

Details on the numerical experiments of

**SPARSE GRIDS VS. RANDOM POINTS
FOR HIGH-DIMENSIONAL
POLYNOMIAL APPROXIMATION**

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ABSTRACT. This document contains detailed information of the conducted numerical experiments in the paper [Eggl, Mindlberger, Ullrich: Sparse grids vs. random points for high-dimensional polynomial approximation].

		Scale 3		Scale 4		Scale 5		Scale 6		Scale 7		Scale 8		Scale 9	
		e_{\max}^{wc}	$e_{\text{mean}}^{\text{wc}}$	e_{\max}^{wc}	$e_{\text{mean}}^{\text{wc}}$	e_{\max}^{wc}	$e_{\text{mean}}^{\text{wc}}$	e_{\max}^{wc}	$e_{\text{mean}}^{\text{wc}}$	e_{\max}^{wc}	$e_{\text{mean}}^{\text{wc}}$	e_{\max}^{wc}	$e_{\text{mean}}^{\text{wc}}$	e_{\max}^{wc}	$e_{\text{mean}}^{\text{wc}}$
Blin. Gauss. $Q = 50$	Smolyak	5.71e-01	2.69e-01	3.66e-01	1.55e-01	1.91e-01	7.14e-02	3.33e-02	9.30e-03	2.47e-03	6.16e-04	8.96e-06	2.44e-06	7.95e-09	2.19e-09
	LS-Uniform	3.51e+00	8.16e-01	5.12e+00	7.82e-01	3.56e+00	3.27e-01	8.19e+00	7.13e-01	3.15e+02	1.41e+01	1.93e+00	6.95e-02	3.03e+04	5.31e+02
	LS-Chebyshev	7.40e-01	2.89e-01	4.20e-01	1.35e-01	1.87e-01	4.82e-02	2.24e-02	6.11e-03	1.52e-03	4.10e-04	6.78e-06	1.50e-06	7.27e-09	1.52e-09
Cont. $Q = 50$	Smolyak	4.40e-02	1.49e-02	2.22e-02	5.15e-03	1.10e-02	1.58e-03	5.23e-03	6.18e-04	2.68e-03	2.17e-04	1.46e-03	8.93e-05	7.65e-04	2.95e-05
	LS-Uniform	2.66e-01	7.08e-02	5.19e-01	7.51e-02	3.64e-01	3.24e-02	6.88e-01	6.27e-02	5.86e+02	2.69e+01	7.77e+02	3.19e+01	8.46e+08	1.50e+07
	LS-Chebyshev	4.80e-02	2.21e-02	1.84e-02	4.06e-03	9.43e-03	1.45e-03	4.69e-03	4.91e-04	2.08e-03	1.85e-04	1.20e-03	6.72e-05	6.47e-04	2.71e-05
Corner Peak $Q = 50$	Smolyak	8.78e-03	4.52e-03	1.48e-03	5.81e-04	1.04e-04	3.31e-05	8.90e-07	2.61e-07	6.08e-09	1.75e-09	2.77e-13	8.54e-14	6.99e-15	9.35e-16
	LS-Uniform	1.54e-02	3.24e-03	3.50e-03	5.50e-04	4.92e-04	5.98e-05	2.30e-04	2.04e-05	6.80e-04	3.05e-05	7.93e-08	2.70e-09	1.66e-02	2.92e-04
	LS-Chebyshev	8.88e-03	3.12e-03	1.42e-03	4.44e-04	8.77e-05	2.65e-05	6.13e-07	1.70e-07	6.32e-09	1.30e-09	2.79e-13	5.98e-14	1.39e-14	1.17e-15
Discont. $Q = 50$	Smolyak	4.73e+00	1.26e+00	3.82e+00	1.08e+00	4.82e+00	8.42e-01	4.56e+00	6.75e-01	4.07e+00	4.26e-01	3.57e+00	3.28e-01	4.70e+00	2.16e-01
	LS-Uniform	3.46e+01	8.99e+00	7.48e+01	1.09e+01	6.02e+01	6.33e+00	3.07e+02	2.72e+01	6.62e+05	2.77e+04	1.79e+06	6.29e+04	6.27e+12	1.10e+11
	LS-Chebyshev	3.47e+00	1.27e+00	3.59e+00	7.15e-01	4.24e+00	6.58e-01	3.44e+00	4.35e-01	3.25e+00	2.92e-01	2.36e+00	2.04e-01	2.92e+00	1.46e-01
Gauss. $Q = 50$	Smolyak	5.81e-01	2.39e-01	3.51e-01	9.72e-02	1.18e-01	3.11e-02	2.17e-02	5.03e-03	1.93e-03	4.27e-04	7.40e-06	1.88e-06	6.31e-09	1.51e-09
	LS-Uniform	3.13e+00	7.97e-01	3.83e+00	5.77e-01	3.44e+00	2.92e-01	7.53e+00	6.71e-01	2.27e+02	1.11e+01	2.62e+00	8.97e-02	1.88e+04	3.30e+02
	LS-Chebyshev	4.84e-01	1.75e-01	3.40e-01	8.12e-02	6.74e-02	1.90e-02	1.67e-02	3.37e-03	1.19e-03	2.87e-04	5.39e-06	1.22e-06	5.51e-09	1.08e-09
Geo. Mean $Q = 50$	Smolyak	1.09e-02	5.50e-03	2.80e-03	7.19e-04	3.70e-04	7.25e-05	4.84e-05	8.72e-06	3.65e-06	6.94e-07	1.36e-07	2.22e-08	2.19e-09	3.61e-10
	LS-Uniform	7.18e-03	1.52e-03	5.70e-03	1.08e-03	1.33e-03	1.36e-04	3.29e-03	2.95e-04	1.07e-01	4.54e-03	3.88e-03	1.23e-04	2.26e+02	3.95e+00
	LS-Chebyshev	9.32e-03	4.52e-03	2.45e-03	9.90e-04	5.63e-04	9.85e-05	3.23e-05	7.11e-06	3.08e-06	5.99e-07	9.87e-08	2.00e-08	2.06e-09	3.03e-10
NOISE $Q = 50$	Smolyak	6.00e-07	3.07e-07	6.46e-07	2.64e-07	7.78e-07	2.43e-07	8.85e-07	2.38e-07	9.21e-07	2.39e-07	1.21e-06	2.51e-07	1.25e-06	2.57e-07
	LS-Uniform	4.11e-06	1.04e-06	2.12e-05	3.07e-06	1.98e-05	2.30e-06	2.21e-04	2.00e-05	5.97e-01	2.69e-02	1.65e+00	5.52e-02	1.05e+07	1.84e+05
	LS-Chebyshev	5.59e-07	2.36e-07	5.80e-07	1.97e-07	7.30e-07	1.51e-07	1.13e-06	1.47e-07	8.96e-07	1.35e-07	7.56e-07	1.28e-07	7.61e-07	1.27e-07
Osc. $Q = 50$	Smolyak	3.97e-02	2.52e-02	2.12e-03	1.06e-03	4.48e-05	1.87e-05	9.42e-09	3.35e-09	5.70e-13	1.95e-13	9.44e-15	2.26e-15	1.97e-14	3.65e-15
	LS-Uniform	1.69e-01	3.71e-02	9.31e-03	1.42e-03	1.97e-04	3.00e-05	2.71e-06	2.40e-07	7.01e-08	3.60e-09	7.54e-09	2.53e-10	5.83e-02	1.02e-03
	LS-Chebyshev	4.57e-02	1.78e-02	2.53e-03	8.35e-04	5.50e-05	1.33e-05	7.69e-09	2.20e-09	5.26e-13	1.40e-13	3.43e-14	2.88e-15	2.81e-14	2.41e-15
Prod. Peak $Q = 50$	Smolyak	1.04e-03	6.17e-04	8.63e-05	3.77e-05	3.51e-06	1.24e-06	2.27e-08	7.41e-09	3.03e-11	8.55e-12	7.77e-15	2.15e-15	1.80e-14	3.63e-15
	LS-Uniform	4.27e-03	9.86e-04	4.65e-04	7.06e-05	1.99e-05	2.55e-06	7.28e-06	6.43e-07	3.65e-06	1.54e-07	3.19e-09	1.14e-10	2.16e-02	3.83e-04
	LS-Chebyshev	1.05e-03	5.03e-04	1.01e-04	3.20e-05	3.45e-06	8.70e-07	1.81e-08	4.66e-09	2.75e-11	6.14e-12	1.55e-14	1.79e-15	2.56e-14	1.73e-15
Ridge Prod. $Q = 50$	Smolyak	2.48e-01	1.05e-01	1.14e-01	3.85e-02	6.89e-02	1.54e-02	3.42e-02	5.39e-03	2.39e-02	2.13e-03	1.18e-02	9.11e-04	7.14e-03	3.33e-04
	LS-Uniform	1.29e+00	3.18e-01	2.37e+00	3.45e-01	1.61e+00	1.67e-01	9.01e+00	7.85e-01	6.36e+03	2.86e+02	5.10e+03	1.54e+02	1.57e+10	2.75e+08
	LS-Chebyshev	1.94e-01	6.80e-02	1.17e-01	4.08e-02	8.27e-02	1.14e-02	4.75e-02	4.39e-03	2.72e-02	1.68e-03	1.14e-02	6.19e-04	5.41e-03	2.38e-04

TABLE 1. The errors e_{\max}^{wc} and $e_{\text{mean}}^{\text{wc}}$ for $d = 2$, each scale for the different algorithms. Lowest value in bold.

			Scale 3		Scale 4		Scale 5		Scale 6		Scale 7		Scale 8		Scale 9	
			e_{\max}^{wc}	$e_{\text{mean}}^{\text{wc}}$	e_{\max}^{wc}	$e_{\text{mean}}^{\text{wc}}$	e_{\max}^{wc}	$e_{\text{mean}}^{\text{wc}}$	e_{\max}^{wc}	$e_{\text{mean}}^{\text{wc}}$	e_{\max}^{wc}	$e_{\text{mean}}^{\text{wc}}$	e_{\max}^{wc}	$e_{\text{mean}}^{\text{wc}}$	e_{\max}^{wc}	$e_{\text{mean}}^{\text{wc}}$
Bin. Gauss. $Q = 50$	Smolyak		7.17e-01	2.93e-01	4.04e-01	1.43e-01	2.98e-01	8.74e-02	1.06e-01	2.28e-02	2.71e-02	5.99e-03	5.82e-03	9.34e-04	6.64e-04	1.12e-04
	LS-Uniform		2.47e+00	4.44e-01	1.33e+00	2.13e-01	8.83e-01	1.27e-01	1.37e+00	6.60e-02	7.62e-01	2.36e-02	1.42e+00	2.36e-02	4.05e+01	4.14e-01
	LS-Chebyshev		6.04e-01	1.77e-01	5.85e-01	1.25e-01	2.25e-01	5.65e-02	7.17e-02	1.27e-02	2.26e-02	2.86e-03	3.83e-03	4.99e-04	3.65e-04	5.14e-05
Cont. $Q = 50$	Smolyak		4.54e-02	1.16e-02	1.57e-02	3.66e-03	1.17e-02	1.68e-03	4.17e-03	4.94e-04	3.22e-03	1.92e-04	1.32e-03	6.54e-05	6.66e-04	2.43e-05
	LS-Uniform		9.00e-02	1.51e-02	2.57e-02	4.94e-03	3.90e-02	2.89e-03	5.98e-02	2.11e-03	5.33e-02	1.49e-03	2.74e-01	4.00e-03	3.04e+01	3.14e-01
	LS-Chebyshev		3.30e-02	9.08e-03	1.90e-02	4.16e-03	7.98e-03	1.43e-03	5.42e-03	5.28e-04	2.11e-03	1.81e-04	1.04e-03	6.09e-05	5.69e-04	2.19e-05
Corner Peak $Q = 50$	Smolyak		5.78e-02	1.58e-02	3.03e-02	5.80e-03	7.48e-03	1.46e-03	1.44e-03	2.55e-04	1.98e-04	2.96e-05	1.35e-05	2.06e-06	3.16e-07	4.17e-08
	LS-Uniform		7.11e-02	1.33e-02	8.54e-03	1.65e-03	6.77e-03	3.90e-04	2.03e-03	1.02e-04	1.40e-03	3.23e-05	6.45e-04	1.13e-05	5.31e-03	5.72e-05
	LS-Chebyshev		3.38e-02	1.34e-02	9.19e-03	2.33e-03	2.31e-03	4.72e-04	3.68e-04	8.59e-05	6.43e-05	1.07e-05	5.24e-06	9.96e-07	1.16e-07	2.05e-08
Discont. $Q = 50$	Smolyak		7.08e+00	1.73e+00	1.11e+01	1.33e+00	7.35e+00	1.03e+00	9.26e+00	8.10e-01	9.99e+00	5.75e-01	8.40e+00	4.81e-01	1.07e+01	3.28e-01
	LS-Uniform		1.70e+01	3.87e+00	1.11e+01	1.66e+00	2.38e+01	1.67e+00	6.09e+01	2.57e+00	2.28e+02	5.88e+00	1.29e+03	2.70e+01	4.47e+05	4.45e+03
	LS-Chebyshev		6.97e+00	1.75e+00	6.09e+00	1.37e+00	5.86e+00	1.04e+00	7.10e+00	7.63e-01	9.30e+00	5.30e-01	6.44e+00	3.77e-01	6.29e+00	2.59e-01
Gauss. $Q = 50$	Smolyak		6.32e-01	2.17e-01	4.54e-01	9.92e-02	2.21e-01	3.99e-02	1.20e-01	1.72e-02	3.31e-02	5.10e-03	7.56e-03	8.69e-04	9.87e-04	1.01e-04
	LS-Uniform		1.66e+00	2.82e-01	7.59e-01	9.70e-02	7.69e-01	6.15e-02	6.03e-01	3.04e-02	6.14e-01	1.82e-02	9.04e-01	1.65e-02	5.56e+01	5.23e-01
	LS-Chebyshev		4.91e-01	1.28e-01	3.66e-01	7.08e-02	2.59e-01	2.74e-02	9.23e-02	8.81e-03	1.72e-02	2.09e-03	2.88e-03	3.73e-04	3.26e-04	4.46e-05
Gro. Mean $Q = 50$	Smolyak		2.27e-02	7.18e-03	3.20e-03	8.34e-04	3.76e-04	6.70e-05	2.58e-05	4.08e-06	2.11e-06	2.89e-07	1.44e-07	1.55e-08	6.94e-09	7.03e-10
	LS-Uniform		4.56e-02	6.02e-03	4.19e-03	5.11e-04	5.38e-04	4.27e-05	5.83e-05	3.24e-06	3.39e-05	9.23e-07	1.81e-06	4.77e-08	6.44e-05	6.38e-07
	LS-Chebyshev		2.22e-02	6.15e-03	3.75e-03	1.06e-03	2.48e-04	7.25e-05	1.39e-05	3.05e-06	9.97e-07	1.49e-07	4.43e-08	6.60e-09	2.38e-09	3.29e-10
NOISE $Q = 50$	Smolyak		9.86e-07	3.43e-07	1.25e-06	3.48e-07	1.44e-06	4.06e-07	1.80e-06	4.04e-07	2.24e-06	4.56e-07	2.73e-06	4.96e-07	3.06e-06	5.57e-07
	LS-Uniform		1.75e-06	3.59e-07	1.39e-06	2.47e-07	3.84e-06	3.29e-07	2.20e-05	8.19e-07	6.91e-05	1.80e-06	6.26e-04	1.09e-05	2.51e-01	2.57e-03
	LS-Chebyshev		6.16e-07	1.70e-07	6.21e-07	1.59e-07	6.35e-07	1.49e-07	8.77e-07	1.46e-07	9.05e-07	1.43e-07	7.14e-07	1.38e-07	7.36e-07	1.31e-07
Oscil. $Q = 50$	Smolyak		4.60e-01	1.54e-01	8.77e-02	3.12e-02	9.48e-03	3.25e-03	5.94e-04	1.77e-04	2.01e-05	4.99e-06	2.82e-07	6.29e-08	7.89e-11	1.71e-11
	LS-Uniform		1.12e+00	2.40e-01	1.43e-01	1.71e-02	2.46e-02	1.93e-03	2.61e-03	1.24e-04	2.75e-04	7.39e-06	4.31e-05	7.63e-07	4.07e-06	3.67e-08
	LS-Chebyshev		6.95e-01	1.94e-01	9.86e-02	2.25e-02	8.00e-03	1.65e-03	3.41e-04	7.41e-05	1.38e-05	2.08e-06	1.45e-07	2.87e-08	4.98e-11	7.59e-12
Prod. Peak $Q = 50$	Smolyak		3.13e-03	1.28e-03	4.30e-04	1.57e-04	4.39e-05	1.56e-05	3.80e-06	1.11e-06	2.04e-07	4.59e-08	5.45e-09	9.51e-10	4.20e-11	7.27e-12
	LS-Uniform		9.12e-03	1.56e-03	5.74e-04	7.52e-05	8.01e-05	8.11e-06	1.44e-05	7.00e-07	2.28e-06	6.98e-08	9.92e-07	1.61e-08	1.46e-06	1.47e-08
	LS-Chebyshev		2.53e-03	1.04e-03	4.28e-04	1.07e-04	3.82e-05	8.14e-06	2.77e-06	5.03e-07	1.50e-07	2.05e-08	2.57e-09	4.36e-10	2.47e-11	3.23e-12
Ridge Prod. $Q = 50$	Smolyak		6.03e-01	1.98e-01	4.75e-01	8.99e-02	2.08e-01	4.03e-02	1.18e-01	1.54e-02	7.33e-02	6.44e-03	3.25e-02	2.47e-03	3.06e-02	9.64e-04
	LS-Uniform		1.09e+00	2.25e-01	4.55e-01	9.45e-02	4.46e-01	4.71e-02	9.80e-01	3.95e-02	2.31e+00	5.32e-02	1.25e+01	2.04e-01	2.03e+03	1.87e+01
	LS-Chebyshev		5.43e-01	1.53e-01	3.43e-01	7.47e-02	1.98e-01	3.08e-02	9.31e-02	1.15e-02	4.65e-02	4.66e-03	3.39e-02	1.74e-03	1.99e-02	6.82e-04

TABLE 2. The errors e_{\max}^{wc} and $e_{\text{mean}}^{\text{wc}}$ for $d = 3$, each scale for the different algorithms. Lowest value in bold.

			Scale 3		Scale 4		Scale 5		Scale 6		Scale 7		Scale 8		Scale 9	
			e_{\max}^{wc}	$e_{\text{mean}}^{\text{wc}}$	e_{\max}^{wc}	$e_{\text{mean}}^{\text{wc}}$	e_{\max}^{wc}	$e_{\text{mean}}^{\text{wc}}$	e_{\max}^{wc}	$e_{\text{mean}}^{\text{wc}}$	e_{\max}^{wc}	$e_{\text{mean}}^{\text{wc}}$	e_{\max}^{wc}	$e_{\text{mean}}^{\text{wc}}$	e_{\max}^{wc}	$e_{\text{mean}}^{\text{wc}}$
Bin. Gauss. $Q = 50$	Smolyak		6.34e-01	1.76e-01	6.14e-01	1.27e-01	3.43e-01	8.48e-02	1.77e-01	4.25e-02	1.01e-01	1.36e-02	3.60e-02	3.18e-03	9.18e-03	8.39e-04
	LS-Uniform		1.40e+00	1.86e-01	1.15e+00	1.23e-01	1.10e+00	9.00e-02	7.49e-01	3.09e-02	4.71e-01	1.36e-02	3.80e-01	5.27e-03	1.39e+00	7.92e-03
	LS-Chebyshev		7.14e-01	1.56e-01	5.03e-01	8.22e-02	2.35e-01	4.60e-02	1.18e-01	1.95e-02	4.77e-02	6.20e-03	1.29e-02	1.29e-03	3.03e-03	2.47e-04
Cont. $Q = 50$	Smolyak		3.06e-02	7.22e-03	1.51e-02	3.07e-03	9.36e-03	1.03e-03	4.98e-03	4.84e-04	2.80e-03	1.64e-04	1.13e-03	5.73e-05	6.70e-04	1.81e-05
	LS-Uniform		6.82e-02	8.94e-03	3.20e-02	3.45e-03	1.81e-02	1.67e-03	1.81e-02	7.44e-04	1.05e-02	3.41e-04	2.03e-02	2.56e-04	2.02e-01	1.17e-03
	LS-Chebyshev		2.80e-02	7.87e-03	1.33e-02	2.72e-03	7.72e-03	9.94e-04	3.78e-03	3.61e-04	2.23e-03	1.29e-04	8.58e-04	4.69e-05	4.37e-04	1.74e-05
Corner Peak $Q = 50$	Smolyak		1.62e-01	2.46e-02	8.14e-02	1.29e-02	5.39e-02	5.69e-03	1.89e-02	2.32e-03	6.42e-03	6.42e-04	1.60e-03	1.37e-04	3.10e-04	2.20e-05
	LS-Uniform		9.45e-02	1.05e-02	6.71e-02	4.08e-03	2.02e-02	1.08e-03	8.63e-03	2.20e-04	6.64e-03	8.51e-05	1.53e-03	2.20e-05	3.41e-03	2.36e-05
	LS-Chebyshev		1.14e-01	2.12e-02	4.08e-02	7.06e-03	1.23e-02	2.09e-03	2.51e-03	4.46e-04	7.63e-04	1.16e-04	1.81e-04	2.37e-05	3.76e-05	4.59e-06
Discont. $Q = 50$	Smolyak		2.40e+01	3.96e+00	3.61e+01	3.29e+00	3.36e+01	2.59e+00	2.92e+01	1.96e+00	2.90e+01	1.40e+00	3.14e+01	1.04e+00	3.13e+01	7.74e-01
	LS-Uniform		2.27e+01	4.40e+00	2.68e+01	3.24e+00	5.96e+01	3.33e+00	4.40e+01	3.05e+00	1.19e+02	2.99e+00	5.04e+02	5.85e+00	1.13e+04	6.19e+01
	LS-Chebyshev		2.40e+01	5.10e+00	2.99e+01	3.74e+00	2.35e+01	2.33e+00	2.49e+01	1.78e+00	1.98e+01	1.33e+00	1.92e+01	1.07e+00	1.81e+01	7.73e-01
Gauss. $Q = 50$	Smolyak		6.42e-01	1.75e-01	4.80e-01	8.93e-02	3.24e-01	4.70e-02	1.98e-01	2.07e-02	1.04e-01	7.66e-03	3.94e-02	3.36e-03	9.55e-03	9.78e-04
	LS-Uniform		8.28e-01	1.35e-01	1.03e+00	7.53e-02	1.01e+00	5.19e-02	1.30e+00	2.99e-02	3.52e-01	9.70e-03	3.41e-01	4.48e-03	1.31e+00	7.83e-03
	LS-Chebyshev		5.90e-01	9.71e-02	4.10e-01	5.53e-02	2.14e-01	2.71e-02	1.08e-01	1.13e-02	3.82e-02	3.73e-03	1.68e-02	1.06e-03	5.26e-03	2.87e-04
Geo. Mean $Q = 50$	Smolyak		8.36e-02	2.25e-02	3.02e-02	5.43e-03	9.51e-03	1.15e-03	2.33e-03	2.05e-04	5.75e-04	3.13e-05	9.34e-05	4.02e-06	1.07e-05	4.43e-07
	LS-Uniform		1.03e-01	1.27e-02	2.90e-02	2.50e-03	1.35e-02	5.94e-04	5.62e-03	1.27e-04	8.27e-04	1.35e-05	1.25e-04	2.09e-06	1.18e-04	9.63e-07
	LS-Chebyshev		1.03e-01	3.31e-02	2.91e-02	7.40e-03	5.91e-03	1.19e-03	8.35e-04	2.12e-04	1.92e-04	2.86e-05	1.68e-05	2.98e-06	3.13e-06	2.92e-07
NOISE $Q = 50$	Smolyak		1.65e-06	4.23e-07	2.09e-06	5.18e-07	2.63e-06	5.34e-07	4.27e-06	6.76e-07	4.65e-06	7.53e-07	5.81e-06	9.02e-07	6.78e-06	1.02e-06
	LS-Uniform		1.45e-06	1.96e-07	1.48e-06	1.88e-07	3.94e-06	2.58e-07	6.85e-06	2.78e-07	1.39e-05	4.01e-07	7.98e-05	9.71e-07	2.00e-03	1.32e-05
	LS-Chebyshev		6.18e-07	1.75e-07	7.31e-07	1.71e-07	7.82e-07	1.49e-07	8.05e-07	1.38e-07	7.41e-07	1.39e-07	7.44e-07	1.37e-07	7.95e-07	1.37e-07
Oscil. $Q = 50$	Smolyak		1.73e+00	3.71e-01	6.40e-01	1.27e-01	1.47e-01	3.57e-02	2.27e-02	5.91e-03	2.29e-03	5.81e-04	1.65e-04	3.36e-05	6.86e-06	1.18e-06
	LS-Uniform		2.57e+00	4.73e-01	1.12e+00	1.18e-01	3.74e-01	2.14e-02	8.01e-02	2.14e-03	6.46e-03	1.75e-04	8.88e-04	1.58e-05	4.14e-04	3.25e-06
	LS-Chebyshev		2.90e+00	6.38e-01	1.04e+00	2.00e-01	1.19e-01	2.60e-02	1.21e-02	2.29e-03	1.06e-03	1.60e-04	6.54e-05	8.09e-06	2.32e-06	2.84e-07
Prod. Peak $Q = 50$	Smolyak		7.49e-03	1.86e-03	1.17e-03	2.97e-04	1.95e-04	4.47e-05	2.37e-05	5.74e-06	2.36e-06	5.40e-07	2.12e-07	3.75e-08	1.43e-08	1.83e-09
	LS-Uniform		7.81e-03	1.60e-03	3.43e-03	1.92e-04	3.59e-04	2.02e-05	7.85e-05	2.11e-06	5.35e-06	1.28e-07	9.85e-07	1.64e-08	8.28e-07	5.47e-09
	LS-Chebyshev		6.16e-03	2.04e-03	1.13e-03	2.97e-04	1.08e-04	2.55e-05	1.09e-05	2.09e-06	1.10e-06	1.52e-07	7.08e-08	9.16e-09	4.38e-09	4.44e-10
Ridge Prod. $Q = 50$	Smolyak		1.70e+00	3.83e-01	1.30e+00	2.05e-01	7.02e-01	8.54e-02	3.40e-01	3.77e-02	2.32e-01	1.58e-02	1.24e-01	6.54e-03	4.96e-02	2.72e-03
	LS-Uniform		2.06e+00	2.88e-01	1.16e+00	1.47e-01	1.02e+00	9.01e-02	1.02e+00	3.59e-02	6.19e-01	2.10e-02	2.03e+00	2.22e-02	1.28e+01	8.16e-02
	LS-Chebyshev		9.88e-01	3.53e-01	5.30e-01	1.27e-01	2.94e-01	5.48e-02	2.30e-01	2.59e-02	1.19e-01	9.61e-03	6.77e-02	3.92e-03	4.23e-02	1.60e-03

TABLE 3. The errors e_{\max}^{wc} and $e_{\text{mean}}^{\text{wc}}$ for $d = 4$, each scale for the different algorithms. Lowest value in bold.

		Scale 3		Scale 4		Scale 5		Scale 6		Scale 7		Scale 8	
		e_{\max}^{wc}	$e_{\text{mean}}^{\text{wc}}$	e_{\max}^{wc}	$e_{\text{mean}}^{\text{wc}}$	e_{\max}^{wc}	$e_{\text{mean}}^{\text{wc}}$	e_{\max}^{wc}	$e_{\text{mean}}^{\text{wc}}$	e_{\max}^{wc}	$e_{\text{mean}}^{\text{wc}}$	e_{\max}^{wc}	$e_{\text{mean}}^{\text{wc}}$
Bin. Gauss. $Q = 50$	Smolyak	6.58e-01	2.19e-01	6.01e-01	9.79e-02	3.44e-01	6.83e-02	2.10e-01	3.35e-02	1.14e-01	1.41e-02	5.70e-02	5.97e-03
	LS-Uniform	9.73e-01	1.46e-01	1.01e+00	8.20e-02	1.05e+00	5.14e-02	1.27e+00	2.76e-02	1.21e+00	1.35e-02	8.27e-01	5.01e-03
	LS-Chebyshev	5.92e-01	1.11e-01	4.63e-01	6.17e-02	2.71e-01	3.30e-02	1.56e-01	1.79e-02	6.91e-02	7.30e-03	2.88e-02	2.14e-03
Cont. $Q = 50$	Smolyak	3.24e-02	6.00e-03	1.60e-02	2.55e-03	8.41e-03	8.77e-04	3.68e-03	3.27e-04	2.18e-03	1.31e-04	8.40e-04	4.66e-05
	LS-Uniform	3.47e-02	7.08e-03	2.26e-02	3.14e-03	1.63e-02	1.17e-03	1.36e-02	4.58e-04	6.62e-03	2.06e-04	7.40e-03	1.04e-04
	LS-Chebyshev	2.80e-02	6.25e-03	1.45e-02	2.23e-03	6.89e-03	7.64e-04	3.13e-03	2.85e-04	1.42e-03	1.02e-04	7.27e-04	3.74e-05
Corner Peak $Q = 50$	Smolyak	1.56e-01	2.07e-02	2.47e-01	1.57e-02	1.25e-01	1.07e-02	7.94e-02	5.60e-03	3.98e-02	2.24e-03	1.65e-02	1.01e-03
	LS-Uniform	4.36e-02	5.32e-03	1.21e-01	4.43e-03	6.88e-02	1.49e-03	1.91e-02	4.54e-04	1.51e-02	1.62e-04	1.33e-02	6.86e-05
	LS-Chebyshev	1.51e-01	3.78e-02	5.15e-02	1.30e-02	2.44e-02	4.49e-03	8.78e-03	1.20e-03	2.72e-03	3.57e-04	1.25e-03	1.15e-04
Discont. $Q = 50$	Smolyak	4.92e+01	7.98e+00	5.33e+01	5.79e+00	5.18e+01	4.33e+00	6.96e+01	3.59e+00	7.05e+01	2.61e+00	7.93e+01	1.92e+00
	LS-Uniform	5.05e+01	9.25e+00	6.57e+01	7.68e+00	7.06e+01	5.40e+00	1.27e+02	4.79e+00	1.84e+02	5.00e+00	3.20e+02	4.61e+00
	LS-Chebyshev	4.93e+01	7.89e+00	6.37e+01	6.57e+00	5.03e+01	4.62e+00	5.36e+01	3.66e+00	4.26e+01	2.71e+00	4.79e+01	2.04e+00
Gauss. $Q = 50$	Smolyak	6.92e-01	1.18e-01	5.59e-01	7.81e-02	4.13e-01	6.14e-02	2.36e-01	2.39e-02	1.11e-01	8.48e-03	4.50e-02	3.53e-03
	LS-Uniform	5.93e-01	9.65e-02	1.05e+00	8.73e-02	1.11e+00	3.93e-02	6.08e-01	1.70e-02	7.12e-01	1.01e-02	8.05e-01	4.63e-03
	LS-Chebyshev	6.71e-01	8.36e-02	4.57e-01	5.40e-02	2.64e-01	2.75e-02	1.49e-01	1.25e-02	7.31e-02	4.62e-03	2.69e-02	1.46e-03
Geo. Mean $Q = 50$	Smolyak	2.20e-02	5.62e-03	5.79e-03	9.61e-04	1.28e-03	1.42e-04	1.84e-04	1.79e-05	3.08e-05	1.64e-06	3.21e-06	1.30e-07
	LS-Uniform	3.12e-02	4.22e-03	6.55e-03	5.47e-04	1.46e-03	6.65e-05	3.74e-04	8.14e-06	5.12e-05	9.06e-07	4.22e-06	6.83e-08
	LS-Chebyshev	2.24e-02	8.09e-03	4.78e-03	7.64e-04	5.93e-04	1.02e-04	6.03e-05	1.21e-05	1.06e-05	1.30e-06	1.13e-06	1.03e-07
NOISE $Q = 50$	Smolyak	1.86e-06	5.11e-07	2.72e-06	6.52e-07	4.71e-06	7.87e-07	6.50e-06	9.68e-07	9.92e-06	1.21e-06	1.06e-05	1.47e-06
	LS-Uniform	1.13e-06	1.76e-07	1.67e-06	1.92e-07	3.03e-06	1.90e-07	7.35e-06	2.27e-07	1.06e-05	2.99e-07	3.63e-05	4.34e-07
	LS-Chebyshev	8.94e-07	1.77e-07	7.39e-07	1.51e-07	7.12e-07	1.48e-07	8.82e-07	1.48e-07	7.43e-07	1.42e-07	8.45e-07	1.43e-07
Osci. $Q = 50$	Smolyak	3.16e+00	6.35e-01	1.96e+00	3.25e-01	8.12e-01	1.37e-01	1.98e-01	3.66e-02	3.98e-02	8.11e-03	5.71e-03	1.11e-03
	LS-Uniform	7.47e+00	8.13e-01	3.53e+00	3.44e-01	1.87e+00	8.95e-02	1.44e+00	2.34e-02	2.71e-01	3.65e-03	3.33e-02	3.65e-04
	LS-Chebyshev	3.93e+00	1.02e+00	2.23e+00	5.68e-01	7.15e-01	1.66e-01	1.56e-01	3.08e-02	2.59e-02	3.41e-03	2.31e-03	2.90e-04
Prod. Peak $Q = 50$	Smolyak	9.44e-03	1.67e-03	1.51e-03	3.14e-04	3.04e-04	5.54e-05	4.84e-05	8.17e-06	6.45e-06	1.09e-06	8.07e-07	1.28e-07
	LS-Uniform	7.66e-03	1.25e-03	2.88e-03	2.89e-04	1.22e-03	3.95e-05	2.85e-04	5.28e-06	2.59e-05	4.58e-07	5.06e-06	4.72e-08
	LS-Chebyshev	9.17e-03	2.19e-03	2.11e-03	3.67e-04	3.62e-04	4.76e-05	4.66e-05	4.97e-06	4.87e-06	4.37e-07	3.99e-07	3.50e-08
Ridge Prod. $Q = 50$	Smolyak	2.92e+00	6.22e-01	2.40e+00	3.19e-01	1.80e+00	1.67e-01	8.60e-01	8.16e-02	4.45e-01	3.77e-02	3.11e-01	1.70e-02
	LS-Uniform	2.32e+00	4.64e-01	2.05e+00	2.24e-01	1.88e+00	1.10e-01	1.40e+00	5.16e-02	9.51e-01	2.92e-02	1.43e+00	1.86e-02
	LS-Chebyshev	2.23e+00	6.17e-01	1.26e+00	2.63e-01	9.49e-01	1.02e-01	3.85e-01	4.24e-02	1.92e-01	1.79e-02	1.58e-01	7.57e-03

TABLE 4. The errors e_{\max}^{wc} and $e_{\text{mean}}^{\text{wc}}$ for $d = 5$, each scale for the different algorithms. Lowest value in bold.

			Scale 3		Scale 4		Scale 5		Scale 6		Scale 7	
			e_{\max}^{wc}	$e_{\text{mean}}^{\text{wc}}$	e_{\max}^{wc}	$e_{\text{mean}}^{\text{wc}}$	e_{\max}^{wc}	$e_{\text{mean}}^{\text{wc}}$	e_{\max}^{wc}	$e_{\text{mean}}^{\text{wc}}$	e_{\max}^{wc}	$e_{\text{mean}}^{\text{wc}}$
Bim. Gauss. $Q = 50$	Smolyak		7.63e-01	1.60e-01	5.55e-01	1.10e-01	4.59e-01	6.18e-02	3.38e-01	2.96e-02	1.88e-01	1.22e-02
	LS-Uniform		9.79e-01	1.16e-01	1.28e+00	7.12e-02	7.96e-01	3.72e-02	8.97e-01	2.03e-02	7.20e-01	1.21e-02
	LS-Chebyshev		7.01e-01	1.23e-01	5.40e-01	6.15e-02	4.19e-01	3.47e-02	2.04e-01	1.63e-02	1.18e-01	7.75e-03
Cont. $Q = 50$	Smolyak		2.46e-02	5.14e-03	1.37e-02	1.90e-03	7.75e-03	7.71e-04	3.31e-03	3.30e-04	1.78e-03	1.10e-04
	LS-Uniform		3.23e-02	5.40e-03	1.68e-02	2.30e-03	1.09e-02	9.02e-04	1.16e-02	3.73e-04	5.36e-03	1.60e-04
	LS-Chebyshev		2.47e-02	5.55e-03	1.38e-02	2.15e-03	6.24e-03	7.46e-04	3.75e-03	2.59e-04	1.58e-03	9.33e-05
Corner Peak $Q = 50$	Smolyak		2.46e-02	2.99e-03	1.80e-01	8.18e-03	2.53e-01	8.59e-03	1.44e-01	5.65e-03	1.07e-01	4.41e-03
	LS-Uniform		2.41e-02	2.87e-03	1.03e-01	2.87e-03	7.82e-02	1.19e-03	7.77e-02	6.63e-04	2.75e-02	1.54e-04
	LS-Chebyshev		7.62e-02	2.10e-02	6.89e-02	9.41e-03	2.50e-02	4.35e-03	1.75e-02	2.11e-03	8.05e-03	7.55e-04
Discont. $Q = 50$	Smolyak		1.19e+02	1.29e+01	1.67e+02	1.14e+01	1.38e+02	7.79e+00	1.31e+02	5.95e+00	1.75e+02	4.24e+00
	LS-Uniform		1.12e+02	1.38e+01	1.27e+02	1.13e+01	1.42e+02	9.37e+00	2.74e+02	8.39e+00	3.37e+02	7.03e+00
	LS-Chebyshev		1.45e+02	4.46e+01	1.37e+02	2.16e+01	1.01e+02	1.07e+01	1.08e+02	7.36e+00	1.01e+02	5.33e+00
Gauss. $Q = 50$	Smolyak		5.74e-01	1.63e-01	5.92e-01	8.11e-02	4.88e-01	4.00e-02	3.05e-01	2.13e-02	1.95e-01	8.53e-03
	LS-Uniform		7.27e-01	1.11e-01	6.73e-01	5.92e-02	9.13e-01	3.42e-02	1.40e+00	1.85e-02	8.09e-01	8.80e-03
	LS-Chebyshev		6.30e-01	9.58e-02	5.16e-01	5.49e-02	3.05e-01	2.89e-02	1.69e-01	1.34e-02	9.95e-02	5.00e-03
Geo. Mean $Q = 50$	Smolyak		9.67e-02	2.23e-02	3.33e-02	4.40e-03	6.61e-03	6.20e-04	1.20e-03	1.12e-04	3.06e-04	1.70e-05
	LS-Uniform		1.24e-01	1.62e-02	2.86e-02	3.13e-03	8.01e-03	3.48e-04	4.62e-03	5.96e-05	5.21e-04	8.06e-06
	LS-Chebyshev		1.04e-01	2.51e-02	2.43e-02	5.71e-03	5.20e-03	9.03e-04	7.05e-04	9.40e-05	1.05e-04	1.40e-05
NOISE $Q = 50$	Smolyak		2.98e-06	6.30e-07	4.34e-06	9.47e-07	8.13e-06	1.08e-06	1.08e-05	1.42e-06	1.53e-05	1.84e-06
	LS-Uniform		1.09e-06	1.62e-07	2.38e-06	1.61e-07	2.97e-06	1.74e-07	8.25e-06	2.14e-07	1.15e-05	2.52e-07
	LS-Chebyshev		7.27e-07	1.73e-07	7.92e-07	1.63e-07	7.93e-07	1.57e-07	8.30e-07	1.49e-07	9.26e-07	1.49e-07
Osci. $Q = 50$	Smolyak		8.22e+00	9.31e-01	5.60e+00	5.52e-01	2.34e+00	2.79e-01	9.08e-01	1.15e-01	2.67e-01	3.63e-02
	LS-Uniform		7.38e+00	9.44e-01	7.51e+00	6.52e-01	6.52e+00	2.55e-01	4.17e+00	8.02e-02	1.47e+00	1.98e-02
	LS-Chebyshev		5.68e+00	1.28e+00	3.63e+00	9.07e-01	2.31e+00	4.57e-01	7.52e-01	1.33e-01	1.63e-01	2.75e-02
Prod. Peak $Q = 50$	Smolyak		9.11e-03	1.58e-03	2.55e-03	2.81e-04	3.29e-04	4.74e-05	5.42e-05	7.37e-06	8.00e-06	1.14e-06
	LS-Uniform		1.96e-02	1.84e-03	4.56e-03	2.72e-04	7.60e-04	2.89e-05	3.73e-04	4.33e-06	8.31e-05	6.08e-07
	LS-Chebyshev		1.50e-02	2.60e-03	3.33e-03	3.93e-04	4.72e-04	4.64e-05	8.14e-05	5.99e-06	1.09e-05	6.35e-07
Ridge Prod. $Q = 50$	Smolyak		6.46e+00	9.21e-01	6.57e+00	5.98e-01	4.06e+00	3.12e-01	1.86e+00	1.60e-01	1.18e+00	7.76e-02
	LS-Uniform		3.24e+00	5.50e-01	4.09e+00	2.63e-01	2.21e+00	1.33e-01	1.91e+00	7.53e-02	1.10e+00	3.85e-02
	LS-Chebyshev		3.78e+00	8.72e-01	3.10e+00	3.98e-01	1.66e+00	1.80e-01	7.14e-01	7.62e-02	4.85e-01	3.25e-02

TABLE 5. The errors e_{\max}^{wc} and $e_{\text{mean}}^{\text{wc}}$ for $d = 6$, each scale for the different algorithms. Lowest value in bold.

		Scale 3		Scale 4		Scale 5		Scale 6		Scale 7	
		e_{\max}^{wc}	$e_{\text{mean}}^{\text{wc}}$	e_{\max}^{wc}	$e_{\text{mean}}^{\text{wc}}$	e_{\max}^{wc}	$e_{\text{mean}}^{\text{wc}}$	e_{\max}^{wc}	$e_{\text{mean}}^{\text{wc}}$	e_{\max}^{wc}	$e_{\text{mean}}^{\text{wc}}$
Bim. Gauss. $Q = 50$	Smolyak	5.74e-01	9.52e-02	5.51e-01	6.38e-02	5.00e-01	4.12e-02	3.37e-01	2.24e-02	2.08e-01	1.29e-02
	LS-Uniform	6.72e-01	7.97e-02	7.04e-01	4.92e-02	2.33e+00	3.49e-02	1.17e+00	2.44e-02	1.85e+00	1.71e-02
	LS-Chebyshev	6.09e-01	8.44e-02	5.50e-01	4.78e-02	4.07e-01	2.92e-02	2.38e-01	1.64e-02	1.49e-01	9.16e-03
Cont. $Q = 50$	Smolyak	3.05e-02	6.08e-03	1.22e-02	1.95e-03	7.33e-03	7.47e-04	4.32e-03	2.83e-04	1.78e-03	1.08e-04
	LS-Uniform	3.32e-02	5.12e-03	1.91e-02	1.94e-03	1.35e-02	8.28e-04	1.17e-02	3.24e-04	6.02e-03	1.35e-04
	LS-Chebyshev	2.13e-02	5.31e-03	1.13e-02	1.78e-03	5.84e-03	6.34e-04	2.99e-03	2.35e-04	1.74e-03	8.47e-05
Corner Peak $Q = 50$	Smolyak	1.68e-02	1.40e-03	3.63e-02	2.11e-03	1.06e-01	3.14e-03	1.36e-01	3.50e-03	1.45e-01	3.08e-03
	LS-Uniform	1.49e-02	7.53e-04	2.17e-02	4.91e-04	3.75e-02	4.28e-04	6.35e-02	4.00e-04	4.31e-02	1.72e-04
	LS-Chebyshev	3.82e-02	1.26e-02	2.98e-02	4.26e-03	2.76e-02	3.68e-03	1.62e-02	1.41e-03	9.84e-03	6.83e-04
Discont. $Q = 50$	Smolyak	3.16e+02	3.26e+01	3.15e+02	2.22e+01	3.21e+02	1.78e+01	2.93e+02	1.29e+01	3.78e+02	9.44e+00
	LS-Uniform	5.30e+02	3.27e+01	3.72e+02	2.34e+01	4.73e+02	1.82e+01	3.96e+02	1.51e+01	7.70e+02	1.33e+01
	LS-Chebyshev	2.55e+02	5.16e+01	2.21e+02	4.58e+01	2.45e+02	2.90e+01	2.71e+02	2.14e+01	3.10e+02	1.54e+01
Gauss. $Q = 50$	Smolyak	8.19e-01	8.12e-02	6.49e-01	9.23e-02	5.66e-01	3.82e-02	4.27e-01	1.90e-02	2.89e-01	1.24e-02
	LS-Uniform	6.16e-01	6.05e-02	6.97e-01	3.99e-02	9.40e-01	2.56e-02	1.90e+00	1.90e-02	1.08e+00	9.96e-03
	LS-Chebyshev	6.45e-01	7.00e-02	5.18e-01	3.70e-02	3.33e-01	2.22e-02	2.45e-01	1.22e-02	1.38e-01	6.03e-03
Geo. Mean $Q = 50$	Smolyak	2.12e-01	3.39e-02	5.98e-02	9.17e-03	1.69e-02	1.66e-03	5.02e-03	3.01e-04	1.22e-03	5.36e-05
	LS-Uniform	2.46e-01	1.96e-02	6.28e-02	5.51e-03	2.67e-02	9.99e-04	5.91e-03	1.25e-04	1.96e-03	1.69e-05
	LS-Chebyshev	1.69e-01	4.66e-02	7.30e-02	1.61e-02	1.53e-02	2.97e-03	2.32e-03	3.96e-04	4.64e-04	5.06e-05
NOISE $Q = 50$	Smolyak	3.89e-06	7.36e-07	7.43e-06	1.18e-06	1.29e-05	1.44e-06	1.83e-05	1.99e-06	2.80e-05	2.59e-06
	LS-Uniform	1.03e-06	1.40e-07	2.18e-06	1.50e-07	4.86e-06	1.72e-07	7.99e-06	1.94e-07	1.59e-05	2.34e-07
	LS-Chebyshev	6.94e-07	1.70e-07	8.48e-07	1.70e-07	9.50e-07	1.58e-07	1.07e-06	1.55e-07	1.12e-06	1.56e-07
Osci. $Q = 50$	Smolyak	1.07e+01	1.22e+00	9.41e+00	9.07e-01	5.95e+00	5.23e-01	3.24e+00	2.56e-01	1.26e+00	1.05e-01
	LS-Uniform	1.10e+01	1.12e+00	1.37e+01	7.95e-01	2.03e+01	4.98e-01	1.55e+01	2.40e-01	8.97e+00	7.95e-02
	LS-Chebyshev	6.02e+00	1.39e+00	4.89e+00	1.17e+00	4.09e+00	7.94e-01	2.75e+00	4.25e-01	8.34e-01	1.35e-01
Prod. Peak $Q = 50$	Smolyak	8.35e-03	1.62e-03	2.36e-03	3.25e-04	5.73e-04	6.41e-05	1.18e-04	1.13e-05	2.85e-05	1.87e-06
	LS-Uniform	1.49e-02	1.82e-03	4.91e-03	3.00e-04	8.68e-04	4.60e-05	3.77e-04	6.61e-06	1.35e-04	1.14e-06
	LS-Chebyshev	1.38e-02	2.93e-03	4.36e-03	5.51e-04	7.29e-04	7.56e-05	1.61e-04	1.14e-05	2.90e-05	1.40e-06
Ridge Prod. $Q = 50$	Smolyak	1.64e+01	1.22e+00	1.34e+01	8.31e-01	9.61e+00	4.93e-01	5.26e+00	2.66e-01	3.23e+00	1.35e-01
	LS-Uniform	7.44e+00	7.64e-01	5.69e+00	4.08e-01	3.73e+00	2.25e-01	3.20e+00	1.20e-01	2.14e+00	6.46e-02
	LS-Chebyshev	6.29e+00	1.45e+00	3.24e+00	7.89e-01	2.17e+00	3.60e-01	1.98e+00	1.62e-01	9.39e-01	7.06e-02

TABLE 6. The errors e_{\max}^{wc} and $e_{\text{mean}}^{\text{wc}}$ for $d = 7$, each scale for the different algorithms. Lowest value in bold.

		Scale 3		Scale 4		Scale 5		Scale 6	
		e_{\max}^{wc}	$e_{\text{mean}}^{\text{wc}}$	e_{\max}^{wc}	$e_{\text{mean}}^{\text{wc}}$	e_{\max}^{wc}	$e_{\text{mean}}^{\text{wc}}$	e_{\max}^{wc}	$e_{\text{mean}}^{\text{wc}}$
Bim. Gauss. $Q = 50$	Smolyak	1.27e+00	3.51e-01	9.95e-01	1.66e-01	4.83e-01	7.72e-02	3.37e-01	3.80e-02
	LS-Uniform	6.17e-01	5.79e-02	6.54e-01	4.06e-02	8.87e-01	2.64e-02	1.06e+00	1.82e-02
	LS-Chebyshev	6.28e-01	6.48e-02	5.80e-01	5.10e-02	4.61e-01	2.74e-02	2.74e-01	1.46e-02
Cont. $Q = 50$	Smolyak	2.35e-02	4.67e-03	1.13e-02	1.89e-03	6.45e-03	6.21e-04	3.25e-03	2.40e-04
	LS-Uniform	2.41e-02	4.44e-03	1.40e-02	1.80e-03	9.09e-03	7.09e-04	5.86e-03	2.80e-04
	LS-Chebyshev	2.22e-02	4.75e-03	1.26e-02	1.65e-03	5.08e-03	6.01e-04	2.85e-03	2.17e-04
Corner Peak $Q = 50$	Smolyak	1.40e-03	6.37e-05	7.79e-03	5.32e-04	4.59e-02	9.43e-04	4.68e-02	1.34e-03
	LS-Uniform	1.29e-03	8.69e-05	1.43e-03	9.33e-05	9.09e-03	7.55e-05	7.43e-03	7.63e-05
	LS-Chebyshev	1.52e-02	3.77e-03	2.64e-02	3.62e-03	2.40e-02	2.21e-03	1.56e-02	9.77e-04
Discont. $Q = 50$	Smolyak	6.16e+02	4.12e+01	4.69e+02	3.33e+01	1.00e+03	2.78e+01	8.55e+02	2.15e+01
	LS-Uniform	4.39e+02	4.64e+01	5.24e+02	4.03e+01	7.67e+02	3.26e+01	8.95e+02	2.75e+01
	LS-Chebyshev	5.56e+02	8.63e+01	4.46e+02	6.56e+01	8.71e+02	4.60e+01	8.14e+02	3.56e+01
Gauss. $Q = 50$	Smolyak	5.52e-01	1.32e-01	6.66e-01	7.20e-02	5.10e-01	4.32e-02	3.42e-01	2.35e-02
	LS-Uniform	5.90e-01	5.49e-02	9.57e-01	3.33e-02	7.75e-01	2.12e-02	8.86e-01	1.30e-02
	LS-Chebyshev	6.10e-01	7.08e-02	5.32e-01	3.60e-02	4.23e-01	2.02e-02	2.45e-01	1.11e-02
Geo. Mean $Q = 50$	Smolyak	7.50e-02	1.68e-02	2.32e-02	3.01e-03	6.07e-03	5.67e-04	1.60e-03	1.05e-04
	LS-Uniform	8.86e-02	1.09e-02	2.59e-02	1.87e-03	3.50e-03	2.15e-04	1.94e-03	2.84e-05
	LS-Chebyshev	1.03e-01	2.53e-02	2.16e-02	4.55e-03	4.17e-03	5.71e-04	3.78e-04	6.69e-05
NOISE $Q = 50$	Smolyak	5.30e-06	8.86e-07	1.10e-05	1.46e-06	1.77e-05	1.94e-06	2.64e-05	2.67e-06
	LS-Uniform	9.18e-07	1.34e-07	1.41e-06	1.42e-07	3.08e-06	1.58e-07	6.80e-06	1.81e-07
	LS-Chebyshev	7.69e-07	1.75e-07	8.40e-07	1.63e-07	9.82e-07	1.61e-07	1.08e-06	1.61e-07
Osci. $Q = 50$	Smolyak	1.74e+01	1.53e+00	1.66e+01	1.13e+00	1.42e+01	7.29e-01	1.29e+01	4.34e-01
	LS-Uniform	1.07e+01	1.20e+00	1.22e+01	9.90e-01	1.71e+01	7.00e-01	2.00e+01	4.00e-01
	LS-Chebyshev	5.75e+00	1.45e+00	5.84e+00	1.27e+00	5.79e+00	1.09e+00	4.52e+00	7.55e-01
Prod. Peak $Q = 50$	Smolyak	1.07e-02	1.34e-03	3.01e-03	2.86e-04	1.05e-03	6.48e-05	2.19e-04	1.16e-05
	LS-Uniform	1.70e-02	1.32e-03	4.71e-03	2.49e-04	1.48e-03	4.39e-05	1.19e-03	7.35e-06
	LS-Chebyshev	1.87e-02	2.34e-03	5.39e-03	4.65e-04	1.11e-03	8.02e-05	2.30e-04	1.16e-05
Ridge Prod. $Q = 50$	Smolyak	2.29e+01	1.85e+00	2.60e+01	1.40e+00	1.89e+01	9.00e-01	1.10e+01	5.33e-01
	LS-Uniform	7.92e+00	1.30e+00	6.54e+00	7.10e-01	5.65e+00	3.95e-01	5.08e+00	2.09e-01
	LS-Chebyshev	9.91e+00	2.75e+00	6.68e+00	1.61e+00	4.08e+00	7.55e-01	2.58e+00	3.53e-01

TABLE 7. The errors e_{\max}^{wc} and $e_{\text{mean}}^{\text{wc}}$ for $d = 8$, each scale for the different algorithms. Lowest value in bold.

		Scale 3		Scale 4		Scale 5		Scale 6	
		e_{\max}^{wc}	$e_{\text{mean}}^{\text{wc}}$	e_{\max}^{wc}	$e_{\text{mean}}^{\text{wc}}$	e_{\max}^{wc}	$e_{\text{mean}}^{\text{wc}}$	e_{\max}^{wc}	$e_{\text{mean}}^{\text{wc}}$
Bim. Gauss. $Q = 50$	Smolyak	8.64e-01	2.63e-01	8.49e-01	1.23e-01	7.20e-01	5.77e-02	3.17e-01	2.91e-02
	LS-Uniform	5.65e-01	6.83e-02	1.42e+00	4.23e-02	1.13e+00	2.90e-02	1.43e+00	1.83e-02
	LS-Chebyshev	6.50e-01	7.35e-02	5.48e-01	4.42e-02	4.36e-01	2.71e-02	2.98e-01	1.59e-02
Cont. $Q = 50$	Smolyak	2.08e-02	4.22e-03	1.26e-02	1.74e-03	4.71e-03	7.08e-04	3.44e-03	2.55e-04
	LS-Uniform	2.26e-02	4.27e-03	1.56e-02	1.86e-03	1.33e-02	7.18e-04	9.70e-03	2.93e-04
	LS-Chebyshev	2.09e-02	4.60e-03	9.93e-03	1.86e-03	5.37e-03	6.31e-04	2.60e-03	2.33e-04
Corner Peak $Q = 50$	Smolyak	1.30e-03	4.80e-05	1.49e-02	4.24e-04	3.07e-02	6.85e-04	4.38e-02	8.63e-04
	LS-Uniform	1.46e-03	5.45e-05	6.29e-03	8.21e-05	5.84e-03	3.80e-05	2.55e-02	8.50e-05
	LS-Chebyshev	3.56e-02	8.37e-03	2.21e-02	3.34e-03	1.46e-02	1.54e-03	1.97e-02	1.03e-03
Discont. $Q = 50$	Smolyak	1.11e+03	7.78e+01	1.84e+03	5.96e+01	1.62e+03	4.44e+01	2.28e+03	3.75e+01
	LS-Uniform	1.25e+03	9.28e+01	1.59e+03	7.37e+01	1.57e+03	5.85e+01	1.85e+03	4.80e+01
	LS-Chebyshev	1.06e+03	1.61e+02	1.98e+03	1.45e+02	1.73e+03	9.91e+01	2.05e+03	7.17e+01
Gauss. $Q = 50$	Smolyak	5.85e-01	1.46e-01	9.05e-01	8.47e-02	5.07e-01	4.52e-02	3.81e-01	2.74e-02
	LS-Uniform	5.43e-01	4.69e-02	8.32e-01	3.36e-02	8.26e-01	2.38e-02	1.39e+00	1.49e-02
	LS-Chebyshev	5.99e-01	4.31e-02	5.93e-01	3.13e-02	4.24e-01	2.10e-02	2.95e-01	1.27e-02
Geo. Mean $Q = 50$	Smolyak	1.54e-01	3.18e-02	5.70e-02	8.03e-03	1.56e-02	1.37e-03	3.57e-03	1.52e-04
	LS-Uniform	1.28e-01	1.48e-02	7.95e-02	4.52e-03	2.44e-02	8.86e-04	4.25e-03	8.22e-05
	LS-Chebyshev	1.58e-01	4.58e-02	6.94e-02	1.42e-02	1.70e-02	2.74e-03	2.24e-03	2.98e-04
NOISE $Q = 50$	Smolyak	5.41e-06	1.18e-06	1.23e-05	1.76e-06	2.59e-05	2.62e-06	3.88e-05	3.57e-06
	LS-Uniform	1.03e-06	1.31e-07	1.69e-06	1.39e-07	3.67e-06	1.53e-07	1.05e-05	1.74e-07
	LS-Chebyshev	7.20e-07	1.85e-07	9.39e-07	1.73e-07	1.14e-06	1.65e-07	1.33e-06	1.65e-07
Osci. $Q = 50$	Smolyak	2.42e+01	2.27e+00	3.33e+01	1.48e+00	2.19e+01	1.12e+00	2.13e+01	7.17e-01
	LS-Uniform	8.54e+00	1.15e+00	1.69e+01	1.07e+00	3.41e+01	8.90e-01	4.13e+01	6.09e-01
	LS-Chebyshev	6.95e+00	1.52e+00	6.84e+00	1.41e+00	6.82e+00	1.24e+00	6.29e+00	9.71e-01
Prod. Peak $Q = 50$	Smolyak	1.05e-02	1.84e-03	3.58e-03	3.43e-04	9.39e-04	6.33e-05	1.40e-04	1.04e-05
	LS-Uniform	1.62e-02	1.52e-03	8.66e-03	3.02e-04	2.92e-03	4.69e-05	9.15e-04	7.01e-06
	LS-Chebyshev	2.39e-02	3.22e-03	6.19e-03	5.25e-04	1.35e-03	8.90e-05	2.73e-04	1.25e-05
Ridge Prod. $Q = 50$	Smolyak	4.61e+01	2.46e+00	4.11e+01	1.85e+00	3.73e+01	1.30e+00	2.35e+01	7.50e-01
	LS-Uniform	1.93e+01	1.52e+00	1.51e+01	8.92e-01	1.48e+01	4.86e-01	8.80e+00	2.75e-01
	LS-Chebyshev	1.54e+01	4.25e+00	1.39e+01	2.17e+00	8.35e+00	1.18e+00	4.76e+00	5.52e-01

TABLE 8. The errors e_{\max}^{wc} and $e_{\text{mean}}^{\text{wc}}$ for $d = 9$, each scale for the different algorithms. Lowest value in bold.

		Scale 3		Scale 4		Scale 5		Scale 6	
		e_{\max}^{wc}	$e_{\text{mean}}^{\text{wc}}$	e_{\max}^{wc}	$e_{\text{mean}}^{\text{wc}}$	e_{\max}^{wc}	$e_{\text{mean}}^{\text{wc}}$	e_{\max}^{wc}	$e_{\text{mean}}^{\text{wc}}$
Bim. Gauss. $Q = 50$	Smolyak	1.21e+00	2.73e-01	9.42e-01	1.45e-01	8.41e-01	6.38e-02	4.33e-01	4.68e-02
	LS-Uniform	5.24e-01	6.01e-02	4.85e-01	4.17e-02	1.13e+00	2.66e-02	1.52e+00	1.71e-02
	LS-Chebyshev	6.88e-01	6.53e-02	5.97e-01	4.47e-02	4.92e-01	2.57e-02	3.66e-01	1.55e-02
Cont. $Q = 50$	Smolyak	2.33e-02	4.16e-03	9.30e-03	1.55e-03	5.37e-03	6.20e-04	2.79e-03	1.92e-04
	LS-Uniform	2.36e-02	3.73e-03	1.20e-02	1.48e-03	1.07e-02	6.01e-04	6.48e-03	2.37e-04
	LS-Chebyshev	2.23e-02	4.08e-03	1.03e-02	1.45e-03	5.61e-03	5.39e-04	2.34e-03	1.95e-04
Corner Peak $Q = 50$	Smolyak	4.08e-04	1.11e-05	5.66e-04	1.08e-05	1.16e-02	1.67e-04	2.88e-02	4.11e-04
	LS-Uniform	3.87e-04	2.09e-05	5.40e-04	1.26e-05	9.89e-04	9.84e-06	2.38e-03	8.90e-06
	LS-Chebyshev	1.62e-03	2.92e-04	1.33e-03	2.53e-04	4.98e-03	3.72e-04	6.19e-03	3.31e-04
Discont. $Q = 50$	Smolyak	4.14e+03	2.24e+02	4.14e+03	1.37e+02	2.92e+03	1.00e+02	5.01e+03	7.81e+01
	LS-Uniform	3.36e+03	1.93e+02	2.89e+03	1.43e+02	2.62e+03	1.13e+02	4.37e+03	9.03e+01
	LS-Chebyshev	3.79e+03	4.90e+02	3.60e+03	3.37e+02	2.60e+03	2.30e+02	3.39e+03	1.75e+02
Gauss. $Q = 50$	Smolyak	7.29e-01	8.59e-02	6.10e-01	5.78e-02	5.69e-01	3.47e-02	4.24e-01	2.04e-02
	LS-Uniform	5.32e-01	4.63e-02	5.28e-01	2.98e-02	1.01e+00	1.82e-02	9.98e-01	1.08e-02
	LS-Chebyshev	7.05e-01	5.57e-02	5.66e-01	3.15e-02	4.67e-01	1.96e-02	3.04e-01	1.07e-02
Geo. Mean $Q = 50$	Smolyak	2.11e-01	3.30e-02	1.24e-01	1.11e-02	5.30e-02	3.61e-03	1.45e-02	1.10e-03
	LS-Uniform	1.33e-01	1.46e-02	7.36e-02	4.07e-03	2.87e-02	8.66e-04	1.27e-02	1.82e-04
	LS-Chebyshev	1.70e-01	3.77e-02	5.84e-02	1.26e-02	1.60e-02	2.92e-03	7.23e-03	8.24e-04
NOISE $Q = 50$	Smolyak	1.06e-05	1.74e-06	2.09e-05	2.14e-06	3.98e-05	3.22e-06	6.21e-05	4.71e-06
	LS-Uniform	7.99e-07	1.25e-07	1.42e-06	1.31e-07	3.67e-06	1.47e-07	5.65e-06	1.65e-07
	LS-Chebyshev	7.85e-07	1.77e-07	1.01e-06	1.76e-07	1.15e-06	1.70e-07	1.29e-06	1.68e-07
Osci. $Q = 50$	Smolyak	4.10e+01	3.10e+00	4.89e+01	2.10e+00	4.38e+01	1.44e+00	4.67e+01	9.71e-01
	LS-Uniform	6.72e+00	1.12e+00	1.70e+01	1.13e+00	4.13e+01	1.09e+00	6.83e+01	8.65e-01
	LS-Chebyshev	6.05e+00	1.46e+00	7.55e+00	1.44e+00	7.39e+00	1.34e+00	8.18e+00	1.20e+00
Prod. Peak $Q = 50$	Smolyak	1.53e-02	1.89e-03	4.83e-03	4.05e-04	1.79e-03	6.80e-05	2.96e-04	1.34e-05
	LS-Uniform	3.22e-02	1.41e-03	8.72e-03	2.71e-04	3.74e-03	5.71e-05	1.61e-03	1.08e-05
	LS-Chebyshev	1.99e-02	2.59e-03	8.62e-03	5.31e-04	2.12e-03	9.73e-05	5.25e-04	1.85e-05
Ridge Prod. $Q = 50$	Smolyak	4.96e+01	2.83e+00	6.16e+01	2.16e+00	4.62e+01	1.64e+00	5.71e+01	1.08e+00
	LS-Uniform	1.98e+01	1.64e+00	1.61e+01	9.34e-01	1.40e+01	5.31e-01	1.15e+01	2.94e-01
	LS-Chebyshev	2.68e+01	6.33e+00	1.98e+01	3.37e+00	1.00e+01	1.63e+00	7.43e+00	7.75e-01

TABLE 9. The errors e_{\max}^{wc} and $e_{\text{mean}}^{\text{wc}}$ for $d = 10$, each scale for the different algorithms. Lowest value in bold.