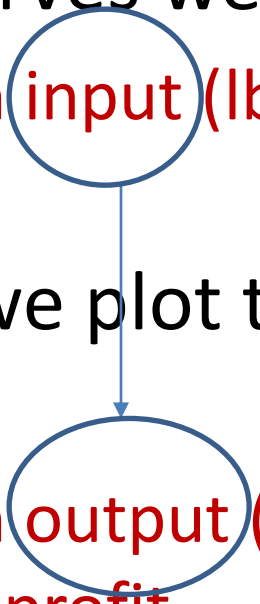
Profit Maximization
One variable input –One Output output side

BLP Chapter 05

Webster Chapter 07

Profit maximization – Output side

- On the input side, we plot the input (fertilizer) on x-axis of all the curves we drew
  - We decide how much input (lbs of fertilizer) to use to maximize profit
- On the output side, we plot the output (corn) on x-axis
  - We decide how much output (bushels of corn) to produce to maximize profit
- Both approaches will yield the same solution



Total Revenue

- Total Revenue (TR)
  - Amount of revenue generated from the total number of units produced/sold

TR = Price of the output per unit * quantity of output produced

e.g. TR from corn = price of corn * bushels of corn produced

$$TR = P_Q * Q$$

Total Cost

- Total Cost (TC)
  - The cost of producing the total number of outputs produced/sold

Total Cost = Total Fixed Cost + Total Variable Cost

$$TC = TFC + TVC$$

e.g. TC = cost of land + (wage rate* number of labor hours used)

$$TC = TFC + w*L = \$1000 + w*L$$

e.g. TC = cost of land + (fertilizer price* lbs of fertilizer used)

$$TC = TFC + P_x*X = \$1000 + P_x*X$$

X is fertilizer use (lbs), Q is Corn (bushels)

P_Q= \$30/bushel, P_x= \$ 100/lb

Decision variable: output (how much output to produce to max profit). Recall, in the input side the decision was how much input to use to max profit.

X	Q	TR = P_Q*Q = 30*Q	TVC = P_X *X = 100*X	TFC	TC = TFC+TVC	Profit
0	0	0		1000	1000	-1000
2	3.7	111	200	1000	1200	-1089
4	13.9	417	400	1000	1400	-983
6	28.8	864	600	1000	1600	-736
8	46.9	1407	800	1000	1800	-393
10	66.7	2001	1000	1000	2000	1
12	86.4	2592	1200	1000	2200	392
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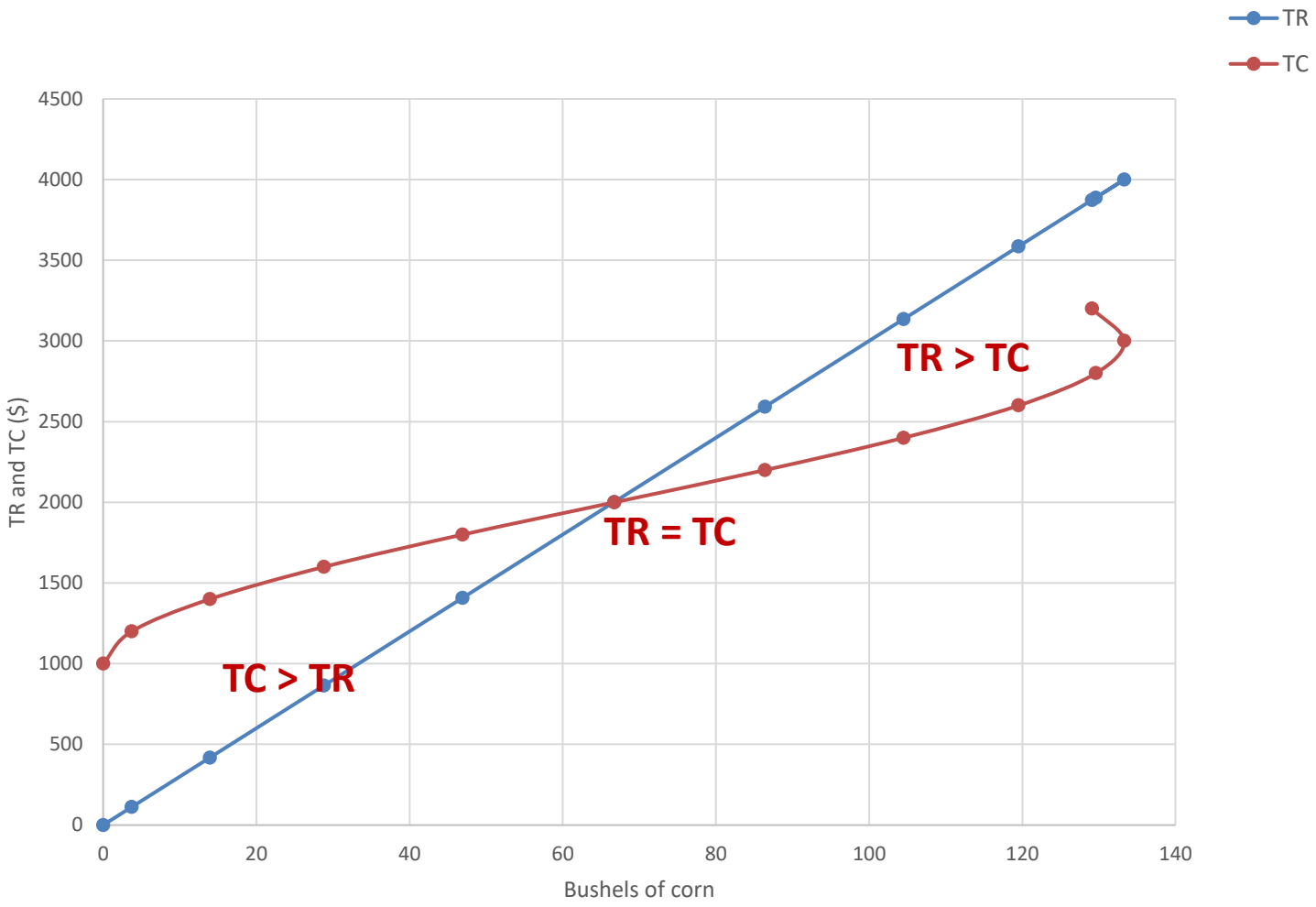
22	129.1	3873	2200	1000	3200	673

TR and TC



Total Fixed Cost (TFC)

Marginal Revenue

- Marginal Revenue (MR)
  - The additional revenue generated from the production of an additional unit of output

MR = slope of the TR curve

$$\text{MR} = \frac{\Delta TR}{\Delta Q} = \frac{\Delta P_Q Q}{\Delta Q} = P_Q \frac{\Delta Q}{\Delta Q} = P_Q$$

  - The additional revenue generated from the production of an additional unit of output is the price of the output

Marginal Cost

- Marginal Cost (MC)
  - The additional cost incurred by producing an additional unit of output

MC = slope of the TC curve

$$\text{MC} = \frac{\Delta TC}{\Delta Q} = \frac{\Delta((P_X X)+TFC)}{\Delta Q} = P_X \frac{\Delta X}{\Delta Q} + \frac{\Delta TFC}{\Delta Q} = P_X \frac{1}{MP_X}$$

MC = price of the input x*inverse of the marginal product of input x

Note that $\frac{\Delta TFC}{\Delta Q}$ is zero because ΔTFC is zero

X is fertilizer use (lbs), Q is Corn (bushels)

P_Q= \$30, P_x= \$ 100

Decision variable: output (how much output to produce to max profit). Recall, in the input side the decision was how much input to use to max profit.

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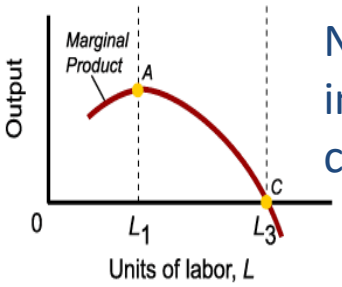
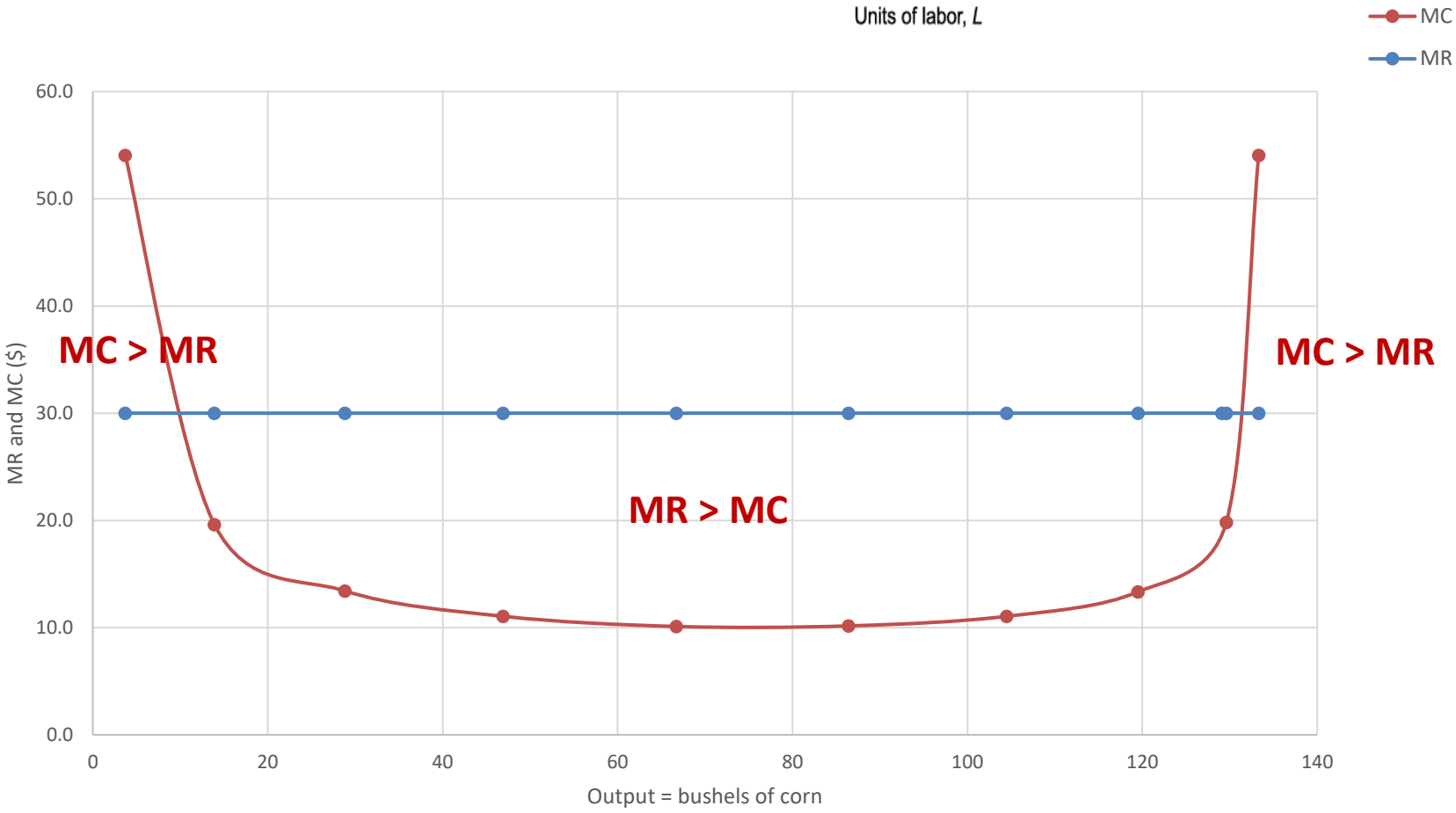
MR and MC

(X-AXIS HAS OUTPUT)

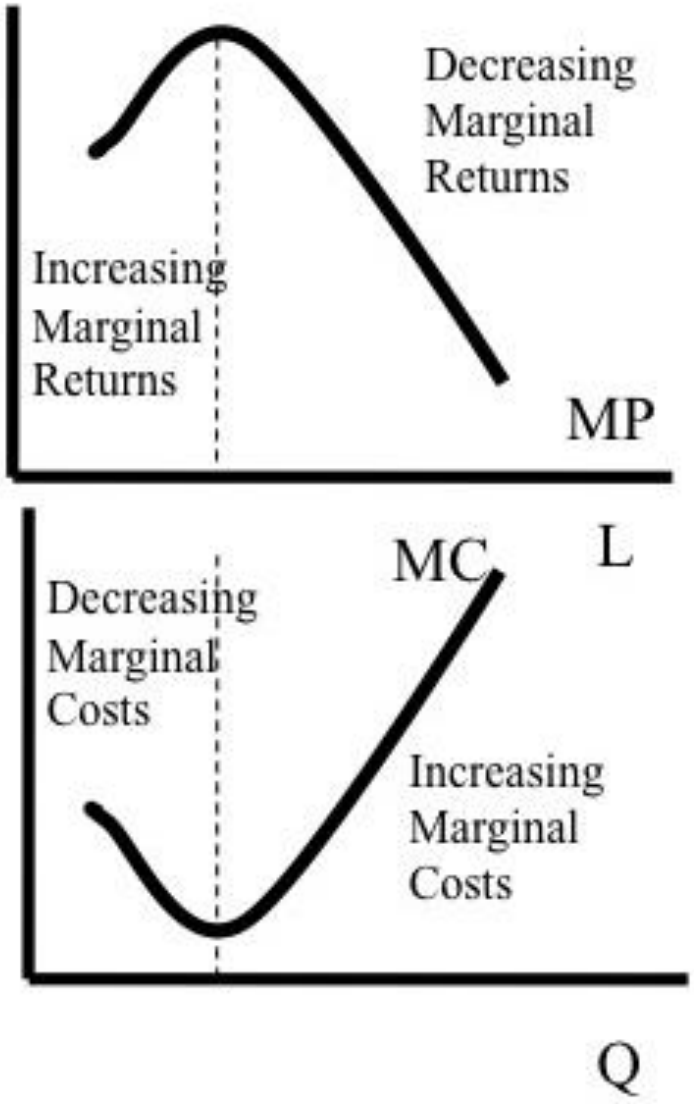
Recall in input side, x-axis had input)



Note that the MC curve is inversely related to the MP curve MC = P_{input}*1/MP



MP and MC



- Recall the MP curve at the early stage of production-it is upward sloping-each additional input is more and more productive.
- So we need less and less input to produce each additional unit of output.
- This means the cost of producing each additional output becomes less and less expensive.
- Then the diminishing marginal returns set in.
- During the later stages of production, each additional unit of input produces less and less output.
- That means we need more and more input to produce each additional unit of output. Hence the cost of producing each output becomes more and more expensive.

Profit maximization – Output side
Optimal level of output to produce

$$Profit = TR - TC = P_Q Q - P_x X - TFC$$

Similar to the input side, the slope of the profit function is zero at the profit max point

Slope of (TR - TC) = 0

Slope of TR– Slope of TC=0

Slope of TR = Slope of TC

Marginal revenue = Marginal cost

MR = MC

Profit maximization
Optimal level of output to produce

$At\ profit$ max MR = MC

What is MR = $\frac{\Delta TR}{\Delta Q} = P_Q = price\ of\ output\ (corn)$

What is MC = $\frac{\Delta TC}{\Delta Q} = P_X \frac{1}{MP} = price\ of\ fertilizer\ * \frac{1}{Marginal\ Product}$

At profit max *Additional revenue generated from the last unit of output = Additional cost of the last unit of output*

X is fertilizer use (lbs), Q is Corn (bushels)

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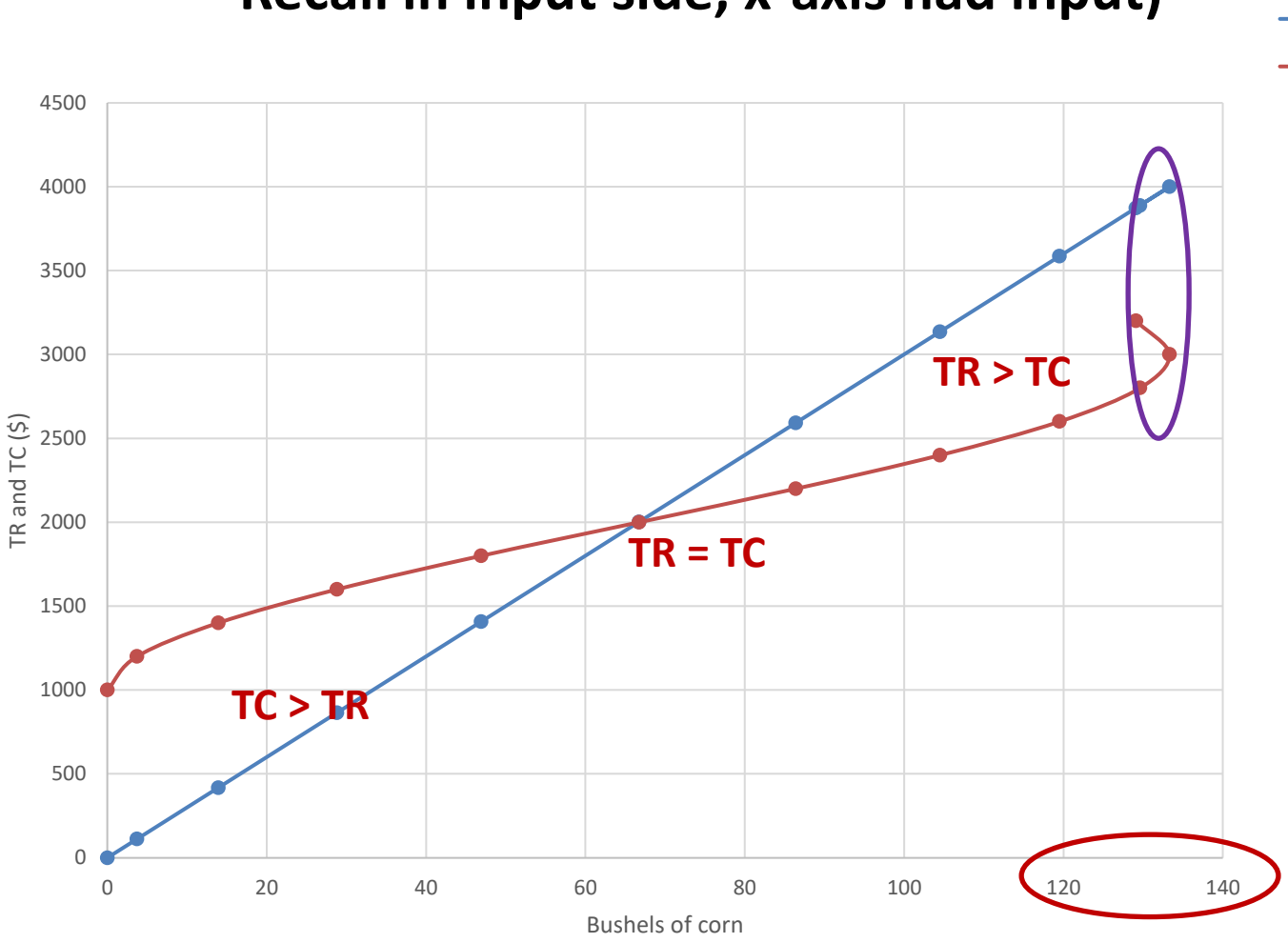
Method 1: TR and TC

(X-AXIS HAS OUTPUT)

Recall in input side, x-axis had input)



Slope of the TR = MR = price of the output

Slope of the TC = MC

Method 2: MR and MC

(X-AXIS HAS OUTPUT)

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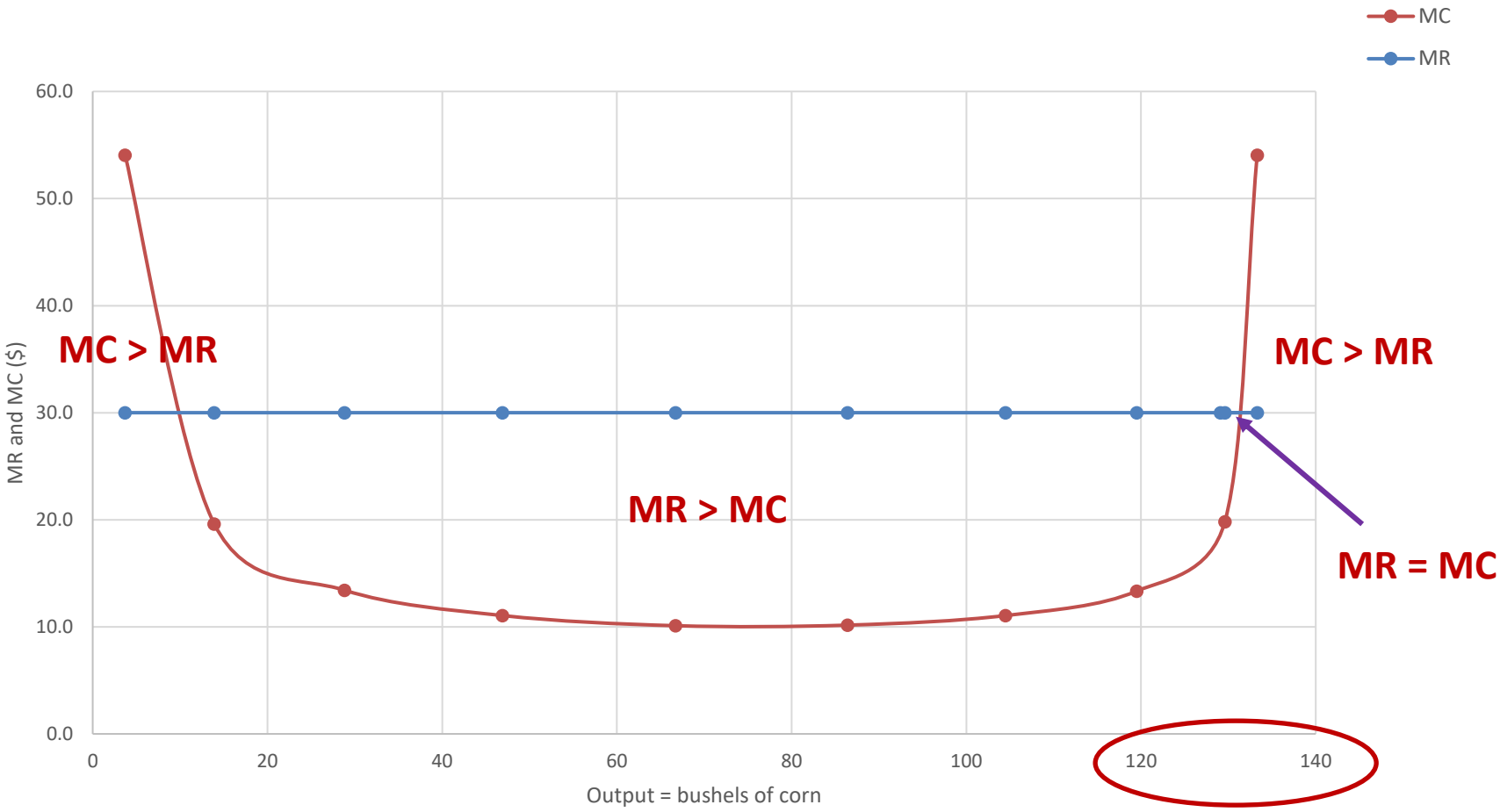


Method 2: MR and MC

Alternatively,

Divide both sides of MR = MC, by MC

$$MR / MC = MC / MC$$

$$MR/MC = 1$$

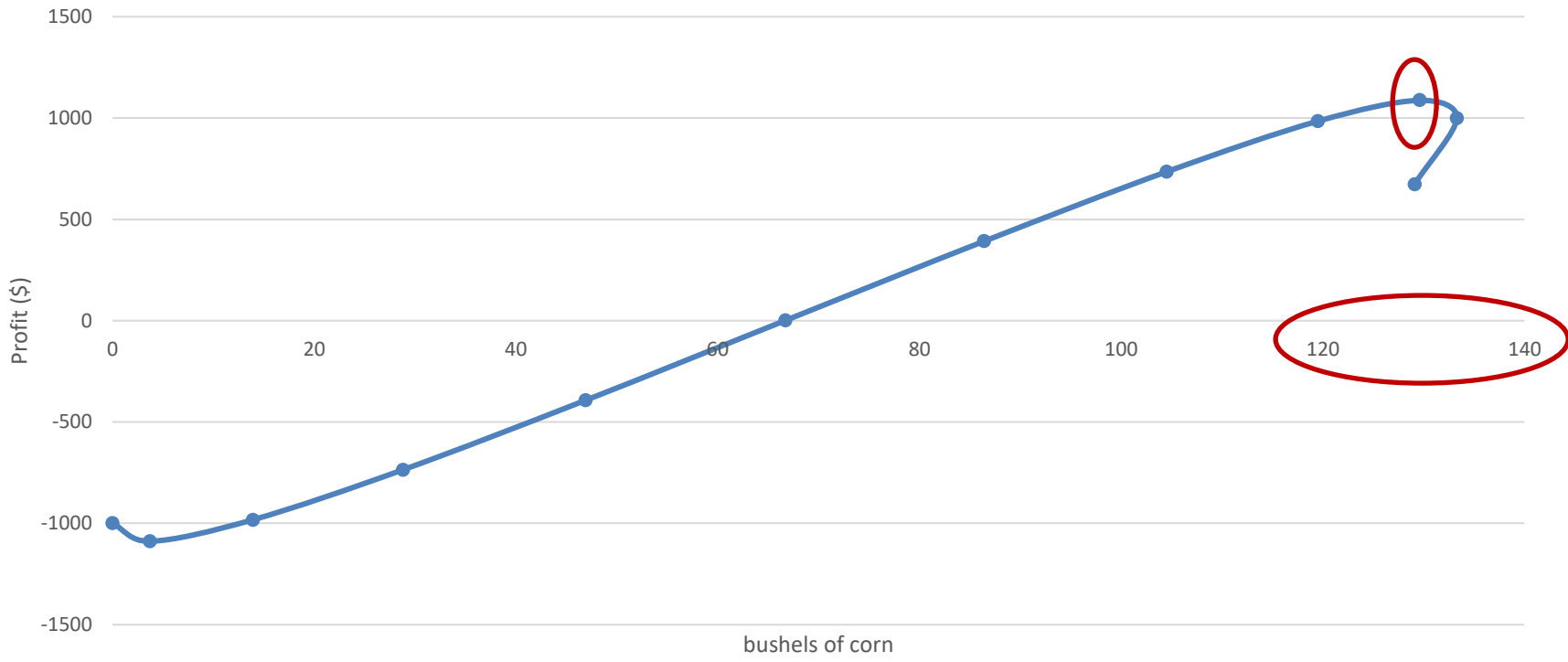
The optimal level of output is where MR/MC = 1

The last dollar spent on producing the last unit of output generated exactly one dollar of additional revenue

The manager will choose the output level at which the above condition holds

Method 3: Profit (X-AXIS HAS OUTPUT)

Recall in input side, x-axis had input)



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