Profit Maximization
One variable input –One Output
Input side

BLP Chapter 05

Webster Chapter 05

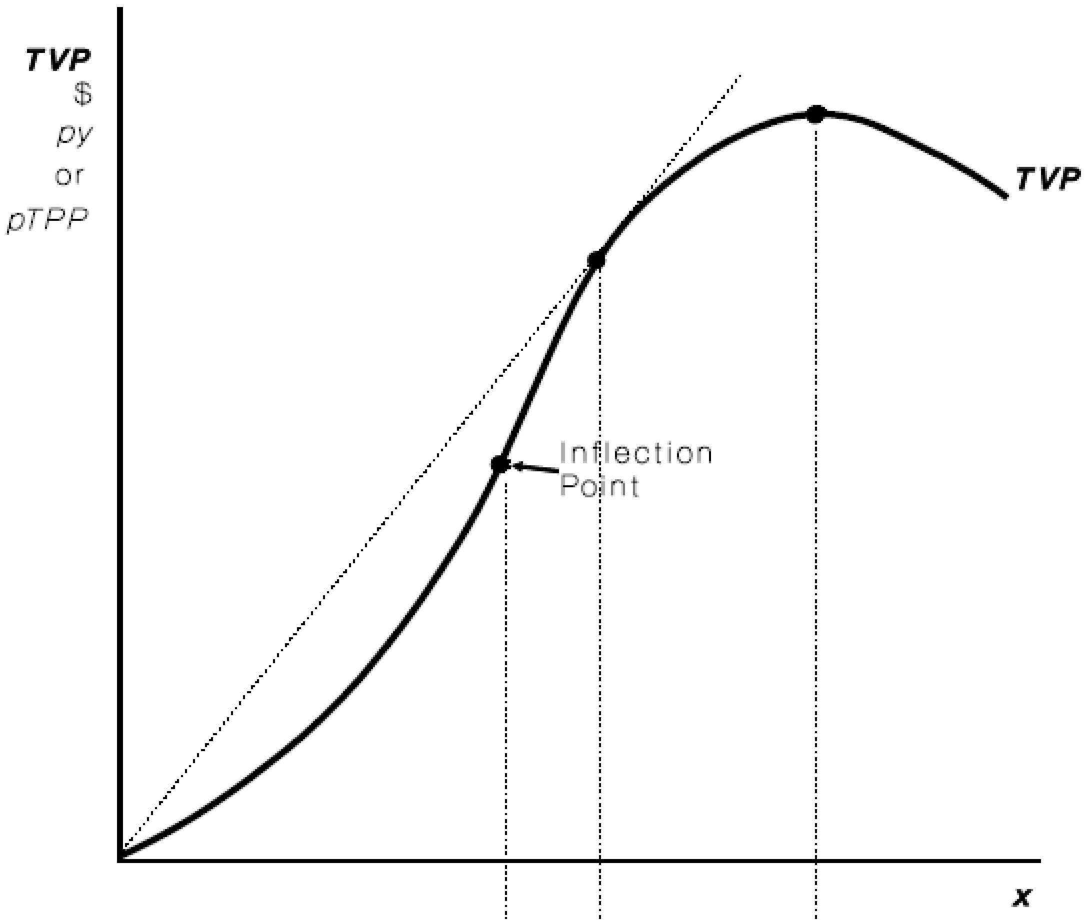
Total Value of Product

- It is a measure of output (*TP*) transformed into dollar terms by multiplying by *price of the output P.*

$$TVP = P*TP = P*Q = P*F(K, L)$$

- For example, for a farmer, it represents the revenue obtained from the sale of a single commodity, such as corn or beef cattle.
- If the output price is constant, the *TVP* function has the same shape as the *TP* function, and only the units on the vertical axis have changed ($)

TVP Vs. amount of input used



Total Factor Cost (TF_cC)

- Cost associated with the purchase of the variable input
- Total Factor Cost of labor is,

$$TF_cC_L = w*L$$

- Total Factor Cost of capital is

$$TF_cC_K = r*K$$

- w and r are the per unit costs of labor and capital (wage and rent), respectively
- TF_cC function has a constant slope

Total Fixed Cost (TFC)

- Total cost of the fixed inputs used in the production
- TFC do not vary with the amount of inputs used in short run

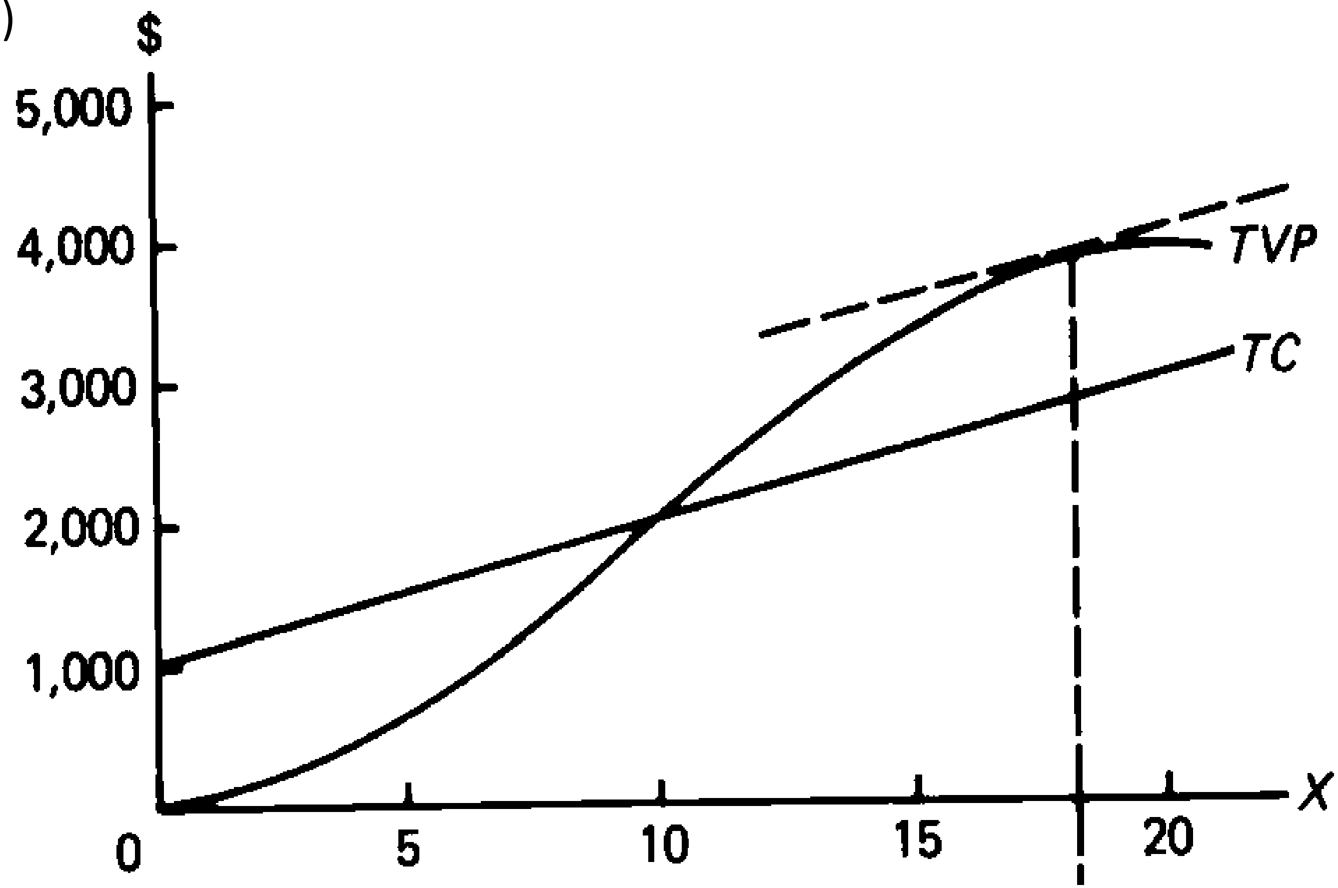
Total Cost (TC)

Total Cost is the sum of total factor cost and total fixed cost

$$TC = TF_cC + TFC$$

TVP and TC Vs. amount of input (X)

TVP ($)
TC ($)



Profit / Net Revenue / Net Value Product / Net Returns

Let's assume capital is fixed and has a total fixed cost of k. labor is the only variable input

$$\text{Profits } (\Pi) = TVP - TC$$
$$= TVP - TF_cC - TFC$$
$$= P^*Q - w^*L - k$$
$$= P^*F(L,K) - w^*L - k$$

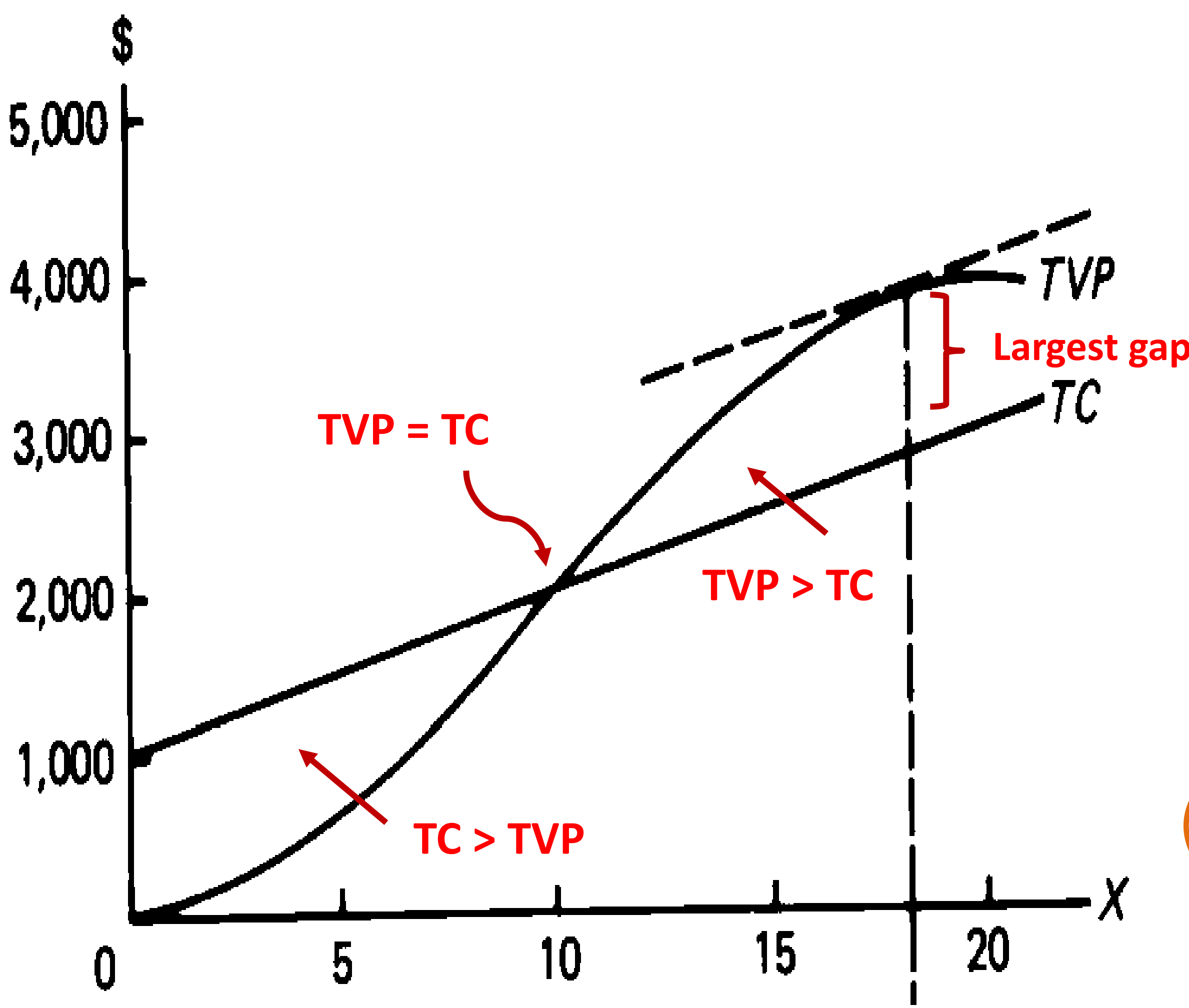
Determining the optimum point of input use (output: corn, input: fertilizer)

X (input)	Q (output)	TVP = P*Q	TC = (TF_cC+TFC)
0	0	0	1000
2	3.7	111	1200
4	13.9	417	1400
6	28.8	864	1600
8	46.9	1407	1800
10	66.7	2001	2000
12	86.4	2592	2200
14	104.5	3135	2400
16	119.5	3585	2600
18	129.6	3888	2800
20	133.3	3999	3000
22	129.1	3873	3200

Optimal amount of input - 1

- Many points of production process with positive profits
- Optimum is when the amount of profit is the largest
- The first way to find the optimum is to find it from TVP and TC graph
- Profits are maximum in a graph when TVP exceeds TC and vertical distance between the two is the maximum and TVP > TC
  - The corresponding amount of input is the optimal amount of input use

TVP and TC Vs. amount of input (X)

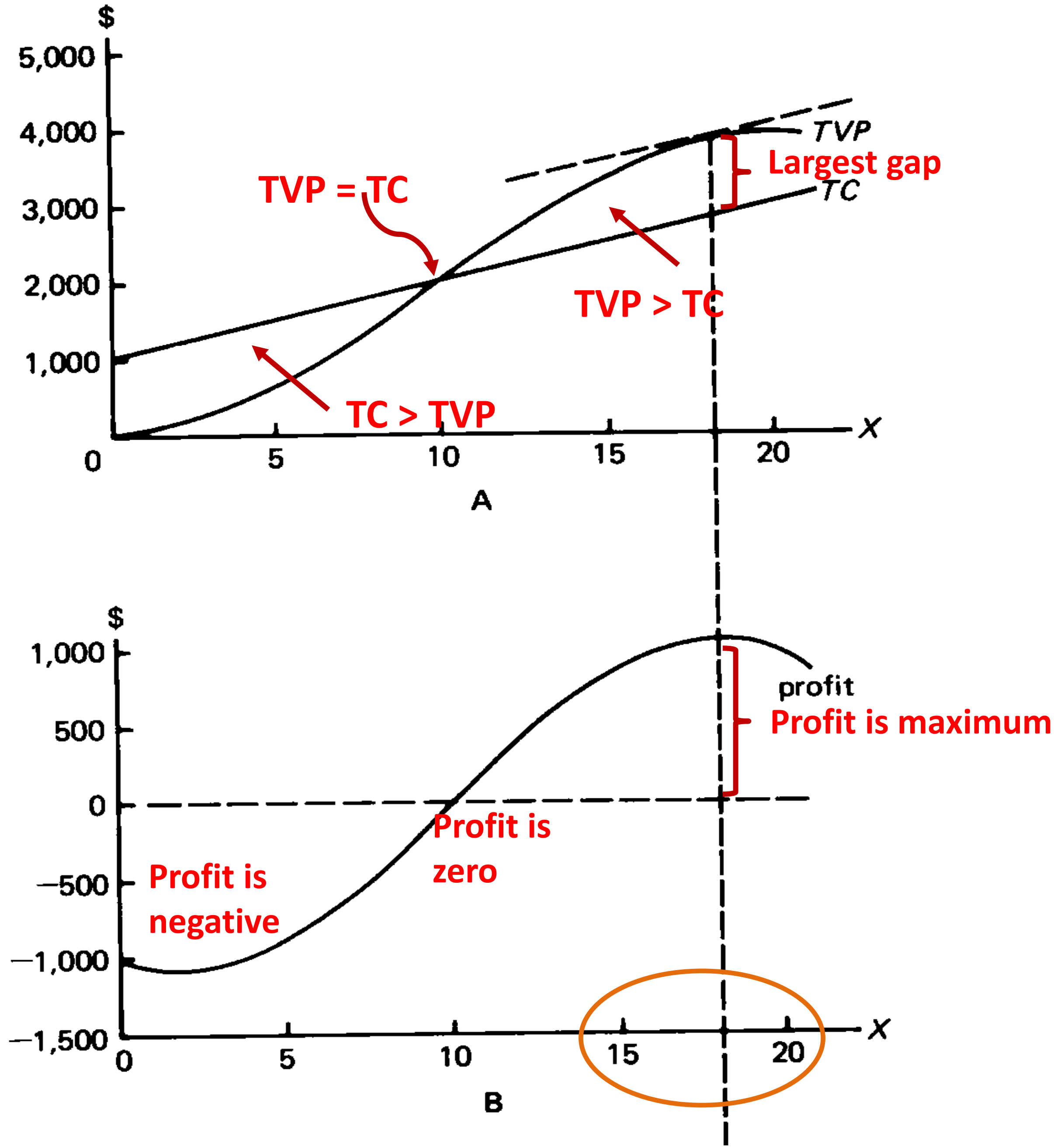


X (input)	TVP	TC
0	0	1000
2	111	1200
4	417	1400
6	864	1600
8	1407	1800
10	2001	2000
12	2592	2200
14	3135	2400
16	3585	2600
18	3888	2800
20	3999	3000
22	3873	3200

Optimal amount of input - 2

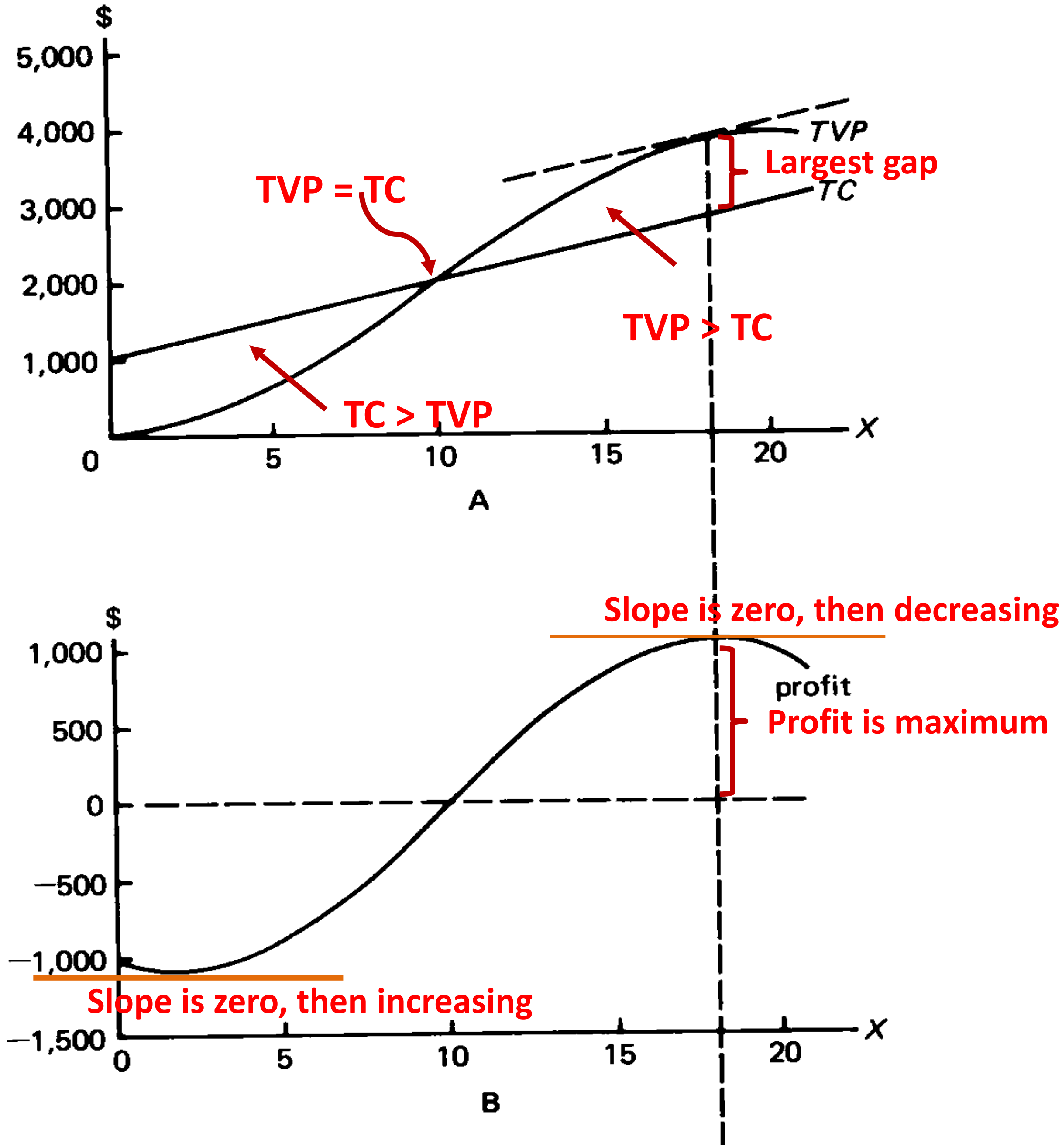
- Another way is to graph the profit function itself
- Profit (TVP-TC) as a function of input
- The optimum amount of input occurs where profit is a maximum

X	TVP	TC	Profit
0	0	1000	-1000
2	111	1200	-1089
4	417	1400	-983
6	864	1600	-736
8	1407	1800	-393
10	2001	2000	1
12	2592	2200	392
14	3135	2400	735
16	3585	2600	985
18	3888	2800	1088
20	3999	3000	999
22	3873	3200	673



Optimal amount of input - 3

- At maximum profit, the slope of the profit function is zero
- "Slope" = "marginal" value of something
- The optimum amount of input occurs where slope of the profit function is zero and <u>profit is decreasing after that point</u>


$
5,000
4,000
3,000
2,000
1,000
0
5
10
15
20
X
TVP
TC
Largest gap
TVP = TC
TVP > TC
TC > TVP
A
$
1,000
500
0
−500
−1,000
−1,500
0
5
10
15
20
X
profit
Slope is zero, then decreasing
Profit is maximum
Slope is zero, then increasing
B

Optimal amount of input - 4

Profits (Π) = TVP – TC = TVP – TF_cC – TFC

At profit max slope of the profit function is zero

Slope of (TVP – TF_cC – TFC) = 0

Slope of TVP – Slope of TF_cC - Slope of a constant = 0

Slope of TVP – Slope of TF_cC - 0 = 0

Slope of TVP = Slope of TF_cC

Slope of TVP = Value of Marginal Product (VMP)

$$Input:\ Fertilizer\ (X),\ Output: Corn\ (Q)$$
$$Price\ of\ Corn = P_Q$$

$$\frac{\Delta(TVP)}{\Delta X} = \frac{\Delta(P_Q Q)}{\Delta X} = PQ\frac{\Delta Q}{\Delta X} = P_Q MP_X = VMP_x$$

This is called **Value of Marginal Product (VMP)**: The additional revenue generated by the last unit of a variable input (fertilizer).

Slope of TF_cC=Marginal Factor Cost (MF_cC)

$$Input:\ Fertilizer\ (X),\ Output:\ Corn\ (Q)$$

$$Price\ of\ fertilizer = P_X$$

$$\frac{\Delta(TF_cC)}{\Delta X} = \frac{\Delta(P_X X)}{\Delta X} = P_X \frac{\Delta X}{\Delta X} = P_X = MF_cC_x$$

This is called **Marginal Factor Cost (MF_cC):** The additional cost associated with the purchase of last unit of the variable input (fertilizer).

Optimal amount of input - 4

At profit max

$$\text{Slope of TVP} = \text{Slope of } TF_cC$$

$$\boxed{VMP = MF_cC}$$

Additional revenue generated from the last unit of input = Additional cost of the last unit of input

The manager will choose the input level at which the above condition holds

Optimal amount of input - 4

Alternatively,

Divide both sides of $VMP = MF_cC$, by MF_cC

$$VMP / MF_cC = MF_cC / MF_cC$$

$$VMP/MF_cC = 1$$

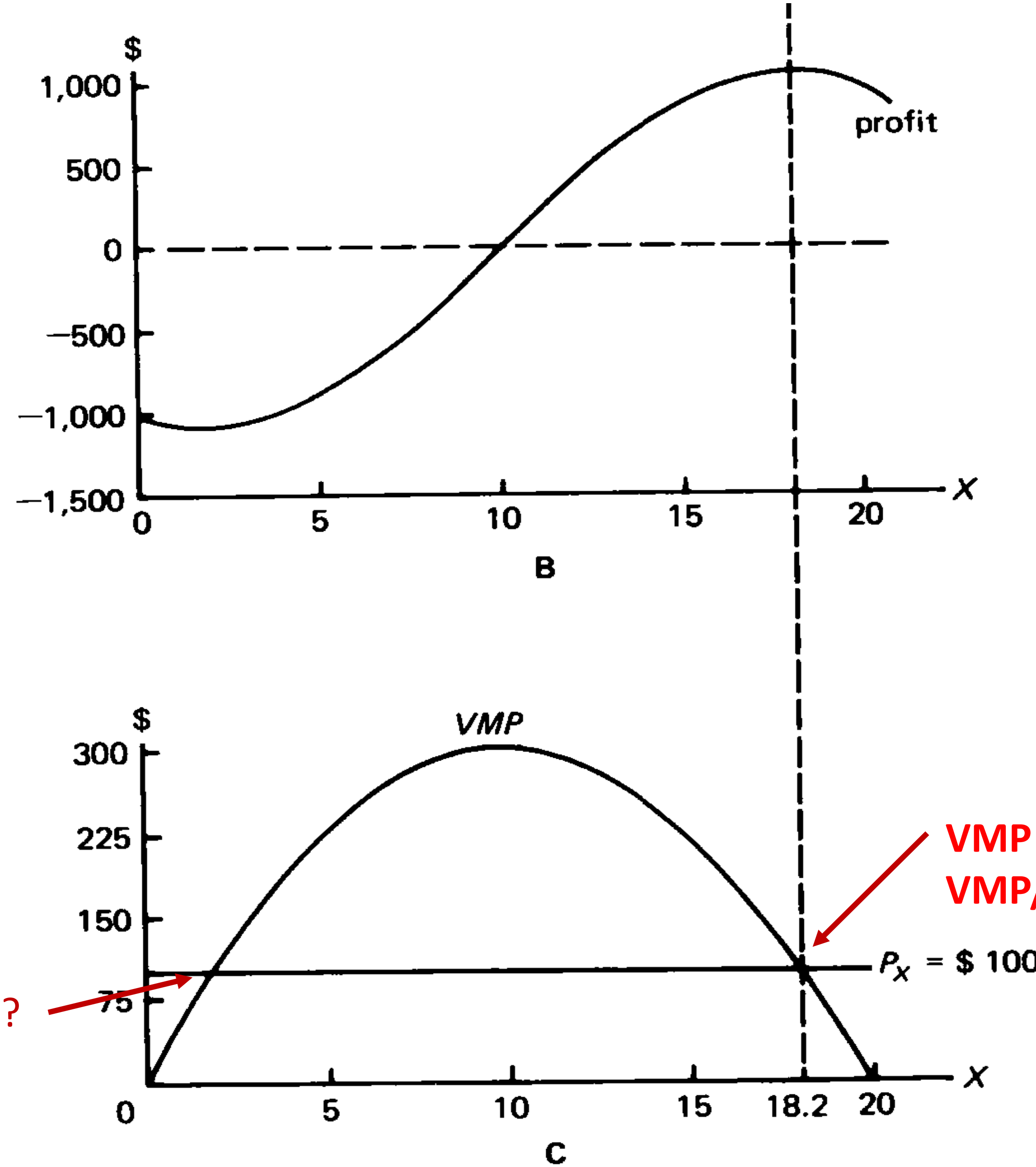
$\boxed{\text{The optimal input use is where } VMP/MF_cC = 1}$

The last dollar spent on input (fertilizer) generated exactly one dollar of additional revenue

The manager will choose the input level at which the above condition holds

Between 18 and 20 units of fertilizer there should be a point where VMP = MF_cC

X	VMP	MF_cC	Profit
0			-1000
2	57	100	-1089
4	153	100	-983
6	225	100	-736
8	273	100	-393
10	297	100	1
12	297	100	392
14	273	100	735
16	225	100	985
18	153	100	1088
20	57	100	999
22	-63	100	673



Why not this?

VMP = MF_cC
VMP/ MF_cC = 1

Instead of fertilizer, labor and capital as inputs- just a word change, still VMP = MF_cC

To maximize profits when labor or capital vary in the short run, the manager will hire:

- Labor (if labor is the variable input) until the value of the marginal product of labor equals the Marginal Factor Cost of labor, the wage rate: $VMP_L/w = 1$
- Capital (if capital is the variable input) until the value of the marginal product of capital equals the Marginal Factor Cost of capital, the rental rate: $VMP_K/r = 1$