Simple Land Surface Scheme

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This land surface scheme solves equations for surface energy balance:

$$C_s \frac{\mathrm{d}T_s}{\mathrm{d}t} = Q_S - Q_L - H - L \tag{1}$$

and the surface moisture balance

$$\frac{\mathrm{d}W}{\mathrm{d}t} = P - E.$$

Expressing soil moisture W in terms of a volumetric soil water content $\phi = W/D$, where D is the soil depth, gives

$$\frac{\mathrm{d}\phi}{\mathrm{d}t} = \frac{P}{D} - \frac{E}{D}.\tag{2}$$

Net longwave and shortwave fluxes must be computed externally and specified. The surface sensible heat flux H and latent heat flux L are parameterized (following ?) as

$$H = \frac{\rho c_p}{r_o} (\theta_s - \theta_{\text{atm}}) \tag{3}$$

and

$$L = \frac{\rho L_v}{r_a + r_s} [q^*(\theta_s) - q_{\text{atm}}]. \tag{4}$$

The surface resistance r_s is modeled (following ?) as a soil moisture dependent ramp

$$r_s = r_s(\phi_{fc}) \frac{\phi_{fc} - \phi_{pwp}}{\phi - \phi_{pwp}}.$$
 (5)

The aerodynamic resistance r_a is computed from the drag coefficient C_k and near-surface wind speed $|\mathbf{u}_{\text{atm}}|$ as

$$r_a = \frac{1}{C_k |\mathbf{u}_{\text{atm}}|}. (6)$$

Finally, vertical momentum fluxes (i.e. surface stresses) are parameterized by a drag law

$$\tau_i = -C_d |\mathbf{u}_{\text{atm}}| u_i. \tag{7}$$

1 Byun (1990) Monin-Obukhov scheme

One option for modelling drag coefficients follows?. The goal of the model is to relate surface fluxes of heat, momentum, and moisture to winds, temperature, and moisture at the surface and the lowest model level; i.e. to calculate

$$C_d = \frac{u_*^2}{|\mathbf{u}_{\text{atm}}|^2}$$

$$C_H = \frac{\overline{w'\theta'}}{|\mathbf{u}_{\text{atm}}|(\theta_s - \theta_{\text{atm}})}$$

and

$$C_L = \frac{\overline{w'q'}}{|\mathbf{u}_{\text{atm}}|[q^*(\theta_s) - q_{\text{atm}}]}.$$

As a simplification, we will assume $C_L = C_H = C_k$ and calculate scalar drag coefficients using potential temperature fields. The first step in the formulation is to define a temperature scale based on the surface heat flux $\overline{w'\theta'}$ and the friction velocity u_* :

$$\theta_* = \frac{\overline{w'\theta'}}{u_*}.$$

The Monin-Obukhov similarity hypothesis assumes that momentum and potential temperature profiles near the surface are described by nondimensional profile functions

$$\frac{kz}{u_*}\frac{\partial u}{\partial z} = \gamma_m(\zeta)$$

and

$$\frac{kz}{\theta_*}\frac{\partial\theta}{\partial z} = \gamma_h(\zeta),$$

where k = 0.4 is the von Karman constant. Common observation-based forms for the profile functions (?) are

$$\gamma_m = \begin{cases} 1 + \alpha_m \zeta & \zeta \ge 0\\ (1 - \beta_m \zeta)^{-1/4} & \zeta < 0 \end{cases}$$

and

$$\gamma_h = \begin{cases} \Pr_0(1 + \alpha_h \zeta) & \zeta \ge 0\\ \Pr_0(1 - \beta_h \zeta)^{-1/2} & \zeta < 0 \end{cases}$$

with $Pr_0=0.74$, $\alpha_m=4.7$, $\alpha_h=\alpha_m/Pr_0$, $\beta_m=15$, and $\beta_h=9$. ζ is the Monin-Obukhov stability parameter

$$\zeta = \frac{z}{L} = \frac{kzg\theta_*}{\theta_s u_*^2}.$$

This parameter is closely related to the gradient Richardson number, which expressed in terms of the profile functions is

$$Ri = \frac{g}{\theta_s} \frac{\partial \theta / \partial z}{(\partial u / \partial z)^2} = \zeta \frac{\gamma_h}{\gamma_m^2}.$$

However, because we have temperatures and velocities only at the surface and the lowest model level, we would prefer to relate ζ to the bulk Richardson number

$$Ri_b = \frac{g}{\theta_s} \frac{(\theta_{\text{atm}} - \theta_s)(z_{\text{atm}} - z_s)}{|\mathbf{u}(z)|^2}.$$
 (8)

 ζ can be expressed in terms of the bulk Richardson number using integrals of the profile functions from a surface roughness length z_0 to $z_{\rm atm}$. (Details are in ?.) Under stable conditions (ζ , ${\rm Ri}_b \geq 0$),

$$\zeta = \frac{1}{2\alpha_h(\alpha_m \operatorname{Ri}_b - 1)} \left[-(2\alpha_h \operatorname{Ri}_b - 1) - \left(1 + \frac{4(\alpha_h - \alpha_m)\operatorname{Ri}_b}{\operatorname{Pr}_0}\right)^{1/2} \right]$$
(9)

$$= \frac{\left(\frac{z}{z-z_0}\right) \ln\left(\frac{z}{z_0}\right)}{2\alpha_h(\alpha_m \operatorname{Ri}_b - 1)} \left[-(2\alpha_h \operatorname{Ri}_b - 1) - \left(1 + \frac{4(\alpha_h - \alpha_m)\operatorname{Ri}_b}{\operatorname{Pr}_0}\right)^{1/2} \right]. \tag{10}$$

Under unstable conditions (ζ , $\mathrm{Ri}_b < 0$), expressing ζ in terms of the bulk Richardson number is difficult, but a very good approximate solution can be found by multiplying an expression for ζ in terms of the gradient Richardson number by $\frac{z}{z-z_0} \ln \left(\frac{z}{z_0}\right)$ and replacing Ri with Ri_b . This gives

$$\zeta = \begin{cases} \frac{z}{z - z_0} \ln\left(\frac{z}{z_0}\right) \left[-2\sqrt{Q_b} \cos\left(\frac{\theta_b}{3}\right) + \frac{1}{3\beta_m} \right] & \operatorname{Ri}_b \le -0.2097\\ \frac{z}{z - z_0} \ln\left(\frac{z}{z_0}\right) \left[-\left(T_b + \frac{Q_b}{T_b}\right) + \frac{1}{3\gamma_m} \right] & -0.2097 < \operatorname{Ri}_b \le 0. \end{cases}$$
(11)

with

$$s_b = \frac{\text{Ri}_b}{\text{Pr}_0}$$

$$Q_b = \frac{1}{9} \left[\frac{1}{\beta_m^2} + 3 \frac{\beta_h}{\beta_m} s_b^2 \right]$$

$$P_b = \frac{1}{54} \left[-\frac{2}{\beta_m^3} + \frac{9}{\beta_m} \left(-\frac{\beta_h}{\beta_m} + 3 \right) s_b^2 \right]$$

$$\theta_b = \text{acos} \left[\frac{P_b}{Q_b^{3/2}} \right]$$

$$T_b = \left[\left(P_b^2 - Q_b^3 \right)^{1/2} + |P_b| \right]^{1/3}.$$

Relationships between u_* , θ_* , and resolved temperature and velocity fields can be derived from integrals of the profile functions:

$$u_* = k |\mathbf{u}_{\text{atm}}| \left[\ln \left(\frac{z}{z_0} \right) - \psi_m \right]^{-1},$$

and

$$\theta_* = \frac{k(\theta_s - \theta_{\text{atm}})}{\text{Pr}_0} \left[\ln \left(\frac{z}{z_0} \right) - \psi_h \right]^{-1}.$$

For stable conditions,

$$\psi_m = -\alpha_m \zeta \left(1 - \frac{z_0}{z} \right)$$

and

$$\psi_h = -\alpha_h \zeta \Big(1 - \frac{z_0}{z} \Big).$$

For unstable conditions,

$$\psi_m = 2 \ln \left(\frac{1+x}{1+x_0} \right) + \ln \left(\frac{1+x^2}{1+x_0^2} \right) - 2 \operatorname{atan}(x) + 2 \operatorname{atan}(x_0)$$

and

$$\psi_h = 2\ln\left(\frac{1+y}{1+y_0}\right)$$

with

$$x = [1 - \beta_m \zeta]^{1/4}$$

$$x_0 = \left[1 - \beta_m \zeta \frac{z_0}{z}\right]^{1/4}$$

$$y = [1 - \beta_h \zeta]^{1/2}$$

$$y_0 = [q - \beta_h \zeta \frac{z_0}{z}]^{1/2}.$$

Combining these expressions with expressions for the surface drag law gives

$$\tau_i = -k^2 \left[\ln \left(\frac{z}{z_0} \right) - \psi_m \right]^{-2} |\mathbf{u}_{\text{atm}}| u_{\text{atm},i}. \tag{12}$$

Similarly, the surface aerodynamic resistance r_a is given by

$$r_a = \frac{\Pr_0\left[\ln\left(\frac{z}{z_0}\right) - \psi_m\right] \left[\ln\left(\frac{z}{z_0}\right) - \psi_h\right]}{k^2 |\mathbf{u}_{\text{atm}}|}.$$
 (13)

The algorithm for computing surface fluxes is as follows

- 1. Calculate Ri_b using Equation 8
- 2. Calculate ζ using Equation 10 or 11

- 3. Calculate τ_i using Equation 12 and r_a using Equation 13
- 4. Calculate r_s from Equation 5
- 5. Calculate H and S from Equations 3 and 4

1.1 Results

Surface exchange and drag coefficients increase as the boundary layer becomes increasingly unstable (Figure 1). For stable boundary layers, exchange coefficients are set to zero above a critical bulk Richardson number of 0.21. Shutting turbulence off completely can have a dramatic impact on simulations run with a diurnal cycle over low heat capacity surfaces: these simulations frequently form nighttime radiation fog In the total absence of a nighttime atmosphere-to-surface moisture flux, but even a very small moisture flux (on the order of 1 $\rm W/m^2$) can be enough to prevent fog from forming.

2 UCLA LES Monin-Obukhov scheme

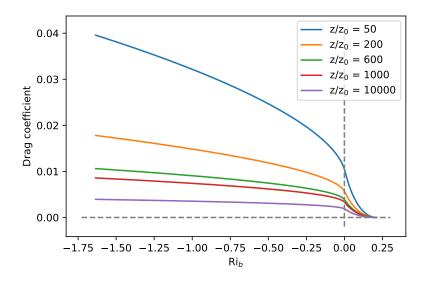
This land surface scheme also includes an option to model drag and exchange coefficients using profile functions from the UCLA LES (?):

$$\gamma_m = \begin{cases} 1 + \alpha_m \zeta & \zeta \ge 0\\ (1 - \beta_m \zeta)^{-1/2} & \zeta < 0 \end{cases}$$

and

$$\gamma_h = \begin{cases} \Pr_0(1 + \alpha_h \zeta) & \zeta \ge 0\\ \Pr_0(1 - \beta_h \zeta)^{-1/2} & \zeta < 0 \end{cases}$$

with $Pr_0 = 0.74$, $\alpha_m = 4.8$, $\alpha_h = 7.8$, $\beta_m = 19.3$, and $\beta_h = 12.0$.



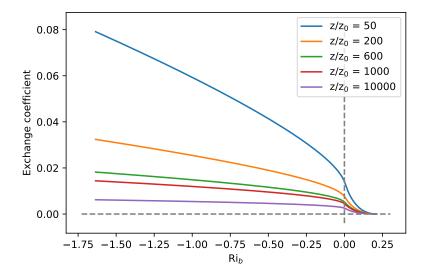


Figure 1: Variation of drag and exchange coefficients with bulk Richardson number for different surface layer depths.