THE RADON TRANSFORM

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Abstract. This is an overview of a project for the subject "Wavelets - Functional Analytical Basics" in the summer semester of 2025 @ JKU Linz.

When a beam of X-Ray photons travels through a homogeneous material, the intensity of photons at the time of entering the material is high compared to the intensity at the moment of exit. This is described by the equation

$$I = I_0 e^{-\mu w}$$

where I_0 is the input intensity and I is the observed intensity after the beam passes the distance w through the material. The *attenuation* coefficent μ depends (among other things) on the density of the material. When the material is inhomogeneous, i.e. μ represents a varying density, the corresponding equation for the observed intensity becomes

$$I = I_0 \exp\left(-\int_L \mu(x, y)\,\mathrm{d}s\right)$$

where L is the beam path parametrised by s. By moving the source of the photons around the material, one obtains a set of line integrals over the density μ and by taking logarithms, one obtains samples of Radon transforms where the d–dimensional Radon transform of an integrable function $f : \mathbb{R}^d \to \mathbb{R}$ is defined as

$$\mathrm{R}\, f(\sigma, \theta) \overset{\mathrm{def}}{=} \int_{\langle x \,|\, \theta\rangle = \sigma} f(x)\,\mathrm{d}x, \qquad (\sigma, \theta) \in \mathbb{R} \times \mathbb{S}^{d-1}.$$

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