Topic 2: ALGEBRAIC EQUATION

LESSON OBJECTIVES:

1. Solving Algebraic equation

2. Construct simple algebraic expressions and equations.

CONTENT: At the end of the lesson, learners should be able to

1. Solve algebraic equation following BODMAS.
2. Rearrange equations and formulae.
3. Solve algebraic fractions.
4. Substitute values into in equations and formulae.
5. Change verbal description to algebraic expression.

TEACHING SCRIPT

LESSON 2

2.1 Solving Equations

An equation is a mathematical expression written such that what is on the left hand side is equal to the left hand side. In other to solve equations, there is an order. The order is what we call BODMAS

B - Brackets

O - Of

D - Division

M - Multiplication

A - Addition

S – Subtraction

Worked example:

a) $7 + 63 \div 9$

To solve this question, we need to follow BODMAS. We have 2 operations in the question: + and ÷. According to BODMAS, division (÷) comes before addition (+).

So, we need to divide 63 by 9 first. So,

$7 + 63 \div 9 = 7 + 7$

Now we can add.

$7 + 63 \div 9 = 7 + 7 = 14$

b) Solve $18 \div 6 + 5 \times (6 - 4)$

$18 \div 6 + 5 \times (6 - 4)$

$= 18 \div 6 + 5 \text{ x } 2$	(bracket done)
$=3 + 5 \text{ x } 2$	(Division done)
$=3 + 5 \text{ x } 2$	(Multiplication done)
$= 3 + 10$	(Addition done)
$=13$	

3. Rearranging to solve equations.

Note: Whenever you are asked to solve an equation, you should work on determining the exact value of the variable in the question. In other to do this, you will need to undo the existing operations used on the variable.

Worked example 5:

a) $Y + 6 = 9$

Y is joined to 6 by the addition operation. To undo this operation, we need to use the inverse operation. Which is subtraction. So, we have to subtract the number 6 from the variable

$Y + 6 – 6 = 9 – 6$ (what ever we do to the LHS must be done to the RHS)

Now we are left with only, since 6 – 6 is zero.

$Y = 3$ (Final answer)

b) $4x + 2 = 10$

$4x + 2 - 2 = 10 - 2$

$4x = 8$

Remember we must determine the exact value of our variable when solving equations. So, we need to make the variable (x) stand-alone by itself on one side of the equation. We do this by checking what operation binds x to 4 and undo the operation.

The operation between 4 and x is multiplication, so we will undo it by using the inverse operation which is division (÷). Since we want to undo the multiplication of 4 and the variable ‘x’, we will have to divide by 4.

$4x \div 4 = 8 \div 4$

$x = 2$ (whatever we do to the LHS must be done to the RHS)

c) $$\frac{2y}{3} + 5 = 13$$

$$= \frac{2y}{3} + 5 - 5 = 13 - 5$$

$$= \frac{2y}{3} = 8$$

$$\frac{2y}{3} \times 3 = 8 \times 3$$

$$2y = 24$$

$$\frac{2y}{2} = \frac{24}{2}$$

$$y = 12$$

2.3 Changing the Subject of formula.

A formula is an equation where letters represent quantities. Changing the subject of formula involve rearranging the arithmetic operations. The subject of the formula is usually written on the left-hand side of the formula.

Worked examples: Make y the subject of formula.

1. y + w = c
2. y + 7t = c + r

Solution

1. y + w = c

   y = c – w

2. y + 7t = c + r

   $y = \frac{c+r}{7t}$

2.4 Solving Algebraic Fractions

When solving algebraic fractions, the first thing to do is to find the common denominator for the fractions.

Worked examples.

$$\frac{2x-3}{2} - \frac{3x+1}{4} = 1$$

Find the lowest common multiple for both fractions. This can be done by finding the equivalent fraction that brings both fractions to have the same denominator. For this example, let us use 8 as the common denominator.

$$\frac{(2x-3)\times 4}{2\ \times 4} - \frac{(3x+1)\times 2}{4\ \times 2} = 1$$

$$= \frac{8x-12}{8} - \frac{6x+2}{8} = 1$$

$$= \frac{8x-6x-12-2}{8} = 1$$

$$= \frac{2x - 14}{8} = 1$$

In other to solve for x, the operation around x needs to be undone.

$$\frac{2x - 14}{8} \times 8 = 1 \times 8$$

$$2x - 14 + 14 = 8 + 14$$

$$2x = 22$$

$$\frac{2x}{2} = \frac{22}{2}$$

$$x = 11$$

2.5 Substitution of values in algebraic equations

Substitution is when you put values/numbers where the letters are.

If x = 3, y = 7, z = - 4

Calculate: a.) $3x + y$ b.) $x^2 + y^2$

a. $3x + y = 3(3) + 7$

$= 9 + 7 = 16$

b. $x^2 + y^2 = 3^2 + 7^2$

$= 9 + 49 = 58$

2.6. Algebraic Word Problems

Under this lesson, we will attempt to change description sentences to algebraic expressions and equations.

Key notes

List of words that mean addition: Sum, Altogether, combine, increase, more, plus, total.

List of words that mean subtraction: Decrease, difference, reduce, minus, less, fewer.

List of words that mean multiplication: Product, times, double (2x), triple (3x), four (4x)

List of words that mean Division: Quotient, divided, share, split

Common phrases used,

The sum of m and n means $m + n$.

m is added to 3 means $m + 3$

x more than y means $y + x$

x less than y means $y – x$

Reduced or decreased by x mean $– x$

the difference of m and n mean $m – n$

Subtract m from n mean $n – m$

The product of m and n mean m X n

The quotient of a and b mean $a \div b$

Result mean =

A certain number can be represented by any letter; a, x, y... e. t. c.

Examples

1. Five times a number increased by seven is equal to forty-seven. What is the number?

   Step 1: Represent the unknown number by x.

   Five times a number will mean 5 multiplied by $x = 5x$

   Step 2: Increased by 7 will mean $5x + 7$

   Step 3: Five times a number increased by seven is equal to forty-seven.

   $5x + 7 = 47$

Step 4: It is time to solve the equation.

$5x + 7 - 7 = 47 - 7$

$$\frac{5x}{5} = \frac{40}{5}$$

$x = 8$

2. Liam bought a bag of 88 Sweets. She ate *a certain number* of the candies. Write the expression that shows how many sweets Liam has left.

   Let the certain number be equal to c,

   Then the expression is $88 - c$

EVALUATION

Exercise 1: Solve the following equations.

1. $x - 5 = 8$
2. $y + 3 = 7$
3. $2x + 5 = 15$
4. $6 - t = 10$
5. $5x = 30$
6. $\frac{y}{2} = -5$
7. $\frac{x}{3} + 4 = 5$
8. $13t - 7 = 33$
9. $m - 2 = 17$
10. $5a - 22 = a - 2$
11. $\frac{x-6}{2} = -1$
12. $\frac{x}{27} + 2 = -5$

Exercise 2: Solve for x in the following equation.

a) $3(x + 1) - 2x = 0$

b) $4 + 2(1 + x) = 12$

c) $3(x - 2) = 2(x - 4)$

d) $2(2x - 4) = 16$

Exercise 3: Make 'a' the subject of formula in the following equations.

1. $ax - t = 8$
2. $$\frac{a}{x} + b = c$$
3. $$v = u + at$$
4. $S(a + b) = t$
5. $2b + 2a = p$

Exercise 4: Solve the following equations.

a) $\frac{x-1}{3} - \frac{3x-1}{5}$

b) $$\frac{x-1}{3} + \frac{x-4}{5} = \frac{1}{4}$$

c) $$\frac{x}{3} + \frac{2x-1}{4} = 1$$

Exercise 5: Solve by substitution.

If a = 2, b = 4, calculate the following;

a. $a^2 + b^2$

b. $$\frac{a+b}{5}$$

c. $2(b-a)$

d. $(3a + 1)^2$

e. $4a + 3b$

Exercise 6: Solve the following.

a. When 9 is subtracted from a number and then divided by 2, the answer is 4. What is the number?

b. Philus, Sipho and Sam have 20 sweets to share. Philus took 8 sweets and Sipho took three times as many as Sam. How many sweets did Sam receive?

c. If the sum of three consecutive numbers is 72, what is the largest number? The largest number is ------------------

d. Rachael is 2 times younger than his sister and his father is 25 years older than her. If the total of their ages is 53 years, what is Rachael's age and his father's age?

e. Lilian is 6 years older than John. If Joh is 2 years, how old is Lilian?