The Modular Structure of an Ontology: Atomic Decomposition

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Ontologies

. . . are finite theories in a (description) logic, e.g.

$$\begin{aligned} \mathsf{Part} &\equiv \exists \mathsf{strict\_part\_of}.\mathsf{Whole} \\ \mathsf{strict\_part\_of} &\sqsubseteq \mathsf{part\_of} \\ \mathsf{Hand} &\sqsubseteq {=}5\,\mathsf{part\_of}.\mathsf{Finger} \end{aligned}$$

OWL (W3C standard, FOL fragment): expressive DL, featuring

- unary and binary predicates
- constructors for unary predicates

  Booleans, $\exists$, $\forall$, counting quantifiers, nominals
- constructors for binary predicates

  inverse, composition
- axioms: inclusions/equivalences of predicates
- global constraints to ensure decidability

Modules

A module $\mathcal{M}(\Sigma, \mathcal{O}) \subseteq \mathcal{O}$ encapsulates knowledge
w.r.t. a signature Σ if $\mathcal{M} \equiv^{c}_{\Sigma} \mathcal{O}$

i.e., for all $C \sqsubseteq D$ with $\mathrm{sig}(C \sqsubseteq D) \subseteq \Sigma$:
$\mathcal{O} \models C \sqsubseteq D$ iff $\mathcal{M}(\Sigma, \mathcal{O}) \models C \sqsubseteq D$

Example: $\mathcal{O} = \texttt{Mereology.owl}$, $\Sigma = \{\mathsf{Part}, \mathsf{Whole}\}$, $\mathcal{M}(\Sigma, \mathcal{O}) =$

$\mathcal{O}$

$\mathcal{M}$

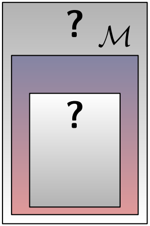
$$\begin{array}{rlrl}
\mathsf{Part} \equiv & \exists \mathsf{strict_part_of}.\mathsf{Whole} & \mathsf{strict_part_of} \equiv & \mathsf{direct_part_of} \\
\mathsf{Part} \sqsubseteq & \forall \mathsf{strict_part_of}.\mathsf{Whole} & \mathsf{direct_part_of} \equiv & \mathsf{direct_part}^{-1} \\
\mathsf{Whole} \equiv & \exists \mathsf{strict_part}.\mathsf{Part} & \mathsf{strict_part} \equiv & \mathsf{direct_part} \\
\mathsf{Whole} \sqsubseteq & \forall \mathsf{strict_part}.\mathsf{Part} & \mathsf{strict_part_of} \equiv & \mathsf{strict_part}^{-1}
\end{array}$$

Modular *structure*

Modules are great: if you know your (seed) signature . . .
and for "module local" tasks such as reuse

Single module extraction does *not* help if you

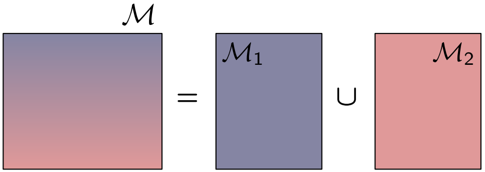
- do *not* know the *right* seed signature
- want to understand *other* modules
- want to understand *axiom dependency structure*



To analyse the modular structure of the ontology:

- significant modules
- significant relations between modules
- . . . which reveals logical dependency between axioms

Are all modules significant?



To understand $\mathcal{M}$, one must understand

- the dependency structure of $\mathcal{M}_1$
- the dependency structure of $\mathcal{M}_2$
- nothing else: $\mathcal{M}_1$ and $\mathcal{M}_2$ have no further dependencies

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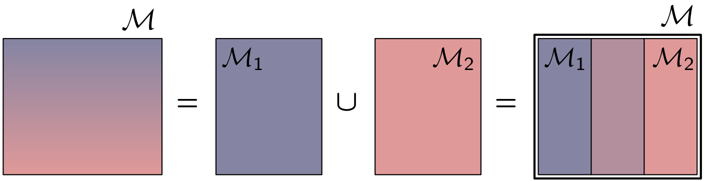

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$\rightsquigarrow$ $\mathcal{M}$ is not significant: it is a fake module

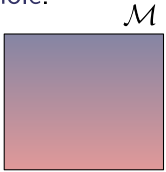
- $\mathcal{M}_1$ and $\mathcal{M}_2$ may be "significant"
- Knowing that $\mathcal{M}$ is "only" a union is important

Are all modules significant?

Consider a module $\mathcal{M}$ that is not fake.

To understand $\mathcal{M}$, one has to understand $\mathcal{M}$ as a whole.

- all axioms in $\mathcal{M}$ logically interact
- in different ways – but interact



"Not fake" implies significant: genuine

Ratio of fake to genuine

Given a set of genuine modules,

- Unions lead to fake modules

$\rightsquigarrow$ The space of fake modules is exponential

- But not *every* union of genuine modules is a module

The number of *all* modules can and does grow exponentially in $|\mathcal{O}|$
[D.,P.,S.,S., KR 2010 & WoMO 2010]

Question 1

Is module growth primarily due to trivial combinations?
I.e., are most modules fake?

Yes!

Theorem 1

Each genuine module is the smallest module that contains α, for some axiom $\alpha \in \mathcal{O}$.

$\leadsto$ The family of genuine modules is linear in $|\mathcal{O}|$.

Most modules are fake!

Proof exploits properties of modules

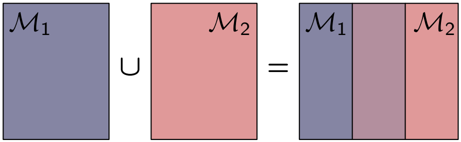
- uniqueness, monotonicity, self-containedness, ...
- which are satisfied by all locality-based modules

Relations between modules

Genuine modules may overlap.

This exposes significant logical dependency between axioms:

Axioms in $\mathcal{M}_1 \setminus \mathcal{M}_2$ depend on axioms in $\mathcal{M}_1 \cap \mathcal{M}_2$



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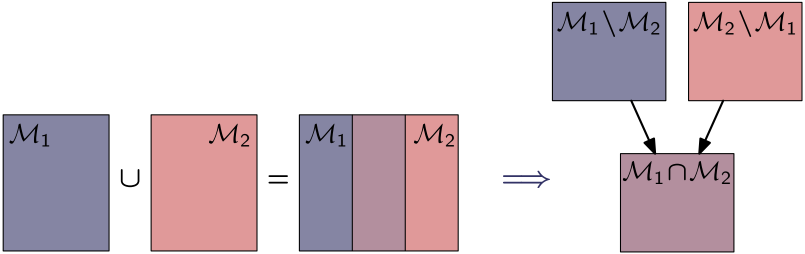
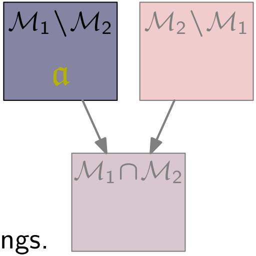


Atoms

An atom is a maximal set $\mathfrak{a} \subseteq \mathcal{O}$ such that, for every module $\mathcal{M}$, either $\mathfrak{a} \subseteq \mathcal{M}$ or $\mathfrak{a} \cap \mathcal{M} = \emptyset$.



- The smallest module for an axiom α contains the whole atom to which α belongs.
- Axioms in an atom are logically interdependent.
- Any two atoms are disjoint.

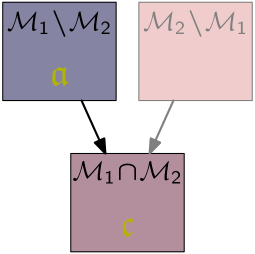
$\rightsquigarrow$ The family of atoms is a partition of the ontology.

- Each module is a disjoint union of atoms.

Proposition

There is a 1-1 correspondence btwn genuine modules and atoms.

Atomic Decomposition



Dependence between atoms:

- $\mathfrak{a} \succeq \mathfrak{c}$ if, for each $\mathcal{M}$: $\mathfrak{a} \subseteq \mathcal{M}$ implies $\mathfrak{c} \subseteq \mathcal{M}$
- Axioms in $\mathfrak{a}$ logically depend on axioms in $\mathfrak{c}$

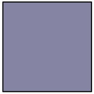
Theorem 2

The relation $\succeq$ is reflexive, antisymmetric, and transitive.

$\rightsquigarrow$ A Hasse diagram exposes 2 logical dependencies: amongst axioms in atoms, between atoms

Mereology ontology

42 axioms
1952 modules



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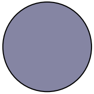

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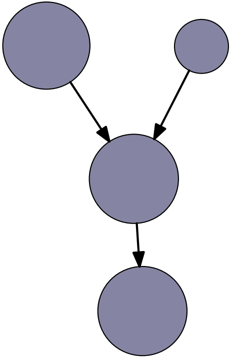

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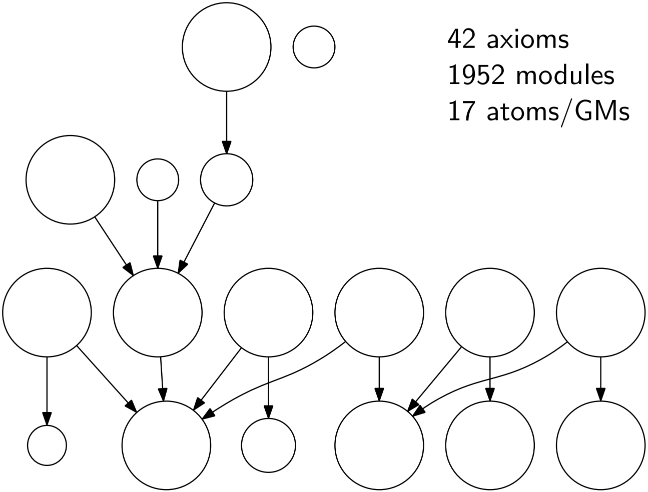

42 axioms
1952 modules
17 atoms/GMs

Feasibility

Question 2

Can we

- compute all genuine modules
- compute all atoms
- compute their dependencies

without computing all modules?!

Yes!

Remember:

Theorem 1

Each genuine module is the smallest module that contains α, for some axiom $\alpha \in \mathcal{O}$.

- Extract $\mathcal{M}(\mathsf{sig}(\alpha), \mathcal{O})$
  ($\leqslant$ linearly many module extractions)
- Atomic decomposition induced by the comparison of only the genuine modules
  (quadratic procedure)

In Reality?

We have decomposed 181 OWL ontologies from NCBO BioPortal

Decomposability:

Average	nr. axioms per atom	1.73
”	max. nr. axioms per atom	86
”	nr. axioms per genuine module	66
”	max. nr. axioms per genuine mod.	143

Summary

- The atomic decomposition (AD) is a linear representation of the potentially exponential set of all modules.
- AD can be computed using a linear number of module extractions.
- AD exposes 2 types of logical dependencies between axioms.

Future work

- Dependency between atoms and *sets* of atoms
- Labels for atoms – different labels for different tasks
- Applications
  - Topicality for ontology comprehension
  - Fast module extraction
  - All module count
  - ...

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Thank you.

Decomposability issues

Ontology $\mathcal{O}$ (ID in BioPortal)	$\#\mathcal{O}$	#max Atom	#Eq. axs.	#Disj. axs.
Nanoparticle Ontology (1083)	16,267	6,425	42	6,106
Breast Tissue Cell Lines Ontology (1438)	2,734	2,201	0	7
IMGT Ontology (1491)	1,112	729	38	594
SNP Ontology (1058)	3,481	598	30	210
Amino Acid Ontology (1054)	477	445	8	190
Comparative Data Analysis (1128)	804	434	8	190
Family Health History (1126)	1,091	378	0	1
Neural Electromagnetic Ontologies (1321)	2,286	259	21	0
Computer-based Patient Record Ontology (1059)	1,454	238	18	20
Basic Formal Ontology (1332)	95	89	13	41
Ontology of Medically-related Social Entities (1565)	138	100	17	41
Ontology for General Medical Science (1414)	194	102	17	41
Cancer Research and Mgmt Acgt Master (1130)	5,435	3,796	16	42