e+00	1.08e+01	2.55e+00	8.43e+00	2.04e+00	1.08e+01	1.26e+00	1.11e+01	**7.98e-01**	7.68e+00	6.82e-01	1.04e+01	5.25e-01	1.02e+01	4.37e-01	9.66e+00
	LS-Uniform	**2.26e+00**	**4.48e+00**	4.20e+00	1.21e+01	4.14e+00	1.59e+01	2.20e+00	1.19e+01	1.87e+00	1.57e+01	1.81e+00	3.90e+01	2.10e+00	6.26e+01	2.61e+00	1.42e+02	3.39e+01	2.76e+03
	LS-Chebyshev	3.63e+00	5.31e+00	**2.37e+00**	**5.88e+00**	**2.36e+00**	**7.11e+00**	**1.59e+00**	**7.53e+00**	**1.24e+00**	**6.59e+00**	8.77e-01	**5.66e+00**	**5.91e-01**	**8.11e+00**	**4.12e-01**	**6.51e+00**	**3.12e-01**	**6.23e+00**
Gaussian $n = 25$	Smolyak	1.19e-01	2.41e-01	3.74e-02	**6.96e-02**	3.93e-03	7.46e-03	2.71e-04	6.07e-04	7.50e-06	2.66e-05	3.10e-07	1.26e-06	6.97e-09	3.23e-08	8.42e-11	4.24e-10	4.94e-14	2.60e-13
	LS-Uniform	**1.04e-01**	**1.98e-01**	8.55e-02	2.36e-01	3.91e-03	2.16e-02	**1.39e-04**	8.76e-04	6.84e-06	5.22e-05	1.84e-07	2.37e-06	5.01e-09	1.19e-07	1.25e-10	4.86e-09	5.54e-13	4.43e-11
	LS-Chebyshev	1.80e-01	2.94e-01	**2.78e-02**	7.21e-02	**2.53e-03**	**6.34e-03**	1.68e-04	**4.87e-04**	**4.85e-06**	**1.83e-05**	**1.26e-07**	**5.29e-07**	**2.83e-09**	**1.48e-08**	**3.76e-11**	**1.91e-10**	**2.25e-14**	**1.46e-13**
G-Function $n = 25$	Smolyak	6.12e-01	1.08e+00	4.34e-01	1.38e+00	1.99e-01	6.79e-01	9.02e-02	3.84e-01	4.39e-02	2.87e-01	1.85e-02	1.80e-01	7.74e-03	8.43e-02	2.88e-03	3.90e-02	**1.07e-03**	2.43e-02
	LS-Uniform	**5.82e-01**	1.17e+00	6.14e-01	1.76e+00	1.97e-01	9.60e-01	**7.56e-02**	4.53e-01	6.39e-02	7.35e-01	4.98e-02	1.13e+00	2.56e-02	6.36e-01	1.58e-02	7.06e-01	5.58e-02	4.08e+00
	LS-Chebyshev	6.84e-01	**9.25e-01**	**3.72e-01**	**8.13e-01**	**1.45e-01**	**4.01e-01**	8.58e-02	**3.01e-01**	**3.75e-02**	**1.60e-01**	**1.62e-02**	**1.06e-01**	**7.48e-03**	**6.97e-02**	**2.84e-03**	**3.44e-02**	1.13e-03	**2.29e-02**
Morokoff Calfisch 1 $n = 25$	Smolyak	1.23e-01	2.46e-01	**2.50e-02**	**5.91e-02**	**2.84e-03**	**9.40e-03**	**2.84e-04**	**1.12e-03**	**1.38e-05**	**8.28e-05**	6.95e-07	5.21e-06	2.64e-08	1.87e-07	4.04e-10	2.97e-09	**5.05e-12**	**4.40e-11**
	LS-Uniform	**5.93e-02**	**1.15e-01**	4.73e-02	1.25e-01	4.34e-03	2.23e-02	3.09e-04	2.80e-03	1.55e-05	2.20e-04	**4.34e-07**	1.09e-05	2.76e-08	1.16e-06	1.10e-09	7.16e-08	6.25e-11	4.44e-09
	LS-Chebyshev	2.45e-01	4.02e-01	2.64e-02	6.54e-02	4.37e-03	1.58e-02	4.47e-04	1.29e-03	1.58e-05	9.10e-05	4.73e-07	**2.42e-06**	**1.96e-08**	**9.58e-08**	**3.11e-10**	**1.64e-09**	7.53e-12	5.78e-11
Morokoff Calfisch 2 $n = 25$	Smolyak	**3.46e-02**	**5.46e-02**	**1.07e-03**	**2.75e-03**	**1.36e-15**	**4.44e-15**	**1.98e-15**	**6.66e-15**	**2.73e-15**	**9.77e-15**	3.74e-15	**1.51e-14**	5.52e-15	**2.26e-14**	7.36e-15	**3.24e-14**	**1.16e-14**	**6.75e-14**
	LS-Uniform	4.33e-02	7.43e-02	1.44e-03	3.87e-03	4.91e-15	1.87e-14	4.81e-15	2.80e-14	3.40e-15	2.31e-14	**3.65e-15**	6.20e-14	**4.50e-15**	1.14e-13	**6.52e-15**	2.97e-13	8.57e-14	8.96e-12
	LS-Chebyshev	1.00e-01	1.63e-01	1.82e-03	3.50e-03	2.91e-15	6.66e-15	7.29e-15	3.31e-14	1.55e-14	6.82e-14	1.22e-14	1.12e-13	1.58e-14	1.10e-13	1.14e-14	1.49e-13	1.24e-14	3.15e-13
Oscillatory $n = 25$	Smolyak	**1.09e-01**	**2.25e-01**	**1.52e-02**	**3.53e-02**	1.16e-03	**2.69e-03**	2.50e-05	5.80e-05	2.64e-07	6.93e-07	1.48e-09	4.68e-09	4.34e-12	1.70e-11	6.15e-15	**2.79e-14**	**4.80e-15**	**2.71e-14**
	LS-Uniform	1.57e-01	2.77e-01	2.08e-02	5.44e-02	1.21e-03	5.95e-03	**1.14e-05**	9.36e-05	1.05e-07	1.02e-06	6.21e-10	7.91e-09	3.02e-12	7.73e-11	9.89e-15	3.66e-13	3.73e-14	4.03e-12
	LS-Chebyshev	3.14e-01	4.91e-01	2.53e-02	4.78e-02	**1.03e-03**	2.84e-03	1.37e-05	**4.49e-05**	**9.48e-08**	**4.74e-07**	**5.48e-10**	**2.57e-09**	**1.73e-12**	**9.71e-12**	**4.48e-15**	7.55e-14	5.75e-15	1.02e-13
Product Peak $n = 25$	Smolyak	**3.41e-01**	**8.35e-01**	**1.29e-02**	**5.80e-02**	**8.83e-16**	**3.11e-15**	**1.36e-15**	**6.22e-15**	**1.88e-15**	**1.15e-14**	**2.72e-15**	**1.87e-14**	**3.95e-15**	**3.02e-14**	**5.04e-15**	**3.64e-14**	**7.45e-15**	**5.33e-14**
	LS-Uniform	5.91e-01	1.01e+00	1.53e-02	6.16e-02	1.99e-14	7.61e-14	1.47e-14	8.57e-14	1.36e-14	9.15e-14	9.60e-15	1.73e-13	7.03e-15	1.68e-13	5.11e-15	2.40e-13	1.03e-13	1.11e-11
	LS-Chebyshev	9.47e-01	1.46e+00	3.29e-02	6.83e-02	1.07e-14	2.35e-14	3.73e-14	1.17e-13	3.78e-14	1.54e-13	4.93e-14	2.01e-13	3.55e-14	1.96e-13	1.77e-14	3.26e-13	2.12e-14	5.28e-13
Roos Arnold $n = 25$	Smolyak	4.54e+00	8.29e+00	2.53e+00	6.31e+00	1.48e+00	3.68e+00	4.41e-01	2.06e+00	1.87e-01	**9.77e-01**	8.06e-02	5.25e-01	2.98e-02	**3.11e-01**	**1.19e-02**	**1.72e-01**	**4.52e-03**	**9.59e-02**
	LS-Uniform	**3.16e+00**	**7.23e+00**	3.50e+00	1.02e+01	1.41e+00	6.82e+00	4.88e-01	2.80e+00	2.17e-01	1.63e+00	1.55e-01	2.42e+00	1.02e-01	3.01e+00	7.21e-02	4.79e+00	4.32e-01	4.76e+01
	LS-Chebyshev	6.32e+00	1.45e+01	**2.07e+00**	**4.93e+00**	**8.67e-01**	**2.97e+00**	**3.48e-01**	**2.00e+00**	**1.34e-01**	1.22e+00	**5.93e-02**	**5.05e-01**	**2.78e-02**	3.76e-01	1.20e-02	2.26e-01	5.04e-03	1.50e-01
Zhou $n = 25$	Smolyak	**1.78e+00**	**2.66e+00**	3.64e-01	**5.92e-01**	2.57e-02	4.64e-02	9.54e-04	**2.08e-03**	1.82e-05	4.77e-05	1.11e-07	**4.01e-07**	8.46e-10	3.64e-09	3.70e-12	1.96e-11	3.07e-13	**1.66e-12**
	LS-Uniform	2.03e+00	3.65e+00	4.20e-01	1.28e+00	2.33e-02	1.46e-01	**4.65e-04**	2.87e-03	1.59e-05	1.59e-04	**5.00e-08**	8.38e-07	6.97e-10	2.97e-08	7.72e-12	3.49e-10	1.69e-12	1.40e-10
	LS-Chebyshev	3.85e+00	6.24e+00	**3.25e-01**	7.17e-01	**1.42e-02**	**4.16e-02**	5.84e-04	2.25e-03	**1.17e-05**	**4.12e-05**	5.03e-08	4.03e-07	**3.33e-10**	**1.95e-09**	**1.92e-12**	**1.06e-11**	**2.09e-13**	4.68e-12

TABLE 2. Dim 3

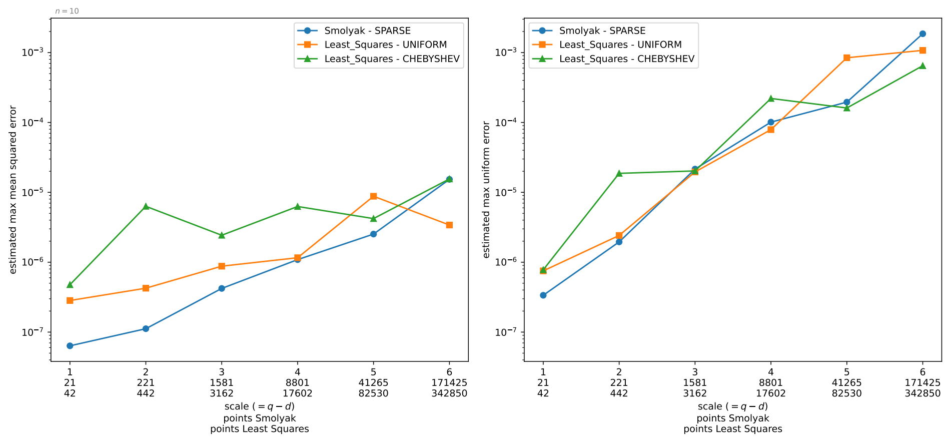

FIGURE 4. Visualization of the results for dim = 10 and various scales tested with $n = 10$ realizations for function class *Corner Peak*. Left plot shows the estimated max mean squared error and the right one shows the estimated max uniform error.

		Scale1 max		Scale2 max		Scale3 max		Scale4 max		Scale5 max		Scale6 max		Scale7 max		Scale8 max		Scale9 max	
		ℓ_2	ℓ_∞	ℓ_2	ℓ_∞	ℓ_2	ℓ_∞	ℓ_2	ℓ_∞	ℓ_2	ℓ_∞	ℓ_2	ℓ_∞	ℓ_2	ℓ_∞	ℓ_2	ℓ_∞	ℓ_2	ℓ_∞
Bratley $n = 25$	Smolyak	**2.41e-01**	**4.02e-01**	**6.99e-02**	**2.34e-01**	**5.29e-03**	**2.57e-02**	**1.79e-15**	**6.88e-15**	**2.63e-15**	**1.02e-14**	3.55e-15	**1.73e-14**	4.60e-15	**2.75e-14**	6.18e-15	**3.15e-14**	8.66e-15	**5.11e-14**
	LS-Uniform	3.00e-01	4.75e-01	1.51e-01	5.16e-01	1.20e-02	6.85e-02	2.30e-15	1.41e-14	6.28e-15	3.69e-14	**1.96e-15**	2.71e-14	**1.84e-15**	4.36e-14	**2.75e-15**	1.04e-13	**5.05e-15**	5.49e-13
	LS-Chebyshev	8.39e-01	1.05e+00	2.10e-01	4.92e-01	1.95e-02	6.84e-02	1.29e-14	6.88e-14	1.10e-14	4.64e-14	1.37e-14	9.71e-14	5.59e-14	2.72e-13	2.87e-14	4.93e-13	2.06e-14	4.87e-13
Continuous $n = 25$	Smolyak	1.69e-01	4.47e-01	6.12e-02	**2.12e-01**	3.75e-02	**1.16e-01**	1.55e-02	**5.44e-02**	5.52e-03	**3.96e-02**	2.15e-03	**2.29e-02**	9.22e-04	**1.42e-02**	3.76e-04	**1.04e-02**	1.76e-04	**4.97e-03**
	LS-Uniform	**1.30e-01**	**3.08e-01**	8.44e-02	3.56e-01	2.70e-02	1.37e-01	1.09e-02	7.07e-02	6.80e-03	7.09e-02	2.87e-03	6.79e-02	1.55e-03	3.32e-02	8.18e-04	4.10e-02	6.25e-04	9.32e-02
	LS-Chebyshev	1.73e-01	4.39e-01	**5.15e-02**	2.17e-01	**2.42e-02**	1.19e-01	**9.02e-03**	6.54e-02	**3.80e-03**	4.28e-02	**1.79e-03**	3.52e-02	**7.78e-04**	1.95e-02	**3.32e-04**	1.55e-02	**1.51e-04**	9.79e-03
Corner Peak $n = 25$	Smolyak	**8.68e-03**	**1.73e-02**	2.62e-02	1.23e-01	2.22e-02	1.64e-01	1.35e-02	8.03e-02	5.65e-03	5.32e-02	2.31e-03	1.81e-02	6.39e-04	6.36e-03	1.35e-04	1.55e-03	2.16e-05	3.04e-04
	LS-Uniform	1.84e-02	3.46e-02	**2.19e-02**	**6.25e-02**	**9.98e-03**	**7.21e-02**	**3.51e-03**	4.68e-02	**1.34e-03**	3.03e-02	**2.63e-04**	1.08e-02	**5.53e-05**	2.61e-03	**1.53e-05**	1.35e-03	**2.87e-06**	2.53e-04
	LS-Chebyshev	8.91e-02	1.58e-01	1.29e-01	2.77e-01	3.67e-02	1.08e-01	8.27e-03	**3.26e-02**	2.20e-03	**1.05e-02**	5.13e-04	**2.84e-03**	1.41e-04	**9.73e-04**	2.97e-05	**1.94e-04**	5.95e-06	**4.72e-05**
Discontinuous $n = 25$	Smolyak	9.48e+00	2.29e+01	5.76e+00	**1.97e+01**	5.20e+00	2.15e+01	2.96e+00	2.32e+01	2.66e+00	2.51e+01	1.73e+00	1.88e+01	1.44e+00	2.16e+01	1.05e+00	2.74e+01	**7.32e-01**	2.60e+01
	LS-Uniform	**7.24e+00**	**2.01e+01**	**5.67e+00**	2.50e+01	4.30e+00	2.31e+01	**2.86e+00**	3.05e+01	3.68e+00	5.69e+01	3.26e+00	7.03e+01	2.47e+00	6.35e+01	2.95e+00	2.06e+02	3.06e+00	3.47e+02
	LS-Chebyshev	9.15e+00	2.45e+01	6.46e+00	2.44e+01	**3.35e+00**	**1.27e+01**	3.79e+00	**1.42e+01**	**2.37e+00**	**1.90e+01**	**1.65e+00**	**1.65e+01**	**1.44e+00**	**1.70e+01**	**1.01e+00**	**1.88e+01**	7.34e-01	**1.85e+01**
Gaussian $n = 25$	Smolyak	1.24e-01	2.27e-01	**3.18e-02**	**7.33e-02**	5.19e-03	1.50e-02	6.17e-04	2.09e-03	6.51e-05	2.41e-04	4.11e-06	1.85e-05	1.63e-07	8.09e-07	5.55e-09	3.34e-08	1.44e-10	1.01e-09
	LS-Uniform	**1.18e-01**	**2.25e-01**	3.72e-02	1.41e-01	**3.68e-03**	3.06e-02	**3.41e-04**	3.84e-03	3.25e-05	6.16e-04	**1.61e-06**	3.73e-05	9.03e-08	3.19e-06	2.95e-09	1.51e-07	6.70e-11	4.31e-09
	LS-Chebyshev	1.80e-01	4.16e-01	4.02e-02	7.70e-02	4.39e-03	**1.24e-02**	3.53e-04	**1.30e-03**	**2.94e-05**	**1.08e-04**	1.68e-06	**8.65e-06**	**6.71e-08**	**3.85e-07**	**1.71e-09**	**1.24e-08**	**4.13e-11**	**2.84e-10**
G-Function $n = 25$	Smolyak	8.16e-01	1.58e+00	**5.69e-01**	2.24e+00	3.74e-01	1.35e+00	1.72e-01	8.06e-01	7.73e-02	6.55e-01	3.29e-02	3.17e-01	1.43e-02	1.61e-01	6.03e-03	1.01e-01	2.51e-03	**4.81e-02**
	LS-Uniform	**5.56e-01**	**1.25e+00**	5.81e-01	2.38e+00	**3.74e-01**	2.64e+00	1.37e-01	1.18e+00	8.55e-02	8.84e-01	3.78e-02	8.67e-01	1.60e-02	6.40e-01	1.19e-02	1.14e+00	7.67e-03	9.26e-01
	LS-Chebyshev	1.05e+00	1.97e+00	6.71e-01	**1.62e+00**	3.90e-01	**1.27e+00**	**1.22e-01**	**5.23e-01**	**5.15e-02**	**4.29e-01**	**2.37e-02**	**2.02e-01**	**1.10e-02**	**8.82e-02**	**4.60e-03**	**7.14e-02**	**1.99e-03**	5.20e-02
Morokoff Calfisch 1 $n = 25$	Smolyak	2.37e-01	4.55e-01	7.05e-02	2.49e-01	2.37e-02	9.16e-02	5.54e-03	2.78e-02	1.45e-03	9.13e-03	3.31e-04	2.67e-03	5.50e-05	5.68e-04	7.10e-06	1.38e-04	**6.67e-07**	**1.55e-05**
	LS-Uniform	**8.55e-02**	**1.59e-01**	**5.74e-02**	2.67e-01	**1.37e-02**	1.38e-01	**3.78e-03**	4.08e-02	**5.87e-04**	7.21e-03	**1.33e-04**	4.33e-03	**3.66e-05**	1.87e-03	**4.92e-06**	3.30e-04	9.31e-07	7.79e-05
	LS-Chebyshev	1.43e-01	2.55e-01	7.43e-02	**2.22e-01**	2.67e-02	**8.95e-02**	4.79e-03	**1.76e-02**	1.15e-03	**7.21e-03**	2.70e-04	**1.38e-03**	6.47e-05	**4.05e-04**	1.26e-05	**1.25e-04**	2.71e-06	1.61e-05
Morokoff Calfisch 2 $n = 25$	Smolyak	**3.30e-02**	**8.03e-02**	**1.46e-03**	**4.81e-03**	**3.39e-05**	**1.65e-04**	3.52e-15	1.42e-14	4.94e-15	**2.22e-14**	6.39e-15	**2.66e-14**	8.55e-15	**4.17e-14**	1.14e-14	**5.24e-14**	1.62e-14	**8.24e-14**
	LS-Uniform	3.89e-02	9.42e-02	3.28e-03	1.10e-02	7.70e-05	4.40e-04	**2.24e-15**	**1.20e-14**	**3.12e-15**	2.66e-14	**3.32e-15**	5.42e-14	**3.03e-15**	8.17e-14	**4.45e-15**	2.20e-13	**9.82e-15**	1.08e-12
	LS-Chebyshev	8.87e-02	1.52e-01	5.11e-03	1.57e-02	1.25e-04	4.39e-04	1.40e-14	7.68e-14	1.01e-14	8.70e-14	1.10e-14	1.31e-13	3.21e-14	2.69e-13	2.78e-14	4.44e-13	2.37e-14	7.15e-13
Oscillatory $n = 25$	Smolyak	**2.19e-01**	**5.82e-01**	**3.83e-02**	**1.38e-01**	**4.20e-03**	**2.17e-02**	3.49e-04	**1.54e-03**	1.10e-05	**3.91e-05**	1.79e-07	6.48e-07	1.63e-09	7.12e-09	8.65e-12	4.10e-11	2.95e-14	**1.70e-13**
	LS-Uniform	2.48e-01	7.03e-01	9.71e-02	3.47e-01	8.49e-03	4.88e-02	**2.63e-04**	2.43e-03	**6.04e-06**	7.96e-05	**5.63e-08**	1.34e-06	**4.88e-10**	1.44e-08	3.03e-12	1.55e-10	1.41e-14	9.29e-13
	LS-Chebyshev	4.17e-01	1.01e+00	1.37e-01	4.04e-01	1.29e-02	4.68e-02	4.23e-04	1.68e-03	7.53e-06	4.46e-05	6.86e-08	**3.79e-07**	5.19e-10	**3.56e-09**	**2.62e-12**	**1.82e-11**	**1.26e-14**	4.01e-13
Product Peak $n = 25$	Smolyak	7.79e-01	2.06e+00	**8.87e-02**	**3.66e-01**	**5.98e-03**	6.06e-02	**3.15e-15**	**1.33e-14**	**4.63e-15**	**3.55e-14**	**5.83e-15**	**3.82e-14**	**8.04e-15**	**4.88e-14**	1.10e-14	**1.09e-13**	1.43e-14	**1.31e-13**
	LS-Uniform	**6.71e-01**	**1.63e+00**	1.11e-01	5.15e-01	6.19e-03	**3.51e-02**	2.95e-14	1.43e-13	2.83e-14	1.78e-13	1.61e-14	2.49e-13	1.13e-14	2.68e-13	**6.70e-15**	2.07e-13	**1.13e-14**	1.27e-12
	LS-Chebyshev	1.52e+00	2.99e+00	2.15e-01	7.50e-01	1.09e-02	4.32e-02	2.26e-13	9.61e-13	9.69e-14	4.34e-13	2.80e-13	1.48e-12	2.75e-13	1.11e-12	1.77e-13	2.40e-12	6.74e-14	1.59e-12
Roos Arnold $n = 25$	Smolyak	2.10e+01	5.31e+01	**8.33e+00**	**1.94e+01**	**3.57e+00**	1.25e+01	1.67e+00	1.02e+01	6.89e-01	4.49e+00	2.88e-01	2.31e+00	**1.18e-01**	**1.35e+00**	**4.84e-02**	8.59e-01	**2.09e-02**	**4.72e-01**
	LS-Uniform	**1.56e+01**	**4.15e+01**	1.02e+01	3.86e+01	3.74e+00	2.66e+01	1.74e+00	1.34e+01	8.79e-01	1.06e+01	4.28e-01	9.58e+00	2.40e-01	4.41e+00	1.36e-01	5.80e+00	8.20e-02	1.16e+01
	LS-Chebyshev	3.49e+01	9.39e+01	1.05e+01	3.08e+01	3.77e+00	**9.94e+00**	**1.64e+00**	**6.40e+00**	**5.88e-01**	**3.77e+00**	**2.73e-01**	**2.17e+00**	1.29e-01	1.39e+00	5.48e-02	**7.43e-01**	2.37e-02	4.95e-01
Zhou $n = 25$	Smolyak	**8.18e+00**	**1.49e+01**	2.22e+00	**3.44e+00**	1.99e-01	**3.85e-01**	1.11e-02	2.88e-02	2.96e-04	**1.01e-03**	9.09e-06	3.63e-05	1.85e-07	7.88e-07	4.21e-09	2.07e-08	6.35e-11	3.75e-10
	LS-Uniform	9.31e+00	2.22e+01	**1.85e+00**	4.59e+00	1.69e-01	1.61e+00	8.41e-03	7.97e-02	3.52e-04	3.82e-03	7.55e-06	3.64e-04	1.00e-07	7.68e-06	1.52e-09	8.93e-08	2.14e-11	1.66e-09
	LS-Chebyshev	1.91e+01	3.32e+01	2.51e+00	5.60e+00	**1.57e-01**	4.67e-01	**7.04e-03**	**2.52e-02**	**2.28e-04**	1.09e-03	**3.43e-06**	**2.47e-05**	**5.93e-08**	**3.43e-07**	**1.13e-09**	**6.99e-09**	**1.76e-11**	**1.52e-10**

TABLE 3. Dim 4

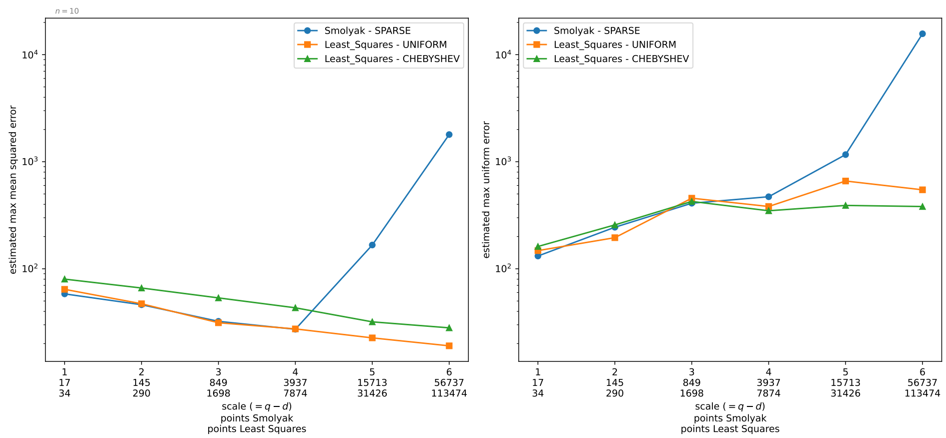

FIGURE 5. Visualization of the results for dim = 8 and various scales tested with $n = 25$ realizations for function class *Discontinuous*. Left plot shows the estimated max mean squared error and the right one shows the estimated max uniform error.

		Scale1 max		Scale2 max		Scale3 max		Scale4 max		Scale5 max		Scale6 max		Scale7 max		Scale8 max	
		ℓ_2	ℓ_∞	ℓ_2	ℓ_∞	ℓ_2	ℓ_∞	ℓ_2	ℓ_∞	ℓ_2	ℓ_∞	ℓ_2	ℓ_∞	ℓ_2	ℓ_∞	ℓ_2	ℓ_∞
Bratley $n = 25$	Smolyak	2.20e-01	5.04e-01	**6.23e-02**	**2.48e-01**	**2.57e-02**	**1.58e-01**	**1.79e-03**	**1.13e-02**	1.02e-13	3.61e-13	1.70e-12	8.84e-12	3.77e-13	2.12e-12	1.60e-10	9.96e-10
	LS-Uniform	**1.82e-01**	**3.83e-01**	1.13e-01	4.61e-01	3.97e-02	2.41e-01	3.26e-03	2.23e-02	**3.53e-15**	**3.22e-14**	**2.64e-15**	**4.57e-14**	**3.31e-15**	**8.05e-14**	**4.31e-15**	**1.37e-13**
	LS-Chebyshev	3.81e-01	8.67e-01	2.28e-01	5.86e-01	8.66e-02	4.19e-01	7.83e-03	4.88e-02	2.49e-14	2.31e-13	5.50e-14	3.42e-13	3.51e-14	4.18e-13	2.37e-14	6.87e-13
Continuous $n = 25$	Smolyak	1.24e-01	2.55e-01	4.79e-02	2.03e-01	2.96e-02	1.53e-01	1.27e-02	**9.53e-02**	3.69e-01	1.67e+00	4.26e-01	3.22e+00	1.28e-01	8.79e-01	3.14e+01	1.86e+02
	LS-Uniform	**1.08e-01**	**2.29e-01**	**4.55e-02**	**1.74e-01**	2.95e-02	1.59e-01	1.60e-02	1.28e-01	6.69e-03	6.55e-02	3.60e-03	6.00e-02	1.71e-03	4.57e-02	8.41e-04	4.29e-02
	LS-Chebyshev	1.37e-01	2.60e-01	5.23e-02	2.69e-01	**2.46e-02**	**1.42e-01**	**1.10e-02**	1.03e-01	**4.96e-03**	**6.10e-02**	**2.26e-03**	**4.49e-02**	**1.03e-03**	**2.44e-02**	**4.45e-04**	**1.30e-02**
Corner Peak $n = 25$	Smolyak	**1.13e-03**	**3.15e-03**	**5.78e-03**	2.98e-02	1.14e-02	8.92e-02	8.63e-03	1.43e-01	6.51e-03	8.86e-02	3.24e-03	7.53e-02	2.14e-03	3.72e-02	1.51e-02	8.48e-02
	LS-Uniform	2.28e-03	4.34e-03	7.08e-03	**1.87e-02**	**3.14e-03**	**2.87e-02**	**2.47e-03**	**4.56e-02**	**6.66e-04**	**1.98e-02**	**3.49e-04**	1.47e-02	**1.48e-04**	1.33e-02	**4.61e-05**	7.17e-03
	LS-Chebyshev	9.80e-03	2.22e-02	2.36e-02	6.71e-02	3.22e-02	1.49e-01	1.24e-02	6.98e-02	4.76e-03	2.46e-02	1.39e-03	**1.14e-02**	4.67e-04	**4.75e-03**	1.41e-04	**1.47e-03**
Discontinuous $n = 25$	Smolyak	1.10e+01	2.93e+01	**7.81e+00**	**3.17e+01**	7.04e+00	4.40e+01	**6.20e+00**	6.06e+01	8.37e+01	3.12e+02	2.73e+02	2.16e+03	4.15e+02	2.25e+03	1.73e+05	1.05e+06
	LS-Uniform	1.04e+01	2.49e+01	9.75e+00	3.34e+01	**6.97e+00**	4.17e+01	6.96e+00	6.86e+01	**4.79e+00**	5.05e+01	4.55e+00	9.36e+01	4.16e+00	1.21e+02	3.56e+00	1.83e+02
	LS-Chebyshev	**7.81e+00**	**1.42e+01**	8.88e+00	3.46e+01	8.53e+00	**3.01e+01**	6.35e+00	**4.80e+01**	5.25e+00	**4.20e+01**	**3.59e+00**	**4.21e+01**	**2.71e+00**	**4.33e+01**	**2.11e+00**	**4.60e+01**
Gaussian $n = 25$	Smolyak	1.74e-01	3.96e-01	4.58e-02	**1.07e-01**	1.10e-02	3.39e-02	1.79e-03	6.75e-03	2.78e-04	1.06e-03	2.65e-05	1.22e-04	2.19e-06	1.01e-05	1.26e-07	7.34e-07
	LS-Uniform	**1.45e-01**	**2.64e-01**	**3.99e-02**	1.71e-01	**7.90e-03**	5.69e-02	**1.23e-03**	1.44e-02	**1.13e-04**	2.11e-03	1.07e-05	3.54e-04	7.73e-07	6.02e-05	5.17e-08	9.34e-06
	LS-Chebyshev	2.08e-01	3.71e-01	5.26e-02	1.65e-01	1.05e-02	**3.20e-02**	1.32e-03	**6.55e-03**	1.41e-04	**6.09e-04**	**1.07e-05**	**5.90e-05**	**6.65e-07**	**4.58e-06**	**3.40e-08**	**2.70e-07**
G-Function $n = 25$	Smolyak	9.34e-01	2.42e+00	9.32e-01	4.26e+00	6.12e-01	3.56e+00	3.10e-01	1.78e+00	1.66e-01	1.34e+00	1.51e+00	1.10e+01	1.08e+00	5.74e+00	3.69e+02	2.36e+03
	LS-Uniform	**7.76e-01**	**1.93e+00**	**6.53e-01**	**2.16e+00**	**3.88e-01**	2.37e+00	**2.00e-01**	1.87e+00	9.20e-02	1.19e+00	4.74e-02	1.15e+00	2.37e-02	1.21e+00	1.32e-02	1.00e+00
	LS-Chebyshev	3.06e+00	4.38e+00	9.59e-01	2.99e+00	4.79e-01	**2.14e+00**	2.08e-01	**1.29e+00**	**8.72e-02**	**6.27e-01**	**4.27e-02**	**3.43e-01**	**2.00e-02**	**2.06e-01**	**9.17e-03**	**1.65e-01**
Morokoff Calfisch 1 $n = 25$	Smolyak	1.92e-01	4.54e-01	8.55e-02	2.74e-01	3.19e-02	1.27e-01	7.81e-03	5.18e-02	8.50e-02	4.56e-01	1.51e-01	8.10e-01	4.72e-02	2.65e-01	1.46e+00	9.02e+00
	LS-Uniform	**9.82e-02**	**1.95e-01**	**5.96e-02**	3.50e-01	**1.75e-02**	1.38e-01	**4.55e-03**	5.66e-02	**1.23e-03**	2.47e-02	**3.56e-04**	8.82e-03	**7.09e-05**	2.22e-03	**1.31e-05**	1.06e-03
	LS-Chebyshev	1.41e-01	2.73e-01	9.40e-02	**2.56e-01**	3.34e-02	**1.18e-01**	9.93e-03	**4.90e-02**	2.95e-03	**1.58e-02**	8.84e-04	**5.89e-03**	2.02e-04	**1.50e-03**	3.74e-05	**2.24e-04**
Morokoff Calfisch 2 $n = 25$	Smolyak	3.10e-02	7.67e-02	**1.22e-03**	**4.43e-03**	**4.04e-05**	**2.20e-04**	**9.97e-07**	**6.29e-06**	8.90e-14	4.64e-13	3.27e-13	2.42e-12	4.75e-13	2.74e-12	3.18e-10	2.08e-09
	LS-Uniform	**3.02e-02**	**6.12e-02**	2.46e-03	8.17e-03	7.14e-05	3.32e-04	1.81e-06	1.24e-05	**2.69e-15**	**2.22e-14**	**3.20e-15**	**3.82e-14**	**4.71e-15**	**1.41e-13**	**7.77e-15**	**3.76e-13**
	LS-Chebyshev	5.22e-02	1.12e-01	4.37e-03	1.35e-02	1.61e-04	5.52e-04	4.35e-06	2.71e-05	2.78e-14	3.45e-13	2.66e-14	4.57e-13	2.70e-14	1.06e-12	3.33e-14	1.70e-12
Oscillatory $n = 25$	Smolyak	**3.15e-01**	9.04e-01	**7.35e-02**	**2.58e-01**	**1.11e-02**	**6.58e-02**	**1.28e-03**	**8.47e-03**	1.25e-04	**6.51e-04**	5.16e-06	2.30e-05	1.15e-07	5.26e-07	1.58e-09	7.71e-09
	LS-Uniform	3.45e-01	**7.68e-01**	1.19e-01	5.17e-01	1.97e-02	1.07e-01	2.26e-03	1.49e-02	**8.43e-05**	1.13e-03	**2.34e-06**	1.37e-04	**3.20e-08**	1.97e-06	**3.53e-10**	3.37e-08
	LS-Chebyshev	5.98e-01	1.24e+00	1.66e-01	4.85e-01	4.24e-02	1.44e-01	5.24e-03	3.39e-02	1.69e-04	7.27e-04	3.18e-06	**1.44e-05**	3.95e-08	**2.22e-07**	3.93e-10	**3.02e-09**
Product Peak $n = 25$	Smolyak	1.96e+00	6.38e+00	4.01e-01	1.93e+00	**2.34e-02**	**1.69e-01**	1.15e-03	2.20e-02	3.65e-13	1.71e-12	2.69e-12	1.12e-11	2.22e-12	1.27e-11	1.60e-09	9.57e-09
	LS-Uniform	**1.42e+00**	**4.42e+00**	**2.61e-01**	**8.35e-01**	2.83e-02	1.74e-01	**1.04e-03**	**1.22e-02**	**5.12e-14**	**4.61e-13**	**2.38e-14**	**3.65e-13**	**3.07e-14**	**1.07e-12**	**7.67e-15**	**3.64e-13**
	LS-Chebyshev	2.08e+00	6.54e+00	5.26e-01	1.37e+00	5.08e-02	1.78e-01	2.58e-03	1.55e-02	5.13e-13	7.77e-12	1.37e-12	9.11e-12	3.24e-13	1.86e-12	1.98e-13	2.18e-12
Roos Arnold $n = 25$	Smolyak	4.81e+01	1.39e+02	2.75e+01	1.11e+02	1.36e+01	5.05e+01	7.56e+00	3.69e+01	5.24e+00	2.67e+01	2.27e+01	1.33e+02	5.31e+01	3.07e+02	1.78e+04	1.01e+05
	LS-Uniform	4.23e+01	9.66e+01	**1.91e+01**	**5.83e+01**	**9.11e+00**	6.26e+01	**4.85e+00**	3.29e+01	2.54e+00	2.96e+01	1.31e+00	2.46e+01	7.41e-01	1.58e+01	3.90e-01	1.84e+01
	LS-Chebyshev	**3.74e+01**	**6.83e+01**	2.34e+01	1.04e+02	1.20e+01	**4.54e+01**	5.02e+00	**2.21e+01**	**2.25e+00**	**1.23e+01**	**1.04e+00**	**7.88e+00**	**4.88e-01**	**6.26e+00**	**2.25e-01**	**3.52e+00**
Zhou $n = 25$	Smolyak	**4.91e+01**	9.92e+01	1.21e+01	**2.33e+01**	1.32e+00	**3.36e+00**	1.18e-01	3.96e-01	8.75e-03	3.14e-02	3.97e-04	1.70e-03	1.00e-05	6.08e-05	2.94e-07	1.90e-06
	LS-Uniform	5.70e+01	**9.60e+01**	**9.13e+00**	3.60e+01	**1.05e+00**	8.94e+00	**7.49e-02**	1.34e+00	**2.75e-03**	4.80e-02	1.44e-04	7.03e-03	4.67e-06	3.84e-04	**1.14e-07**	8.62e-06
	LS-Chebyshev	6.06e+01	1.10e+02	1.22e+01	3.29e+01	1.20e+00	4.06e+00	7.96e-02	**2.78e-01**	3.53e-03	**1.71e-02**	**1.38e-04**	**7.55e-04**	**4.10e-06**	**2.93e-05**	1.18e-07	**1.04e-06**

TABLE 4. Dim 5

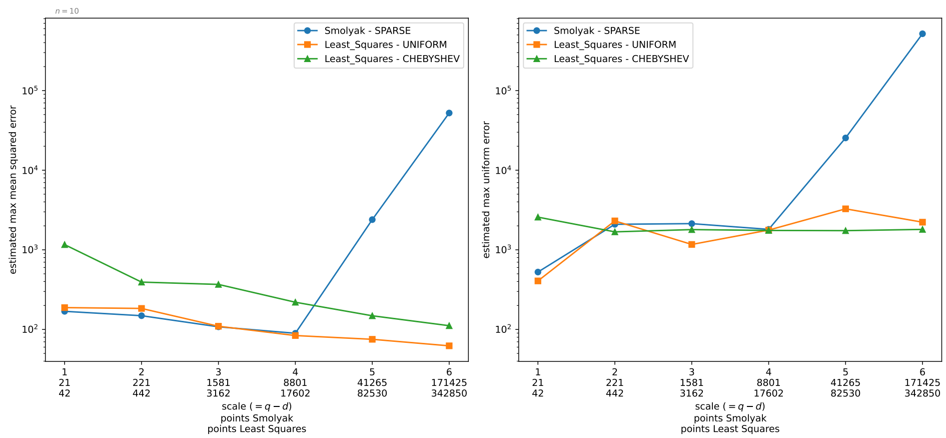

FIGURE 6. Visualization of the results for dim = 10 and various scales tested with $n = 10$ realizations for function class *Corner Peak*. Left plot shows the estimated max mean squared error and the right one shows the estimated max uniform error.

		Scale1 max ℓ_2	Scale1 max ℓ_∞	Scale2 max ℓ_2	Scale2 max ℓ_∞	Scale3 max ℓ_2	Scale3 max ℓ_∞	Scale4 max ℓ_2	Scale4 max ℓ_∞	Scale5 max ℓ_2	Scale5 max ℓ_∞	Scale6 max ℓ_2	Scale6 max ℓ_∞	Scale7 max ℓ_2	Scale7 max ℓ_∞
Bratley $n = 25$	Smolyak	**4.00e-01**	**9.16e-01**	**8.18e-02**	**3.02e-01**	**1.62e-02**	**1.00e-01**	**5.85e-03**	**5.23e-02**	**3.99e-04**	**4.80e-03**	5.21e-13	2.95e-12	2.57e-12	1.59e-11
	LS-Uniform	7.54e-01	1.69e+00	1.66e-01	6.66e-01	2.41e-02	1.67e-01	9.28e-03	9.77e-02	6.86e-04	8.63e-03	**3.10e-15**	**4.57e-14**	**2.26e-15**	**8.78e-14**
	LS-Chebyshev	9.47e-01	1.96e+00	2.21e-01	6.32e-01	6.27e-02	2.18e-01	2.41e-02	1.22e-01	2.58e-03	1.31e-02	5.38e-13	2.60e-12	2.52e-13	1.15e-12
Continuous $n = 25$	Smolyak	1.30e-01	**2.61e-01**	6.84e-02	**2.24e-01**	5.04e-02	1.57e-01	1.54e-02	1.25e-01	3.25e-01	8.38e-01	8.54e-01	4.18e+00	5.50e-01	3.29e+00
	LS-Uniform	**1.13e-01**	3.31e-01	4.96e-02	2.40e-01	2.27e-02	**1.49e-01**	9.41e-03	**8.68e-02**	4.82e-03	**5.43e-02**	2.69e-03	6.78e-02	1.37e-03	7.07e-02
	LS-Chebyshev	1.23e-01	3.71e-01	**4.95e-02**	3.36e-01	**2.12e-02**	2.17e-01	**8.18e-03**	1.39e-01	**3.98e-03**	9.18e-02	**1.87e-03**	**5.23e-02**	**8.57e-04**	**3.40e-02**
Corner Peak $n = 25$	Smolyak	**1.14e-03**	**4.11e-03**	**1.54e-03**	**1.22e-02**	2.81e-03	**2.15e-02**	4.71e-03	1.12e-01	4.98e-03	1.72e-01	4.10e-03	1.03e-01	3.07e-03	9.44e-02
	LS-Uniform	1.58e-03	4.29e-03	4.75e-03	2.00e-02	**2.71e-03**	2.87e-02	**1.45e-03**	**4.05e-02**	**6.61e-04**	3.78e-02	**3.29e-04**	2.24e-02	**1.25e-04**	1.15e-02
	LS-Chebyshev	3.58e-02	6.67e-02	1.85e-02	5.02e-02	6.43e-03	2.22e-02	5.10e-03	6.38e-02	4.18e-03	**2.31e-02**	1.90e-03	**1.81e-02**	7.66e-04	**9.90e-03**
Discontinuous $n = 25$	Smolyak	2.48e+01	**6.34e+01**	1.88e+01	9.58e+01	**1.22e+01**	1.32e+02	**8.69e+00**	1.22e+02	9.32e+02	2.87e+03	1.11e+03	7.38e+03	3.01e+03	1.97e+04
	LS-Uniform	**2.30e+01**	7.01e+01	**1.70e+01**	6.93e+01	1.38e+01	1.21e+02	1.10e+01	**8.31e+01**	9.22e+00	1.17e+02	7.75e+00	1.70e+02	6.76e+00	2.26e+02
	LS-Chebyshev	6.53e+01	1.04e+02	1.79e+01	**6.35e+01**	1.65e+01	**9.97e+01**	1.31e+01	9.02e+01	**8.91e+00**	**8.68e+01**	**6.67e+00**	**9.65e+01**	**5.49e+00**	**8.63e+01**
Gaussian $n = 25$	Smolyak	**1.14e-01**	3.09e-01	3.63e-02	**9.34e-02**	1.03e-02	**4.42e-02**	2.68e-03	**1.11e-02**	3.82e-04	2.08e-03	6.13e-05	3.13e-04	7.11e-06	4.26e-05
	LS-Uniform	1.53e-01	**2.91e-01**	**3.61e-02**	1.47e-01	**8.16e-03**	6.35e-02	**1.66e-03**	3.59e-02	**1.84e-04**	4.80e-03	3.21e-05	2.89e-03	**2.51e-06**	2.09e-04
	LS-Chebyshev	2.16e-01	4.85e-01	6.59e-02	1.93e-01	1.41e-02	5.01e-02	2.27e-03	1.16e-02	2.78e-04	**1.75e-03**	**3.07e-05**	**2.23e-04**	2.68e-06	**2.03e-05**
G-Function $n = 25$	Smolyak	2.27e+00	7.69e+00	1.23e+00	7.26e+00	7.87e-01	6.08e+00	4.96e-01	6.13e+00	8.36e-01	4.94e+00	1.50e+00	9.23e+00	4.10e+00	2.53e+01
	LS-Uniform	**1.63e+00**	**5.40e+00**	**1.08e+00**	**3.30e+00**	**6.38e-01**	3.14e+00	**3.14e-01**	**2.76e+00**	**1.41e-01**	2.04e+00	**8.01e-02**	1.63e+00	4.10e-02	1.15e+00
	LS-Chebyshev	5.40e+00	1.07e+01	2.04e+00	5.57e+00	8.05e-01	**2.95e+00**	4.24e-01	2.81e+00	1.83e-01	**1.45e+00**	8.16e-02	**6.10e-01**	**3.84e-02**	**5.31e-01**
Morokoff Calfisch 1 $n = 25$	Smolyak	2.30e-01	5.74e-01	8.29e-02	3.30e-01	3.12e-02	1.70e-01	1.09e-02	8.55e-02	9.26e-03	5.01e-02	3.08e-02	1.86e-01	2.86e-01	1.69e+00
	LS-Uniform	**1.01e-01**	**3.10e-01**	**3.11e-02**	**1.14e-01**	**1.25e-02**	**1.02e-01**	**2.65e-03**	**2.43e-02**	**7.37e-04**	2.02e-02	**2.57e-04**	7.12e-03	**6.80e-05**	4.16e-03
	LS-Chebyshev	2.41e-01	5.67e-01	6.65e-02	2.00e-01	2.80e-02	1.40e-01	8.45e-03	4.01e-02	2.52e-03	**1.10e-02**	7.87e-04	**4.84e-03**	2.16e-04	**1.63e-03**
Morokoff Calfisch 2 $n = 25$	Smolyak	**2.83e-02**	**7.71e-02**	**1.32e-03**	**5.28e-03**	**4.23e-05**	**3.38e-04**	**1.12e-06**	**1.24e-05**	**1.96e-08**	**2.37e-07**	1.95e-13	1.44e-12	1.21e-12	7.30e-12
	LS-Uniform	3.51e-02	7.94e-02	2.43e-03	8.95e-03	6.84e-05	4.06e-04	1.94e-06	2.34e-05	3.38e-08	4.26e-07	**4.94e-15**	**1.26e-13**	**6.78e-15**	**1.99e-13**
	LS-Chebyshev	6.65e-02	1.62e-01	4.36e-03	1.83e-02	1.74e-04	5.79e-04	4.79e-06	2.66e-05	1.27e-07	6.47e-07	8.07e-14	7.55e-13	6.57e-14	1.05e-12
Oscillatory $n = 25$	Smolyak	**3.65e-01**	1.22e+00	**1.13e-01**	**4.72e-01**	**2.11e-02**	**1.86e-01**	**3.82e-03**	**4.68e-02**	**3.88e-04**	**4.44e-03**	3.26e-05	**3.19e-04**	1.73e-06	1.33e-05
	LS-Uniform	4.78e-01	**1.05e+00**	1.97e-01	8.69e-01	3.77e-02	2.85e-01	6.30e-03	7.86e-02	4.91e-04	7.57e-03	**2.50e-05**	7.49e-04	**7.39e-07**	5.65e-05
	LS-Chebyshev	8.97e-01	2.40e+00	3.08e-01	9.02e-01	7.57e-02	2.97e-01	1.66e-02	7.36e-02	1.71e-03	1.04e-02	6.38e-05	3.89e-04	1.54e-06	**1.00e-05**
Product Peak $n = 25$	Smolyak	**2.07e+00**	**6.82e+00**	5.25e-01	4.40e+00	1.01e-01	1.44e+00	8.24e-03	2.01e-01	**3.46e-04**	1.08e-02	1.11e-11	8.36e-11	1.07e-10	6.57e-10
	LS-Uniform	2.07e+00	7.00e+00	**5.15e-01**	3.59e+00	**8.96e-02**	9.74e-01	**6.66e-03**	8.63e-02	4.53e-04	**6.30e-03**	**9.89e-14**	**1.36e-12**	**1.08e-13**	**4.15e-12**
	LS-Chebyshev	3.29e+00	8.02e+00	1.19e+00	**2.48e+00**	1.71e-01	**7.10e-01**	2.08e-02	**7.64e-02**	1.49e-03	6.52e-03	1.88e-11	1.05e-10	7.04e-12	5.61e-11
Roos Arnold $n = 25$	Smolyak	8.54e+01	2.66e+02	4.90e+01	2.23e+02	3.52e+01	2.20e+02	1.92e+01	1.21e+02	1.93e+01	7.50e+01	8.65e+01	4.64e+02	4.81e+02	2.93e+03
	LS-Uniform	**6.62e+01**	**1.86e+02**	**3.82e+01**	**1.83e+02**	**1.85e+01**	**1.03e+02**	**9.70e+00**	**7.18e+01**	**5.43e+00**	8.88e+01	3.09e+00	6.94e+01	1.72e+00	7.18e+01
	LS-Chebyshev	2.85e+02	4.46e+02	7.91e+01	2.32e+02	2.90e+01	1.46e+02	1.37e+01	8.41e+01	6.28e+00	**4.12e+01**	**2.82e+00**	**2.47e+01**	**1.41e+00**	**1.50e+01**
Zhou $n = 25$	Smolyak	**1.81e+02**	**4.06e+02**	4.78e+01	**1.14e+02**	7.94e+00	**2.70e+01**	1.06e+00	3.16e+00	8.33e-02	2.79e-01	3.81e-03	1.93e-02	2.34e-04	1.13e-03
	LS-Uniform	3.26e+02	7.14e+02	**3.90e+01**	1.92e+02	**5.70e+00**	6.33e+01	**4.40e-01**	7.11e+00	**2.69e-02**	6.56e-01	**1.59e-03**	6.75e-02	**8.66e-05**	7.67e-03
	LS-Chebyshev	5.56e+02	1.12e+03	6.57e+01	1.98e+02	8.92e+00	2.84e+01	6.47e-01	**2.56e+00**	3.81e-02	**1.82e-01**	2.26e-03	**1.19e-02**	1.09e-04	**8.66e-04**

TABLE 5. Dim 6

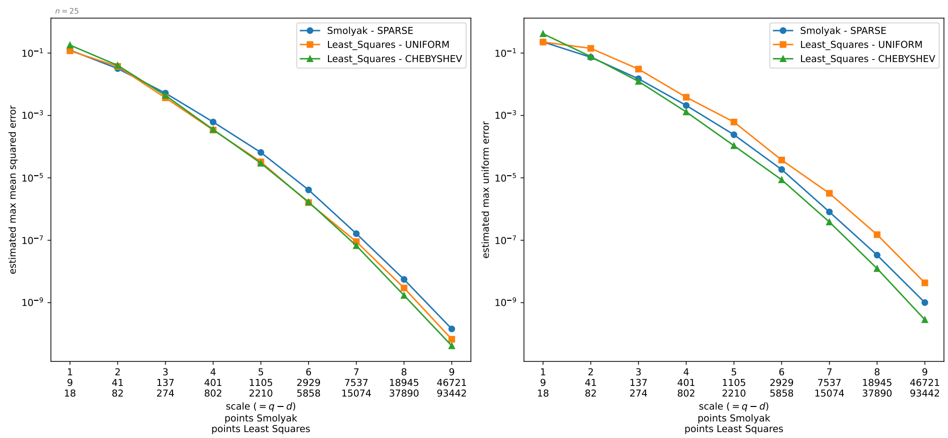

FIGURE 7. Visualization of the results for dim = 4 and various scales tested with $n = 25$ realizations for function class *Gaussian*. Left plot shows the estimated max mean squared error and the right one shows the estimated max uniform error.

		Scale1 max ℓ_2	Scale1 max ℓ_∞	Scale2 max ℓ_2	Scale2 max ℓ_∞	Scale3 max ℓ_2	Scale3 max ℓ_∞	Scale4 max ℓ_2	Scale4 max ℓ_∞	Scale5 max ℓ_2	Scale5 max ℓ_∞	Scale6 max ℓ_2	Scale6 max ℓ_∞	Scale7 max ℓ_2	Scale7 max ℓ_∞
Bratley $n = 20$	Smolyak	**6.31e-01**	**1.77e+00**	**1.68e-01**	**6.59e-01**	**2.53e-02**	**2.08e-01**	**6.48e-03**	**8.83e-02**	**1.14e-03**	**2.16e-02**	**1.30e-04**	**3.11e-03**	**8.31e-11**	**7.60e-10**
	LS-Uniform	7.98e-01	2.74e+00	1.99e-01	1.02e+00	4.17e-02	3.51e-01	1.21e-02	1.92e-01	2.08e-03	4.26e-02	2.52e-04	8.44e-03	3.80e-09	9.67e-07
	LS-Chebyshev	1.31e+00	2.66e+00	4.26e-01	1.38e+00	7.72e-02	3.80e-01	2.23e-02	1.34e-01	4.73e-03	5.62e-02	6.55e-04	1.90e-02	9.29e-08	1.81e-06
Continuous $n = 20$	Smolyak	1.18e-01	3.24e-01	5.94e-02	**1.93e-01**	3.58e-02	1.76e-01	1.64e-02	1.49e-01	2.01e-01	1.64e+00	2.67e-01	2.24e+00	3.08e+00	2.67e+01
	LS-Uniform	**6.66e-02**	**1.88e-01**	**3.29e-02**	2.18e-01	**1.53e-02**	**1.51e-01**	7.28e-03	**1.14e-01**	3.65e-03	9.41e-02	1.89e-03	**4.96e-02**	9.26e-04	4.39e-02
	LS-Chebyshev	1.74e-01	3.22e-01	4.53e-02	2.08e-01	1.72e-02	2.01e-01	**7.02e-03**	1.55e-01	**3.33e-03**	**8.45e-02**	**1.59e-03**	6.32e-02	**7.71e-04**	**4.04e-02**
Corner Peak $n = 20$	Smolyak	**4.58e-04**	**2.47e-03**	**9.86e-04**	1.47e-02	9.43e-04	2.78e-02	4.04e-03	7.65e-02	6.07e-03	1.22e-01	4.61e-03	1.09e-01	2.90e-03	1.59e-01
	LS-Uniform	4.58e-04	2.48e-03	1.10e-03	**8.19e-03**	**8.01e-04**	**2.66e-02**	**4.65e-04**	**1.12e-02**	**4.04e-04**	5.40e-02	**1.83e-04**	4.35e-02	**1.42e-04**	4.51e-02
	LS-Chebyshev	1.21e-03	2.83e-03	2.42e-03	1.17e-02	1.08e-03	2.77e-02	9.72e-04	1.21e-02	6.60e-04	**5.06e-02**	3.31e-04	**1.68e-02**	2.22e-04	**1.03e-02**
Discontinuous $n = 20$	Smolyak	**2.61e+01**	1.30e+02	**3.01e+01**	2.32e+02	**2.31e+01**	3.28e+02	**1.75e+01**	2.84e+02	2.20e+02	1.74e+03	1.12e+03	8.56e+03	2.06e+04	1.79e+05
	LS-Uniform	4.48e+01	1.14e+02	3.56e+01	**1.52e+02**	2.80e+01	**2.17e+02**	2.13e+01	2.95e+02	**1.86e+01**	5.64e+02	**1.53e+01**	5.81e+02	**1.24e+01**	4.82e+02
	LS-Chebyshev	3.98e+01	**1.04e+02**	5.63e+01	1.77e+02	5.05e+01	3.24e+02	3.33e+01	**2.11e+02**	2.84e+01	**4.65e+02**	2.19e+01	**3.15e+02**	1.96e+01	**2.96e+02**
Gaussian $n = 20$	Smolyak	**1.27e-01**	5.09e-01	**5.25e-02**	**1.98e-01**	2.04e-02	8.19e-02	5.99e-03	2.55e-02	1.03e-03	5.48e-03	1.04e-04	7.67e-04	1.09e-05	9.11e-05
	LS-Uniform	1.64e-01	4.80e-01	5.76e-02	3.60e-01	**1.43e-02**	1.76e-01	**3.02e-03**	5.33e-02	**4.11e-04**	1.40e-02	**4.38e-05**	3.14e-03	**3.70e-06**	4.87e-04
	LS-Chebyshev	2.16e-01	**4.32e-01**	9.13e-02	3.07e-01	1.97e-02	**7.26e-02**	3.65e-03	**1.72e-02**	4.95e-04	**4.03e-03**	4.84e-05	**5.75e-04**	5.06e-06	**7.89e-05**
G-Function $n = 20$	Smolyak	2.66e+00	1.03e+01	2.83e+00	3.10e+01	1.33e+00	1.92e+01	8.36e-01	1.01e+01	5.09e-01	7.42e+00	8.11e-01	7.16e+00	3.41e+01	2.99e+02
	LS-Uniform	**1.64e+00**	**7.09e+00**	**1.42e+00**	1.52e+01	**6.32e-01**	**4.80e+00**	3.28e-01	4.31e+00	1.75e-01	3.53e+00	9.32e-02	2.01e+00	4.96e-02	2.84e+00
	LS-Chebyshev	5.62e+00	1.07e+01	2.09e+00	**1.32e+01**	7.16e-01	4.88e+00	**3.12e-01**	**2.53e+00**	**1.50e-01**	**2.01e+00**	**7.47e-02**	**1.05e+00**	**3.74e-02**	**6.95e-01**
Morokoff Calfisch 1 $n = 20$	Smolyak	1.84e-01	4.04e-01	3.86e-02	1.27e-01	1.01e-02	4.78e-02	1.42e-03	1.25e-02	3.84e-03	2.53e-02	1.88e-03	1.41e-02	5.08e-03	4.83e-02
	LS-Uniform	**1.03e-01**	**3.60e-01**	**2.60e-02**	1.61e-01	**5.38e-03**	6.23e-02	**1.05e-03**	1.55e-02	**1.77e-04**	4.89e-03	**2.95e-05**	8.68e-04	**4.20e-06**	2.16e-04
	LS-Chebyshev	1.48e-01	3.91e-01	3.69e-02	**1.26e-01**	8.74e-03	**4.46e-02**	1.38e-03	**8.73e-03**	3.17e-04	**2.45e-03**	7.67e-05	**6.24e-04**	1.08e-05	**1.53e-04**
Morokoff Calfisch 2 $n = 20$	Smolyak	**1.15e-02**	**3.17e-02**	**6.20e-04**	4.55e-03	**2.48e-05**	**3.31e-04**	**7.38e-07**	**1.32e-05**	**1.65e-08**	**2.89e-07**	**1.97e-10**	**3.94e-09**	**1.30e-11**	**1.05e-10**
	LS-Uniform	1.68e-02	4.60e-02	8.95e-04	**4.33e-03**	3.76e-05	6.02e-04	1.25e-06	1.67e-05	3.04e-08	7.25e-07	3.72e-10	1.09e-08	4.18e-11	2.60e-09
	LS-Chebyshev	2.28e-02	5.42e-02	1.40e-03	7.15e-03	6.60e-05	9.22e-04	2.49e-06	2.90e-05	6.96e-08	2.74e-06	1.79e-09	2.85e-08	2.62e-09	4.70e-08
Oscillatory $n = 20$	Smolyak	**3.66e-01**	1.70e+00	**1.30e-01**	1.05e+00	**3.83e-02**	**4.20e-01**	**6.64e-03**	**1.33e-01**	**1.22e-03**	**2.06e-02**	**1.22e-04**	**1.83e-03**	9.23e-06	**1.46e-04**
	LS-Uniform	5.29e-01	**1.49e+00**	2.02e-01	**9.42e-01**	5.51e-02	6.55e-01	1.20e-02	1.58e-01	2.33e-03	5.04e-02	1.56e-04	6.58e-03	**5.53e-06**	5.14e-04
	LS-Chebyshev	1.02e+00	1.98e+00	3.74e-01	1.75e+00	9.50e-02	9.65e-01	2.44e-02	2.34e-01	4.75e-03	1.75e-01	3.18e-04	1.09e-02	1.30e-05	3.79e-04
Product Peak $n = 20$	Smolyak	**1.85e-02**	**5.98e-02**	**5.82e-03**	**2.65e-02**	**1.56e-03**	**1.11e-02**	3.50e-04	3.71e-03	7.07e-05	7.34e-04	1.22e-05	1.47e-04	1.86e-06	2.38e-05
	LS-Uniform	2.83e-02	8.55e-02	7.92e-03	3.06e-02	1.78e-03	2.08e-02	**2.65e-04**	5.21e-03	**4.18e-05**	1.39e-03	**5.64e-06**	4.92e-04	**8.04e-07**	9.14e-05
	LS-Chebyshev	3.13e-02	8.76e-02	1.32e-02	5.36e-02	2.66e-03	1.59e-02	4.31e-04	**2.68e-03**	6.17e-05	**5.02e-04**	7.73e-06	**8.25e-05**	8.51e-07	**1.27e-05**
Roos Arnold $n = 20$	Smolyak	1.74e+02	8.26e+02	**8.71e+01**	1.12e+03	6.68e+01	1.03e+03	3.97e+01	4.99e+02	2.21e+01	3.81e+02	1.37e+02	1.12e+03	2.12e+03	1.90e+04
	LS-Uniform	**1.60e+02**	7.44e+02	9.45e+01	6.56e+02	**4.17e+01**	3.81e+02	**2.33e+01**	**2.79e+02**	**1.28e+01**	2.80e+02	**6.60e+00**	1.58e+02	3.49e+00	1.70e+02
	LS-Chebyshev	3.08e+02	**7.12e+02**	1.58e+02	**5.14e+02**	6.37e+01	**3.54e+02**	3.26e+01	2.86e+02	1.42e+01	**1.50e+02**	6.65e+00	**9.01e+01**	**3.24e+00**	**7.67e+01**
Zhou $n = 20$	Smolyak	**7.56e+02**	**2.32e+03**	**1.89e+02**	**6.25e+02**	3.75e+01	**1.28e+02**	5.00e+00	**2.11e+01**	5.54e-01	2.35e+00	5.79e-02	2.62e-01	4.21e-03	2.06e-02
	LS-Uniform	1.03e+03	2.62e+03	2.51e+02	1.67e+03	**2.61e+01**	2.49e+02	**2.94e+00**	7.44e+01	**2.96e-01**	1.17e+01	**2.44e-02**	1.39e+00	**1.34e-03**	8.32e-02
	LS-Chebyshev	1.97e+03	4.34e+03	3.16e+02	9.56e+02	3.96e+01	1.62e+02	4.44e+00	2.27e+01	3.97e-01	**2.11e+00**	2.89e-02	**1.83e-01**	1.64e-03	**1.12e-02**

TABLE 6. Dim 7

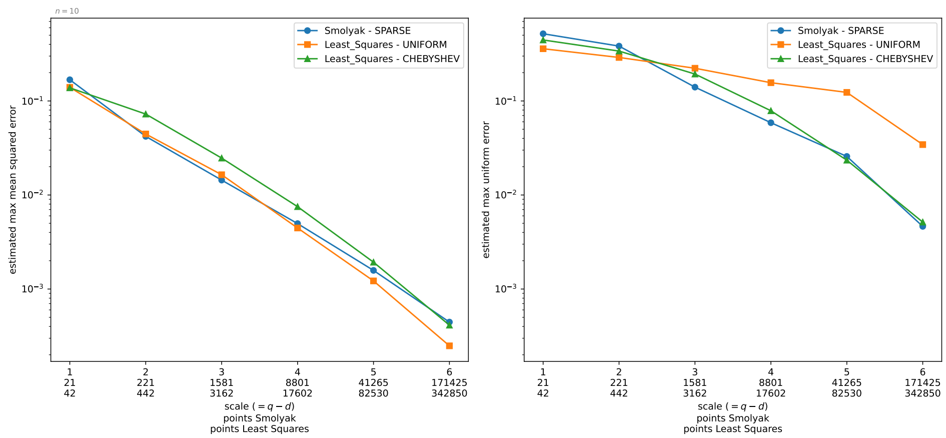

FIGURE 8. Visualization of the results for dim = 10 and various scales tested with $n = 10$ realizations for function class *Gaussian*. Left plot shows the estimated max mean squared error and the right one shows the estimated max uniform error.

		Scale1 max ℓ_2	Scale1 max ℓ_∞	Scale2 max ℓ_2	Scale2 max ℓ_∞	Scale3 max ℓ_2	Scale3 max ℓ_∞	Scale4 max ℓ_2	Scale4 max ℓ_∞	Scale5 max ℓ_2	Scale5 max ℓ_∞	Scale6 max ℓ_2	Scale6 max ℓ_∞
Bratley $n = 10$	Smolyak	**2.21e-01**	**6.10e-01**	**9.08e-02**	**3.36e-01**	**2.45e-02**	2.73e-01	**9.25e-03**	**1.58e-01**	**1.77e-03**	**3.56e-02**	**2.55e-04**	**7.25e-03**
	LS-Uniform	3.56e-01	9.09e-01	1.45e-01	8.21e-01	3.58e-02	**2.27e-01**	1.52e-02	2.24e-01	3.06e-03	6.61e-02	4.65e-04	1.89e-02
	LS-Chebyshev	7.81e-01	1.77e+00	2.26e-01	7.64e-01	6.60e-02	4.70e-01	2.91e-02	3.87e-01	7.25e-03	1.25e-01	1.29e-03	1.68e-02
Continuous $n = 10$	Smolyak	**8.52e-02**	**3.01e-01**	**3.04e-02**	2.49e-01	2.08e-02	2.16e-01	8.93e-03	1.35e-01	1.19e-01	7.79e-01	3.17e-01	2.58e+00
	LS-Uniform	1.17e-01	3.28e-01	3.19e-02	**2.21e-01**	**1.62e-02**	**1.42e-01**	8.46e-03	**9.53e-02**	4.25e-03	**8.13e-02**	2.17e-03	**6.28e-02**
	LS-Chebyshev	1.25e-01	4.27e-01	3.16e-02	2.85e-01	1.62e-02	1.92e-01	**7.84e-03**	1.58e-01	**3.58e-03**	1.49e-01	**1.80e-03**	7.73e-02
Corner Peak $n = 10$	Smolyak	**3.79e-05**	2.09e-04	**3.90e-05**	6.27e-04	**1.91e-05**	**6.36e-04**	7.45e-05	3.16e-03	2.98e-04	2.20e-02	7.17e-04	8.41e-02
	LS-Uniform	3.21e-04	7.33e-04	3.05e-04	2.25e-03	1.42e-04	2.31e-03	**6.64e-05**	**2.30e-03**	**5.85e-05**	9.05e-03	**3.90e-05**	7.56e-03
	LS-Chebyshev	4.89e-05	**1.96e-04**	1.20e-04	**6.06e-04**	2.30e-04	1.76e-03	1.55e-04	5.73e-03	1.62e-04	**4.95e-03**	1.53e-04	**4.48e-03**
Discontinuous $n = 10$	Smolyak	**5.83e+01**	**1.32e+02**	**4.62e+01**	2.44e+02	3.23e+01	**4.10e+02**	**2.72e+01**	4.71e+02	1.67e+02	1.17e+03	1.79e+03	1.57e+04
	LS-Uniform	6.43e+01	1.48e+02	4.72e+01	**1.95e+02**	**3.13e+01**	4.57e+02	2.74e+01	3.82e+02	**2.26e+01**	6.61e+02	**1.91e+01**	5.47e+02
	LS-Chebyshev	8.01e+01	1.62e+02	6.62e+01	2.57e+02	5.35e+01	4.26e+02	4.32e+01	**3.48e+02**	3.20e+01	**3.91e+02**	2.81e+01	**3.82e+02**
Gaussian $n = 10$	Smolyak	**1.43e-01**	4.33e-01	**4.47e-02**	**2.08e-01**	1.66e-02	**9.36e-02**	5.26e-03	**2.73e-02**	1.38e-03	7.80e-03	2.54e-04	1.54e-03
	LS-Uniform	1.47e-01	**4.30e-01**	6.12e-02	4.46e-01	**1.65e-02**	1.85e-01	**3.64e-03**	9.65e-02	**6.60e-04**	3.80e-02	**9.44e-05**	6.00e-03
	LS-Chebyshev	1.68e-01	5.81e-01	6.78e-02	2.90e-01	2.39e-02	1.51e-01	5.44e-03	3.81e-02	9.19e-04	**6.05e-03**	1.26e-04	**1.25e-03**
G-Function $n = 10$	Smolyak	**2.57e+00**	1.11e+01	2.51e+00	2.38e+01	1.49e+00	2.39e+01	1.19e+00	1.84e+01	7.56e-01	1.61e+01	9.98e-01	1.02e+01
	LS-Uniform	4.69e+00	**1.09e+01**	**1.76e+00**	1.73e+01	**8.08e-01**	**7.67e+00**	**3.99e-01**	5.33e+00	**2.15e-01**	4.03e+00	**1.20e-01**	5.01e+00
	LS-Chebyshev	7.42e+00	1.54e+01	2.65e+00	**1.05e+01**	1.08e+00	9.30e+00	5.18e-01	**4.22e+00**	2.56e-01	**3.29e+00**	1.43e-01	**1.73e+00**
Morokoff Calfisch 1 $n = 10$	Smolyak	1.01e-01	2.95e-01	1.94e-02	7.63e-02	2.75e-03	2.04e-02	4.38e-04	5.73e-03	6.61e-05	1.14e-03	1.53e-05	2.17e-04
	LS-Uniform	**6.37e-02**	**1.81e-01**	**1.10e-02**	**4.12e-02**	**1.54e-03**	2.05e-02	**1.58e-04**	**1.85e-03**	**1.49e-05**	3.44e-04	**1.67e-06**	9.00e-05
	LS-Chebyshev	7.90e-02	1.98e-01	1.65e-02	6.66e-02	2.39e-03	**1.12e-02**	2.38e-04	2.05e-03	2.64e-05	**2.83e-04**	3.77e-06	**4.51e-05**
Morokoff Calfisch 2 $n = 10$	Smolyak	**1.03e-02**	**3.80e-02**	**5.32e-04**	**2.91e-03**	**2.04e-05**	**1.53e-04**	**5.76e-07**	**1.08e-05**	**1.49e-08**	**4.69e-07**	**2.91e-10**	**8.54e-09**
	LS-Uniform	2.02e-02	5.98e-02	9.42e-04	3.76e-03	3.30e-05	2.58e-04	9.52e-07	1.34e-05	2.73e-08	7.68e-07	5.46e-10	2.39e-08
	LS-Chebyshev	2.53e-02	8.65e-02	1.50e-03	7.68e-03	6.37e-05	4.61e-04	2.12e-06	4.99e-05	6.23e-08	1.62e-06	3.51e-09	3.92e-08
Oscillatory $n = 10$	Smolyak	**4.61e-01**	2.22e+00	**1.60e-01**	**1.14e+00**	**4.33e-02**	**4.17e-01**	**1.18e-02**	**2.84e-01**	**1.77e-03**	**6.05e-02**	**2.47e-04**	**7.27e-03**
	LS-Uniform	6.25e-01	**1.93e+00**	2.68e-01	1.50e+00	6.82e-02	5.12e-01	1.88e-02	4.30e-01	3.04e-03	1.13e-01	4.47e-04	2.42e-02
	LS-Chebyshev	6.61e-01	2.08e+00	3.85e-01	1.93e+00	1.01e-01	7.33e-01	4.09e-02	8.28e-01	5.46e-03	1.99e-01	1.14e-03	3.26e-02
Product Peak $n = 10$	Smolyak	3.89e-02	1.24e-01	**7.72e-03**	**4.67e-02**	1.84e-03	**1.24e-02**	3.26e-04	**4.62e-03**	5.97e-05	**7.33e-04**	1.09e-05	1.99e-04
	LS-Uniform	**3.25e-02**	**1.10e-01**	9.21e-03	8.89e-02	**1.47e-03**	2.12e-02	**2.42e-04**	4.79e-03	**4.20e-05**	1.94e-03	**7.57e-06**	7.92e-04
	LS-Chebyshev	4.17e-02	1.32e-01	1.11e-02	4.68e-02	2.20e-03	1.97e-02	3.89e-04	4.74e-03	6.18e-05	9.90e-04	9.33e-06	**1.85e-04**
Roos Arnold $n = 10$	Smolyak	1.42e+03	8.09e+03	**5.08e+02**	4.07e+03	**2.67e+02**	**2.49e+03**	1.62e+02	2.72e+03	9.16e+01	1.88e+03	3.18e+02	2.78e+03
	LS-Uniform	1.43e+03	**7.00e+03**	5.84e+02	3.85e+03	2.79e+02	2.99e+03	**1.22e+02**	2.37e+03	**5.59e+01**	1.25e+03	**2.91e+01**	7.71e+02
	LS-Chebyshev	**1.35e+03**	7.02e+03	7.12e+02	**3.23e+03**	3.30e+02	2.66e+03	1.49e+02	**1.21e+03**	6.79e+01	**9.95e+02**	3.16e+01	**6.20e+02**
Zhou $n = 10$	Smolyak	**3.91e+03**	**1.46e+04**	1.08e+03	**2.74e+03**	2.26e+02	**7.50e+02**	2.93e+01	1.58e+02	3.46e+00	1.85e+01	3.97e-01	2.08e+00
	LS-Uniform	6.87e+03	1.93e+04	**1.02e+03**	5.53e+03	**2.10e+02**	1.76e+03	**2.08e+01**	4.64e+02	**1.73e+00**	6.85e+01	**1.51e-01**	1.97e+01
	LS-Chebyshev	5.76e+03	1.47e+04	1.37e+03	4.51e+03	2.35e+02	1.26e+03	2.76e+01	**1.20e+02**	2.48e+00	**1.70e+01**	2.07e-01	**1.51e+00**

TABLE 7. Dim 8

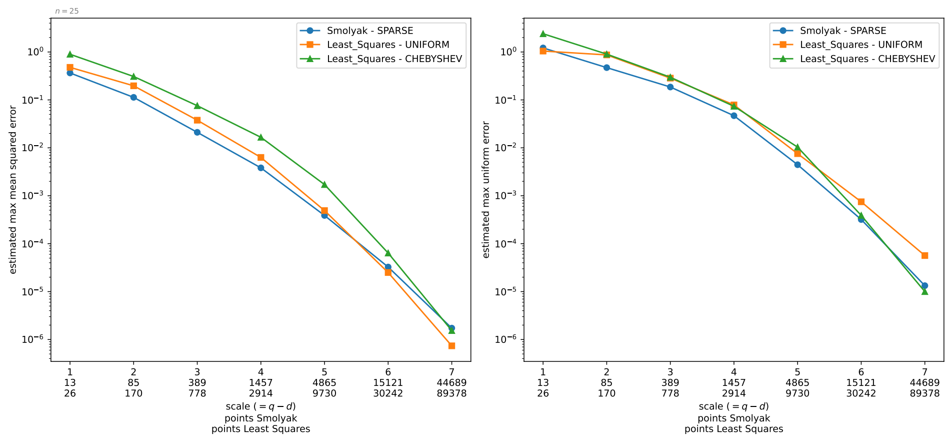

FIGURE 9. Visualization of the results for dim = 6 and various scales tested with $n = 25$ realizations for function class *Oscillatory*. Left plot shows the estimated max mean squared error and the right one shows the estimated max uniform error.

		Scale1 max ℓ_2	Scale1 max ℓ_∞	Scale2 max ℓ_2	Scale2 max ℓ_∞	Scale3 max ℓ_2	Scale3 max ℓ_∞	Scale4 max ℓ_2	Scale4 max ℓ_∞	Scale5 max ℓ_2	Scale5 max ℓ_∞	Scale6 max ℓ_2	Scale6 max ℓ_∞
Bratley $n = 25$	Smolyak	**4.23e-01**	**7.52e-01**	**1.29e-01**	**6.43e-01**	**4.45e-02**	**2.98e-01**	**9.67e-03**	**1.14e-01**	**2.23e-03**	**4.88e-02**	**3.82e-04**	**1.50e-02**
	LS-Uniform	5.79e-01	1.69e+00	2.07e-01	9.54e-01	6.53e-02	4.35e-01	1.56e-02	1.80e-01	3.71e-03	9.71e-02	6.86e-04	2.69e-02
	LS-Chebyshev	1.65e+00	4.10e+00	4.29e-01	1.28e+00	1.83e-01	6.48e-01	5.27e-02	2.70e-01	1.54e-02	8.49e-02	3.57e-03	2.84e-02
Continuous $n = 25$	Smolyak	**5.84e-02**	1.84e-01	3.22e-02	2.26e-01	3.09e-02	2.10e-01	1.37e-02	2.31e-01	6.23e-03	1.65e-01	3.45e-03	1.06e-01
	LS-Uniform	7.41e-02	**1.75e-01**	**2.96e-02**	**2.15e-01**	**1.44e-02**	**1.87e-01**	**7.63e-03**	**1.48e-01**	**3.81e-03**	**9.10e-02**	2.01e-03	**9.37e-02**
	LS-Chebyshev	1.20e-01	2.15e-01	3.37e-02	2.81e-01	1.75e-02	2.30e-01	8.10e-03	2.39e-01	3.83e-03	1.36e-01	**1.83e-03**	9.73e-02
Corner Peak $n = 25$	Smolyak	3.04e-06	1.33e-05	**4.70e-06**	**4.94e-05**	**9.96e-06**	3.36e-04	**3.59e-05**	2.78e-03	7.27e-05	3.67e-03	3.50e-04	3.70e-02
	LS-Uniform	**3.01e-06**	**1.30e-05**	1.40e-05	1.11e-04	1.12e-05	**2.87e-04**	4.41e-05	**2.55e-03**	**2.14e-05**	**2.34e-03**	**3.66e-05**	**1.15e-02**
	LS-Chebyshev	4.45e-04	6.74e-04	2.04e-04	6.48e-04	3.54e-04	1.29e-03	1.07e-03	1.03e-02	1.03e-03	1.08e-02	1.57e-03	2.96e-02
Discontinuous $n = 25$	Smolyak	**1.86e+02**	**6.52e+02**	**1.09e+02**	6.76e+02	**7.13e+01**	8.03e+02	**5.40e+01**	**8.61e+02**	**4.59e+01**	1.58e+03	**3.54e+01**	1.53e+03
	LS-Uniform	2.43e+02	9.66e+02	1.39e+02	7.04e+02	8.33e+01	**6.98e+02**	7.50e+01	1.00e+03	6.16e+01	1.48e+03	5.24e+01	1.74e+03
	LS-Chebyshev	5.89e+02	1.18e+03	1.59e+02	**6.21e+02**	1.30e+02	1.36e+03	9.55e+01	1.05e+03	7.68e+01	**1.20e+03**	5.85e+01	**1.17e+03**
Gaussian $n = 25$	Smolyak	1.74e-01	5.90e-01	6.51e-02	3.16e-01	2.33e-02	1.87e-01	7.40e-03	6.08e-02	1.72e-03	1.89e-02	4.46e-04	4.04e-03
	LS-Uniform	**1.38e-01**	**3.03e-01**	**5.18e-02**	**2.05e-01**	**1.60e-02**	1.54e-01	**4.36e-03**	9.49e-02	**8.34e-04**	3.40e-02	**1.46e-04**	1.21e-02
	LS-Chebyshev	2.42e-01	4.86e-01	8.55e-02	3.43e-01	2.57e-02	**1.37e-01**	6.30e-03	**4.01e-02**	1.33e-03	**1.02e-02**	2.25e-04	**2.25e-03**
G-Function $n = 25$	Smolyak	**2.07e+00**	7.90e+00	2.69e+00	2.18e+01	1.94e+00	2.43e+01	1.50e+00	3.58e+01	1.05e+00	3.46e+01	6.75e-01	1.99e+01
	LS-Uniform	3.96e+00	**7.81e+00**	**1.82e+00**	**9.22e+00**	**1.00e+00**	**1.02e+01**	**5.77e-01**	**8.25e+00**	**3.34e-01**	7.85e+00	**1.88e-01**	6.19e+00
	LS-Chebyshev	1.73e+01	3.14e+01	6.00e+00	1.85e+01	3.21e+00	1.22e+01	1.74e+00	9.25e+00	8.41e-01	**5.52e+00**	4.05e-01	**3.97e+00**
Morokoff Calfisch 1 $n = 25$	Smolyak	2.90e-01	7.01e-01	8.59e-02	2.55e-01	2.87e-02	1.44e-01	8.41e-03	7.06e-02	2.36e-03	2.26e-02	6.38e-04	7.12e-03
	LS-Uniform	**7.18e-02**	**1.86e-01**	**3.45e-02**	**1.35e-01**	**1.33e-02**	**1.13e-01**	**2.92e-03**	3.67e-02	**7.66e-04**	3.91e-02	**1.91e-04**	1.36e-02
	LS-Chebyshev	3.28e-01	5.50e-01	1.03e-01	2.64e-01	3.69e-02	1.32e-01	8.70e-03	**3.63e-02**	2.48e-03	**1.20e-02**	7.74e-04	**5.69e-03**
Morokoff Calfisch 2 $n = 25$	Smolyak	**2.08e-02**	**6.39e-02**	**8.67e-04**	**5.64e-03**	**3.03e-05**	**3.66e-04**	**7.92e-07**	**1.68e-05**	**1.76e-08**	**6.13e-07**	**3.21e-10**	**1.19e-08**
	LS-Uniform	2.45e-02	8.13e-02	1.56e-03	7.83e-03	4.88e-05	3.83e-04	1.32e-06	2.15e-05	2.98e-08	7.43e-07	5.92e-10	2.16e-08
	LS-Chebyshev	6.49e-02	1.59e-01	2.86e-03	9.68e-03	1.32e-04	6.62e-04	4.74e-06	3.12e-05	1.26e-07	1.26e-06	2.96e-09	2.03e-08
Oscillatory $n = 25$	Smolyak	7.99e-01	2.40e+00	**2.31e-01**	**1.32e+00**	**7.70e-02**	1.09e+00	**1.77e-02**	**4.51e-01**	**3.87e-03**	**1.19e-01**	**5.91e-04**	**1.79e-02**
	LS-Uniform	**6.98e-01**	**1.90e+00**	2.72e-01	1.61e+00	1.18e-01	**1.08e+00**	2.90e-02	5.07e-01	6.59e-03	2.58e-01	9.69e-04	4.29e-02
	LS-Chebyshev	1.39e+00	2.46e+00	6.19e-01	2.44e+00	2.71e-01	1.14e+00	8.26e-02	5.14e-01	2.34e-02	1.58e-01	4.42e-03	2.92e-02
Product Peak $n = 25$	Smolyak	**1.64e+00**	**5.55e+00**	**2.12e+00**	2.26e+01	5.05e-01	9.76e+00	9.90e-02	4.03e+00	8.57e-03	5.69e-01	1.37e-03	2.19e-01
	LS-Uniform	4.75e+00	9.52e+00	2.13e+00	**1.51e+01**	**4.45e-01**	**4.75e+00**	**5.95e-02**	1.38e+00	**5.81e-03**	2.27e-01	**8.40e-04**	9.11e-02
	LS-Chebyshev	1.04e+01	2.27e+01	4.92e+00	1.52e+01	1.39e+00	5.80e+00	2.24e-01	**1.17e+00**	2.81e-02	**1.73e-01**	3.60e-03	**3.10e-02**
Roos Arnold $n = 25$	Smolyak	6.79e+02	2.27e+03	9.78e+02	1.05e+04	4.05e+02	7.91e+03	2.57e+02	5.04e+03	1.79e+02	3.97e+03	1.03e+02	2.48e+03
	LS-Uniform	**6.49e+02**	**1.53e+03**	**7.46e+02**	7.82e+03	**2.68e+02**	3.65e+03	**1.62e+02**	**2.27e+03**	**9.05e+01**	1.36e+03	**5.17e+01**	1.63e+03
	LS-Chebyshev	3.36e+03	6.23e+03	1.60e+03	**6.39e+03**	6.98e+02	**2.80e+03**	3.57e+02	2.60e+03	1.76e+02	**1.25e+03**	8.42e+01	**8.59e+02**
Zhou $n = 25$	Smolyak	**2.15e+04**	5.01e+04	**4.49e+03**	**1.93e+04**	9.62e+02	**4.32e+03**	2.01e+02	**6.98e+02**	2.87e+01	**1.48e+02**	3.34e+00	2.06e+01
	LS-Uniform	2.28e+04	**4.93e+04**	4.90e+03	2.27e+04	**9.29e+02**	1.01e+04	**1.20e+02**	2.54e+03	**1.20e+01**	4.91e+02	**1.14e+00**	7.11e+01
	LS-Chebyshev	2.78e+04	7.08e+04	7.48e+03	2.20e+04	1.45e+03	4.96e+03	2.08e+02	1.00e+03	2.21e+01	1.59e+02	1.87e+00	**1.61e+01**

TABLE 8. Dim 9

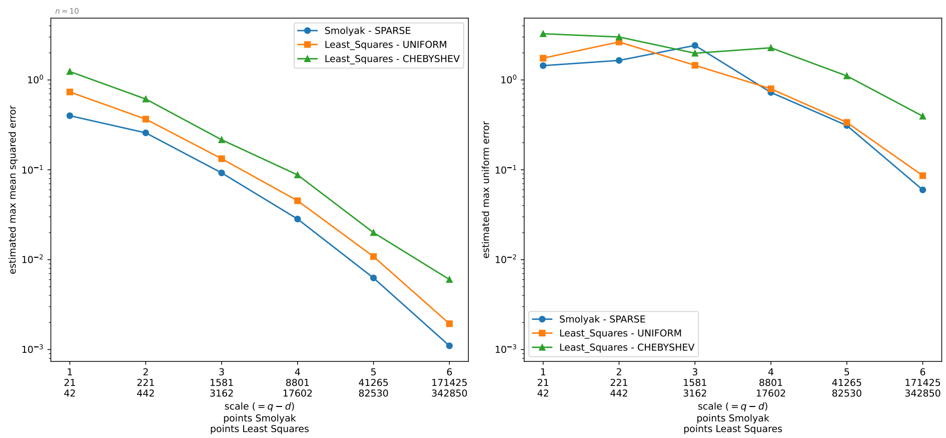

FIGURE 10. Visualization of the results for dim = 10 and various scales tested with $n = 10$ realizations for function class *Oscillatory*. Left plot shows the estimated max mean squared error and the right one shows the estimated max uniform error.

		Scale1 max		Scale2 max		Scale3 max		Scale4 max		Scale5 max		Scale6 max	
		ℓ_2	ℓ_∞	ℓ_2	ℓ_∞	ℓ_2	ℓ_∞	ℓ_2	ℓ_∞	ℓ_2	ℓ_∞	ℓ_2	ℓ_∞
Bratley $n = 10$	Smolyak	**4.76e-01**	**1.18e+00**	**5.79e-02**	**4.21e-01**	**2.69e-02**	**3.25e-01**	**6.81e-03**	**1.19e-01**	**1.50e-03**	**3.81e-02**	**3.50e-04**	**1.34e-02**
	LS-Uniform	7.95e-01	2.03e+00	7.01e-02	4.62e-01	4.14e-02	3.51e-01	1.09e-02	1.27e-01	2.54e-03	5.67e-02	6.21e-04	3.17e-02
	LS-Chebyshev	1.03e+00	3.86e+00	1.89e-01	6.08e-01	9.14e-02	3.80e-01	2.75e-02	2.69e-01	6.75e-03	7.61e-02	1.85e-03	3.81e-02
Continuous $n = 10$	Smolyak	5.19e-02	**2.21e-01**	**2.03e-02**	**1.39e-01**	1.69e-02	**1.22e-01**	8.83e-03	1.37e-01	1.72e-01	2.12e+00	1.76e+00	1.71e+01
	LS-Uniform	**5.17e-02**	2.76e-01	2.06e-02	1.67e-01	**9.44e-03**	1.24e-01	**4.85e-03**	**9.10e-02**	2.66e-03	**7.49e-02**	1.42e-03	**7.87e-02**
	LS-Chebyshev	6.37e-02	2.68e-01	2.40e-02	2.06e-01	1.06e-02	1.99e-01	5.17e-03	1.17e-01	**2.62e-03**	1.19e-01	**1.31e-03**	1.21e-01
Corner Peak $n = 10$	Smolyak	**6.36e-08**	**3.36e-07**	**1.12e-07**	**1.95e-06**	**4.22e-07**	2.15e-05	**1.09e-06**	1.01e-04	**2.53e-06**	1.95e-04	1.53e-05	1.86e-03
	LS-Uniform	2.83e-07	7.51e-07	4.25e-07	2.41e-06	8.78e-07	**1.96e-05**	1.16e-06	**7.89e-05**	8.78e-06	8.42e-04	**3.40e-06**	1.07e-03
	LS-Chebyshev	4.76e-07	7.79e-07	6.31e-06	1.87e-05	2.43e-06	2.02e-05	6.27e-06	2.20e-04	4.21e-06	**1.61e-04**	1.55e-05	**6.48e-04**
Discontinuous $n = 10$	Smolyak	**1.69e+02**	5.26e+02	**1.49e+02**	2.09e+03	**1.08e+02**	2.13e+03	8.96e+01	1.81e+03	2.40e+03	2.54e+04	5.23e+04	5.17e+05
	LS-Uniform	1.88e+02	**4.07e+02**	1.84e+02	2.31e+03	1.10e+02	**1.17e+03**	**8.38e+01**	1.77e+03	**7.52e+01**	3.27e+03	**6.23e+01**	2.23e+03
	LS-Chebyshev	1.16e+03	2.57e+03	3.93e+02	**1.68e+03**	3.68e+02	1.80e+03	2.20e+02	**1.75e+03**	1.49e+02	**1.74e+03**	1.11e+02	**1.81e+03**
Gaussian $n = 10$	Smolyak	1.68e-01	5.18e-01	**4.20e-02**	3.83e-01	**1.44e-02**	**1.40e-01**	4.96e-03	**5.87e-02**	1.58e-03	2.57e-02	4.45e-04	**4.64e-03**
	LS-Uniform	1.40e-01	**3.59e-01**	4.45e-02	**2.90e-01**	1.64e-02	2.23e-01	**4.46e-03**	1.56e-01	**1.22e-03**	1.23e-01	**2.49e-04**	3.44e-02
	LS-Chebyshev	**1.38e-01**	4.45e-01	7.25e-02	3.39e-01	2.47e-02	1.93e-01	7.52e-03	7.87e-02	1.93e-03	**2.35e-02**	4.13e-04	5.15e-03
G-Function $n = 10$	Smolyak	**4.80e+00**	2.13e+01	3.85e+00	3.74e+01	3.36e+00	5.23e+01	2.98e+00	9.17e+01	2.08e+00	6.29e+01	3.80e+00	5.50e+01
	LS-Uniform	5.16e+00	**1.41e+01**	**2.06e+00**	**2.04e+01**	**1.42e+00**	1.81e+01	**8.82e-01**	2.21e+01	**5.11e-01**	1.01e+01	**2.95e-01**	1.06e+01
	LS-Chebyshev	1.01e+01	2.43e+01	6.59e+00	2.11e+01	3.07e+00	**1.80e+01**	1.56e+00	**1.50e+01**	8.10e-01	**9.99e+00**	5.45e-01	**8.00e+00**
Morokoff Calfisch 1 $n = 10$	Smolyak	2.15e-01	6.01e-01	6.60e-02	2.27e-01	1.91e-02	9.96e-02	4.83e-03	4.73e-02	3.85e-02	3.54e-01	2.51e-02	2.80e-01
	LS-Uniform	**8.17e-02**	**2.59e-01**	**3.26e-02**	**1.37e-01**	**9.92e-03**	9.09e-02	**1.95e-03**	3.76e-02	**5.93e-04**	**1.28e-02**	**1.48e-04**	8.75e-03
	LS-Chebyshev	1.42e-01	3.07e-01	5.72e-02	2.23e-01	1.73e-02	**7.04e-02**	6.10e-03	**3.23e-02**	1.92e-03	1.40e-02	4.81e-04	**4.57e-03**
Morokoff Calfisch 2 $n = 10$	Smolyak	**9.91e-03**	**2.99e-02**	**5.43e-04**	**4.68e-03**	**2.32e-05**	3.25e-04	**6.91e-07**	**1.15e-05**	**1.21e-08**	**6.23e-07**	**2.10e-10**	**1.21e-08**
	LS-Uniform	1.55e-02	4.95e-02	7.66e-04	4.89e-03	3.59e-05	**3.17e-04**	1.10e-06	1.49e-05	2.03e-08	7.06e-07	3.79e-10	2.29e-08
	LS-Chebyshev	1.75e-02	3.84e-02	1.16e-03	6.71e-03	7.10e-05	5.42e-04	2.68e-06	3.75e-05	6.19e-08	2.45e-06	4.89e-08	7.28e-07
Oscillatory $n = 10$	Smolyak	**4.00e-01**	**1.44e+00**	**2.58e-01**	**1.65e+00**	**9.24e-02**	2.42e+00	**2.83e-02**	**7.26e-01**	**6.27e-03**	**3.11e-01**	**1.10e-03**	**5.98e-02**
	LS-Uniform	7.35e-01	1.75e+00	3.66e-01	2.65e+00	1.33e-01	**1.45e+00**	4.53e-02	7.97e-01	1.08e-02	3.37e-01	1.94e-03	8.61e-02
	LS-Chebyshev	1.24e+00	3.27e+00	6.13e-01	3.01e+00	2.16e-01	1.98e+00	8.77e-02	2.28e+00	2.00e-02	1.11e+00	6.02e-03	3.95e-01
Product Peak $n = 10$	Smolyak	2.21e-02	5.26e-02	4.35e-03	3.51e-02	1.02e-03	**7.00e-03**	2.20e-04	3.37e-03	4.36e-05	1.05e-03	7.98e-06	**1.91e-04**
	LS-Uniform	**1.77e-02**	**3.89e-02**	**2.97e-03**	**1.61e-02**	**6.12e-04**	8.38e-03	**1.47e-04**	1.02e-02	**2.63e-05**	2.67e-03	**3.56e-06**	4.04e-04
	LS-Chebyshev	2.29e-02	5.99e-02	4.85e-03	2.69e-02	9.90e-04	8.92e-03	1.98e-04	**2.37e-03**	3.71e-05	**8.10e-04**	7.24e-06	2.04e-04
Roos Arnold $n = 10$	Smolyak	9.78e+02	5.89e+03	6.01e+02	1.15e+04	5.72e+02	1.01e+04	4.95e+02	1.66e+04	3.99e+02	1.35e+04	1.27e+03	1.28e+04
	LS-Uniform	**9.42e+02**	**4.49e+03**	**4.64e+02**	**8.87e+03**	**3.06e+02**	4.31e+03	**2.12e+02**	**4.74e+03**	**1.25e+02**	5.31e+03	**7.29e+01**	**2.97e+03**
	LS-Chebyshev	2.46e+03	5.99e+03	1.33e+03	8.92e+03	8.08e+02	**3.92e+03**	5.08e+02	5.50e+03	2.68e+02	**4.14e+03**	1.46e+02	3.33e+03
Zhou $n = 10$	Smolyak	7.84e+04	2.55e+05	**1.05e+04**	1.11e+05	3.02e+03	3.27e+04	5.91e+02	5.95e+03	1.06e+02	9.28e+02	1.62e+01	1.62e+02
	LS-Uniform	**6.76e+04**	**1.57e+05**	1.26e+04	**9.18e+04**	**2.67e+03**	3.70e+04	**4.42e+02**	1.57e+04	**5.77e+01**	2.03e+03	**7.06e+00**	6.20e+02
	LS-Chebyshev	8.19e+04	3.01e+05	2.43e+04	1.05e+05	4.52e+03	**2.38e+04**	7.89e+02	**5.77e+03**	1.07e+02	**7.63e+02**	1.25e+01	**1.36e+02**

TABLE 9. Dim 10

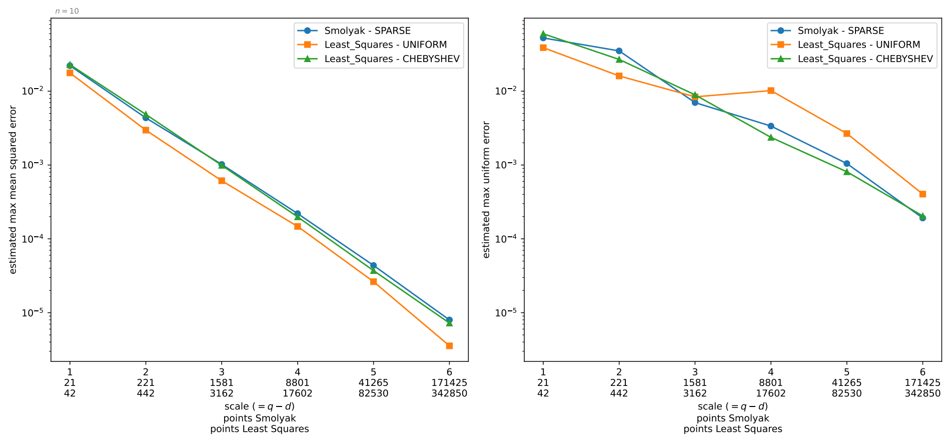

FIGURE 11. Visualization of the results for dim = 10 and various scales tested with $n = 10$ realizations for function class *Product-Peak*. Left plot shows the estimated max mean squared error and the right one shows the estimated max uniform error.

Add more detailed description mentioning whether Smolyak or Least Squares is preferable.

		Scale1		Scale2		Scale3		Scale4		Scale5		Scale6		Scale7		Scale8		Scale9	
		max		max		max		max		max		max		max		max		max	
		ℓ_2	ℓ_∞	ℓ_2	ℓ_∞	ℓ_2	ℓ_∞	ℓ_2	ℓ_∞	ℓ_2	ℓ_∞	ℓ_2	ℓ_∞	ℓ_2	ℓ_∞	ℓ_2	ℓ_∞	ℓ_2	ℓ_∞
Dim 2 $n = 25$	Smolyak	1.55e-01	2.28e-01	1.01e-01	1.97e-01	2.50e-02	9.26e-02	**7.88e-03**	3.37e-02	**2.72e-03**	**1.92e-02**	**1.35e-03**	1.35e-02	**4.62e-04**	**5.29e-03**	**1.97e-04**	**2.42e-03**	**7.01e-05**	**1.22e-03**
	LS-Uniform	**9.13e-02**	**1.35e-01**	**4.27e-02**	**1.09e-01**	7.79e-02	2.84e-01	6.99e-02	4.70e-01	2.56e-02	2.24e-01	5.99e-03	7.54e-02	5.35e-03	8.21e-02	1.44e-02	5.47e-01	7.67e+00	4.39e+02
	LS-Chebyshev	2.16e-01	3.24e-01	5.49e-02	1.20e-01	**1.90e-02**	**6.58e-02**	9.06e-03	**3.05e-02**	3.58e-03	2.46e-02	1.92e-03	**1.30e-02**	6.80e-04	6.77e-03	2.50e-04	3.94e-03	1.06e-04	2.05e-03
Dim 3 $n = 25$	Smolyak	1.45e-01	2.48e-01	5.89e-02	1.90e-01	2.87e-02	**8.67e-02**	1.19e-02	5.77e-02	4.54e-03	**2.78e-02**	1.80e-03	**1.70e-02**	**6.65e-04**	1.07e-02	**2.69e-04**	**4.95e-03**	**1.04e-04**	**3.40e-03**
	LS-Uniform	**9.91e-02**	**1.51e-01**	1.67e-01	4.57e-01	5.12e-02	2.39e-01	1.48e-02	1.07e-01	7.14e-03	6.28e-02	6.91e-03	1.87e-01	2.41e-03	7.40e-02	1.31e-03	7.25e-02	9.36e-03	8.41e-01
	LS-Chebyshev	1.54e-01	2.93e-01	**5.14e-02**	**1.56e-01**	**1.86e-02**	9.44e-02	**8.80e-03**	**4.80e-02**	**3.55e-03**	2.93e-02	**1.63e-03**	1.72e-02	6.98e-04	**9.45e-03**	3.01e-04	7.23e-03	1.24e-04	3.79e-03
Dim 4 $n = 25$	Smolyak	1.69e-01	4.47e-01	6.12e-02	**2.12e-01**	3.75e-02	**1.16e-01**	1.55e-02	**5.44e-02**	5.52e-03	**3.96e-02**	2.15e-03	**2.29e-02**	9.22e-04	**1.42e-02**	3.76e-04	**1.04e-02**	1.76e-04	**4.97e-03**
	LS-Uniform	**1.30e-01**	**3.08e-01**	8.44e-02	3.56e-01	2.70e-02	1.37e-01	1.09e-02	7.07e-02	6.80e-03	7.09e-02	2.87e-03	6.79e-02	1.55e-03	3.32e-02	8.18e-04	4.10e-02	6.25e-04	9.32e-02
	LS-Chebyshev	1.73e-01	4.39e-01	**5.15e-02**	2.17e-01	**2.42e-02**	1.19e-01	**9.02e-03**	6.54e-02	**3.80e-03**	4.28e-02	**1.79e-03**	3.52e-02	**7.78e-04**	1.95e-02	**3.32e-04**	1.55e-02	**1.51e-04**	9.79e-03
Dim 5 $n = 25$	Smolyak	1.24e-01	2.55e-01	4.79e-02	2.03e-01	2.96e-02	1.53e-01	1.27e-02	**9.53e-02**	3.69e-01	1.67e+00	4.26e-01	3.22e+00	1.28e-01	8.79e-01	3.14e+01	1.86e+02		
	LS-Uniform	**1.08e-01**	**2.29e-01**	**4.55e-02**	**1.74e-01**	2.95e-02	1.59e-01	1.60e-02	1.28e-01	6.69e-03	6.55e-02	3.60e-03	6.00e-02	1.71e-03	4.57e-02	8.41e-04	4.29e-02		
	LS-Chebyshev	1.37e-01	2.60e-01	5.23e-02	2.69e-01	**2.46e-02**	**1.42e-01**	**1.10e-02**	1.03e-01	**4.96e-03**	**6.10e-02**	**2.26e-03**	**4.49e-02**	**1.03e-03**	**2.44e-02**	**4.45e-04**	**1.30e-02**		
Dim 6 $n = 25$	Smolyak	1.30e-01	**2.61e-01**	6.84e-02	**2.24e-01**	5.04e-02	1.57e-01	1.54e-02	1.25e-01	3.25e-01	8.38e-01	8.54e-01	4.18e+00	5.50e-01	3.29e+00				
	LS-Uniform	**1.13e-01**	3.31e-01	4.96e-02	2.40e-01	2.27e-02	**1.49e-01**	9.41e-03	**8.68e-02**	4.82e-03	**5.43e-02**	2.69e-03	6.78e-02	1.37e-03	7.07e-02				
	LS-Chebyshev	1.23e-01	3.71e-01	**4.95e-02**	3.36e-01	**2.12e-02**	2.17e-01	**8.18e-03**	1.39e-01	**3.98e-03**	9.18e-02	**1.87e-03**	**5.23e-02**	**8.57e-04**	**3.40e-02**				
Dim 7 $n = 20$	Smolyak	1.18e-01	3.24e-01	5.94e-02	**1.93e-01**	3.58e-02	1.76e-01	1.64e-02	1.49e-01	2.01e-01	1.64e+00	2.67e-01	2.24e+00	3.08e+00	2.67e+01				
	LS-Uniform	**6.66e-02**	**1.88e-01**	**3.29e-02**	2.18e-01	**1.53e-02**	**1.51e-01**	7.28e-03	**1.14e-01**	3.65e-03	9.41e-02	1.89e-03	**4.96e-02**	9.26e-04	4.39e-02				
	LS-Chebyshev	1.74e-01	3.22e-01	4.53e-02	2.08e-01	1.72e-02	2.01e-01	**7.02e-03**	1.55e-01	**3.33e-03**	**8.45e-02**	**1.59e-03**	6.32e-02	**7.71e-04**	**4.04e-02**				
Dim 8 $n = 10$	Smolyak	**8.52e-02**	**3.01e-01**	**3.04e-02**	2.49e-01	2.08e-02	2.16e-01	8.93e-03	1.35e-01	1.19e-01	7.79e-01	3.17e-01	2.58e+00						
	LS-Uniform	1.17e-01	3.28e-01	3.19e-02	**2.21e-01**	**1.62e-02**	**1.42e-01**	8.46e-03	**9.53e-02**	4.25e-03	**8.13e-02**	2.17e-03	**6.28e-02**						
	LS-Chebyshev	1.25e-01	4.27e-01	3.16e-02	2.85e-01	1.62e-02	1.92e-01	**7.84e-03**	1.58e-01	**3.58e-03**	1.49e-01	**1.80e-03**	7.73e-02						
Dim 9 $n = 25$	Smolyak	**5.84e-02**	1.84e-01	3.22e-02	2.26e-01	3.09e-02	2.10e-01	1.37e-02	2.31e-01	6.23e-03	1.65e-01	3.45e-03	1.06e-01						
	LS-Uniform	7.41e-02	**1.75e-01**	**2.96e-02**	**2.15e-01**	**1.44e-02**	**1.87e-01**	**7.63e-03**	**1.48e-01**	**3.81e-03**	**9.10e-02**	2.01e-03	**9.37e-02**						
	LS-Chebyshev	1.20e-01	2.15e-01	3.37e-02	2.81e-01	1.75e-02	2.30e-01	8.10e-03	2.39e-01	3.83e-03	1.36e-01	**1.83e-03**	9.73e-02						
Dim 10 $n = 10$	Smolyak	5.19e-02	**2.21e-01**	**2.03e-02**	**1.39e-01**	1.69e-02	**1.22e-01**	8.83e-03	1.37e-01	1.72e-01	2.12e+00	1.76e+00	1.71e+01						
	LS-Uniform	**5.17e-02**	2.76e-01	2.06e-02	1.67e-01	**9.44e-03**	1.24e-01	**4.85e-03**	**9.10e-02**	2.66e-03	**7.49e-02**	1.42e-03	**7.87e-02**						
	LS-Chebyshev	6.37e-02	2.68e-01	2.40e-02	2.06e-01	1.06e-02	1.99e-01	5.17e-03	1.17e-01	**2.62e-03**	1.19e-01	**1.31e-03**	1.21e-01						

TABLE 10. Visualization of the results for function class *Continuous* for various dimensions and scales, tested with n realizations. Best algorithm per dimension and scale is depicted bold.

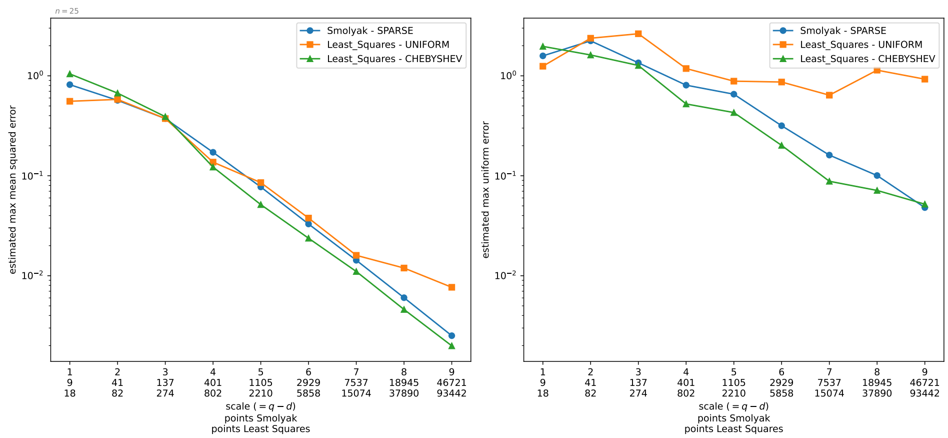

FIGURE 12. Visualization of the results for dim = 4 and various scales tested with $n = 25$ realizations for function class *G-Function*. Left plot shows the estimated max mean squared error and the right one shows the estimated max uniform error.

		Scale1 max		Scale2 max		Scale3 max		Scale4 max		Scale5 max		Scale6 max		Scale7 max		Scale8 max		Scale9 max	
		ℓ_2	ℓ_∞	ℓ_2	ℓ_∞	ℓ_2	ℓ_∞	ℓ_2	ℓ_∞	ℓ_2	ℓ_∞	ℓ_2	ℓ_∞	ℓ_2	ℓ_∞	ℓ_2	ℓ_∞	ℓ_2	ℓ_∞
Dim 2 $n = 25$	Smolyak	7.91e-02	1.17e-01	1.67e-02	3.26e-02	4.52e-03	8.68e-03	5.81e-04	**1.48e-03**	3.31e-05	1.04e-04	2.55e-07	**8.90e-07**	1.75e-09	6.08e-09	7.94e-14	**2.56e-13**	**8.06e-16**	**6.22e-15**
	LS-Uniform	**3.50e-02**	**4.91e-02**	**6.20e-03**	**1.21e-02**	3.23e-03	1.54e-02	5.19e-04	3.31e-03	5.36e-05	4.50e-04	4.37e-07	4.13e-06	1.07e-08	2.01e-07	8.45e-13	2.33e-11	1.38e-11	7.92e-10
	LS-Chebyshev	1.47e-01	2.28e-01	3.10e-02	5.44e-02	**2.62e-03**	**8.10e-03**	**3.80e-04**	1.94e-03	**1.72e-05**	**5.50e-05**	**1.86e-07**	8.95e-07	**1.10e-09**	**4.69e-09**	**6.70e-14**	2.87e-13	1.89e-15	2.48e-14
Dim 3 $n = 25$	Smolyak	**4.34e-02**	**6.73e-02**	3.96e-02	1.57e-01	1.36e-02	5.59e-02	5.80e-03	3.01e-02	1.43e-03	7.14e-03	2.46e-04	1.34e-03	2.77e-05	1.84e-04	1.88e-06	1.24e-05	4.08e-08	3.12e-07
	LS-Uniform	5.36e-02	9.50e-02	**1.69e-02**	**4.45e-02**	1.20e-02	5.84e-02	**1.65e-03**	**9.08e-03**	**3.77e-04**	7.16e-03	**6.96e-05**	1.03e-03	1.36e-05	5.16e-04	2.32e-06	8.73e-05	2.51e-07	1.69e-05
	LS-Chebyshev	2.22e-01	3.36e-01	4.79e-02	1.26e-01	**1.12e-02**	**3.55e-02**	3.33e-03	1.16e-02	5.53e-04	**2.93e-03**	8.54e-05	**4.00e-04**	**1.15e-05**	**6.56e-05**	**9.61e-07**	**4.63e-06**	**2.12e-08**	**1.34e-07**
Dim 4 $n = 25$	Smolyak	**8.68e-03**	**1.73e-02**	2.62e-02	1.23e-01	2.22e-02	1.64e-01	1.35e-02	8.03e-02	5.65e-03	5.32e-02	2.31e-03	1.81e-02	6.39e-04	6.36e-03	1.35e-04	1.55e-03	2.16e-05	3.04e-04
	LS-Uniform	1.84e-02	3.46e-02	**2.19e-02**	**6.25e-02**	**9.98e-03**	**7.21e-02**	**3.51e-03**	4.68e-02	**1.34e-03**	3.03e-02	**2.63e-04**	1.08e-02	**5.53e-05**	2.61e-03	**1.53e-05**	1.35e-03	**2.87e-06**	2.53e-04
	LS-Chebyshev	8.91e-02	1.58e-01	1.29e-01	2.77e-01	3.67e-02	1.08e-01	8.27e-03	**3.26e-02**	2.20e-03	**1.05e-02**	5.13e-04	**2.84e-03**	1.41e-04	**9.73e-04**	2.97e-05	**1.94e-04**	5.95e-06	**4.72e-05**
Dim 5 $n = 25$	Smolyak	**1.13e-03**	**3.15e-03**	**5.78e-03**	2.98e-02	1.14e-02	8.92e-02	8.63e-03	1.43e-01	6.51e-03	8.86e-02	3.24e-03	7.53e-02	2.14e-03	3.72e-02	1.51e-02	8.48e-02		
	LS-Uniform	2.28e-03	4.34e-03	7.08e-03	**1.87e-02**	**3.14e-03**	**2.87e-02**	**2.47e-03**	**4.56e-02**	**6.66e-04**	**1.98e-02**	**3.49e-04**	1.47e-02	**1.48e-04**	1.33e-02	**4.61e-05**	7.17e-03		
	LS-Chebyshev	9.80e-03	2.22e-02	2.36e-02	6.71e-02	3.22e-02	1.49e-01	1.24e-02	6.98e-02	4.76e-03	2.46e-02	1.39e-03	**1.14e-02**	4.67e-04	**4.75e-03**	1.41e-04	**1.47e-03**		
Dim 6 $n = 25$	Smolyak	**1.14e-03**	**4.11e-03**	**1.54e-03**	**1.22e-02**	2.81e-03	**2.15e-02**	4.71e-03	1.12e-01	4.98e-03	1.72e-01	4.10e-03	1.03e-01	3.07e-03	9.44e-02				
	LS-Uniform	1.58e-03	4.29e-03	4.75e-03	2.00e-02	**2.71e-03**	2.87e-02	**1.45e-03**	**4.05e-02**	**6.61e-04**	3.78e-02	**3.29e-04**	2.24e-02	**1.25e-04**	1.15e-02				
	LS-Chebyshev	3.58e-02	6.67e-02	1.85e-02	5.02e-02	6.43e-03	2.22e-02	5.10e-03	6.38e-02	4.18e-03	**2.31e-02**	1.90e-03	**1.81e-02**	7.66e-04	**9.90e-03**				
Dim 7 $n = 20$	Smolyak	**4.58e-04**	**2.47e-03**	**9.86e-04**	1.47e-02	9.43e-04	2.78e-02	4.04e-03	7.65e-02	6.07e-03	1.22e-01	4.61e-03	1.09e-01	2.90e-03	1.59e-01				
	LS-Uniform	4.58e-04	2.48e-03	1.10e-03	**8.19e-03**	**8.01e-04**	**2.66e-02**	**4.65e-04**	**1.12e-02**	**4.04e-04**	5.40e-02	**1.83e-04**	4.35e-02	**1.42e-04**	4.51e-02				
	LS-Chebyshev	1.21e-03	2.83e-03	2.42e-03	1.17e-02	1.08e-03	2.77e-02	9.72e-04	1.21e-02	6.60e-04	**5.06e-02**	3.31e-04	**1.68e-02**	2.22e-04	**1.03e-02**				
Dim 8 $n = 10$	Smolyak	**3.79e-05**	2.09e-04	**3.90e-05**	6.27e-04	**1.91e-05**	**6.36e-04**	7.45e-05	3.16e-03	2.98e-04	2.20e-02	7.17e-04	8.41e-02						
	LS-Uniform	3.21e-04	7.33e-04	3.05e-04	2.25e-03	1.42e-04	2.31e-03	**6.64e-05**	**2.30e-03**	**5.85e-05**	9.05e-03	**3.90e-05**	7.56e-03						
	LS-Chebyshev	4.89e-05	**1.96e-04**	1.20e-04	**6.06e-04**	2.30e-04	1.76e-03	1.55e-04	5.73e-03	1.62e-04	**4.95e-03**	1.53e-04	**4.48e-03**						
Dim 9 $n = 25$	Smolyak	3.04e-06	1.33e-05	**4.70e-06**	**4.94e-05**	**9.96e-06**	3.36e-04	**3.59e-05**	2.78e-03	7.27e-05	3.67e-03	3.50e-04	3.70e-02						
	LS-Uniform	**3.01e-06**	**1.30e-05**	1.40e-05	1.11e-04	1.12e-05	**2.87e-04**	4.41e-05	**2.55e-03**	**2.14e-05**	**2.34e-03**	**3.66e-05**	**1.15e-02**						
	LS-Chebyshev	4.45e-04	6.74e-04	2.04e-04	6.48e-04	3.54e-04	1.29e-03	1.07e-03	1.03e-02	1.03e-03	1.08e-02	1.57e-03	2.96e-02						
Dim 10 $n = 10$	Smolyak	**6.36e-08**	**3.36e-07**	**1.12e-07**	**1.95e-06**	**4.22e-07**	2.15e-05	**1.09e-06**	1.01e-04	**2.53e-06**	1.95e-04	1.53e-05	1.86e-03						
	LS-Uniform	2.83e-07	7.51e-07	4.25e-07	2.41e-06	8.78e-07	**1.96e-05**	1.16e-06	**7.89e-05**	8.78e-06	8.42e-04	**3.40e-06**	1.07e-03						
	LS-Chebyshev	4.76e-07	7.79e-07	6.31e-06	1.87e-05	2.43e-06	2.02e-05	6.27e-06	2.20e-04	4.21e-06	**1.61e-04**	1.55e-05	**6.48e-04**						

TABLE 11. Visualization of the results for function class *Corner Peak* for various dimensions and scales, tested with n realizations. Best algorithm per dimension and scale is depicted bold.

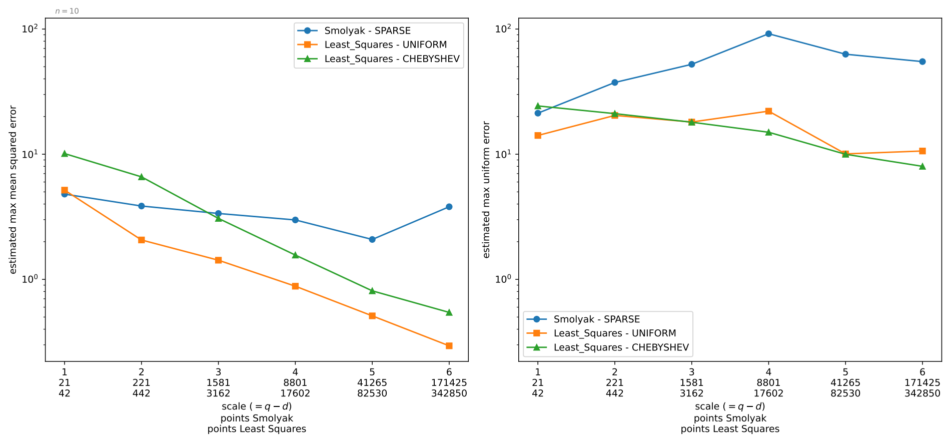

FIGURE 13. Visualization of the results for dim = 10 and various scales tested with $n = 10$ realizations for function class *G-Function*. Left plot shows the estimated max mean squared error and the right one shows the estimated max uniform error.

		Scale1 max		Scale2 max		Scale3 max		Scale4 max		Scale5 max		Scale6 max		Scale7 max		Scale8 max		Scale9 max	
		ℓ_2	ℓ_∞	ℓ_2	ℓ_∞	ℓ_2	ℓ_∞	ℓ_2	ℓ_∞	ℓ_2	ℓ_∞	ℓ_2	ℓ_∞	ℓ_2	ℓ_∞	ℓ_2	ℓ_∞	ℓ_2	ℓ_∞
Dim 2 $n = 25$	Smolyak	**1.41e+00**	**2.44e+00**	2.23e+00	5.65e+00	1.12e+00	**2.83e+00**	8.57e-01	3.83e+00	6.10e-01	4.35e+00	3.93e-01	4.18e+00	**2.89e-01**	3.33e+00	2.68e-01	4.30e+00	1.67e-01	3.31e+00
	LS-Uniform	2.32e+00	3.51e+00	**1.27e+00**	**3.36e+00**	2.65e+00	1.07e+01	1.60e+01	1.06e+02	5.31e+00	4.74e+01	2.15e+00	3.57e+01	2.82e+00	5.96e+01	9.45e+00	3.56e+02	7.90e+03	4.49e+05
	LS-Chebyshev	1.55e+00	2.98e+00	1.80e+00	3.45e+00	**1.06e+00**	4.16e+00	**8.25e-01**	**3.66e+00**	**5.28e-01**	**3.07e+00**	**3.67e-01**	**2.95e+00**	2.91e-01	**3.31e+00**	**2.00e-01**	**3.70e+00**	**1.54e-01**	**2.96e+00**
Dim 3 $n = 25$	Smolyak	2.70e+00	4.58e+00	3.31e+00	1.08e+01	2.55e+00	8.43e+00	2.04e+00	1.08e+01	1.26e+00	1.11e+01	**7.98e-01**	7.68e+00	6.82e-01	1.04e+01	5.25e-01	1.02e+01	4.37e-01	9.66e+00
	LS-Uniform	**2.26e+00**	**4.48e+00**	4.20e+00	1.21e+01	4.14e+00	1.59e+01	2.20e+00	1.19e+01	1.87e+00	1.57e+01	1.81e+00	3.90e+01	2.10e+00	6.26e+01	2.61e+00	1.42e+02	3.39e+01	2.76e+03
	LS-Chebyshev	3.63e+00	5.31e+00	**2.37e+00**	**5.88e+00**	**2.36e+00**	**7.11e+00**	**1.59e+00**	**7.53e+00**	**1.24e+00**	**6.59e+00**	8.77e-01	**5.66e+00**	**5.91e-01**	**8.11e+00**	**4.12e-01**	**6.51e+00**	**3.12e-01**	**6.23e+00**
Dim 4 $n = 25$	Smolyak	9.48e+00	2.29e+01	5.76e+00	**1.97e+01**	5.20e+00	2.15e+01	2.96e+00	2.32e+01	2.66e+00	2.51e+01	1.73e+00	1.88e+01	1.44e+00	2.16e+01	1.05e+00	2.74e+01	**7.32e-01**	2.60e+01
	LS-Uniform	**7.24e+00**	**2.01e+01**	**5.67e+00**	2.50e+01	4.30e+00	2.31e+01	**2.86e+00**	3.05e+01	3.68e+00	5.69e+01	3.26e+00	7.03e+01	2.47e+00	6.35e+01	2.95e+00	2.06e+02	3.06e+00	3.47e+02
	LS-Chebyshev	9.15e+00	2.45e+01	6.46e+00	2.44e+01	**3.35e+00**	**1.27e+01**	3.79e+00	**1.42e+01**	**2.37e+00**	**1.90e+01**	**1.65e+00**	**1.65e+01**	**1.44e+00**	**1.70e+01**	**1.01e+00**	**1.88e+01**	7.34e-01	**1.85e+01**
Dim 5 $n = 25$	Smolyak	1.10e+01	2.93e+01	**7.81e+00**	**3.17e+01**	7.04e+00	4.40e+01	**6.20e+00**	6.06e+01	8.37e+01	3.12e+02	2.73e+02	2.16e+03	4.15e+02	2.25e+03	1.73e+05	1.05e+06		
	LS-Uniform	1.04e+01	2.49e+01	9.75e+00	3.34e+01	**6.97e+00**	4.17e+01	6.96e+00	6.86e+01	**4.79e+00**	5.05e+01	4.55e+00	9.36e+01	4.16e+00	1.21e+02	3.56e+00	1.83e+02		
	LS-Chebyshev	**7.81e+00**	**1.42e+01**	8.88e+00	3.46e+01	8.53e+00	**3.01e+01**	6.35e+00	**4.80e+01**	5.25e+00	**4.20e+01**	**3.59e+00**	**4.21e+01**	**2.71e+00**	**4.33e+01**	**2.11e+00**	**4.60e+01**		
Dim 6 $n = 25$	Smolyak	2.48e+01	**6.34e+01**	1.88e+01	9.58e+01	**1.22e+01**	1.32e+02	**8.69e+00**	1.22e+02	9.32e+02	2.87e+03	1.11e+03	7.38e+03	3.01e+03	1.97e+04				
	LS-Uniform	**2.30e+01**	7.01e+01	**1.70e+01**	6.93e+01	1.38e+01	1.21e+02	1.10e+01	**8.31e+01**	9.22e+00	1.17e+02	7.75e+00	1.70e+02	6.76e+00	2.26e+02				
	LS-Chebyshev	6.53e+01	1.04e+02	1.79e+01	**6.35e+01**	1.65e+01	**9.97e+01**	1.31e+01	9.02e+01	**8.91e+00**	**8.68e+01**	**6.67e+00**	**9.65e+01**	**5.49e+00**	**8.63e+01**				
Dim 7 $n = 20$	Smolyak	**2.61e+01**	1.30e+02	**3.01e+01**	2.32e+02	**2.31e+01**	3.28e+02	**1.75e+01**	2.84e+02	2.20e+02	1.74e+03	1.12e+03	8.56e+03	2.06e+04	1.79e+05				
	LS-Uniform	4.48e+01	1.14e+02	3.56e+01	**1.52e+02**	2.80e+01	**2.17e+02**	2.13e+01	2.95e+02	**1.86e+01**	5.64e+02	**1.53e+01**	5.81e+02	**1.24e+01**	4.82e+02				
	LS-Chebyshev	3.98e+01	**1.04e+02**	5.63e+01	1.77e+02	5.05e+01	3.24e+02	3.33e+01	**2.11e+02**	2.84e+01	**4.65e+02**	2.19e+01	**3.15e+02**	1.96e+01	**2.96e+02**				
Dim 8 $n = 10$	Smolyak	**5.83e+01**	**1.32e+02**	**4.62e+01**	2.44e+02	3.23e+01	**4.10e+02**	**2.72e+01**	4.71e+02	1.67e+02	1.17e+03	1.79e+03	1.57e+04						
	LS-Uniform	6.43e+01	1.48e+02	4.72e+01	**1.95e+02**	**3.13e+01**	4.57e+02	2.74e+01	3.82e+02	**2.26e+01**	6.61e+02	**1.91e+01**	5.47e+02						
	LS-Chebyshev	8.01e+01	1.62e+02	6.62e+01	2.57e+02	5.35e+01	4.26e+02	4.32e+01	**3.48e+02**	3.20e+01	**3.91e+02**	2.81e+01	**3.82e+02**						
Dim 9 $n = 25$	Smolyak	**1.86e+02**	**6.52e+02**	**1.09e+02**	6.76e+02	**7.13e+01**	8.03e+02	**5.40e+01**	**8.61e+02**	**4.59e+01**	1.58e+03	**3.54e+01**	1.53e+03						
	LS-Uniform	2.43e+02	9.66e+02	1.39e+02	7.04e+02	8.33e+01	**6.98e+02**	7.50e+01	1.00e+03	6.16e+01	1.48e+03	5.24e+01	1.74e+03						
	LS-Chebyshev	5.89e+02	1.18e+03	1.59e+02	**6.21e+02**	1.30e+02	1.36e+03	9.55e+01	1.05e+03	7.68e+01	**1.20e+03**	5.85e+01	**1.17e+03**						
Dim 10 $n = 10$	Smolyak	**1.69e+02**	5.26e+02	**1.49e+02**	2.09e+03	**1.08e+02**	2.13e+03	8.96e+01	1.81e+03	2.40e+03	2.54e+04	5.23e+04	5.17e+05						
	LS-Uniform	1.88e+02	**4.07e+02**	1.84e+02	2.31e+03	1.10e+02	**1.17e+03**	**8.38e+01**	1.77e+03	**7.52e+01**	3.27e+03	**6.23e+01**	2.23e+03						
	LS-Chebyshev	1.16e+03	2.57e+03	3.93e+02	**1.68e+03**	3.68e+02	1.80e+03	2.20e+02	**1.75e+03**	1.49e+02	**1.74e+03**	1.11e+02	**1.81e+03**						

TABLE 12. Visualization of the results for function class *Discontinuous* for various dimensions and scales, tested with n realizations. Best algorithm per dimension and scale is depicted bold.

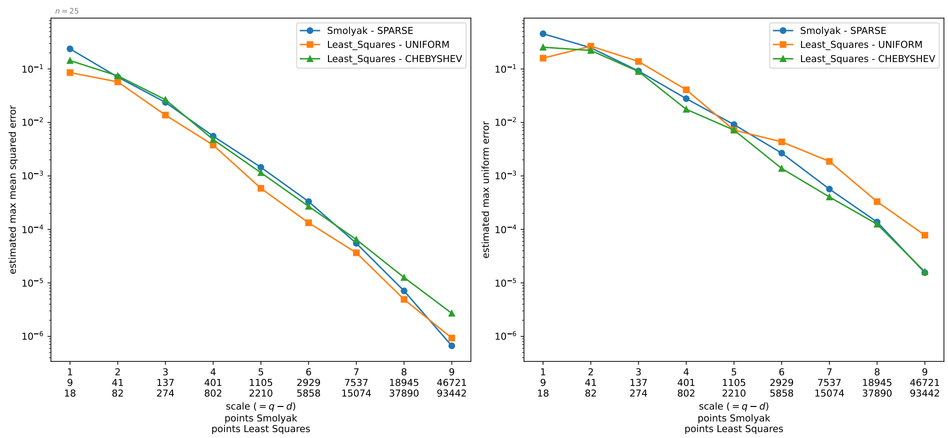

FIGURE 14. Visualization of the results for dim = 4 and various scales tested with $n = 25$ realizations for function class *Morokoff Calfisch 1*. Left plot shows the estimated max mean squared error and the right one shows the estimated max uniform error.

		Scale1 max		Scale2 max		Scale3 max		Scale4 max		Scale5 max		Scale6 max		Scale7 max		Scale8 max		Scale9 max	
		ℓ_2	ℓ_∞	ℓ_2	ℓ_∞	ℓ_2	ℓ_∞	ℓ_2	ℓ_∞	ℓ_2	ℓ_∞	ℓ_2	ℓ_∞	ℓ_2	ℓ_∞	ℓ_2	ℓ_∞	ℓ_2	ℓ_∞
Dim 2 $n = 25$	Smolyak	1.01e-01	1.20e-01	8.34e-03	1.29e-02	5.85e-04	8.95e-04	1.95e-05	**3.77e-05**	3.01e-07	7.14e-07	1.23e-10	**3.57e-10**	1.59e-14	4.77e-14	**2.21e-15**	**8.10e-15**	3.76e-15	**2.07e-14**
	LS-Uniform	**7.49e-02**	**1.15e-01**	**4.30e-03**	**8.14e-03**	8.71e-04	3.82e-03	1.43e-05	8.85e-05	4.97e-07	3.64e-06	1.91e-10	1.58e-09	1.15e-13	2.26e-12	1.46e-14	4.63e-13	5.66e-11	3.25e-09
	LS-Chebyshev	1.53e-01	2.34e-01	1.22e-02	2.79e-02	**2.86e-04**	**7.18e-04**	**1.37e-05**	6.92e-05	**1.68e-07**	**4.77e-07**	**8.37e-11**	3.88e-10	**9.38e-15**	**3.20e-14**	2.38e-15	1.70e-14	**2.14e-15**	2.94e-14
Dim 3 $n = 25$	Smolyak	1.19e-01	2.41e-01	3.74e-02	**6.96e-02**	3.93e-03	7.46e-03	2.71e-04	6.07e-04	7.50e-06	2.66e-05	3.10e-07	1.26e-06	6.97e-09	3.23e-08	8.42e-11	4.24e-10	4.94e-14	2.60e-13
	LS-Uniform	**1.04e-01**	**1.98e-01**	8.55e-02	2.36e-01	3.91e-03	2.16e-02	**1.39e-04**	8.76e-04	6.84e-06	5.22e-05	1.84e-07	2.37e-06	5.01e-09	1.19e-07	1.25e-10	4.86e-09	5.54e-13	4.43e-11
	LS-Chebyshev	1.80e-01	2.94e-01	**2.78e-02**	7.21e-02	**2.53e-03**	**6.34e-03**	1.68e-04	**4.87e-04**	**4.85e-06**	**1.83e-05**	**1.26e-07**	**5.29e-07**	**2.83e-09**	**1.48e-08**	**3.76e-11**	**1.91e-10**	**2.25e-14**	**1.46e-13**
Dim 4 $n = 25$	Smolyak	1.24e-01	2.27e-01	**3.18e-02**	**7.33e-02**	5.19e-03	1.50e-02	6.17e-04	2.09e-03	6.51e-05	2.41e-04	4.11e-06	1.85e-05	1.63e-07	8.09e-07	5.55e-09	3.34e-08	1.44e-10	1.01e-09
	LS-Uniform	**1.18e-01**	**2.25e-01**	3.72e-02	1.41e-01	**3.68e-03**	3.06e-02	**3.41e-04**	3.84e-03	3.25e-05	6.16e-04	**1.61e-06**	3.73e-05	9.03e-08	3.19e-06	2.95e-09	1.51e-07	6.70e-11	4.31e-09
	LS-Chebyshev	1.80e-01	4.16e-01	4.02e-02	7.70e-02	4.39e-03	**1.24e-02**	3.53e-04	**1.30e-03**	**2.94e-05**	**1.08e-04**	1.68e-06	**8.65e-06**	**6.71e-08**	**3.85e-07**	**1.71e-09**	**1.24e-08**	**4.13e-11**	**2.84e-10**
Dim 5 $n = 25$	Smolyak	1.74e-01	3.96e-01	4.58e-02	**1.07e-01**	1.10e-02	3.39e-02	1.79e-03	6.75e-03	2.78e-04	1.06e-03	2.65e-05	1.22e-04	2.19e-06	1.01e-05	1.26e-07	7.34e-07		
	LS-Uniform	**1.45e-01**	**2.64e-01**	**3.99e-02**	1.71e-01	**7.90e-03**	5.69e-02	**1.23e-03**	1.44e-02	**1.13e-04**	2.11e-03	1.07e-05	3.54e-04	7.73e-07	6.02e-05	5.17e-08	9.34e-06		
	LS-Chebyshev	2.08e-01	3.71e-01	5.26e-02	1.65e-01	1.05e-02	**3.20e-02**	1.32e-03	**6.55e-03**	1.41e-04	**6.09e-04**	**1.07e-05**	**5.90e-05**	**6.65e-07**	**4.58e-06**	**3.40e-08**	**2.70e-07**		
Dim 6 $n = 25$	Smolyak	**1.14e-01**	3.09e-01	3.63e-02	**9.34e-02**	1.03e-02	**4.42e-02**	2.68e-03	**1.11e-02**	3.82e-04	2.08e-03	6.13e-05	3.13e-04	7.11e-06	4.26e-05				
	LS-Uniform	1.53e-01	**2.91e-01**	**3.61e-02**	1.47e-01	**8.16e-03**	6.35e-02	**1.66e-03**	3.59e-02	**1.84e-04**	4.80e-03	3.21e-05	2.89e-03	**2.51e-06**	2.09e-04				
	LS-Chebyshev	2.16e-01	4.85e-01	6.59e-02	1.93e-01	1.41e-02	5.01e-02	2.27e-03	1.16e-02	2.78e-04	**1.75e-03**	**3.07e-05**	**2.23e-04**	2.68e-06	**2.03e-05**				
Dim 7 $n = 20$	Smolyak	**1.27e-01**	5.09e-01	**5.25e-02**	**1.98e-01**	2.04e-02	8.19e-02	5.99e-03	2.55e-02	1.03e-03	5.48e-03	1.04e-04	7.67e-04	1.09e-05	9.11e-05				
	LS-Uniform	1.64e-01	4.80e-01	5.76e-02	3.60e-01	**1.43e-02**	1.76e-01	**3.02e-03**	5.33e-02	**4.11e-04**	1.40e-02	**4.38e-05**	3.14e-03	**3.70e-06**	4.87e-04				
	LS-Chebyshev	2.16e-01	**4.32e-01**	9.13e-02	3.07e-01	1.97e-02	**7.26e-02**	3.65e-03	**1.72e-02**	4.95e-04	**4.03e-03**	4.84e-05	**5.75e-04**	5.06e-06	**7.89e-05**				
Dim 8 $n = 10$	Smolyak	**1.43e-01**	4.33e-01	**4.47e-02**	**2.08e-01**	1.66e-02	**9.36e-02**	5.26e-03	**2.73e-02**	1.38e-03	7.80e-03	2.54e-04	1.54e-03						
	LS-Uniform	1.47e-01	**4.30e-01**	6.12e-02	4.46e-01	**1.65e-02**	1.85e-01	**3.64e-03**	9.65e-02	**6.60e-04**	3.80e-02	**9.44e-05**	6.00e-03						
	LS-Chebyshev	1.68e-01	5.81e-01	6.78e-02	2.90e-01	2.39e-02	1.51e-01	5.44e-03	3.81e-02	9.19e-04	**6.05e-03**	1.26e-04	**1.25e-03**						
Dim 9 $n = 25$	Smolyak	1.74e-01	5.90e-01	6.51e-02	3.16e-01	2.33e-02	1.87e-01	7.40e-03	6.08e-02	1.72e-03	1.89e-02	4.46e-04	4.04e-03						
	LS-Uniform	**1.38e-01**	**3.03e-01**	**5.18e-02**	**2.05e-01**	**1.60e-02**	1.54e-01	**4.36e-03**	9.49e-02	**8.34e-04**	3.40e-02	**1.46e-04**	1.21e-02						
	LS-Chebyshev	2.42e-01	4.86e-01	8.55e-02	3.43e-01	2.57e-02	**1.37e-01**	6.30e-03	**4.01e-02**	1.33e-03	**1.02e-02**	2.25e-04	**2.25e-03**						
Dim 10 $n = 10$	Smolyak	1.68e-01	5.18e-01	**4.20e-02**	3.83e-01	**1.44e-02**	**1.40e-01**	4.96e-03	**5.87e-02**	1.58e-03	2.57e-02	4.45e-04	**4.64e-03**						
	LS-Uniform	1.40e-01	**3.59e-01**	4.45e-02	**2.90e-01**	1.64e-02	2.23e-01	**4.46e-03**	1.56e-01	**1.22e-03**	1.23e-01	**2.49e-04**	3.44e-02						
	LS-Chebyshev	**1.38e-01**	4.45e-01	7.25e-02	3.39e-01	2.47e-02	1.93e-01	7.52e-03	7.87e-02	1.93e-03	**2.35e-02**	4.13e-04	5.15e-03						

TABLE 13. Visualization of the results for function class *Gaussian* for various dimensions and scales, tested with n realizations. Best algorithm per dimension and scale is depicted bold.

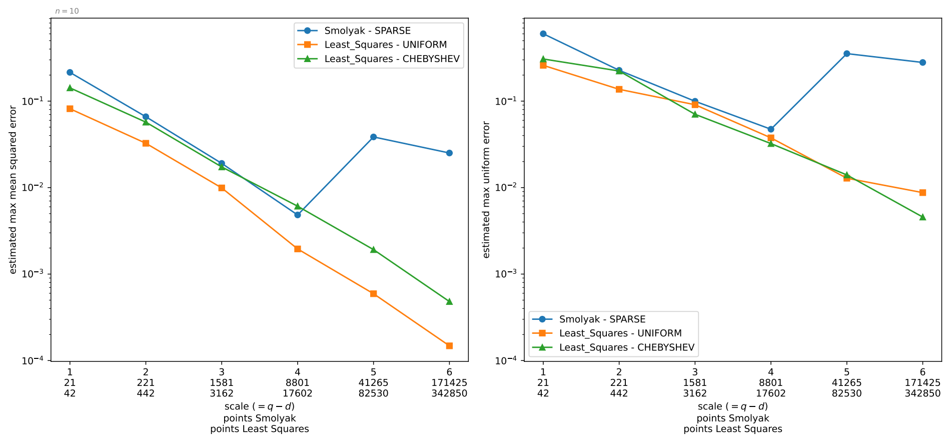

FIGURE 15. Visualization of the results for dim = 10 and various scales tested with $n = 10$ realizations for function class *Morokoff Calfisch 1*. Left plot shows the estimated max mean squared error and the right one shows the estimated max uniform error.

		Scale1 max ℓ_2	Scale1 max ℓ_∞	Scale2 max ℓ_2	Scale2 max ℓ_∞	Scale3 max ℓ_2	Scale3 max ℓ_∞	Scale4 max ℓ_2	Scale4 max ℓ_∞	Scale5 max ℓ_2	Scale5 max ℓ_∞	Scale6 max ℓ_2	Scale6 max ℓ_∞	Scale7 max ℓ_2	Scale7 max ℓ_∞	Scale8 max ℓ_2	Scale8 max ℓ_∞	Scale9 max ℓ_2	Scale9 max ℓ_∞
Dim 2 $n = 25$	Smolyak	**7.28e-02**	**9.21e-02**	3.22e-03	5.02e-03	4.66e-05	**7.06e-05**	2.02e-07	**4.00e-07**	3.76e-10	8.91e-10	**1.24e-15**	**4.11e-15**	**1.62e-15**	**7.44e-15**	**2.33e-15**	**8.33e-15**	3.89e-15	**1.98e-14**
	LS-Uniform	8.09e-02	1.01e-01	**9.64e-04**	**2.00e-03**	5.94e-05	2.74e-04	2.32e-07	1.52e-06	5.88e-10	4.00e-09	2.19e-15	2.13e-14	3.93e-15	7.05e-14	1.38e-14	5.34e-13	3.69e-11	2.13e-09
	LS-Chebyshev	1.84e-01	2.88e-01	4.61e-03	1.07e-02	**2.43e-05**	7.37e-05	**1.43e-07**	6.98e-07	**2.14e-10**	**6.16e-10**	1.48e-15	7.11e-15	3.68e-15	1.91e-14	2.42e-15	1.63e-14	**1.64e-15**	2.38e-14
Dim 3 $n = 25$	Smolyak	**1.09e-01**	**2.25e-01**	**1.52e-02**	**3.53e-02**	1.16e-03	**2.69e-03**	2.50e-05	5.80e-05	2.64e-07	6.93e-07	1.48e-09	4.68e-09	4.34e-12	1.70e-11	6.15e-15	**2.79e-14**	**4.80e-15**	**2.71e-14**
	LS-Uniform	1.57e-01	2.77e-01	2.08e-02	5.44e-02	1.21e-03	5.95e-03	**1.14e-05**	9.36e-05	1.05e-07	1.02e-06	6.21e-10	7.91e-09	3.02e-12	7.73e-11	9.89e-15	3.66e-13	3.73e-14	4.03e-12
	LS-Chebyshev	3.14e-01	4.91e-01	2.53e-02	4.78e-02	**1.03e-03**	2.84e-03	1.37e-05	**4.49e-05**	**9.48e-08**	**4.74e-07**	**5.48e-10**	**2.57e-09**	**1.73e-12**	**9.71e-12**	**4.48e-15**	7.55e-14	5.75e-15	1.02e-13
Dim 4 $n = 25$	Smolyak	**2.19e-01**	**5.82e-01**	**3.83e-02**	**1.38e-01**	**4.20e-03**	**2.17e-02**	3.49e-04	**1.54e-03**	1.10e-05	**3.91e-05**	1.79e-07	6.48e-07	1.63e-09	7.12e-09	8.65e-12	4.10e-11	2.95e-14	**1.70e-13**
	LS-Uniform	2.48e-01	7.03e-01	9.71e-02	3.47e-01	8.49e-03	4.88e-02	**2.63e-04**	2.43e-03	**6.04e-06**	7.96e-05	**5.63e-08**	1.34e-06	**4.88e-10**	1.44e-08	3.03e-12	1.55e-10	1.41e-14	9.29e-13
	LS-Chebyshev	4.17e-01	1.01e+00	1.37e-01	4.04e-01	1.29e-02	4.68e-02	4.23e-04	1.68e-03	7.53e-06	4.46e-05	6.86e-08	**3.79e-07**	5.19e-10	**3.56e-09**	**2.62e-12**	**1.82e-11**	**1.26e-14**	4.01e-13
Dim 5 $n = 25$	Smolyak	**3.15e-01**	9.04e-01	**7.35e-02**	**2.58e-01**	**1.11e-02**	**6.58e-02**	**1.28e-03**	**8.47e-03**	1.25e-04	**6.51e-04**	5.16e-06	2.30e-05	1.15e-07	5.26e-07	1.58e-09	7.71e-09		
	LS-Uniform	3.45e-01	**7.68e-01**	1.19e-01	5.17e-01	1.97e-02	1.07e-01	2.26e-03	1.49e-02	**8.43e-05**	1.13e-03	**2.34e-06**	1.37e-04	**3.20e-08**	1.97e-06	**3.53e-10**	3.37e-08		
	LS-Chebyshev	5.98e-01	1.24e+00	1.66e-01	4.85e-01	4.24e-02	1.44e-01	5.24e-03	3.39e-02	1.69e-04	7.27e-04	3.18e-06	**1.44e-05**	3.95e-08	**2.22e-07**	3.93e-10	**3.02e-09**		
Dim 6 $n = 25$	Smolyak	**3.65e-01**	1.22e+00	**1.13e-01**	**4.72e-01**	**2.11e-02**	**1.86e-01**	**3.82e-03**	**4.68e-02**	**3.88e-04**	**4.44e-03**	3.26e-05	**3.19e-04**	1.73e-06	1.33e-05				
	LS-Uniform	4.78e-01	**1.05e+00**	1.97e-01	8.69e-01	3.77e-02	2.85e-01	6.30e-03	7.86e-02	4.91e-04	7.57e-03	**2.50e-05**	7.49e-04	**7.39e-07**	5.65e-05				
	LS-Chebyshev	8.97e-01	2.40e+00	3.08e-01	9.02e-01	7.57e-02	2.97e-01	1.66e-02	7.36e-02	1.71e-03	1.04e-02	6.38e-05	3.89e-04	1.54e-06	**1.00e-05**				
Dim 7 $n = 20$	Smolyak	**3.66e-01**	1.70e+00	**1.30e-01**	1.05e+00	**3.83e-02**	**4.20e-01**	**6.64e-03**	**1.33e-01**	**1.22e-03**	**2.06e-02**	**1.22e-04**	**1.83e-03**	9.23e-06	**1.46e-04**				
	LS-Uniform	5.29e-01	**1.49e+00**	2.02e-01	**9.42e-01**	5.51e-02	6.55e-01	1.20e-02	1.58e-01	2.33e-03	5.04e-02	1.56e-04	6.58e-03	**5.53e-06**	5.14e-04				
	LS-Chebyshev	1.02e+00	1.98e+00	3.74e-01	1.75e+00	9.50e-02	9.65e-01	2.44e-02	2.34e-01	4.75e-03	1.75e-01	3.18e-04	1.09e-02	1.30e-05	3.79e-04				
Dim 8 $n = 10$	Smolyak	**4.61e-01**	2.22e+00	**1.60e-01**	**1.14e+00**	**4.33e-02**	**4.17e-01**	**1.18e-02**	**2.84e-01**	**1.77e-03**	**6.05e-02**	**2.47e-04**	**7.27e-03**						
	LS-Uniform	6.25e-01	**1.93e+00**	2.68e-01	1.50e+00	6.82e-02	5.12e-01	1.88e-02	4.30e-01	3.04e-03	1.13e-01	4.47e-04	2.42e-02						
	LS-Chebyshev	6.61e-01	2.08e+00	3.85e-01	1.93e+00	1.01e-01	7.33e-01	4.09e-02	8.28e-01	5.46e-03	1.99e-01	1.14e-03	3.26e-02						
Dim 9 $n = 25$	Smolyak	7.99e-01	2.40e+00	**2.31e-01**	**1.32e+00**	**7.70e-02**	1.09e+00	**1.77e-02**	**4.51e-01**	**3.87e-03**	**1.19e-01**	**5.91e-04**	**1.79e-02**						
	LS-Uniform	**6.98e-01**	**1.90e+00**	2.72e-01	1.61e+00	1.18e-01	**1.08e+00**	2.90e-02	5.07e-01	6.59e-03	2.58e-01	9.69e-04	4.29e-02						
	LS-Chebyshev	1.39e+00	2.46e+00	6.19e-01	2.44e+00	2.71e-01	1.14e+00	8.26e-02	5.14e-01	2.34e-02	1.58e-01	4.42e-03	2.92e-02						
Dim 10 $n = 10$	Smolyak	**4.00e-01**	**1.44e+00**	**2.58e-01**	**1.65e+00**	**9.24e-02**	2.42e+00	**2.83e-02**	**7.26e-01**	**6.27e-03**	**3.11e-01**	**1.10e-03**	**5.98e-02**						
	LS-Uniform	7.35e-01	1.75e+00	3.66e-01	2.65e+00	1.33e-01	**1.45e+00**	4.53e-02	7.97e-01	1.08e-02	3.37e-01	1.94e-03	8.61e-02						
	LS-Chebyshev	1.24e+00	3.27e+00	6.13e-01	3.01e+00	2.16e-01	1.98e+00	8.77e-02	2.28e+00	2.00e-02	1.11e+00	6.02e-03	3.95e-01						

TABLE 14. Visualization of the results for function class *Oscillatory* for various dimensions and scales, tested with n realizations. Best algorithm per dimension and scale is depicted bold.

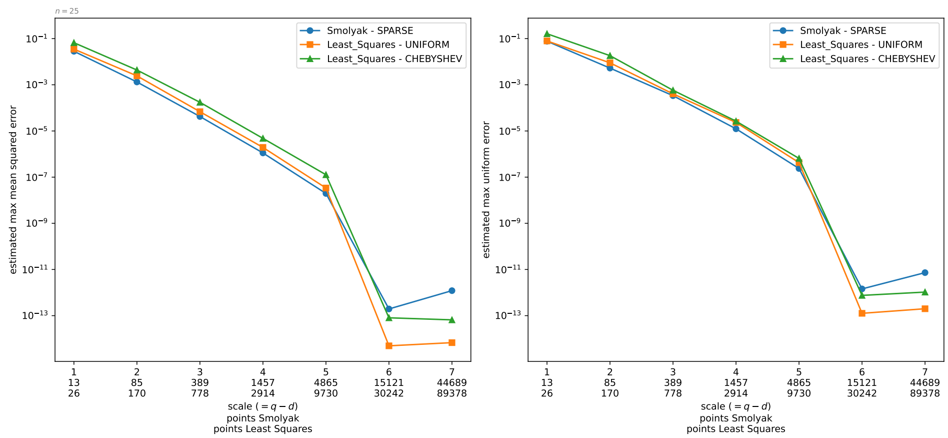

FIGURE 16. Visualization of the results for dim = 6 and various scales tested with $n = 25$ realizations for function class *Morokoff Calfisch 2*. Left plot shows the estimated max mean squared error and the right one shows the estimated max uniform error.

		Scale1 max ℓ_2	Scale1 max ℓ_∞	Scale2 max ℓ_2	Scale2 max ℓ_∞	Scale3 max ℓ_2	Scale3 max ℓ_∞	Scale4 max ℓ_2	Scale4 max ℓ_∞	Scale5 max ℓ_2	Scale5 max ℓ_∞
Dim 2 $n = 25$	Smolyak	**6.06e-02**	**9.32e-02**	**4.82e-16**	**1.11e-15**	**4.52e-16**	**1.33e-15**	**8.43e-16**	**2.44e-15**	**1.14e-15**	**3.77e-15**
	LS-Uniform	7.11e-02	9.82e-02	2.44e-15	4.66e-15	2.17e-15	7.55e-15	4.26e-14	2.82e-13	5.88e-15	4.12e-14
	LS-Chebyshev	1.10e-01	1.90e-01	7.19e-15	1.44e-14	8.31e-15	2.15e-14	1.22e-14	3.43e-14	5.17e-15	2.04e-14
Dim 3 $n = 25$	Smolyak	**3.41e-01**	**8.35e-01**	**1.29e-02**	**5.80e-02**	**8.83e-16**	**3.11e-15**	**1.36e-15**	**6.22e-15**	**1.88e-15**	**1.15e-14**
	LS-Uniform	5.91e-01	1.01e+00	1.53e-02	6.16e-02	1.99e-14	7.61e-14	1.47e-14	8.57e-14	1.36e-14	9.15e-14
	LS-Chebyshev	9.47e-01	1.46e+00	3.29e-02	6.83e-02	1.07e-14	2.35e-14	3.73e-14	1.17e-13	3.78e-14	1.54e-13
Dim 4 $n = 25$	Smolyak	7.79e-01	2.06e+00	**8.87e-02**	**3.66e-01**	**5.98e-03**	6.06e-02	**3.15e-15**	**1.33e-14**	**4.63e-15**	**3.55e-14**
	LS-Uniform	**6.71e-01**	**1.63e+00**	1.11e-01	5.15e-01	6.19e-03	**3.51e-02**	2.95e-14	1.43e-13	2.83e-14	1.78e-13
	LS-Chebyshev	1.52e+00	2.99e+00	2.15e-01	7.50e-01	1.09e-02	4.32e-02	2.26e-13	9.61e-13	9.69e-14	4.34e-13
Dim 5 $n = 25$	Smolyak	1.96e+00	6.38e+00	4.01e-01	1.93e+00	**2.34e-02**	**1.69e-01**	1.15e-03	2.20e-02	3.65e-13	1.71e-12
	LS-Uniform	**1.42e+00**	**4.42e+00**	**2.61e-01**	**8.35e-01**	2.83e-02	1.74e-01	**1.04e-03**	**1.22e-02**	**5.12e-14**	**4.61e-13**
	LS-Chebyshev	2.08e+00	6.54e+00	5.26e-01	1.37e+00	5.08e-02	1.78e-01	2.58e-03	1.55e-02	5.13e-13	7.77e-12
Dim 6 $n = 25$	Smolyak	**2.07e+00**	**6.82e+00**	5.25e-01	4.40e+00	1.01e-01	1.44e+00	8.24e-03	2.01e-01	**3.46e-04**	1.08e-02
	LS-Uniform	2.07e+00	7.00e+00	**5.15e-01**	3.59e+00	**8.96e-02**	9.74e-01	**6.66e-03**	8.63e-02	4.53e-04	**6.30e-03**
	LS-Chebyshev	3.29e+00	8.02e+00	1.19e+00	**2.48e+00**	1.71e-01	**7.10e-01**	2.08e-02	**7.64e-02**	1.49e-03	6.52e-03
Dim 7 $n = 20$	Smolyak	**1.85e-02**	**5.98e-02**	**5.82e-03**	**2.65e-02**	**1.56e-03**	**1.11e-02**	3.50e-04	3.71e-03	7.07e-05	7.34e-04
	LS-Uniform	2.83e-02	8.55e-02	7.92e-03	3.06e-02	1.78e-03	2.08e-02	**2.65e-04**	5.21e-03	**4.18e-05**	1.39e-03
	LS-Chebyshev	3.13e-02	8.76e-02	1.32e-02	5.36e-02	2.66e-03	1.59e-02	4.31e-04	**2.68e-03**	6.17e-05	**5.02e-04**
Dim 8 $n = 10$	Smolyak	3.89e-02	1.24e-01	**7.72e-03**	**4.67e-02**	1.84e-03	**1.24e-02**	3.26e-04	**4.62e-03**	5.97e-05	**7.33e-04**
	LS-Uniform	**3.25e-02**	**1.10e-01**	9.21e-03	8.89e-02	**1.47e-03**	2.12e-02	**2.42e-04**	4.79e-03	**4.20e-05**	1.94e-03
	LS-Chebyshev	4.17e-02	1.32e-01	1.11e-02	4.68e-02	2.20e-03	1.97e-02	3.89e-04	4.74e-03	6.18e-05	9.90e-04
Dim 9 $n = 25$	Smolyak	**1.64e+00**	**5.55e+00**	**2.12e+00**	2.26e+01	5.05e-01	9.76e+00	9.90e-02	4.03e+00	8.57e-03	5.69e-01
	LS-Uniform	4.75e+00	9.52e+00	2.13e+00	**1.51e+01**	**4.45e-01**	**4.75e+00**	**5.95e-02**	1.38e+00	**5.81e-03**	2.27e-01
	LS-Chebyshev	1.04e+01	2.27e+01	4.92e+00	1.52e+01	1.39e+00	5.80e+00	2.24e-01	**1.17e+00**	2.81e-02	**1.73e-01**
Dim 10 $n = 10$	Smolyak	2.21e-02	5.26e-02	4.35e-03	3.51e-02	1.02e-03	**7.00e-03**	2.20e-04	3.37e-03	4.36e-05	1.05e-03
	LS-Uniform	**1.77e-02**	**3.89e-02**	**2.97e-03**	**1.61e-02**	**6.12e-04**	8.38e-03	**1.47e-04**	1.02e-02	**2.63e-05**	2.67e-03
	LS-Chebyshev	2.29e-02	5.99e-02	4.85e-03	2.69e-02	9.90e-04	8.92e-03	1.98e-04	**2.37e-03**	3.71e-05	**8.10e-04**

		Scale6 max ℓ_2	Scale6 max ℓ_∞	Scale7 max ℓ_2	Scale7 max ℓ_∞	Scale8 max ℓ_2	Scale8 max ℓ_∞	Scale9 max ℓ_2	Scale9 max ℓ_∞
Dim 2 $n = 25$	Smolyak	**1.71e-15**	**6.22e-15**	**2.50e-15**	**1.33e-14**	**3.56e-15**	**1.55e-14**	5.65e-15	**2.62e-14**
	LS-Uniform	4.23e-15	3.73e-14	7.79e-15	1.42e-13	2.34e-14	8.82e-13	5.96e-11	3.43e-09
	LS-Chebyshev	3.52e-15	1.60e-14	7.11e-15	4.31e-14	6.33e-15	6.73e-14	**4.77e-15**	5.48e-14
Dim 3 $n = 25$	Smolyak	**2.72e-15**	**1.87e-14**	**3.95e-15**	**3.02e-14**	**5.04e-15**	**3.64e-14**	**7.45e-15**	**5.33e-14**
	LS-Uniform	9.60e-15	1.73e-13	7.03e-15	1.68e-13	5.11e-15	2.40e-13	1.03e-13	1.11e-11
	LS-Chebyshev	4.93e-14	2.01e-13	3.55e-14	1.96e-13	1.77e-14	3.26e-13	2.12e-14	5.28e-13
Dim 4 $n = 25$	Smolyak	**5.83e-15**	**3.82e-14**	**8.04e-15**	**4.88e-14**	1.10e-14	**1.09e-13**	1.43e-14	**1.31e-13**
	LS-Uniform	1.61e-14	2.49e-13	1.13e-14	2.68e-13	**6.70e-15**	2.07e-13	**1.13e-14**	1.27e-12
	LS-Chebyshev	2.80e-13	1.48e-12	2.75e-13	1.11e-12	1.77e-13	2.40e-12	6.74e-14	1.59e-12
Dim 5 $n = 25$	Smolyak	2.69e-12	1.12e-11	2.22e-12	1.27e-11	1.60e-09	9.57e-09		
	LS-Uniform	**2.38e-14**	**3.65e-13**	**3.07e-14**	**1.07e-12**	**7.67e-15**	**3.64e-13**		
	LS-Chebyshev	1.37e-12	9.11e-12	3.24e-13	1.86e-12	1.98e-13	2.18e-12		
Dim 6 $n = 25$	Smolyak	1.11e-11	8.36e-11	1.07e-10	6.57e-10				
	LS-Uniform	**9.89e-14**	**1.36e-12**	**1.08e-13**	**4.15e-12**				
	LS-Chebyshev	1.88e-11	1.05e-10	7.04e-12	5.61e-11				
Dim 7 $n = 20$	Smolyak	1.22e-05	1.47e-04	1.86e-06	2.38e-05				
	LS-Uniform	**5.64e-06**	4.92e-04	**8.04e-07**	9.14e-05				
	LS-Chebyshev	7.73e-06	**8.25e-05**	8.51e-07	**1.27e-05**				
Dim 8 $n = 10$	Smolyak	1.09e-05	1.99e-04						
	LS-Uniform	**7.57e-06**	7.92e-04						
	LS-Chebyshev	9.33e-06	**1.85e-04**						
Dim 9 $n = 25$	Smolyak	1.37e-03	2.19e-01						
	LS-Uniform	**8.40e-04**	9.11e-02						
	LS-Chebyshev	3.60e-03	**3.10e-02**						
Dim 10 $n = 10$	Smolyak	7.98e-06	**1.91e-04**						
	LS-Uniform	**3.56e-06**	4.04e-04						
	LS-Chebyshev	7.24e-06	2.04e-04						

TABLE 15. Visualization of the results for function class *Product Peak* for various dimensions and scales, tested with n realizations. Best algorithm per dimension and scale is depicted bold.

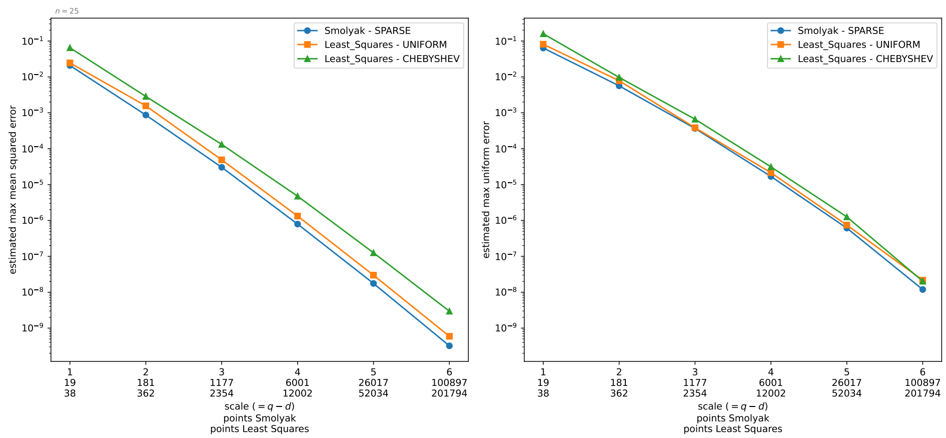

FIGURE 17. Visualization of the results for dim = 9 and various scales tested with $n = 25$ realizations for function class *Morokoff Calfisch 2*. Left plot shows the estimated max mean squared error and the right one shows the estimated max uniform error.

		Scale1 max ℓ_2	Scale1 max ℓ_∞	Scale2 max ℓ_2	Scale2 max ℓ_∞	Scale3 max ℓ_2	Scale3 max ℓ_∞	Scale4 max ℓ_2	Scale4 max ℓ_∞	Scale5 max ℓ_2	Scale5 max ℓ_∞	Scale6 max ℓ_2	Scale6 max ℓ_∞	Scale7 max ℓ_2	Scale7 max ℓ_∞	Scale8 max ℓ_2	Scale8 max ℓ_∞	Scale9 max ℓ_2	Scale9 max ℓ_∞
Dim 2 $n = 25$	Smolyak	3.93e-01	**4.99e-01**	1.99e-01	3.67e-01	8.16e-02	2.13e-01	**3.26e-02**	**1.10e-01**	**1.11e-02**	7.23e-02	**4.40e-03**	**3.70e-02**	**1.80e-03**	**2.05e-02**	**8.20e-04**	**1.25e-02**	**2.77e-04**	**6.82e-03**
	LS-Uniform	**3.41e-01**	6.53e-01	**1.19e-01**	**2.95e-01**	3.64e-01	1.48e+00	3.63e-01	2.42e+00	1.11e-01	9.17e-01	2.37e-02	2.70e-01	2.67e-02	5.62e-01	3.48e-02	1.19e+00	3.54e+01	2.01e+03
	LS-Chebyshev	7.29e-01	1.01e+00	1.42e-01	4.01e-01	**6.08e-02**	**1.44e-01**	3.41e-02	1.25e-01	1.32e-02	**6.48e-02**	5.15e-03	3.77e-02	2.08e-03	2.43e-02	9.49e-04	1.81e-02	3.63e-04	9.36e-03
Dim 3 $n = 25$	Smolyak	6.12e-01	1.08e+00	4.34e-01	1.38e+00	1.99e-01	6.79e-01	9.02e-02	3.84e-01	4.39e-02	2.87e-01	1.85e-02	1.80e-01	7.74e-03	8.43e-02	2.88e-03	3.90e-02	**1.07e-03**	2.43e-02
	LS-Uniform	**5.82e-01**	1.17e+00	6.14e-01	1.76e+00	1.97e-01	9.60e-01	**7.56e-02**	4.53e-01	6.39e-02	7.35e-01	4.98e-02	1.13e+00	2.56e-02	6.36e-01	1.58e-02	7.06e-01	5.58e-02	4.08e+00
	LS-Chebyshev	6.84e-01	**9.25e-01**	**3.72e-01**	**8.13e-01**	**1.45e-01**	**4.01e-01**	8.58e-02	**3.01e-01**	**3.75e-02**	**1.60e-01**	**1.62e-02**	**1.06e-01**	**7.48e-03**	**6.97e-02**	**2.84e-03**	**3.44e-02**	1.13e-03	**2.29e-02**
Dim 4 $n = 25$	Smolyak	8.16e-01	1.58e+00	**5.69e-01**	2.24e+00	3.74e-01	1.35e+00	1.72e-01	8.06e-01	7.73e-02	6.55e-01	3.29e-02	3.17e-01	1.43e-02	1.61e-01	6.03e-03	1.01e-01	2.51e-03	**4.81e-02**
	LS-Uniform	**5.56e-01**	**1.25e+00**	5.81e-01	2.38e+00	**3.74e-01**	2.64e+00	1.37e-01	1.18e+00	8.55e-02	8.84e-01	3.78e-02	8.67e-01	1.60e-02	6.40e-01	1.19e-02	1.14e+00	7.67e-03	9.26e-01
	LS-Chebyshev	1.05e+00	1.97e+00	6.71e-01	**1.62e+00**	3.90e-01	**1.27e+00**	**1.22e-01**	**5.23e-01**	**5.15e-02**	**4.29e-01**	**2.37e-02**	**2.02e-01**	**1.10e-02**	**8.82e-02**	**4.60e-03**	**7.14e-02**	**1.99e-03**	5.20e-02
Dim 5 $n = 25$	Smolyak	9.34e-01	2.42e+00	9.32e-01	4.26e+00	6.12e-01	3.56e+00	3.10e-01	1.78e+00	1.66e-01	1.34e+00	1.51e+00	1.10e+01	1.08e+00	5.74e+00	3.69e+02	2.36e+03		
	LS-Uniform	**7.76e-01**	**1.93e+00**	**6.53e-01**	**2.16e+00**	**3.88e-01**	2.37e+00	**2.00e-01**	1.87e+00	9.20e-02	1.19e+00	4.74e-02	1.15e+00	2.37e-02	1.21e+00	1.32e-02	1.00e+00		
	LS-Chebyshev	3.06e+00	4.38e+00	9.59e-01	2.99e+00	4.79e-01	**2.14e+00**	2.08e-01	**1.29e+00**	**8.72e-02**	**6.27e-01**	**4.27e-02**	**3.43e-01**	**2.00e-02**	**2.06e-01**	**9.17e-03**	**1.65e-01**		
Dim 6 $n = 25$	Smolyak	2.27e+00	7.69e+00	1.23e+00	7.26e+00	7.87e-01	6.08e+00	4.96e-01	6.13e+00	8.36e-01	4.94e+00	1.50e+00	9.23e+00	4.10e+00	2.53e+01				
	LS-Uniform	**1.63e+00**	**5.40e+00**	**1.08e+00**	**3.30e+00**	**6.38e-01**	3.14e+00	**3.14e-01**	**2.76e+00**	**1.41e-01**	2.04e+00	**8.01e-02**	1.63e+00	4.10e-02	1.15e+00				
	LS-Chebyshev	5.40e+00	1.07e+01	2.04e+00	5.57e+00	8.05e-01	**2.95e+00**	4.24e-01	2.81e+00	1.83e-01	**1.45e+00**	8.16e-02	**6.10e-01**	**3.84e-02**	**5.31e-01**				
Dim 7 $n = 20$	Smolyak	2.66e+00	1.03e+01	2.83e+00	3.10e+01	1.33e+00	1.92e+01	8.36e-01	1.01e+01	5.09e-01	7.42e+00	8.11e-01	7.16e+00	3.41e+01	2.99e+02				
	LS-Uniform	**1.64e+00**	**7.09e+00**	**1.42e+00**	1.52e+01	**6.32e-01**	**4.80e+00**	3.28e-01	4.31e+00	1.75e-01	3.53e+00	9.32e-02	2.01e+00	4.96e-02	2.84e+00				
	LS-Chebyshev	5.62e+00	1.07e+01	2.09e+00	**1.32e+01**	7.16e-01	4.88e+00	**3.12e-01**	**2.53e+00**	**1.50e-01**	**2.01e+00**	**7.47e-02**	**1.05e+00**	**3.74e-02**	**6.95e-01**				
Dim 8 $n = 10$	Smolyak	**2.57e+00**	1.11e+01	2.51e+00	2.38e+01	1.49e+00	2.39e+01	1.19e+00	1.84e+01	7.56e-01	1.61e+01	9.98e-01	1.02e+01						
	LS-Uniform	4.69e+00	**1.09e+01**	**1.76e+00**	1.73e+01	**8.08e-01**	**7.67e+00**	**3.99e-01**	5.33e+00	**2.15e-01**	4.03e+00	**1.20e-01**	5.01e+00						
	LS-Chebyshev	7.42e+00	1.54e+01	2.65e+00	**1.05e+01**	1.08e+00	9.30e+00	5.18e-01	**4.22e+00**	2.56e-01	**3.29e+00**	1.43e-01	**1.73e+00**						
Dim 9 $n = 25$	Smolyak	**2.07e+00**	7.90e+00	2.69e+00	2.18e+01	1.94e+00	2.43e+01	1.50e+00	3.58e+01	1.05e+00	3.46e+01	6.75e-01	1.99e+01						
	LS-Uniform	3.96e+00	**7.81e+00**	**1.82e+00**	**9.22e+00**	**1.00e+00**	**1.02e+01**	**5.77e-01**	**8.25e+00**	**3.34e-01**	7.85e+00	**1.88e-01**	6.19e+00						
	LS-Chebyshev	1.73e+01	3.14e+01	6.00e+00	1.85e+01	3.21e+00	1.22e+01	1.74e+00	9.25e+00	8.41e-01	**5.52e+00**	4.05e-01	**3.97e+00**						
Dim 10 $n = 10$	Smolyak	**4.80e+00**	2.13e+01	3.85e+00	3.74e+01	3.36e+00	5.23e+01	2.98e+00	9.17e+01	2.08e+00	6.29e+01	3.80e+00	5.50e+01						
	LS-Uniform	5.16e+00	**1.41e+01**	**2.06e+00**	**2.04e+01**	**1.42e+00**	1.81e+01	**8.82e-01**	2.21e+01	**5.11e-01**	1.01e+01	**2.95e-01**	1.06e+01						
	LS-Chebyshev	1.01e+01	2.43e+01	6.59e+00	2.11e+01	3.07e+00	**1.80e+01**	1.56e+00	**1.50e+01**	8.10e-01	**9.99e+00**	5.45e-01	**8.00e+00**						

TABLE 16. Visualization of the results for function class *G-Function* for various dimensions and scales, tested with n realizations. Best algorithm per dimension and scale is depicted bold.

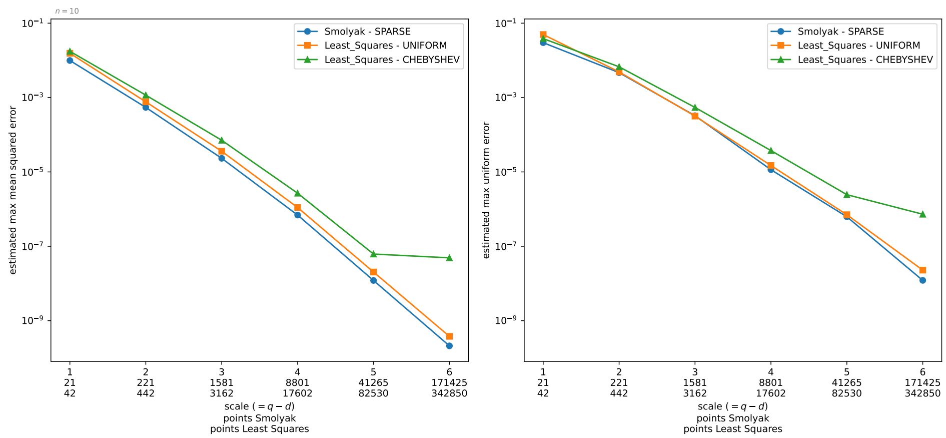

FIGURE 18. Visualization of the results for dim = 10 and various scales tested with $n = 10$ realizations for function class *Morokoff Calfisch 2*. Left plot shows the estimated max mean squared error and the right one shows the estimated max uniform error.

		Scale1		Scale2		Scale3		Scale4		Scale5		Scale6		Scale7		Scale8		Scale9	
		max		max		max		max		max		max		max		max		max	
		ℓ_2	ℓ_∞	ℓ_2	ℓ_∞	ℓ_2	ℓ_∞	ℓ_2	ℓ_∞	ℓ_2	ℓ_∞	ℓ_2	ℓ_∞	ℓ_2	ℓ_∞	ℓ_2	ℓ_∞	ℓ_2	ℓ_∞
Dim 2 $n=25$	Smolyak	**4.43e-02**	**6.43e-02**	1.31e-02	2.59e-02	1.66e-03	3.21e-03	**1.02e-04**	**3.05e-04**	**3.02e-06**	**9.59e-06**	**7.42e-08**	**2.66e-07**	**1.15e-09**	**4.95e-09**	**2.37e-12**	**1.10e-11**	**1.21e-14**	**6.17e-14**
	LS-Uniform	6.26e-02	7.99e-02	**2.14e-03**	**3.95e-03**	**4.82e-04**	**1.93e-03**	3.77e-04	2.33e-03	7.05e-06	5.76e-05	9.23e-08	7.85e-07	6.04e-09	1.41e-07	1.92e-10	7.21e-09	1.09e-09	6.19e-08
	LS-Chebyshev	1.45e-01	2.06e-01	1.06e-02	2.72e-02	1.82e-03	3.73e-03	2.03e-04	5.55e-04	3.40e-06	1.16e-05	9.34e-08	4.62e-07	2.03e-09	9.98e-09	3.13e-11	1.45e-10	4.23e-13	1.88e-12
Dim 3 $n=25$	Smolyak	1.23e-01	2.46e-01	**2.50e-02**	**5.91e-02**	**2.84e-03**	**9.40e-03**	**2.84e-04**	**1.12e-03**	**1.38e-05**	**8.28e-05**	6.95e-07	5.21e-06	2.64e-08	1.87e-07	4.04e-10	2.97e-09	**5.05e-12**	**4.40e-11**
	LS-Uniform	**5.93e-02**	**1.15e-01**	4.73e-02	1.25e-01	4.34e-03	2.23e-02	3.09e-04	2.80e-03	1.55e-05	2.20e-04	**4.34e-07**	1.09e-05	2.76e-08	1.16e-06	1.10e-09	7.16e-08	6.25e-11	4.44e-09
	LS-Chebyshev	2.45e-01	4.02e-01	2.64e-02	6.54e-02	4.37e-03	1.58e-02	4.47e-04	1.29e-03	1.58e-05	9.10e-05	4.73e-07	**2.42e-06**	**1.96e-08**	**9.58e-08**	**3.11e-10**	**1.64e-09**	7.53e-12	5.78e-11
Dim 4 $n=25$	Smolyak	2.37e-01	4.55e-01	7.05e-02	2.49e-01	2.37e-02	9.16e-02	5.54e-03	2.78e-02	1.45e-03	9.13e-03	3.31e-04	2.67e-03	5.50e-05	5.68e-04	7.10e-06	1.38e-04	**6.67e-07**	**1.55e-05**
	LS-Uniform	**8.55e-02**	**1.59e-01**	**5.74e-02**	2.67e-01	**1.37e-02**	1.38e-01	**3.78e-03**	4.08e-02	**5.87e-04**	7.21e-03	**1.33e-04**	4.33e-03	**3.66e-05**	1.87e-03	**4.92e-06**	3.30e-04	9.31e-07	7.79e-05
	LS-Chebyshev	1.43e-01	2.55e-01	7.43e-02	**2.22e-01**	2.67e-02	**8.95e-02**	4.79e-03	**1.76e-02**	1.15e-03	**7.21e-03**	2.70e-04	**1.38e-03**	6.47e-05	**4.05e-04**	1.26e-05	**1.25e-04**	2.71e-06	1.61e-05
Dim 5 $n=25$	Smolyak	1.92e-01	4.54e-01	8.55e-02	2.74e-01	3.19e-02	1.27e-01	7.81e-03	5.18e-02	8.50e-02	4.56e-01	1.51e-01	8.10e-01	4.72e-02	2.65e-01	1.46e+00	9.02e+00		
	LS-Uniform	**9.82e-02**	**1.95e-01**	**5.96e-02**	3.50e-01	**1.75e-02**	1.38e-01	**4.55e-03**	5.66e-02	**1.23e-03**	2.47e-02	**3.56e-04**	8.82e-03	**7.09e-05**	2.22e-03	**1.31e-05**	1.06e-03		
	LS-Chebyshev	1.41e-01	2.73e-01	9.40e-02	**2.56e-01**	3.34e-02	**1.18e-01**	9.93e-03	**4.90e-02**	2.95e-03	**1.58e-02**	8.84e-04	**5.89e-03**	2.02e-04	**1.50e-03**	3.74e-05	**2.24e-04**		
Dim 6 $n=25$	Smolyak	2.30e-01	5.74e-01	8.29e-02	3.30e-01	3.12e-02	1.70e-01	1.09e-02	8.55e-02	9.26e-03	5.01e-02	3.08e-02	1.86e-01	2.86e-01	1.69e+00				
	LS-Uniform	**1.01e-01**	**3.10e-01**	**3.11e-02**	**1.14e-01**	**1.25e-02**	**1.02e-01**	**2.65e-03**	**2.43e-02**	**7.37e-04**	2.02e-02	**2.57e-04**	7.12e-03	**6.80e-05**	4.16e-03				
	LS-Chebyshev	2.41e-01	5.67e-01	6.65e-02	2.00e-01	2.80e-02	1.40e-01	8.45e-03	4.01e-02	2.52e-03	**1.10e-02**	7.87e-04	**4.84e-03**	2.16e-04	**1.63e-03**				
Dim 7 $n=20$	Smolyak	1.84e-01	4.04e-01	3.86e-02	1.27e-01	1.01e-02	4.78e-02	1.42e-03	1.25e-02	3.84e-03	2.53e-02	1.88e-03	1.41e-02	5.08e-03	4.83e-02				
	LS-Uniform	**1.03e-01**	**3.60e-01**	**2.60e-02**	1.61e-01	**5.38e-03**	6.23e-02	**1.05e-03**	1.55e-02	**1.77e-04**	4.89e-03	**2.95e-05**	8.68e-04	**4.20e-06**	2.16e-04				
	LS-Chebyshev	1.48e-01	3.91e-01	3.69e-02	**1.26e-01**	8.74e-03	**4.46e-02**	1.38e-03	**8.73e-03**	3.17e-04	**2.45e-03**	7.67e-05	**6.24e-04**	1.08e-05	**1.53e-04**				
Dim 8 $n=10$	Smolyak	1.01e-01	2.95e-01	1.94e-02	7.63e-02	2.75e-03	2.04e-02	4.38e-04	5.73e-03	6.61e-05	1.14e-03	1.53e-05	2.17e-04						
	LS-Uniform	**6.37e-02**	**1.81e-01**	**1.10e-02**	**4.12e-02**	**1.54e-03**	2.05e-02	**1.58e-04**	**1.85e-03**	**1.49e-05**	3.44e-04	**1.67e-06**	9.00e-05						
	LS-Chebyshev	7.90e-02	1.98e-01	1.65e-02	6.66e-02	2.39e-03	**1.12e-02**	2.38e-04	2.05e-03	2.64e-05	**2.83e-04**	3.77e-06	**4.51e-05**						
Dim 9 $n=25$	Smolyak	2.90e-01	7.01e-01	8.59e-02	2.55e-01	2.87e-02	1.44e-01	8.41e-03	7.06e-02	2.36e-03	2.26e-02	6.38e-04	7.12e-03						
	LS-Uniform	**7.18e-02**	**1.86e-01**	**3.45e-02**	**1.35e-01**	**1.33e-02**	**1.13e-01**	**2.92e-03**	3.67e-02	**7.66e-04**	3.91e-02	**1.91e-04**	1.36e-02						
	LS-Chebyshev	3.28e-01	5.50e-01	1.03e-01	2.64e-01	3.69e-02	1.32e-01	8.70e-03	**3.63e-02**	2.48e-03	**1.20e-02**	7.74e-04	**5.69e-03**						
Dim 10 $n=10$	Smolyak	2.15e-01	6.01e-01	6.60e-02	2.27e-01	1.91e-02	9.96e-02	4.83e-03	4.73e-02	3.85e-02	3.54e-01	2.51e-02	2.80e-01						
	LS-Uniform	**8.17e-02**	**2.59e-01**	**3.26e-02**	**1.37e-01**	**9.92e-03**	9.09e-02	**1.95e-03**	3.76e-02	**5.93e-04**	**1.28e-02**	**1.48e-04**	8.75e-03						
	LS-Chebyshev	1.42e-01	3.07e-01	5.72e-02	2.23e-01	1.73e-02	**7.04e-02**	6.10e-03	**3.23e-02**	1.92e-03	1.40e-02	4.81e-04	**4.57e-03**						

TABLE 17. Visualization of the results for function class *Morokoff Calfisch 1* for various dimensions and scales, tested with n realizations. Best algorithm per dimension and scale is depicted bold.

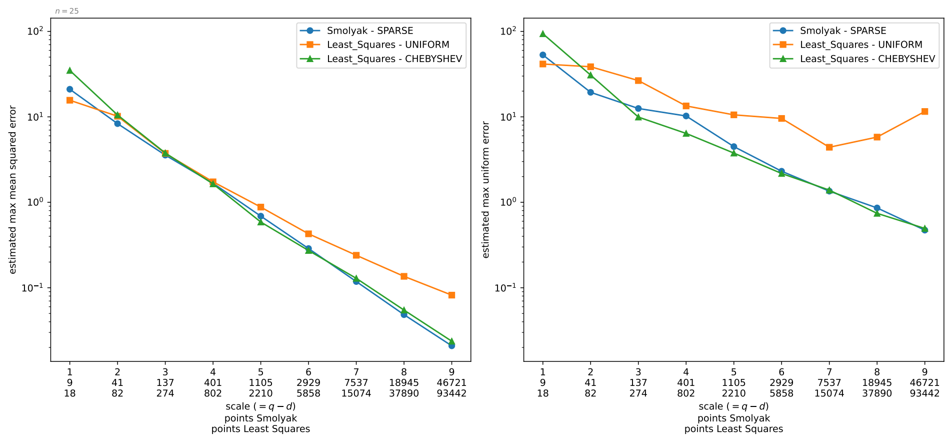

FIGURE 19. Visualization of the results for dim = 4 and various scales tested with $n = 25$ realizations for function class *Roos Arnold*. Left plot shows the estimated max mean squared error and the right one shows the estimated max uniform error.

		Scale1 max		Scale2 max		Scale3 max		Scale4 max		Scale5 max		Scale6 max		Scale7 max		Scale8 max		Scale9 max	
		ℓ_2	ℓ_∞	ℓ_2	ℓ_∞	ℓ_2	ℓ_∞	ℓ_2	ℓ_∞	ℓ_2	ℓ_∞	ℓ_2	ℓ_∞	ℓ_2	ℓ_∞	ℓ_2	ℓ_∞	ℓ_2	ℓ_∞
Dim 2 $n = 25$	Smolyak	**3.35e-02**	**4.25e-02**	**7.84e-16**	**1.78e-15**	**7.91e-16**	**1.78e-15**	**1.35e-15**	**3.55e-15**	**2.06e-15**	**5.77e-15**	**2.69e-15**	**9.33e-15**	**4.36e-15**	**1.60e-14**	**6.31e-15**	**2.09e-14**	1.04e-14	**5.28e-14**
	LS-Uniform	3.70e-02	4.65e-02	1.59e-15	3.11e-15	1.93e-15	7.11e-15	3.78e-14	2.50e-13	7.75e-15	6.08e-14	6.06e-15	5.55e-14	8.93e-15	1.74e-13	3.18e-14	1.07e-12	9.76e-11	5.62e-09
	LS-Chebyshev	8.56e-02	1.33e-01	5.59e-15	1.07e-14	5.53e-15	1.47e-14	7.19e-15	2.02e-14	4.98e-15	1.69e-14	2.94e-15	1.29e-14	9.84e-15	5.73e-14	8.39e-15	8.28e-14	**6.89e-15**	9.59e-14
Dim 3 $n = 25$	Smolyak	**3.46e-02**	**5.46e-02**	**1.07e-03**	**2.75e-03**	**1.36e-15**	**4.44e-15**	**1.98e-15**	**6.66e-15**	**2.73e-15**	**9.77e-15**	3.74e-15	**1.51e-14**	5.52e-15	**2.26e-14**	7.36e-15	**3.24e-14**	**1.16e-14**	**6.75e-14**
	LS-Uniform	4.33e-02	7.43e-02	1.44e-03	3.87e-03	4.91e-15	1.87e-14	4.81e-15	2.80e-14	3.40e-15	2.31e-14	**3.65e-15**	6.20e-14	**4.50e-15**	1.14e-13	**6.52e-15**	2.97e-13	8.57e-14	8.96e-12
	LS-Chebyshev	1.00e-01	1.63e-01	1.82e-03	3.50e-03	2.91e-15	6.66e-15	7.29e-15	3.31e-14	1.55e-14	6.82e-14	1.22e-14	1.12e-13	1.58e-14	1.10e-13	1.14e-14	1.49e-13	1.24e-14	3.15e-13
Dim 4 $n = 25$	Smolyak	**3.30e-02**	**8.03e-02**	**1.46e-03**	**4.81e-03**	**3.39e-05**	**1.65e-04**	3.52e-15	1.42e-14	4.94e-15	**2.22e-14**	6.39e-15	**2.66e-14**	8.55e-15	**4.17e-14**	1.14e-14	**5.24e-14**	1.62e-14	**8.24e-14**
	LS-Uniform	3.89e-02	9.42e-02	3.28e-03	1.10e-02	7.70e-05	4.40e-04	**2.24e-15**	**1.20e-14**	**3.12e-15**	2.66e-14	**3.32e-15**	5.42e-14	**3.03e-15**	8.17e-14	**4.45e-15**	2.20e-13	**9.82e-15**	1.08e-12
	LS-Chebyshev	8.87e-02	1.52e-01	5.11e-03	1.57e-02	1.25e-04	4.39e-04	1.40e-14	7.68e-14	1.01e-14	8.70e-14	1.10e-14	1.31e-13	3.21e-14	2.69e-13	2.78e-14	4.44e-13	2.37e-14	7.15e-13
Dim 5 $n = 25$	Smolyak	3.10e-02	7.67e-02	**1.22e-03**	**4.43e-03**	**4.04e-05**	**2.20e-04**	**9.97e-07**	**6.29e-06**	8.90e-14	4.64e-13	3.27e-13	2.42e-12	4.75e-13	2.74e-12	3.18e-10	2.08e-09		
	LS-Uniform	**3.02e-02**	**6.12e-02**	2.46e-03	8.17e-03	7.14e-05	3.32e-04	1.81e-06	1.24e-05	**2.69e-15**	**2.22e-14**	**3.20e-15**	**3.82e-14**	**4.71e-15**	**1.41e-13**	**7.77e-15**	**3.76e-13**		
	LS-Chebyshev	5.22e-02	1.12e-01	4.37e-03	1.35e-02	1.61e-04	5.52e-04	4.35e-06	2.71e-05	2.78e-14	3.45e-13	2.66e-14	4.57e-13	2.70e-14	1.06e-12	3.33e-14	1.70e-12		
Dim 6 $n = 25$	Smolyak	**2.83e-02**	**7.71e-02**	**1.32e-03**	**5.28e-03**	**4.23e-05**	**3.38e-04**	**1.12e-06**	**1.24e-05**	**1.96e-08**	**2.37e-07**	1.95e-13	1.44e-12	1.21e-12	7.30e-12				
	LS-Uniform	3.51e-02	7.94e-02	2.43e-03	8.95e-03	6.84e-05	4.06e-04	1.94e-06	2.34e-05	3.38e-08	4.26e-07	**4.94e-15**	**1.26e-13**	**6.78e-15**	**1.99e-13**				
	LS-Chebyshev	6.65e-02	1.62e-01	4.36e-03	1.83e-02	1.74e-04	5.79e-04	4.79e-06	2.66e-05	1.27e-07	6.47e-07	8.07e-14	7.55e-13	6.57e-14	1.05e-12				
Dim 7 $n = 20$	Smolyak	**1.15e-02**	**3.17e-02**	**6.20e-04**	4.55e-03	**2.48e-05**	**3.31e-04**	**7.38e-07**	**1.32e-05**	**1.65e-08**	**2.89e-07**	**1.97e-10**	**3.94e-09**	**1.30e-11**	**1.05e-10**				
	LS-Uniform	1.68e-02	4.60e-02	8.95e-04	**4.33e-03**	3.76e-05	6.02e-04	1.25e-06	1.67e-05	3.04e-08	7.25e-07	3.72e-10	1.09e-08	4.18e-11	2.60e-09				
	LS-Chebyshev	2.28e-02	5.42e-02	1.40e-03	7.15e-03	6.60e-05	9.22e-04	2.49e-06	2.90e-05	6.96e-08	2.74e-06	1.79e-09	2.85e-08	2.62e-09	4.70e-08				
Dim 8 $n = 10$	Smolyak	**1.03e-02**	**3.80e-02**	**5.32e-04**	**2.91e-03**	**2.04e-05**	**1.53e-04**	**5.76e-07**	**1.08e-05**	**1.49e-08**	**4.69e-07**	**2.91e-10**	**8.54e-09**						
	LS-Uniform	2.02e-02	5.98e-02	9.42e-04	3.76e-03	3.30e-05	2.58e-04	9.52e-07	1.34e-05	2.73e-08	7.68e-07	5.46e-10	2.39e-08						
	LS-Chebyshev	2.53e-02	8.65e-02	1.50e-03	7.68e-03	6.37e-05	4.61e-04	2.12e-06	4.99e-05	6.23e-08	1.62e-06	3.51e-09	3.92e-08						
Dim 9 $n = 25$	Smolyak	**2.08e-02**	**6.39e-02**	**8.67e-04**	**5.64e-03**	**3.03e-05**	**3.66e-04**	**7.92e-07**	**1.68e-05**	**1.76e-08**	**6.13e-07**	**3.21e-10**	**1.19e-08**						
	LS-Uniform	2.45e-02	8.13e-02	1.56e-03	7.83e-03	4.88e-05	3.83e-04	1.32e-06	2.15e-05	2.98e-08	7.43e-07	5.92e-10	2.16e-08						
	LS-Chebyshev	6.49e-02	1.59e-01	2.86e-03	9.68e-03	1.32e-04	6.62e-04	4.74e-06	3.12e-05	1.26e-07	1.26e-06	2.96e-09	2.03e-08						
Dim 10 $n = 10$	Smolyak	**9.91e-03**	**2.99e-02**	**5.43e-04**	**4.68e-03**	**2.32e-05**	3.25e-04	**6.91e-07**	**1.15e-05**	**1.21e-08**	**6.23e-07**	**2.10e-10**	**1.21e-08**						
	LS-Uniform	1.55e-02	4.95e-02	7.66e-04	4.89e-03	3.59e-05	**3.17e-04**	1.10e-06	1.49e-05	2.03e-08	7.06e-07	3.79e-10	2.29e-08						
	LS-Chebyshev	1.75e-02	3.84e-02	1.16e-03	6.71e-03	7.10e-05	5.42e-04	2.68e-06	3.75e-05	6.19e-08	2.45e-06	4.89e-08	7.28e-07						

TABLE 18. Visualization of the results for function class *Morokoff Calfisch 2* for various dimensions and scales, tested with n realizations. Best algorithm per dimension and scale is depicted bold.

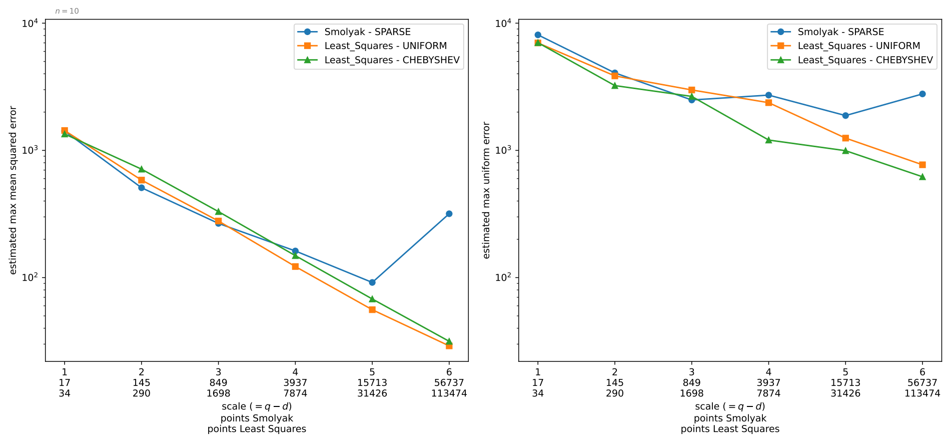

FIGURE 20. Visualization of the results for dim = 8 and various scales tested with $n = 25$ realizations for function class *Roos Arnold*. Left plot shows the estimated max mean squared error and the right one shows the estimated max uniform error.

		Scale1 max		Scale2 max		Scale3 max		Scale4 max		Scale5 max		Scale6 max		Scale7 max		Scale8 max		Scale9 max	
		ℓ_2	ℓ_∞	ℓ_2	ℓ_∞	ℓ_2	ℓ_∞	ℓ_2	ℓ_∞	ℓ_2	ℓ_∞	ℓ_2	ℓ_∞	ℓ_2	ℓ_∞	ℓ_2	ℓ_∞	ℓ_2	ℓ_∞
Dim 2 $n = 25$	Smolyak	1.41e+00	2.15e+00	9.44e-01	1.95e+00	2.99e-01	7.45e-01	**1.10e-01**	**4.09e-01**	4.15e-02	2.38e-01	**1.53e-02**	**1.04e-01**	**5.97e-03**	**5.52e-02**	**2.10e-03**	**3.87e-02**	**8.52e-04**	**1.80e-02**
	LS-Uniform	**1.40e+00**	**1.90e+00**	**4.45e-01**	**9.78e-01**	1.23e+00	5.07e+00	1.15e+00	7.74e+00	3.65e-01	3.51e+00	6.03e-02	7.97e-01	1.32e-01	2.77e+00	9.68e-02	3.67e+00	1.33e+02	7.64e+03
	LS-Chebyshev	2.48e+00	3.77e+00	8.71e-01	2.04e+00	**2.41e-01**	**6.78e-01**	1.24e-01	5.05e-01	**4.00e-02**	**1.98e-01**	2.08e-02	1.66e-01	7.15e-03	7.05e-02	2.74e-03	4.05e-02	1.07e-03	2.87e-02
Dim 3 $n = 25$	Smolyak	4.54e+00	8.29e+00	2.53e+00	6.31e+00	1.48e+00	3.68e+00	4.41e-01	2.06e+00	1.87e-01	**9.77e-01**	8.06e-02	5.25e-01	2.98e-02	**3.11e-01**	**1.19e-02**	**1.72e-01**	**4.52e-03**	**9.59e-02**
	LS-Uniform	**3.16e+00**	**7.23e+00**	3.50e+00	1.02e+01	1.41e+00	6.82e+00	4.88e-01	2.80e+00	2.17e-01	1.63e+00	1.55e-01	2.42e+00	1.02e-01	3.01e+00	7.21e-02	4.79e+00	4.32e-01	4.76e+01
	LS-Chebyshev	6.32e+00	1.45e+01	**2.07e+00**	**4.93e+00**	**8.67e-01**	**2.97e+00**	**3.48e-01**	**2.00e+00**	**1.34e-01**	1.22e+00	**5.93e-02**	**5.05e-01**	**2.78e-02**	3.76e-01	1.20e-02	2.26e-01	5.04e-03	1.50e-01
Dim 4 $n = 25$	Smolyak	2.10e+01	5.31e+01	**8.33e+00**	**1.94e+01**	**3.57e+00**	1.25e+01	1.67e+00	1.02e+01	6.89e-01	4.49e+00	2.88e-01	2.31e+00	**1.18e-01**	**1.35e+00**	**4.84e-02**	8.59e-01	**2.09e-02**	**4.72e-01**
	LS-Uniform	**1.56e+01**	**4.15e+01**	1.02e+01	3.86e+01	3.74e+00	2.66e+01	1.74e+00	1.34e+01	8.79e-01	1.06e+01	4.28e-01	9.58e+00	2.40e-01	4.41e+00	1.36e-01	5.80e+00	8.20e-02	1.16e+01
	LS-Chebyshev	3.49e+01	9.39e+01	1.05e+01	3.08e+01	3.77e+00	**9.94e+00**	**1.64e+00**	**6.40e+00**	**5.88e-01**	**3.77e+00**	**2.73e-01**	**2.17e+00**	1.29e-01	1.39e+00	5.48e-02	**7.43e-01**	2.37e-02	4.95e-01
Dim 5 $n = 25$	Smolyak	4.81e+01	1.39e+02	2.75e+01	1.11e+02	1.36e+01	5.05e+01	7.56e+00	3.69e+01	5.24e+00	2.67e+01	2.27e+01	1.33e+02	5.31e+01	3.07e+02	1.78e+04	1.01e+05		
	LS-Uniform	4.23e+01	9.66e+01	**1.91e+01**	**5.83e+01**	**9.11e+00**	6.26e+01	**4.85e+00**	3.29e+01	2.54e+00	2.96e+01	1.31e+00	2.46e+01	7.41e-01	1.58e+01	3.90e-01	1.84e+01		
	LS-Chebyshev	**3.74e+01**	**6.83e+01**	2.34e+01	1.04e+02	1.20e+01	**4.54e+01**	5.02e+00	**2.21e+01**	**2.25e+00**	**1.23e+01**	**1.04e+00**	**7.88e+00**	**4.88e-01**	**6.26e+00**	**2.25e-01**	**3.52e+00**		
Dim 6 $n = 25$	Smolyak	8.54e+01	2.66e+02	4.90e+01	2.23e+02	3.52e+01	2.20e+02	1.92e+01	1.21e+02	1.93e+01	7.50e+01	8.65e+01	4.64e+02	4.81e+02	2.93e+03				
	LS-Uniform	**6.62e+01**	**1.86e+02**	**3.82e+01**	**1.83e+02**	**1.85e+01**	**1.03e+02**	**9.70e+00**	**7.18e+01**	**5.43e+00**	8.88e+01	3.09e+00	6.94e+01	1.72e+00	7.18e+01				
	LS-Chebyshev	2.85e+02	4.46e+02	7.91e+01	2.32e+02	2.90e+01	1.46e+02	1.37e+01	8.41e+01	6.28e+00	**4.12e+01**	**2.82e+00**	**2.47e+01**	**1.41e+00**	**1.50e+01**				
Dim 7 $n = 20$	Smolyak	1.74e+02	8.26e+02	**8.71e+01**	1.12e+03	6.68e+01	1.03e+03	3.97e+01	4.99e+02	2.21e+01	3.81e+02	1.37e+02	1.12e+03	2.12e+03	1.90e+04				
	LS-Uniform	**1.60e+02**	7.44e+02	9.45e+01	6.56e+02	**4.17e+01**	3.81e+02	**2.33e+01**	**2.79e+02**	**1.28e+01**	2.80e+02	**6.60e+00**	1.58e+02	3.49e+00	1.70e+02				
	LS-Chebyshev	3.08e+02	**7.12e+02**	1.58e+02	**5.14e+02**	6.37e+01	**3.54e+02**	3.26e+01	2.86e+02	1.42e+01	**1.50e+02**	6.65e+00	**9.01e+01**	**3.24e+00**	**7.67e+01**				
Dim 8 $n = 10$	Smolyak	1.42e+03	8.09e+03	**5.08e+02**	4.07e+03	**2.67e+02**	**2.49e+03**	1.62e+02	2.72e+03	9.16e+01	1.88e+03	3.18e+02	2.78e+03						
	LS-Uniform	1.43e+03	**7.00e+03**	5.84e+02	3.85e+03	2.79e+02	2.99e+03	**1.22e+02**	2.37e+03	**5.59e+01**	1.25e+03	**2.91e+01**	7.71e+02						
	LS-Chebyshev	**1.35e+03**	7.02e+03	7.12e+02	**3.23e+03**	3.30e+02	2.66e+03	1.49e+02	**1.21e+03**	6.79e+01	**9.95e+02**	3.16e+01	**6.20e+02**						
Dim 9 $n = 25$	Smolyak	6.79e+02	2.27e+03	9.78e+02	1.05e+04	4.05e+02	7.91e+03	2.57e+02	5.04e+03	1.79e+02	3.97e+03	1.03e+02	2.48e+03						
	LS-Uniform	**6.49e+02**	**1.53e+03**	**7.46e+02**	7.82e+03	**2.68e+02**	3.65e+03	**1.62e+02**	**2.27e+03**	**9.05e+01**	1.36e+03	**5.17e+01**	1.63e+03						
	LS-Chebyshev	3.36e+03	6.23e+03	1.60e+03	**6.39e+03**	6.98e+02	**2.80e+03**	3.57e+02	2.60e+03	1.76e+02	**1.25e+03**	8.42e+01	**8.59e+02**						
Dim 10 $n = 10$	Smolyak	9.78e+02	5.89e+03	6.01e+02	1.15e+04	5.72e+02	1.01e+04	4.95e+02	1.66e+04	3.99e+02	1.35e+04	1.27e+03	1.28e+04						
	LS-Uniform	**9.42e+02**	**4.49e+03**	**4.64e+02**	**8.87e+03**	**3.06e+02**	4.31e+03	**2.12e+02**	**4.74e+03**	**1.25e+02**	5.31e+03	**7.29e+01**	**2.97e+03**						
	LS-Chebyshev	2.46e+03	5.99e+03	1.33e+03	8.92e+03	8.08e+02	**3.92e+03**	5.08e+02	5.50e+03	2.68e+02	**4.14e+03**	1.46e+02	3.33e+03						

TABLE 19. Visualization of the results for function class *Roos Arnold* for various dimensions and scales, tested with n realizations. Best algorithm per dimension and scale is depicted bold.

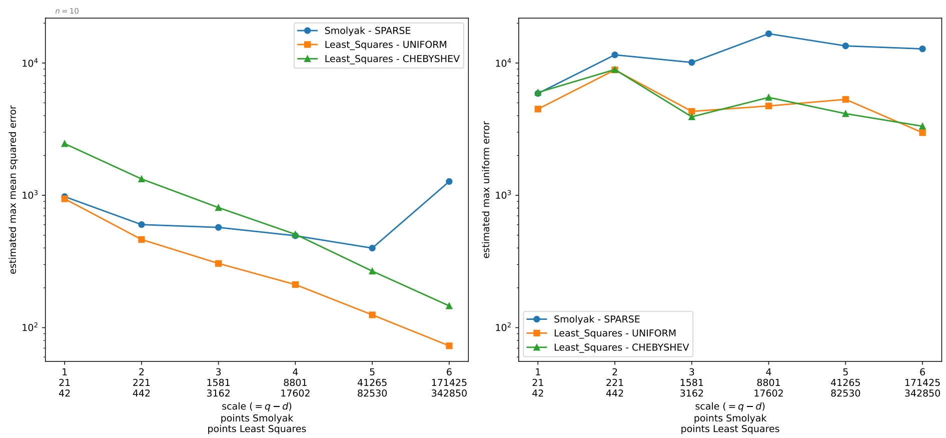

FIGURE 21. Visualization of the results for dim = 10 and various scales tested with $n = 10$ realizations for function class *Roos Arnold*. Left plot shows the estimated max mean squared error and the right one shows the estimated max uniform error.

		Scale1 max ℓ_2	Scale1 max ℓ_∞	Scale2 max ℓ_2	Scale2 max ℓ_∞	Scale3 max ℓ_2	Scale3 max ℓ_∞	Scale4 max ℓ_2	Scale4 max ℓ_∞	Scale5 max ℓ_2	Scale5 max ℓ_∞	Scale6 max ℓ_2	Scale6 max ℓ_∞	Scale7 max ℓ_2	Scale7 max ℓ_∞	Scale8 max ℓ_2	Scale8 max ℓ_∞	Scale9 max ℓ_2	Scale9 max ℓ_∞
Dim 2 $n = 25$	Smolyak	**7.53e-02**	**9.57e-02**	**3.25e-16**	**6.66e-16**	**6.27e-16**	**1.33e-15**	**7.09e-16**	**2.22e-15**	**1.16e-15**	**4.88e-15**	**1.61e-15**	**6.22e-15**	**2.44e-15**	**1.15e-14**	**3.38e-15**	**1.55e-14**	5.34e-15	**2.66e-14**
	LS-Uniform	8.32e-02	1.05e-01	7.53e-16	1.33e-15	1.11e-15	3.55e-15	1.79e-14	1.19e-13	4.14e-15	2.71e-14	2.62e-15	2.80e-14	5.71e-15	7.59e-14	2.04e-14	7.78e-13	3.48e-11	1.97e-09
	LS-Chebyshev	1.92e-01	2.99e-01	3.05e-15	6.11e-15	2.45e-15	7.11e-15	4.02e-15	1.55e-14	2.29e-15	9.33e-15	2.58e-15	8.88e-15	4.24e-15	2.27e-14	3.67e-15	3.57e-14	**2.99e-15**	2.82e-14
Dim 3 $n = 25$	Smolyak	**1.57e-01**	**2.57e-01**	**1.69e-02**	**4.33e-02**	**7.02e-16**	**2.22e-15**	**1.07e-15**	**3.11e-15**	**1.76e-15**	**9.33e-15**	2.37e-15	**1.11e-14**	3.26e-15	**1.38e-14**	**4.44e-15**	**2.66e-14**	**6.74e-15**	**4.71e-14**
	LS-Uniform	1.88e-01	3.37e-01	2.27e-02	6.09e-02	1.97e-15	7.33e-15	2.00e-15	1.02e-14	2.68e-15	1.83e-14	**1.91e-15**	2.96e-14	**2.01e-15**	5.76e-14	4.50e-15	1.84e-13	2.81e-14	1.68e-12
	LS-Chebyshev	4.85e-01	7.74e-01	2.86e-02	5.51e-02	1.84e-15	5.55e-15	5.32e-15	1.80e-14	8.33e-15	3.15e-14	7.42e-15	5.87e-14	8.79e-15	5.16e-14	8.00e-15	8.22e-14	1.19e-14	1.33e-13
Dim 4 $n = 25$	Smolyak	**2.41e-01**	**4.02e-01**	**6.99e-02**	**2.34e-01**	**5.29e-03**	**2.57e-02**	**1.79e-15**	**6.88e-15**	**2.63e-15**	**1.02e-14**	3.55e-15	**1.73e-14**	4.60e-15	**2.75e-14**	6.18e-15	**3.15e-14**	8.66e-15	**5.11e-14**
	LS-Uniform	3.00e-01	4.75e-01	1.51e-01	5.16e-01	1.20e-02	6.85e-02	2.30e-15	1.41e-14	6.28e-15	3.69e-14	**1.96e-15**	2.71e-14	**1.84e-15**	4.36e-14	**2.75e-15**	1.04e-13	**5.05e-15**	5.49e-13
	LS-Chebyshev	8.39e-01	1.05e+00	2.10e-01	4.92e-01	1.95e-02	6.84e-02	1.29e-14	6.88e-14	1.10e-14	4.64e-14	1.37e-14	9.71e-14	5.59e-14	2.72e-13	2.87e-14	4.93e-13	2.06e-14	4.87e-13
Dim 5 $n = 25$	Smolyak	2.20e-01	5.04e-01	**6.23e-02**	**2.48e-01**	**2.57e-02**	**1.58e-01**	**1.79e-03**	**1.13e-02**	1.02e-13	3.61e-13	1.70e-12	8.84e-12	3.77e-13	2.12e-12	1.60e-10	9.96e-10		
	LS-Uniform	**1.82e-01**	**3.83e-01**	1.13e-01	4.61e-01	3.97e-02	2.41e-01	3.26e-03	2.23e-02	**3.53e-15**	**3.22e-14**	**2.64e-15**	**4.57e-14**	**3.31e-15**	**8.05e-14**	**4.31e-15**	**1.37e-13**		
	LS-Chebyshev	3.81e-01	8.67e-01	2.28e-01	5.86e-01	8.66e-02	4.19e-01	7.83e-03	4.88e-02	2.49e-14	2.31e-13	5.50e-14	3.42e-13	3.51e-14	4.18e-13	2.37e-14	6.87e-13		
Dim 6 $n = 25$	Smolyak	**4.00e-01**	**9.16e-01**	**8.18e-02**	**3.02e-01**	**1.62e-02**	**1.00e-01**	**5.85e-03**	**5.23e-02**	**3.99e-04**	**4.80e-03**	5.21e-13	2.95e-12	2.57e-12	1.59e-11				
	LS-Uniform	7.54e-01	1.69e+00	1.66e-01	6.66e-01	2.41e-02	1.67e-01	9.28e-03	9.77e-02	6.86e-04	8.63e-03	**3.10e-15**	**4.57e-14**	**2.26e-15**	**8.78e-14**				
	LS-Chebyshev	9.47e-01	1.96e+00	2.21e-01	6.32e-01	6.27e-02	2.18e-01	2.41e-02	1.22e-01	2.58e-03	1.31e-02	5.38e-13	2.60e-12	2.52e-13	1.15e-12				
Dim 7 $n = 20$	Smolyak	**6.31e-01**	**1.77e+00**	**1.68e-01**	**6.59e-01**	**2.53e-02**	**2.08e-01**	**6.48e-03**	**8.83e-02**	**1.14e-03**	**2.16e-02**	**1.30e-04**	**3.11e-03**	**8.31e-11**	**7.60e-10**				
	LS-Uniform	7.98e-01	2.74e+00	1.99e-01	1.02e+00	4.17e-02	3.51e-01	1.21e-02	1.92e-01	2.08e-03	4.26e-02	2.52e-04	8.44e-03	3.80e-09	9.67e-07				
	LS-Chebyshev	1.31e+00	2.66e+00	4.26e-01	1.38e+00	7.72e-02	3.80e-01	2.23e-02	1.34e-01	4.73e-03	5.62e-02	6.55e-04	1.90e-02	9.29e-08	1.81e-06				
Dim 8 $n = 10$	Smolyak	**2.21e-01**	**6.10e-01**	**9.08e-02**	**3.36e-01**	**2.45e-02**	2.73e-01	**9.25e-03**	**1.58e-01**	**1.77e-03**	**3.56e-02**	**2.55e-04**	**7.25e-03**						
	LS-Uniform	3.56e-01	9.09e-01	1.45e-01	8.21e-01	3.58e-02	**2.27e-01**	1.52e-02	2.24e-01	3.06e-03	6.61e-02	4.65e-04	1.89e-02						
	LS-Chebyshev	7.81e-01	1.77e+00	2.26e-01	7.64e-01	6.60e-02	4.70e-01	2.91e-02	3.87e-01	7.25e-03	1.25e-01	1.29e-03	1.68e-02						
Dim 9 $n = 25$	Smolyak	**4.23e-01**	**7.52e-01**	**1.29e-01**	**6.43e-01**	**4.45e-02**	**2.98e-01**	**9.67e-03**	**1.14e-01**	**2.23e-03**	**4.88e-02**	**3.82e-04**	**1.50e-02**						
	LS-Uniform	5.79e-01	1.69e+00	2.07e-01	9.54e-01	6.53e-02	4.35e-01	1.56e-02	1.80e-01	3.71e-03	9.71e-02	6.86e-04	2.69e-02						
	LS-Chebyshev	1.65e+00	4.10e+00	4.29e-01	1.28e+00	1.83e-01	6.48e-01	5.27e-02	2.70e-01	1.54e-02	8.49e-02	3.57e-03	2.84e-02						
Dim 10 $n = 10$	Smolyak	**4.76e-01**	**1.18e+00**	**5.79e-02**	**4.21e-01**	**2.69e-02**	**3.25e-01**	**6.81e-03**	**1.19e-01**	**1.50e-03**	**3.81e-02**	**3.50e-04**	**1.34e-02**						
	LS-Uniform	7.95e-01	2.03e+00	7.01e-02	4.62e-01	4.14e-02	3.51e-01	1.09e-02	1.27e-01	2.54e-03	5.67e-02	6.21e-04	3.17e-02						
	LS-Chebyshev	1.03e+00	3.86e+00	1.89e-01	6.08e-01	9.14e-02	3.80e-01	2.75e-02	2.69e-01	6.75e-03	7.61e-02	1.85e-03	3.81e-02						

TABLE 20. Visualization of the results for function class *Bratley* for various dimensions and scales, tested with n realizations. Best algorithm per dimension and scale is depicted bold.

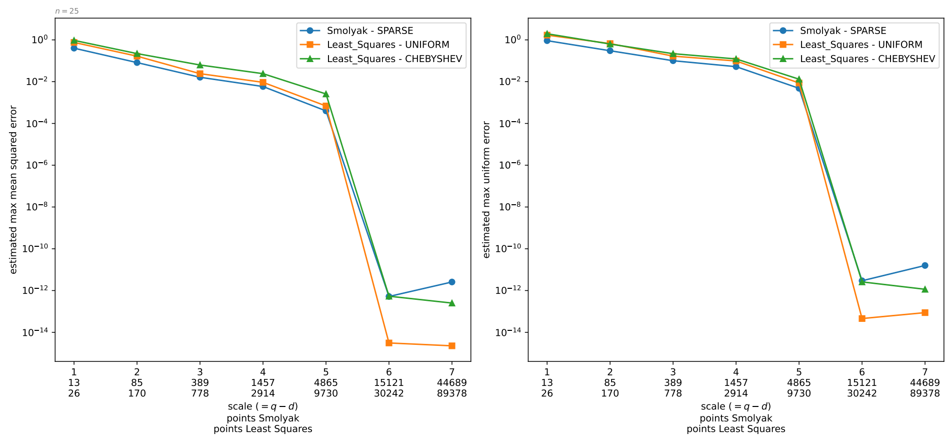

FIGURE 22. Visualization of the results for dim = 6 and various scales tested with $n = 25$ realizations for function class *Bratley*. Left plot shows the estimated max mean squared error and the right one shows the estimated max uniform error.

		Scale1 max ℓ_2	Scale1 max ℓ_∞	Scale2 max ℓ_2	Scale2 max ℓ_∞	Scale3 max ℓ_2	Scale3 max ℓ_∞	Scale4 max ℓ_2	Scale4 max ℓ_∞	Scale5 max ℓ_2	Scale5 max ℓ_∞	Scale6 max ℓ_2	Scale6 max ℓ_∞	Scale7 max ℓ_2	Scale7 max ℓ_∞	Scale8 max ℓ_2	Scale8 max ℓ_∞	Scale9 max ℓ_2	Scale9 max ℓ_∞
Dim 2 $n = 25$	Smolyak	2.44e-01	2.85e-01	2.03e-02	2.93e-02	9.62e-04	1.46e-03	1.74e-05	**3.47e-05**	1.51e-07	3.60e-07	1.43e-11	4.15e-11	**2.49e-14**	**1.14e-13**	**3.56e-14**	**1.37e-13**	6.11e-14	**3.13e-13**
	LS-Uniform	**2.04e-01**	**2.50e-01**	**6.21e-03**	**1.13e-02**	9.46e-04	4.19e-03	1.81e-05	1.19e-04	2.42e-07	1.68e-06	2.45e-11	2.47e-10	5.85e-14	1.24e-12	1.69e-13	6.49e-12	6.28e-10	3.62e-08
	LS-Chebyshev	4.56e-01	7.18e-01	2.94e-02	7.10e-02	**4.76e-04**	**1.20e-03**	**1.24e-05**	6.05e-05	**8.58e-08**	**2.46e-07**	**9.42e-12**	**4.02e-11**	4.92e-14	2.74e-13	4.09e-14	3.55e-13	**3.66e-14**	6.13e-13
Dim 3 $n = 25$	Smolyak	**1.78e+00**	**2.66e+00**	3.64e-01	**5.92e-01**	2.57e-02	4.64e-02	9.54e-04	**2.08e-03**	1.82e-05	4.77e-05	1.11e-07	**4.01e-07**	8.46e-10	3.64e-09	3.70e-12	1.96e-11	3.07e-13	**1.66e-12**
	LS-Uniform	2.03e+00	3.65e+00	4.20e-01	1.28e+00	2.33e-02	1.46e-01	**4.65e-04**	2.87e-03	1.59e-05	1.59e-04	**5.00e-08**	8.38e-07	6.97e-10	2.97e-08	7.72e-12	3.49e-10	1.69e-12	1.40e-10
	LS-Chebyshev	3.85e+00	6.24e+00	**3.25e-01**	7.17e-01	**1.42e-02**	**4.16e-02**	5.84e-04	2.25e-03	**1.17e-05**	**4.12e-05**	5.03e-08	4.03e-07	**3.33e-10**	**1.95e-09**	**1.92e-12**	**1.06e-11**	**2.09e-13**	4.68e-12
Dim 4 $n = 25$	Smolyak	**8.18e+00**	**1.49e+01**	2.22e+00	**3.44e+00**	1.99e-01	**3.85e-01**	1.11e-02	2.88e-02	2.96e-04	**1.01e-03**	9.09e-06	3.63e-05	1.85e-07	7.88e-07	4.21e-09	2.07e-08	6.35e-11	3.75e-10
	LS-Uniform	9.31e+00	2.22e+01	**1.85e+00**	4.59e+00	1.69e-01	1.61e+00	8.41e-03	7.97e-02	3.52e-04	3.82e-03	7.55e-06	3.64e-04	1.00e-07	7.68e-06	1.52e-09	8.93e-08	2.14e-11	1.66e-09
	LS-Chebyshev	1.91e+01	3.32e+01	2.51e+00	5.60e+00	**1.57e-01**	4.67e-01	**7.04e-03**	**2.52e-02**	**2.28e-04**	1.09e-03	**3.43e-06**	**2.47e-05**	**5.93e-08**	**3.43e-07**	**1.13e-09**	**6.99e-09**	**1.76e-11**	**1.52e-10**
Dim 5 $n = 25$	Smolyak	**4.91e+01**	9.92e+01	1.21e+01	**2.33e+01**	1.32e+00	**3.36e+00**	1.18e-01	3.96e-01	8.75e-03	3.14e-02	3.97e-04	1.70e-03	1.00e-05	6.08e-05	2.94e-07	1.90e-06		
	LS-Uniform	5.70e+01	**9.60e+01**	**9.13e+00**	3.60e+01	**1.05e+00**	8.94e+00	**7.49e-02**	1.34e+00	**2.75e-03**	4.80e-02	1.44e-04	7.03e-03	4.67e-06	3.84e-04	**1.14e-07**	8.62e-06		
	LS-Chebyshev	6.06e+01	1.10e+02	1.22e+01	3.29e+01	1.20e+00	4.06e+00	7.96e-02	**2.78e-01**	3.53e-03	**1.71e-02**	**1.38e-04**	**7.55e-04**	**4.10e-06**	**2.93e-05**	1.18e-07	**1.04e-06**		
Dim 6 $n = 25$	Smolyak	**1.81e+02**	**4.06e+02**	4.78e+01	**1.14e+02**	7.94e+00	**2.70e+01**	1.06e+00	3.16e+00	8.33e-02	2.79e-01	3.81e-03	1.93e-02	2.34e-04	1.13e-03				
	LS-Uniform	3.26e+02	7.14e+02	**3.90e+01**	1.92e+02	**5.70e+00**	6.33e+01	**4.40e-01**	7.11e+00	**2.69e-02**	6.56e-01	**1.59e-03**	6.75e-02	**8.66e-05**	7.67e-03				
	LS-Chebyshev	5.56e+02	1.12e+03	6.57e+01	1.98e+02	8.92e+00	2.84e+01	6.47e-01	**2.56e+00**	3.81e-02	**1.82e-01**	2.26e-03	**1.19e-02**	1.09e-04	**8.66e-04**				
Dim 7 $n = 20$	Smolyak	**7.56e+02**	**2.32e+03**	**1.89e+02**	**6.25e+02**	3.75e+01	**1.28e+02**	5.00e+00	**2.11e+01**	5.54e-01	2.35e+00	5.79e-02	2.62e-01	4.21e-03	2.06e-02				
	LS-Uniform	1.03e+03	2.62e+03	2.51e+02	1.67e+03	**2.61e+01**	2.49e+02	**2.94e+00**	7.44e+01	**2.96e-01**	1.17e+01	**2.44e-02**	1.39e+00	**1.34e-03**	8.32e-02				
	LS-Chebyshev	1.97e+03	4.34e+03	3.16e+02	9.56e+02	3.96e+01	1.62e+02	4.44e+00	2.27e+01	3.97e-01	**2.11e+00**	2.89e-02	**1.83e-01**	1.64e-03	**1.12e-02**				
Dim 8 $n = 10$	Smolyak	**3.91e+03**	**1.46e+04**	1.08e+03	**2.74e+03**	2.26e+02	**7.50e+02**	2.93e+01	1.58e+02	3.46e+00	1.85e+01	3.97e-01	2.08e+00						
	LS-Uniform	6.87e+03	1.93e+04	**1.02e+03**	5.53e+03	**2.10e+02**	1.76e+03	**2.08e+01**	4.64e+02	**1.73e+00**	6.85e+01	**1.51e-01**	1.97e+01						
	LS-Chebyshev	5.76e+03	1.47e+04	1.37e+03	4.51e+03	2.35e+02	1.26e+03	2.76e+01	**1.20e+02**	2.48e+00	**1.70e+01**	2.07e-01	**1.51e+00**						
Dim 9 $n = 25$	Smolyak	**2.15e+04**	5.01e+04	**4.49e+03**	**1.93e+04**	9.62e+02	**4.32e+03**	2.01e+02	**6.98e+02**	2.87e+01	**1.48e+02**	3.34e+00	2.06e+01						
	LS-Uniform	2.28e+04	**4.93e+04**	4.90e+03	2.27e+04	**9.29e+02**	1.01e+04	**1.20e+02**	2.54e+03	**1.20e+01**	4.91e+02	**1.14e+00**	7.11e+01						
	LS-Chebyshev	2.78e+04	7.08e+04	7.48e+03	2.20e+04	1.45e+03	4.96e+03	2.08e+02	1.00e+03	2.21e+01	1.59e+02	1.87e+00	**1.61e+01**						
Dim 10 $n = 10$	Smolyak	7.84e+04	2.55e+05	**1.05e+04**	1.11e+05	3.02e+03	3.27e+04	5.91e+02	5.95e+03	1.06e+02	9.28e+02	1.62e+01	1.62e+02						
	LS-Uniform	**6.76e+04**	**1.57e+05**	1.26e+04	**9.18e+04**	**2.67e+03**	3.70e+04	**4.42e+02**	1.57e+04	**5.77e+01**	2.03e+03	**7.06e+00**	6.20e+02						
	LS-Chebyshev	8.19e+04	3.01e+05	2.43e+04	1.05e+05	4.52e+03	**2.38e+04**	7.89e+02	**5.77e+03**	1.07e+02	**7.63e+02**	1.25e+01	**1.36e+02**						

TABLE 21. Visualization of the results for function class *Zhou* for various dimensions and scales, tested with n realizations. Best algorithm per dimension and scale is depicted bold.

As soon as the pictures are fixed, align them properly!

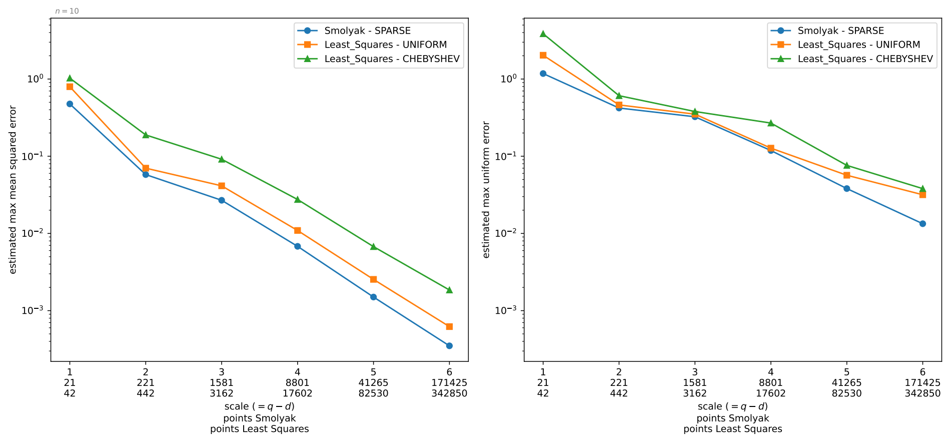

FIGURE 23. Visualization of the results for dim = 10 and various scales tested with $n = 10$ realizations for function class *Bratley*. Left plot shows the estimated max mean squared error and the right one shows the estimated max uniform error.

7. Conclusion

Needs to be formalized

Ideas

- Same (or usually not a worse—mostly better) order of decay
- $2n$ points seem to suffice compared to the number of the paper
- In some cases, Least Squares outperforms Smolyak a lot
- Tasmainian: Sometimes bad performance: Bad implementation or maybe sometimes Smolyak really bad (Mario: Approximation of the 0-Function). People might not be aware of the fact that the approximation quality might be extra-poor
- Write that np.linalg.solve was inaccurate → That's why we used lstsq

Acknowledgements

What to write here?

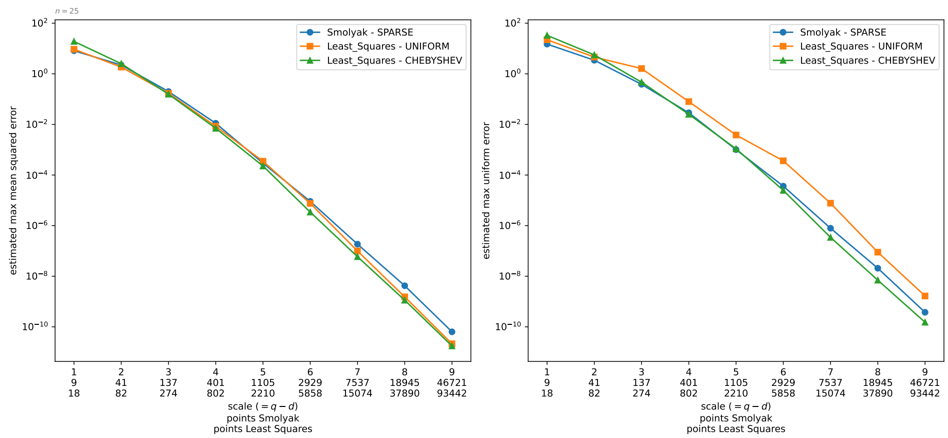

FIGURE 24. Visualization of the results for dim = 4 and various scales tested with $n = 25$ realizations for function class *Zhou*. Left plot shows the estimated max mean squared error and the right one shows the estimated max uniform error.

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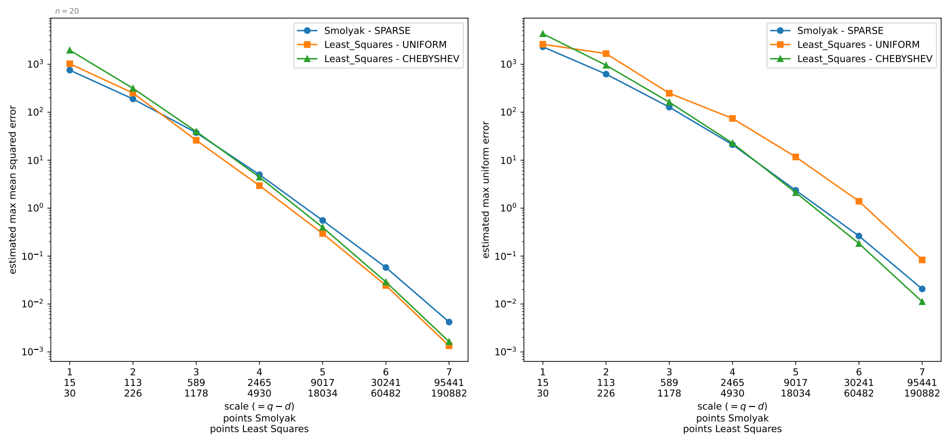

FIGURE 25. Visualization of the results for dim = 7 and various scales tested with $n = 20$ realizations for function class *Zhou*. Left plot shows the estimated max mean squared error and the right one shows the estimated max uniform error.

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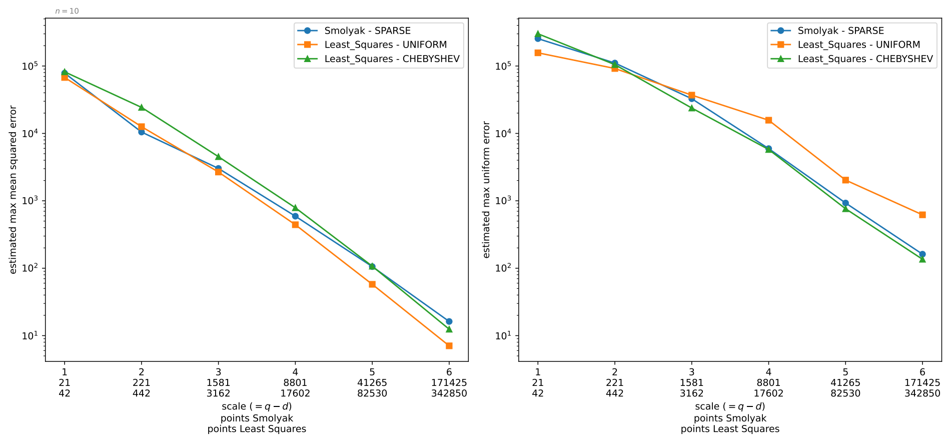

FIGURE 26. Visualization of the results for dim = 10 and various scales tested with $n = 10$ realizations for function class *Zhou*. Left plot shows the estimated max mean squared error and the right one shows the estimated max uniform error.

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