

Topic 1: ALGEBRAIC MANIPULATION

LESSON OBJECTIVES:

1. Simplifying problems involving algebraic expressions
2. Solving problems involving algebraic expressions

CONTENT: At the end of the lesson, learners should be able to

1. Understand algebraic terms.
2. Identify like and unlike terms.
3. Simplify Algebraic expressions using like terms.
4. Adding and subtracting with brackets
5. Multiplying and dividing algebraic expressions

TEACHING SCRIPT

1.1 Algebraic expression

Algebraic expression consists of variables, coefficients, constants, operations (+, -, \div , \times).

An algebraic expression can be made up of one or more term. A term is a part of the expression that is separated by – or +.

Worked example:

a) $5x + 2$

This algebraic expression has two terms (5x and 2)

The **variable** is **x** (This is the letter that represents an unknown value in the expression).

Where **5** is the **coefficient** of X. Coefficient is a number or symbol that is multiplied with a variable, this also includes the sign in front of it.

The **constant** in this expression is **2** (A stand-alone number that makes a term on its own)

Like terms: These are terms with the same variable. They can be added or subtracted.

Worked example:

b) $2x + 5p + 3x + 7p$

$2x$ and $3x$ have the same variable, so they are like terms. $5p$ and $7p$ are also like terms.

So, we can solve;

$$\begin{aligned}2x + 5p + 3x + 7p &= 2x + 3x + 5p + 7p \\ &= 5x + 12p\end{aligned}$$

Note that.

An expression with **one term** is called **monomial**.

An expression with **two term** is called **binomial**.

An expression with **three term** is called **trinomial**.

An expression with **more than two terms** is called **polynomial**.

1.2 Simplifying Expressions

Rule:

- Only add and subtract like terms.
- Do not change the name of variable, only the coefficient can change.
- The sign in front of the term is for the term.

Now imagine we have some set of books and glues. We will call the books 'b' and the glues 'g'.

Let us solve this expression: $5b + 7g - 2b + 4b$

First, we need to check the like terms. We will put the terms with 'b' together and the term with 'g' together. As we move each term, we pick them with the sign in front of them.

So, $5b + 7g - 2b + 4b$

$$= 5b - 2b + 7g + 4g$$

$$= 3b + 11g$$

Do you think, we can add $3b + 11g$?

No? That's right.

Remember it will not be possible to add 3 books with 11 glues?

Yes, you are correct. After we are done with how many books and glues we have. Our answer will be reported as 3 books and 11 glues. Because they cannot be added together.

Worked example:

- a. $6x + 4x = 10x$ (In this example, we have a new coefficient but the variable 'x' is the same. Remember the 2nd rule)

b. $5ac + 2b - 2ac + b$

$$= 5ac - 2ac + 2b + b \text{ (collecting like terms together)}$$

$$= 3ac + 3b$$

1.3 Adding and subtracting algebraic expressions with brackets.

Note: Use these rules when multiplying terms with different signs.

$$- \times - = +$$

$$+ \times - = -$$

$$+ \times + = +$$

$$- \times + = -$$

This means that; $-2a \times 3$ will be equal to $-6a$ because $-2a$ multiplies $+3$, our result will be $-6a$.

Worked Example

- a. Add $(2a^2 - 4a + 3)$ and $(a^2 + 4a - 4)$


$$(2a^2 - 4a + 3) + (a^2 + 4a - 4)$$

$$= 2a^2 - 4a + 3 + a^2 + 4a - 4 \quad \text{(Remove the bracket)}$$

$$= 2a^2 + a^2 - 4a + 4a + 3 - 4 \quad (\text{Collect like terms})$$

$$= 3a^2 - 1 \quad (\text{Simplify by adding like terms})$$

b. $(5b^2 + 3b - 6) - (4b^2 - 2b + 3)$ (Remove the bracket)



Note that the subtraction multiplies the signs in the second bracket.

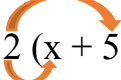
$$= 5b^2 + 3b - 6 - 4b^2 + 2b - 3$$

$$= 5b^2 - 4b^2 + 3b + 2b - 6 - 3 \quad (\text{Collect like terms})$$


$$= b^2 + 5b - 9 \quad (\text{Simplify by adding like terms})$$

1.4 Multiplying and dividing algebraic expressions

When there is a bracket in an algebraic expression, we remove the bracket by multiplying the term outside with each of the term inside.

$$2(x + 5) = 2x + 10$$


When there is a minus outside the brackets.

$$-2(3x^2 - x + 1) = -6x^2 + 2x - 2$$


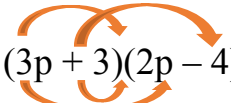
When there is multiply double bracket, use the FOIL

F = Multiply first terms together

O = multiply the outer terms together

I = Multiply the inner terms together

L = Multiply the last term together

$$(3p + 3)(2p - 4)$$


$$\begin{aligned}
 (3p + 3)(2p - 4) &= (3p)(2p) + (3p)(-4) + (3)(2p) + (3)(-4) \\
 &= 6p^2 - 12p + 6p - 12 \\
 &= 6p^2 - 6p - 12
 \end{aligned}$$

Dividing algebraic expressions

$$\begin{aligned}
 &\frac{2x^4 - 8x^3 + 4x^2}{2x^2} \\
 &= \frac{2x^4}{2x^2} + \frac{8x^3}{2x^2} + \frac{4x^2}{2x^2} \\
 &= x^2 + 4x + 2
 \end{aligned}$$

EVALUATION

Exercise 1:

1. $a + a + a$
2. $4a + 3a - 2$
3. $p + q - 4p + 2q$
4. $2a + 3b + 7b$
5. $7y^2 + 3y^2$
6. $5x^2 + 3x + 2x^2$

Exercise 2:

1. $6b + 4c - 13$
 - a. What is the coefficient b.
 - b. What is the constant in this algebraic expression?
 - c. How many terms does the expression have? It is called a

2. How many terms are in the following expressions.

a. $4x^2 + 2x + 9$

b. $3x - 75$

c. $p + \frac{p - 2q}{3}$

2.3 Write down if these expressions are monomials, binomials, or trinomials.

a.) $2x^2 + 11y$

b.) $17abc - 28bpd + 122fgh$

Exercise 3: Add the following polynomials.

a. $(13x^2 - 10) + (2x^2 - 11x + 5)$

b. $(7x^2 - 8x) + (x^2 - 5) + (2x^2 + 8x + 5)$

c. $(3x^2 + 3xy - 6y^2) - (4x^2 - 2xy + 6y^2)$

Exercise 4: Simplify the following.

a. $(2x + 1)(x - 5)$

b. $(a - b)(3a + 7b)$

c. $-3(2x + 4)$

d. $(5x - 8)^2$

e. $(6x + 1)^2$

f. $2a(a - 4)(3a + 5)$

g. $\frac{9x^4 - 6x^3 + 3x^2 + 12x}{3x}$

h. $\frac{10x^2 - 5a}{5a}$