Simple Land Surface Scheme

Tristan Abbott

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This land surface scheme solves equations for surface energy balance:

$$C_s \frac{\mathrm{d}T_s}{\mathrm{d}t} = Q_S - Q_L - H - L \tag{1}$$

and the surface moisture balance

$$\frac{\mathrm{d}W}{\mathrm{d}t} = P - E.$$

Expressing soil moisture W in terms of a volumetric soil water content $\phi = W/D$, where D is the soil depth, gives

$$\frac{\mathrm{d}\phi}{\mathrm{d}t} = \frac{P}{D} - \frac{E}{D}. \tag{2}$$

Net longwave and shortwave fluxes must be computed externally and specified. The surface sensible heat flux H and latent heat flux L are parameterized (following **?**) as

$$H = \frac{\rho c_p}{r_a}(\theta_s - \theta_{\mathrm{atm}}) \tag{3}$$

and

$$L = \frac{\rho L_v}{r_a + r_s}[q^*(\theta_s) - q_{\mathrm{atm}}]. \tag{4}$$

The surface resistance r_s is modeled (following **?**) as a soil moisture dependent ramp

$$r_s = r_s(\phi_{fc}) \frac{\phi_{fc} - \phi_{pwp}}{\phi - \phi_{pwp}}. \tag{5}$$

The aerodynamic resistance r_a is computed from the drag coefficient C_k and near-surface wind speed $|\mathbf{u}_{\mathrm{atm}}|$ as

$$r_a = \frac{1}{C_k|\mathbf{u}_{\mathrm{atm}}|}. \tag{6}$$

Finally, vertical momentum fluxes (i.e. surface stresses) are parameterized by a drag law

$$\tau_i = -C_d|\mathbf{u}_{\mathrm{atm}}|u_i. \tag{7}$$

The drag coefficients are modeled using Monin-Obukhov similarity theory following **?**. The goal of the model is to relate surface fluxes of heat, momentum, and moisture to winds, temperature, and moisture at the surface and the lowest model level; i.e. to calculate

$$C_d = \frac{u_*^2}{|\mathbf{u}_{\rm atm}|^2}$$

$$C_H = \frac{\overline{w'\theta'}}{|\mathbf{u}_{\rm atm}|(\theta_s - \theta_{\rm atm})}$$

and

$$C_L = \frac{\overline{w'q'}}{|\mathbf{u}_{\rm atm}|[q^*(\theta_s) - q_{\rm atm}]}.$$

As a simplification, we will assume $C_L = C_H = C_k$ and calculate scalar drag coefficients using potential temperature fields. The first step in the formulation is to define a temperature scale based on the surface heat flux $\overline{w'\theta'}$ and the friction velocity u_*:

$$\theta_* = \frac{\overline{w'\theta'}}{u_*}.$$

The Monin-Obukhov similarity hypothesis assumes that momentum and potential temperature profiles near the surface are described by nondimensional profile functions

$$\frac{kz}{u_*}\frac{\partial u}{\partial z} = \gamma_m(\zeta)$$

and

$$\frac{kz}{\theta_*}\frac{\partial \theta}{\partial z} = \gamma_h(\zeta),$$

where $k = 0.4$ is the von Karman constant. Common observation-based forms for the profile functions (**?**) are

$$\gamma_m = \begin{cases} 1 + \alpha_m \zeta & \zeta \geq 0 \\ (1 - \beta_m \zeta)^{-1/4} & \zeta < 0 \end{cases}$$

and

$$\gamma_h = \begin{cases} \mathrm{Pr}_0(1 + \alpha_h \zeta) & \zeta \geq 0 \\ \mathrm{Pr}_0(1 - \beta_h \zeta)^{-1/2} & \zeta < 0 \end{cases}$$

with $\mathrm{Pr}_0 = 0.74$, $\alpha_m = 4.7$, $\alpha_h = \alpha_m/\mathrm{Pr}_0$, $\beta_m = 15$, and $\beta_h = 9$. ζ is the Monin-Obukhov stability parameter

$$\zeta = \frac{z}{L} = \frac{kzg\theta_*}{\theta_s u_*^2}.$$

This parameter is closely related to the gradient Richardson number, which expressed in terms of the profile functions is

$$\mathrm{Ri} = \frac{g}{\theta_s}\frac{\partial\theta/\partial z}{(\partial u/\partial z)^2} = \zeta\frac{\gamma_h}{\gamma_m^2}.$$

However, because we have temperatures and velocities only at the surface and the lowest model level, we would prefer to relate ζ to the bulk Richardson number

$$Ri_b = \frac{g}{\theta_s}\frac{(\theta_{\mathrm{atm}} - \theta_s)(z_{\mathrm{atm}} - z_s)}{|\mathbf{u}(z)|^2}. \tag{8}$$

ζ can be expressed in terms of the bulk Richardson number using integrals of the profile functions from a surface roughness length z_0 to z_{atm}. (Details are in ?.) Under stable conditions ($\zeta, \mathrm{Ri}_b \geq 0$),

$$\begin{aligned}
\zeta &= \frac{1}{2\alpha_h(\alpha_m \mathrm{Ri}_b - 1)}\left[-(2\alpha_h \mathrm{Ri}_b - 1) - \left(1 + \frac{4(\alpha_h - \alpha_m)\mathrm{Ri}_b}{\mathrm{Pr}_0}\right)^{1/2}\right] && (9)\\
&= \frac{\left(\frac{z}{z-z_0}\right)\ln\left(\frac{z}{z_0}\right)}{2\alpha_h(\alpha_m \mathrm{Ri}_b - 1)}\left[-(2\alpha_h \mathrm{Ri}_b - 1) - \left(1 + \frac{4(\alpha_h - \alpha_m)\mathrm{Ri}_b}{\mathrm{Pr}_0}\right)^{1/2}\right]. && (10)
\end{aligned}$$

Under unstable conditions ($\zeta, \mathrm{Ri}_b < 0$), expressing ζ in terms of the bulk Richardson number is difficult, but a very good approximate solution can be found by multiplying an expression for ζ in terms of the gradient Richardson number by $\frac{z}{z-z_0}\ln\left(\frac{z}{z_0}\right)$ and replacing Ri with Ri_b. This gives

$$\zeta = \begin{cases} \frac{z}{z-z_0}\ln\left(\frac{z}{z_0}\right)\left[-2\sqrt{Q_b}\cos\left(\frac{\theta_b}{3}\right) + \frac{1}{3\beta_m}\right] & \mathrm{Ri}_b \leq -0.2097 \\ \frac{z}{z-z_0}\ln\left(\frac{z}{z_0}\right)\left[-\left(T_b + \frac{Q_b}{T_b}\right) + \frac{1}{3\gamma_m}\right] & -0.2097 < \mathrm{Ri}_b \leq 0. \end{cases} \tag{11}$$

with

$$\begin{aligned}
s_b &= \frac{\mathrm{Ri}_b}{\mathrm{Pr}_0}\\
Q_b &= \frac{1}{9}\left[\frac{1}{\beta_m^2} + 3\frac{\beta_h}{\beta_m}s_b^2\right]\\
P_b &= \frac{1}{54}\left[-\frac{2}{\beta_m^3} + \frac{9}{\beta_m}\left(-\frac{\beta_h}{\beta_m} + 3\right)s_b^2\right]\\
\theta_b &= \mathrm{acos}\left[\frac{P_b}{Q_b^{3/2}}\right]\\
T_b &= \left[\left(P_b^2 - Q_b^3\right)^{1/2} + |P_b|\right]^{1/3}.
\end{aligned}$$

Relationships between u_*, θ_*, and resolved temperature and velocity fields can be derived from integrals of the profile functions:

$$u_* = k|\mathbf{u}_{\mathrm{atm}}|\left[\ln\left(\frac{z}{z_0}\right) - \psi_m\right]^{-1},$$

and

$$\theta_* = \frac{k(\theta_s - \theta_{\mathrm{atm}})}{\mathrm{Pr}_0}\left[\ln\left(\frac{z}{z_0}\right) - \psi_h\right]^{-1}.$$

For stable conditions,

$$\psi_m = -\alpha_m \zeta \Big(1 - \frac{z_0}{z}\Big)$$

and

$$\psi_h = -\alpha_h \zeta \Big(1 - \frac{z_0}{z}\Big).$$

For unstable conditions,

$$\psi_m = 2\ln\left(\frac{1+x}{1+x_0}\right) + \ln\left(\frac{1+x^2}{1+x_0^2}\right) - 2\,\mathrm{atan}\,(x) + 2\,\mathrm{atan}\,(x_0)$$

and

$$\psi_h = 2\ln\left(\frac{1+y}{1+y_0}\right)$$

with

$$\begin{aligned} x &= [1 - \beta_m \zeta]^{1/4} \\ x_0 &= \Big[1 - \beta_m \zeta \frac{z_0}{z}\Big]^{1/4} \\ y &= [1 - \beta_h \zeta]^{1/2} \\ y_0 &= [q - \beta_h \zeta \frac{z_0}{z}]^{1/2}. \end{aligned}$$

Combining these expressions with expressions for the surface drag law gives

$$\tau_i = -k^2 \left[\ln\left(\frac{z}{z_0}\right) - \psi_m\right]^{-2} |\mathbf{u}_{\mathrm{atm}}| u_{\mathrm{atm},i}. \tag{12}$$

Similarly, the surface aerodynamic resistance r_a is given by

$$r_a = \frac{\mathrm{Pr}_0 \left[\ln\left(\frac{z}{z_0}\right) - \psi_m\right]\left[\ln\left(\frac{z}{z_0}\right) - \psi_h\right]}{k^2 |\mathbf{u}_{\mathrm{atm}}|}. \tag{13}$$

The algorithm for computing surface fluxes is as follows

1. Calculate Ri_b using Equation 8
2. Calculate ζ using Equation 10 or 11
3. Calculate τ_i using Equation 12 and r_a using Equation 13
4. Calculate r_s from Equation 5
5. Calculate H and S from Equations 3 and 4