Simple Land Surface Scheme

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This land surface scheme solves equations for surface energy balance:

$$C_s \frac{\mathrm{d}T_s}{\mathrm{d}t} = Q_S - Q_L - H - L \tag{1}$$

and the surface moisture balance

$$\frac{\mathrm{d}W}{\mathrm{d}t} = P - E.$$

Expressing soil moisture W in terms of a volumetric soil water content $\phi = W/D$, where D is the soil depth, gives

$$\frac{\mathrm{d}\phi}{\mathrm{d}t} = \frac{P}{D} - \frac{E}{D}. \tag{2}$$

Net longwave and shortwave fluxes must be computed externally and specified. The surface sensible heat flux H and latent heat flux L are parameterized (following ?) as

$$H = \frac{\rho c_p}{r_a}(\theta_s - \theta_{\mathrm{atm}}) \tag{3}$$

and

$$L = \frac{\rho L_v}{r_a + r_s}[q^*(\theta_s) - q_{\mathrm{atm}}]. \tag{4}$$

The surface resistance r_s is modeled (following ?) as a soil moisture dependent ramp

$$r_s = r_s(\phi_{fc})\frac{\phi_{fc} - \phi_{pwp}}{\phi - \phi_{pwp}}. \tag{5}$$

The aerodynamic resistance r_a is computed from the drag coefficient C_k and near-surface wind speed $|\mathbf{u}_{\mathrm{atm}}|$ as

$$r_a = \frac{1}{C_k|\mathbf{u}_{\mathrm{atm}}|}. \tag{6}$$

Finally, vertical momentum fluxes (i.e. surface stresses) are parameterized by a drag law

$$\tau_i = -C_d|\mathbf{u}_{\mathrm{atm}}|u_i. \tag{7}$$

1 Byun (1990) Monin-Obukhov scheme

One option for modelling drag coefficients follows **?**. The goal of the model is to relate surface fluxes of heat, momentum, and moisture to winds, temperature, and moisture at the surface and the lowest model level; i.e. to calculate

$$C_d = \frac{u_*^2}{|\mathbf{u}_{\text{atm}}|^2}$$

$$C_H = \frac{\overline{w'\theta'}}{|\mathbf{u}_{\text{atm}}|(\theta_s - \theta_{\text{atm}})}$$

and

$$C_L = \frac{\overline{w'q'}}{|\mathbf{u}_{\text{atm}}|[q^*(\theta_s) - q_{\text{atm}}]}.$$

As a simplification, we will assume $C_L = C_H = C_k$ and calculate scalar drag coefficients using potential temperature fields. The first step in the formulation is to define a temperature scale based on the surface heat flux $\overline{w'\theta'}$ and the friction velocity u_*:

$$\theta_* = \frac{\overline{w'\theta'}}{u_*}.$$

The Monin-Obukhov similarity hypothesis assumes that momentum and potential temperature profiles near the surface are described by nondimensional profile functions

$$\frac{kz}{u_*}\frac{\partial u}{\partial z} = \gamma_m(\zeta)$$

and

$$\frac{kz}{\theta_*}\frac{\partial \theta}{\partial z} = \gamma_h(\zeta),$$

where $k = 0.4$ is the von Karman constant. Common observation-based forms for the profile functions (**?**) are

$$\gamma_m = \begin{cases} 1 + \alpha_m\zeta & \zeta \geq 0 \\ (1 - \beta_m\zeta)^{-1/4} & \zeta < 0 \end{cases}$$

and

$$\gamma_h = \begin{cases} \text{Pr}_0(1 + \alpha_h\zeta) & \zeta \geq 0 \\ \text{Pr}_0(1 - \beta_h\zeta)^{-1/2} & \zeta < 0 \end{cases}$$

with $\text{Pr}_0 = 0.74$, $\alpha_m = 4.7$, $\alpha_h = \alpha_m/\text{Pr}_0$, $\beta_m = 15$, and $\beta_h = 9$. ζ is the Monin-Obukhov stability parameter

$$\zeta = \frac{z}{L} = \frac{kzg\theta_*}{\theta_s u_*^2}.$$

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This parameter is closely related to the gradient Richardson number, which expressed in terms of the profile functions is

$$\text{Ri} = \frac{g}{\theta_s} \frac{\partial\theta/\partial z}{(\partial u/\partial z)^2} = \zeta \frac{\gamma_h}{\gamma_m^2}.$$

However, because we have temperatures and velocities only at the surface and the lowest model level, we would prefer to relate ζ to the bulk Richardson number

$$Ri_b = \frac{g}{\theta_s} \frac{(\theta_{\text{atm}} - \theta_s)(z_{\text{atm}} - z_s)}{|\mathbf{u}(z)|^2}. \tag{8}$$

ζ can be expressed in terms of the bulk Richardson number using integrals of the profile functions from a surface roughness length z_0 to z_{atm}. (Details are in ?.) Under stable conditions $(\zeta, \text{Ri}_b \geq 0)$,

$$\zeta = \frac{1}{2\alpha_h(\alpha_m \text{Ri}_b - 1)} \left[-(2\alpha_h \text{Ri}_b - 1) - \left(1 + \frac{4(\alpha_h - \alpha_m)\text{Ri}_b}{\text{Pr}_0}\right)^{1/2} \right] \tag{9}$$

$$= \frac{\left(\frac{z}{z-z_0}\right) \ln\left(\frac{z}{z_0}\right)}{2\alpha_h(\alpha_m \text{Ri}_b - 1)} \left[-(2\alpha_h \text{Ri}_b - 1) - \left(1 + \frac{4(\alpha_h - \alpha_m)\text{Ri}_b}{\text{Pr}_0}\right)^{1/2} \right]. \tag{10}$$

Under unstable conditions $(\zeta, \text{Ri}_b < 0)$, expressing ζ in terms of the bulk Richardson number is difficult, but a very good approximate solution can be found by multiplying an expression for ζ in terms of the gradient Richardson number by $\frac{z}{z-z_0} \ln\left(\frac{z}{z_0}\right)$ and replacing Ri with Ri_b. This gives

$$\zeta = \begin{cases} \frac{z}{z-z_0} \ln\left(\frac{z}{z_0}\right) \left[-2\sqrt{Q_b} \cos\left(\frac{\theta_b}{3}\right) + \frac{1}{3\beta_m} \right] & \text{Ri}_b \leq -0.2097 \\ \frac{z}{z-z_0} \ln\left(\frac{z}{z_0}\right) \left[-\left(T_b + \frac{Q_b}{T_b}\right) + \frac{1}{3\gamma_m} \right] & -0.2097 < \text{Ri}_b \leq 0. \end{cases} \tag{11}$$

with

$$s_b = \frac{\text{Ri}_b}{\text{Pr}_0}$$

$$Q_b = \frac{1}{9}\left[\frac{1}{\beta_m^2} + 3\frac{\beta_h}{\beta_m} s_b^2 \right]$$

$$P_b = \frac{1}{54}\left[-\frac{2}{\beta_m^3} + \frac{9}{\beta_m}\left(-\frac{\beta_h}{\beta_m} + 3\right) s_b^2 \right]$$

$$\theta_b = \text{acos}\left[\frac{P_b}{Q_b^{3/2}} \right]$$

$$T_b = \left[\left(P_b^2 - Q_b^3\right)^{1/2} + |P_b| \right]^{1/3}.$$

Relationships between u_*, θ_*, and resolved temperature and velocity fields can be derived from integrals of the profile functions:

$$u_* = k|\mathbf{u}_{\text{atm}}| \left[\ln\left(\frac{z}{z_0}\right) - \psi_m \right]^{-1},$$

and

$$\theta_* = \frac{k(\theta_s - \theta_{\text{atm}})}{\text{Pr}_0} \left[\ln\left(\frac{z}{z_0}\right) - \psi_h \right]^{-1}.$$

For stable conditions,

$$\psi_m = -\alpha_m \zeta \left(1 - \frac{z_0}{z}\right)$$

and

$$\psi_h = -\alpha_h \zeta \left(1 - \frac{z_0}{z}\right).$$

For unstable conditions,

$$\psi_m = 2\ln\left(\frac{1+x}{1+x_0}\right) + \ln\left(\frac{1+x^2}{1+x_0^2}\right) - 2\operatorname{atan}(x) + 2\operatorname{atan}(x_0)$$

and

$$\psi_h = 2\ln\left(\frac{1+y}{1+y_0}\right)$$

with

$$x = [1 - \beta_m \zeta]^{1/4}$$
$$x_0 = \left[1 - \beta_m \zeta \frac{z_0}{z}\right]^{1/4}$$
$$y = [1 - \beta_h \zeta]^{1/2}$$
$$y_0 = [q - \beta_h \zeta \frac{z_0}{z}]^{1/2}.$$

Combining these expressions with expressions for the surface drag law gives

$$\tau_i = -k^2 \left[\ln\left(\frac{z}{z_0}\right) - \psi_m \right]^{-2} |\mathbf{u}_{\text{atm}}| u_{\text{atm},i}. \tag{12}$$

Similarly, the surface aerodynamic resistance r_a is given by

$$r_a = \frac{\text{Pr}_0 \left[\ln\left(\frac{z}{z_0}\right) - \psi_m \right] \left[\ln\left(\frac{z}{z_0}\right) - \psi_h \right]}{k^2 |\mathbf{u}_{\text{atm}}|}. \tag{13}$$

The algorithm for computing surface fluxes is as follows

1. Calculate Ri_b using Equation 8

2. Calculate ζ using Equation 10 or 11

3. Calculate τ_i using Equation 12 and r_a using Equation 13

4. Calculate r_s from Equation 5

5. Calculate H and S from Equations 3 and 4

1.1   Results

Surface exchange and drag coefficients increase as the boundary layer becomes increasingly unstable (Figure 1). For stable boundary layers, exchange coefficients are set to zero above a critical bulk Richardson number of 0.21. Shutting turbulence off completely can have a dramatic impact on simulations run with a diurnal cycle over low heat capacity surfaces: these simulations frequently form nighttime radiation fog In the total absence of a nighttime atmosphere-to-surface moisture flux, but even a very small moisture flux (on the order of $1 \ \text{W/m}^2$) can be enough to prevent fog from forming. If fog is undesirable, a common way to avoid this issue is to set a lower bound on exchange coefficients or an upper bound on ζ.
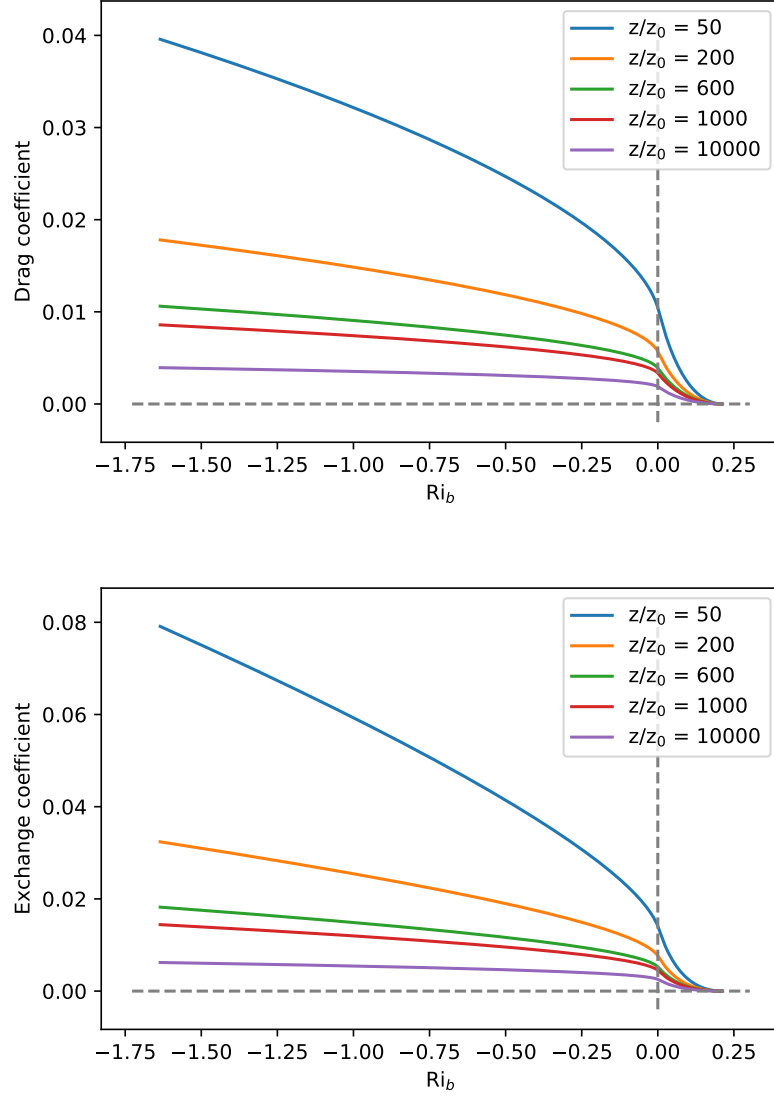
Figure 1: Variation of drag and exchange coefficients with bulk Richardson number for different surface layer depths.