

# Ontology Partitioning Using $\mathcal{E}$ -Connections Revisited



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# Introduction

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# Modularity

Large ontologies with 100,000s of axioms

e.g.



## Challenges

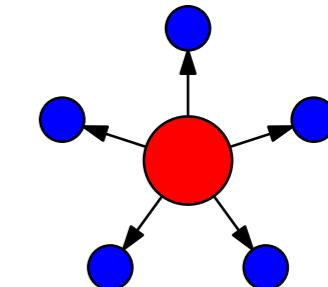
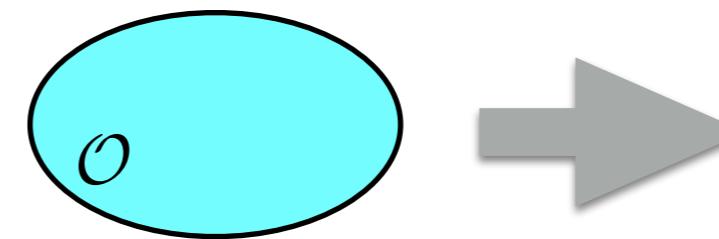
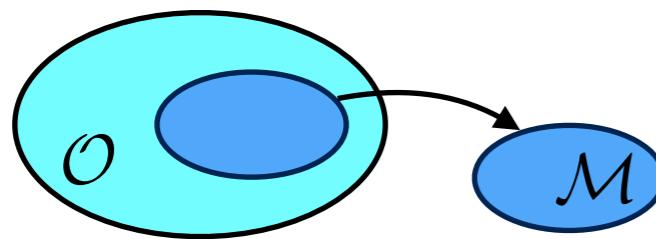
- Loading, navigation
- Understanding the logical structure (comprehension)
- Efficient automated reasoning
- Efficient re-use
- Versioning and more ...

## Modularity helps:

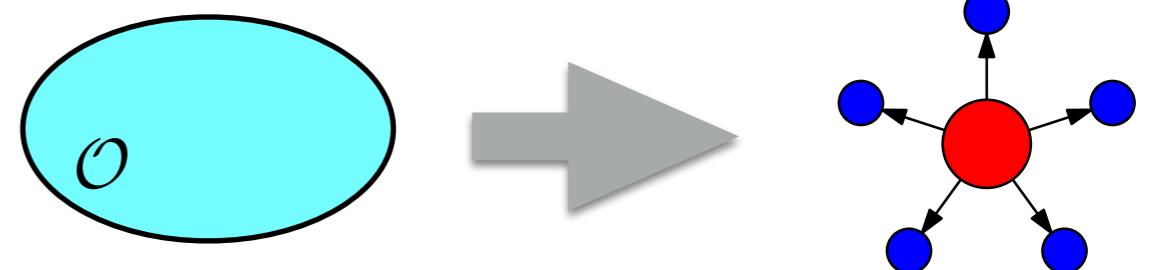
Module extraction

and

## Decomposition

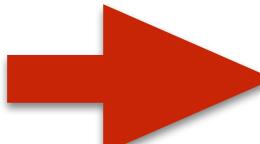


# Decomposition



## Existing approaches

- Signature splitting [Parikh '99]
- Signature  $\Delta$ -decomposition [Konev et al. '10]

-  Partitions based on  $\mathcal{E}$ -connections [Cuenca Grau et al. '06]
- Atomic decomposition [Del Vescovo et al. '11]
  - Structure-based partitioning  
[Stuckenschmidt & Klein '04, Amato et al. '15]

# $\mathcal{E}$ -Partitions in a Nutshell

**Aim:** Automatic and efficient partitioning of an ontology;  
parts are connected via “semantic links” in the style of  $\mathcal{E}$ -connections

**$\mathcal{E}$ -connections ...** [Kutz et al. 2004]

- combine (heterogeneous) logical theories via *link relations*
- semantics via partitioned interpretations

**An  $\mathcal{E}$ -partition of an ontology  $\mathcal{O}$  ...** [Cuenca Grau et al. 2006]

- is the unique maximal  $\mathcal{E}$ -connection equivalent to  $\mathcal{O}$   
(with link relations from  $\mathcal{O}$ 's role names)
- can be computed in polytime for  $\mathcal{O}$  in DLs up to  $SHOIQ(\mathcal{D})$
- its components are logically encapsulating

**e.g.:**

Fracture  $\sqsubseteq$  Disorder  $\sqcap \exists \text{affects} . \text{Bone}$   
Bone  $\sqsubseteq$  BodyStructure

Fracture  $\sqsubseteq$  Disorder  $\sqcap \exists \text{affects} . \text{Bone}$

Bone  $\sqsubseteq$  BodyStructure



## Where we started

Understand algorithm?

Fix bugs in original implementation?

## Where we got

- ★ found a **simpler** algorithm that runs in **linear time**
- ★ **simplified** notation and proofs
- ★ extended the approach to (almost) **OWL**
- ★ identified potential for **extension** beyond OWL  
and **limits**

## Work in progress!

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# $\varepsilon$ -Connections and $\varepsilon$ -Partitions for OWL

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# Indexing the Vocabulary

Let  $S$  be an arbitrary index set.

## Index function $\iota$

- (concept names)  $A \mapsto$  index  $\iota(A) \in S$
- (role names)  $r \mapsto$  pair of indices  $\iota(r)$
- is extended to complex concepts:

e.g.,  $\iota(\exists r.D) = i$  if  $\iota(r) = (i, j)$  and  $\iota(D) = j$

(and many more cases)

$\exists r.D$  is  $\iota$ -wellformed

- is extended to axioms:
- e.g.,  $\iota(C \sqsubseteq D) = i$  if  $\iota(C) = \iota(D) = i$
- and thus determines a partitioning of ontologies

## Two views on an ontology:

- as a monolithic ontology
- as a  $\iota$ -ontology – an  $\mathcal{E}$ -connection!

## $\iota$ -interpretations

- Domain  $\Delta^{\mathcal{I}}$  is partitioned into  $(\Delta_i^{\mathcal{I}})_{i \in S}$
- Concept names  $A$  with  $\iota(A) = i$  are interpreted within  $\Delta_i^{\mathcal{I}}$ , analogously for role names
- Extension to complex concepts as usual  
**except negation:**  $(\neg C)^{\mathcal{I}, \iota} = \Delta_{\underline{\iota(C)}}^{\mathcal{I}} \setminus C^{\mathcal{I}, \iota}$

## Two views on semantics:

- Standard semantics, denoted  $\mathcal{I} \models \mathcal{O}$
- Semantics w.r.t. indexing  $\iota$ , denoted  $\mathcal{I} \models^{\iota} \mathcal{O}$

# Compatibility and Equivalence

Let  $\mathcal{O}$  be an ontology and  $\mathbb{O} = (\mathcal{O}_i)_{i \in S}$  a  $\iota$ -ontology.

## Important relationships between $\mathcal{O}$ and $\mathbb{O}$ :

- $\mathcal{O}$  and  $\mathbb{O}$  are **compatible**, written  $\mathcal{O} \sim \mathbb{O}$ , if  $\mathcal{O} = \bigcup_{i \in S} \mathcal{O}_i$ .
- $\mathcal{O}$  and  $\mathbb{O}$  are **equivalent**, written  $\mathcal{O} \equiv \mathbb{O}$ , if for all  $\iota$ -interpret.  $\mathcal{I}$ :

$$\mathcal{I} \models \mathcal{O} \quad \text{iff} \quad \mathcal{I} \models^\iota \mathbb{O}$$

Apparently, compatibility and equivalence do **not** imply each other!

# Domain-Independence

Well-known notion from database theory relates compatibility & equivalence:

$\mathcal{O}$  is **domain-independent (DI)**

if for all interpretations  $\mathcal{I}, \mathcal{J}$  with  $X^{\mathcal{I}} = X^{\mathcal{J}}$  for all terms  $X$ :

$$\mathcal{I} \models \mathcal{O} \text{ iff } \mathcal{J} \models \mathcal{O}$$

**Nice characterization of all DI concepts** [Cuenca Grau et al. 2006]

allows to check DI in linear time; additionally gives:

- If  $C$  is **not** DI and  $\mathcal{I}, \mathcal{J}$  are as above with  $\Delta^{\mathcal{J}} = \Delta^{\mathcal{I}} \uplus S$ ,  
then  $C^{\mathcal{J}} = C^{\mathcal{I}} \cup S$ .

Holds for all of OWL **except the universal role**.

# Domain-Independence

Previous characterization is crucial in the proof of the following:

## Theorem.

1. If  $\mathcal{O}$  is DI and  $\mathcal{O} \sim \mathbb{O}$ , then  $\mathcal{O} \approx \mathbb{O}$ .
2. If additionally  $\mathcal{O}$  is consistent, then so is  $\mathbb{O}$ .

## Consequence

For DI ontologies,  
it suffices to compute the minimal **compatible** E-connection.

- From now on, we assume that the input ontology  $\mathcal{O}$  is DI.

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# The New Partitioning Algorithm and First Tests

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# A Simple Algorithm

## Idea:

For input ontology  $\mathcal{O}$ ,

find index set  $S$  of **maximal** cardinality and index function  $\iota$   
such that all concepts and axioms in  $\mathcal{O}$  are  $\iota$ -wellformed

## The Algorithm:

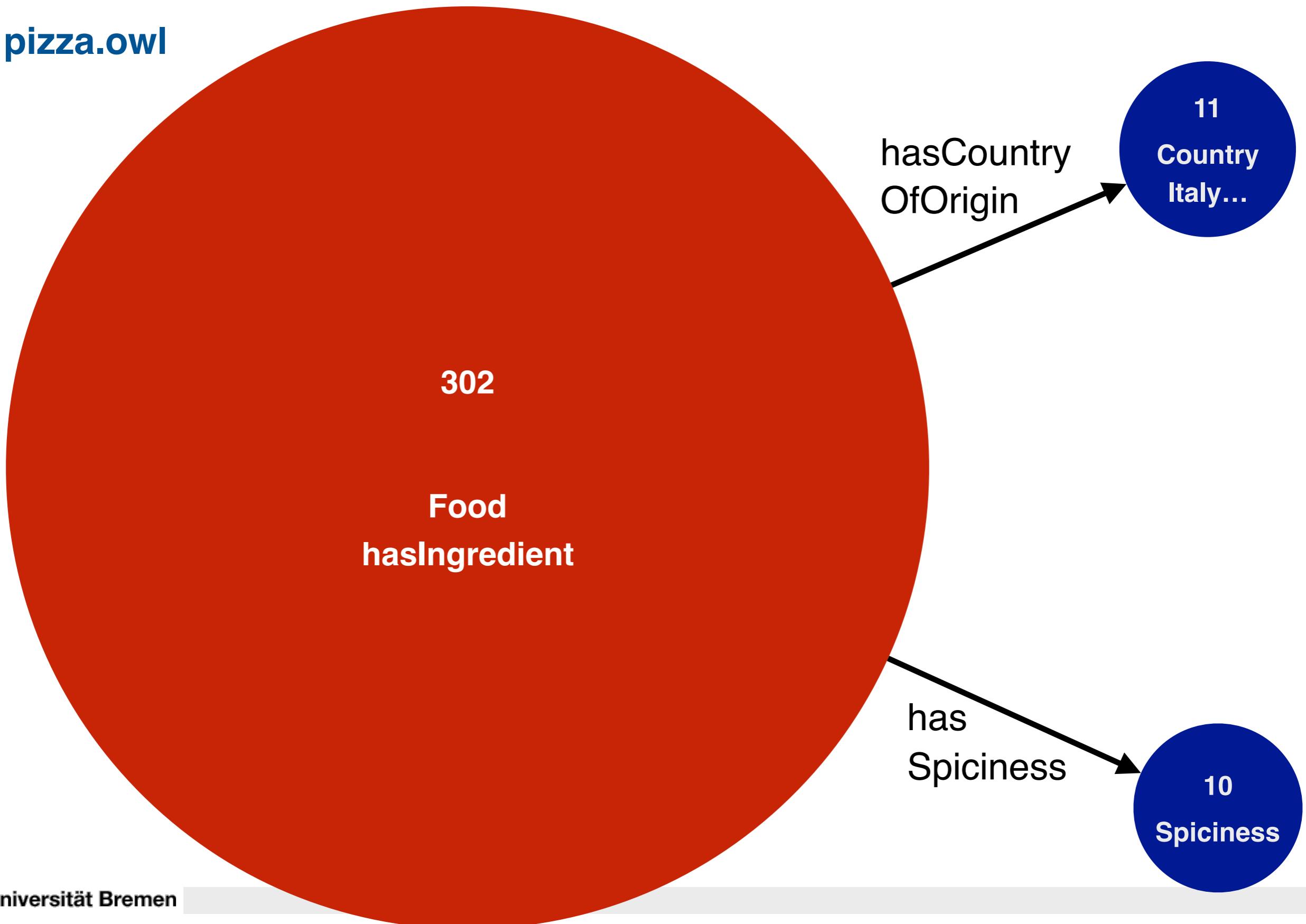
1. Collect wellformedness constraints in an undirected graph  $G$ 
  - nodes: one per (complex) concept, 2 per role name
  - edges = constraints
2.  $G$ 's connected components induce  $S, \iota, \mathcal{O}$

Both steps easy to implement in **linear time**.

Correctness and maximality are straightforward to show:  
algo mimics wellformedness definition!

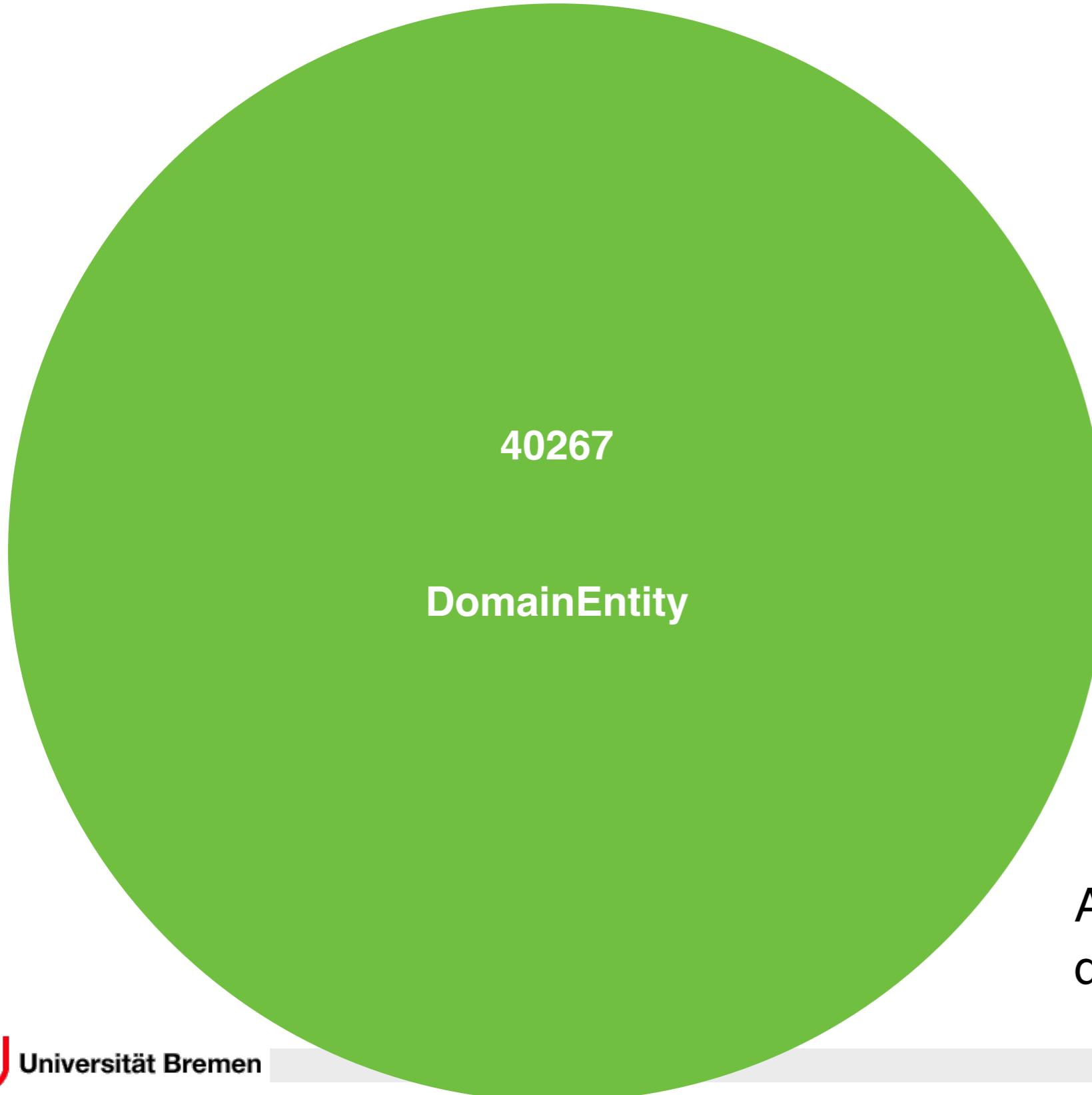
# Example Decomposition: Pizza Ont.

pizza.owl



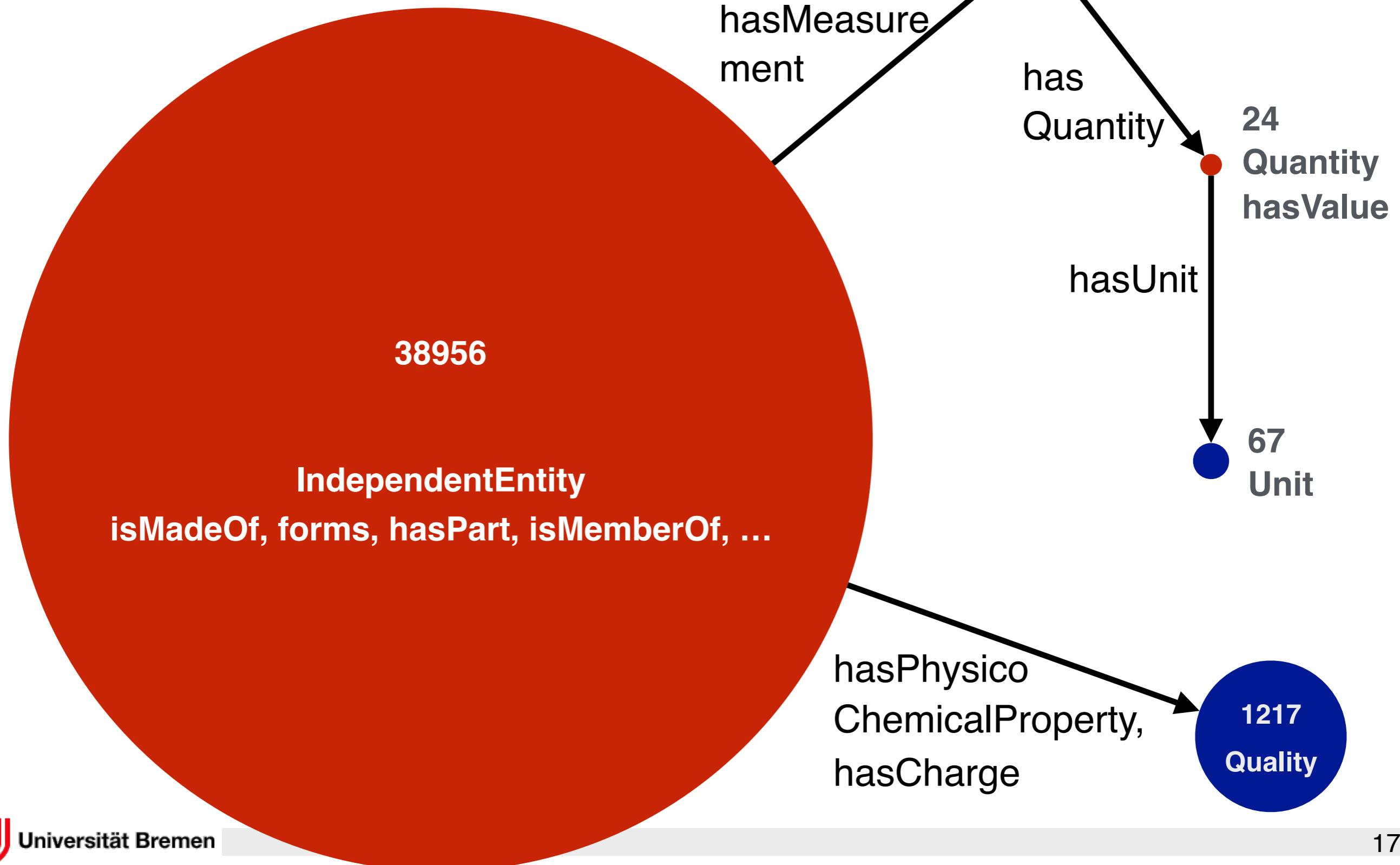
# Example Decomposition: PTO

## Periodic Table Ontology by Robert Stevens



# Decomposition: PTO with 3 levels removed

## Periodic Table Ontology by Robert Stevens





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# Outlook

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## Coming soon:

- Systematic evaluation
- Heuristics for ontologies that don't decompose well
- Extensions: TGDs, UNFO?

# The End

Questions?

¿Preguntas?

Fragen?

Vragen?

Pytania?

**Thank you.**

Vrae?

Kysymyksiä?

Ερωτήσεις;

Întrebări?

Вопросы?

Questões?