Step of a 3rd Order State Space System

The previous ODE45 comparison study of a step input to a PSS model was altered to test a variety of variable step MATLAB ode solvers. Simulation time was set to one minute so that step time variations could be observed after a disturbance. It should be noted that maximum step size was limited to 20 seconds (20,000 ms).

Summary

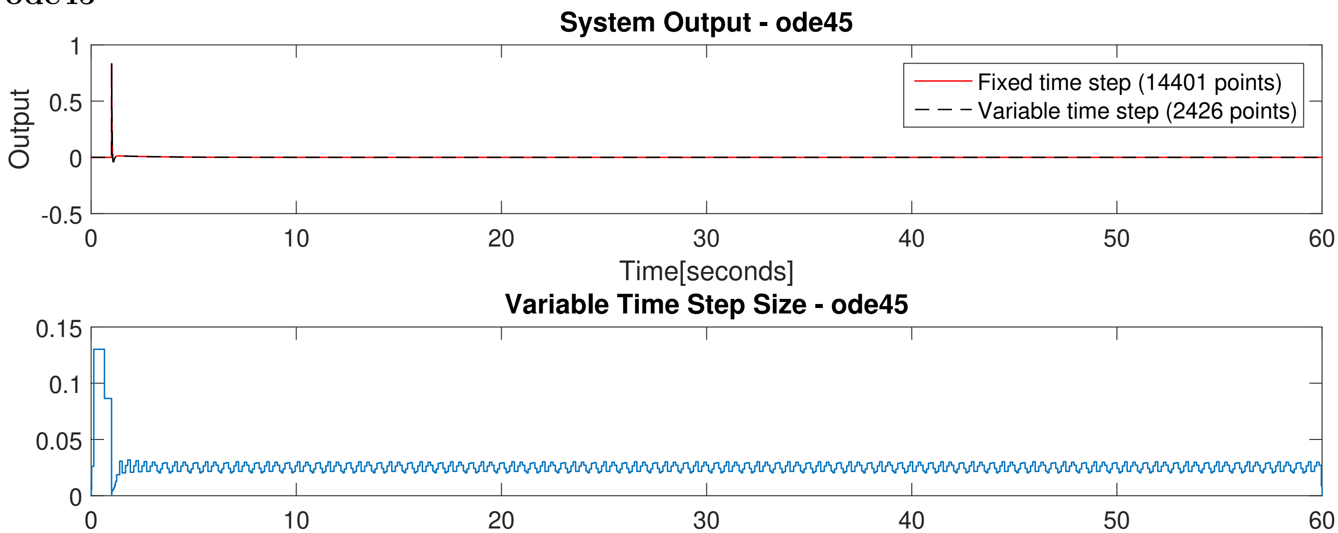
The table below shows the number of steps each method took for a 60 second simulation, the maximum step size post-step event, the magnitude of the steady state error, and if the method is Octave compatible. Full result plots are presented in the following pages with MATLAB code at the end of this document.

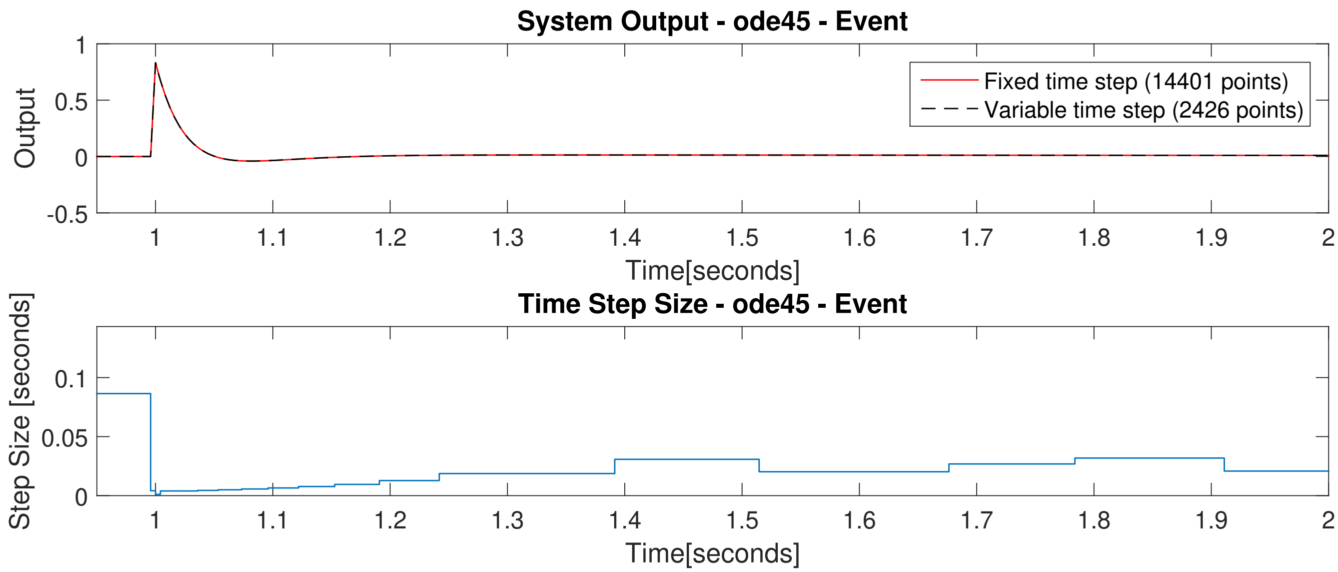
Method	Number of Steps	Max Step Post Disturbance [ms]	SS Error Magnitude	Octave compatible
Fixed	14,401	≈4	-	*
ODE45	2,426	30	10^{-4}	*
ODE23	803	75	10^{-4}	*
ODE113	1,298	75	10^{-4}	
ODE15s	71	20,000	10^{-8}	*
ODE23s	45	20,000	10^{-9}	
ODE23t	72	≈17,000	10^{-9}	
ODE23tb	49	20,000	10^{-8}	

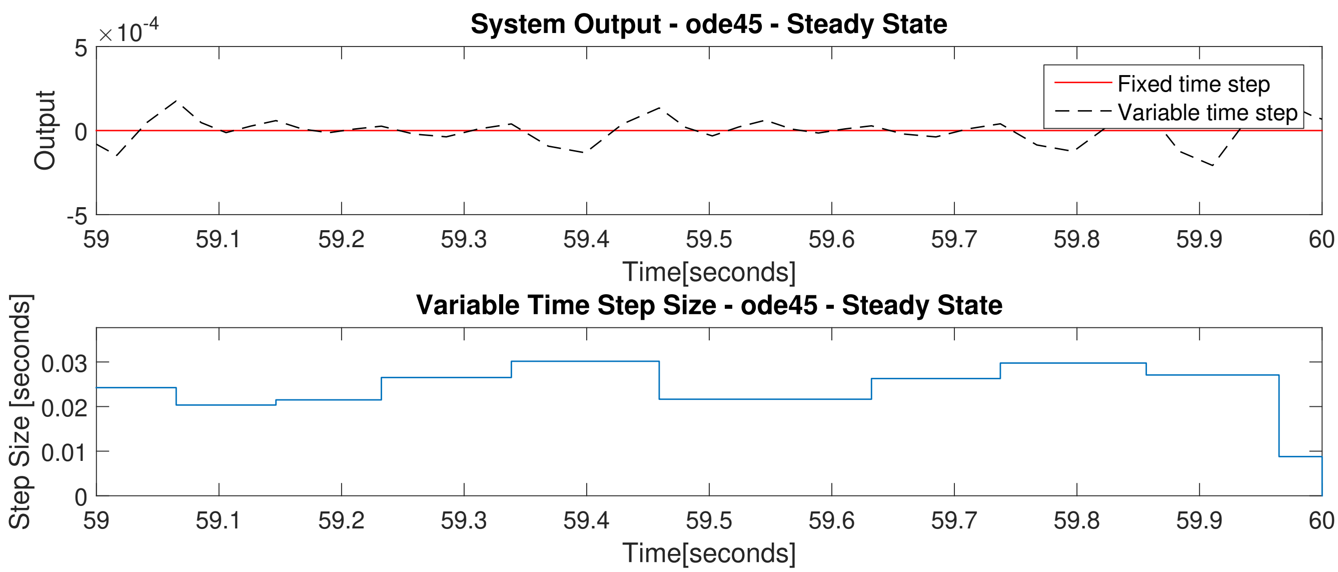
Observations of Note

1. All ODE methods greatly reduce the number of required steps.
2. ODE15s, ODE23s, and ODE23tb reached the maximum allowed step size of 20 seconds.
3. Steady state error for ODE45, ODE23, and ODE113 was approximately 4 orders of magnitude larger than all other methods and step size stayed below 75 ms.
4. ODE23s used the least amount of steps and had one of the smallest steady state errors.
5. ODE15s appears to be the most appropriate Octave compatible solver.

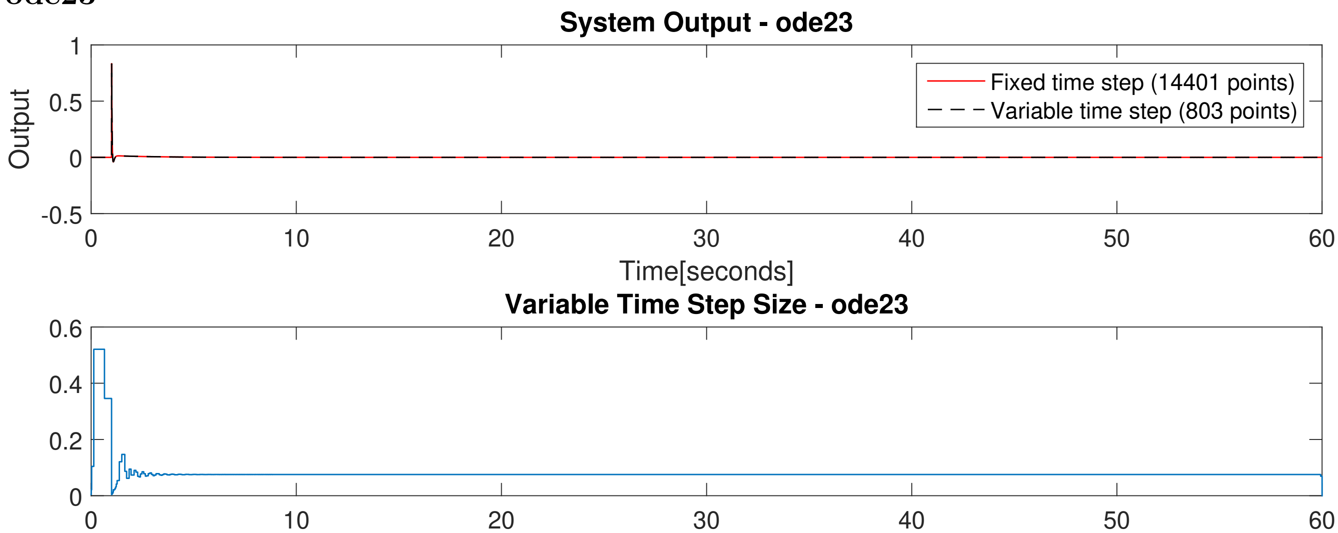
ode45

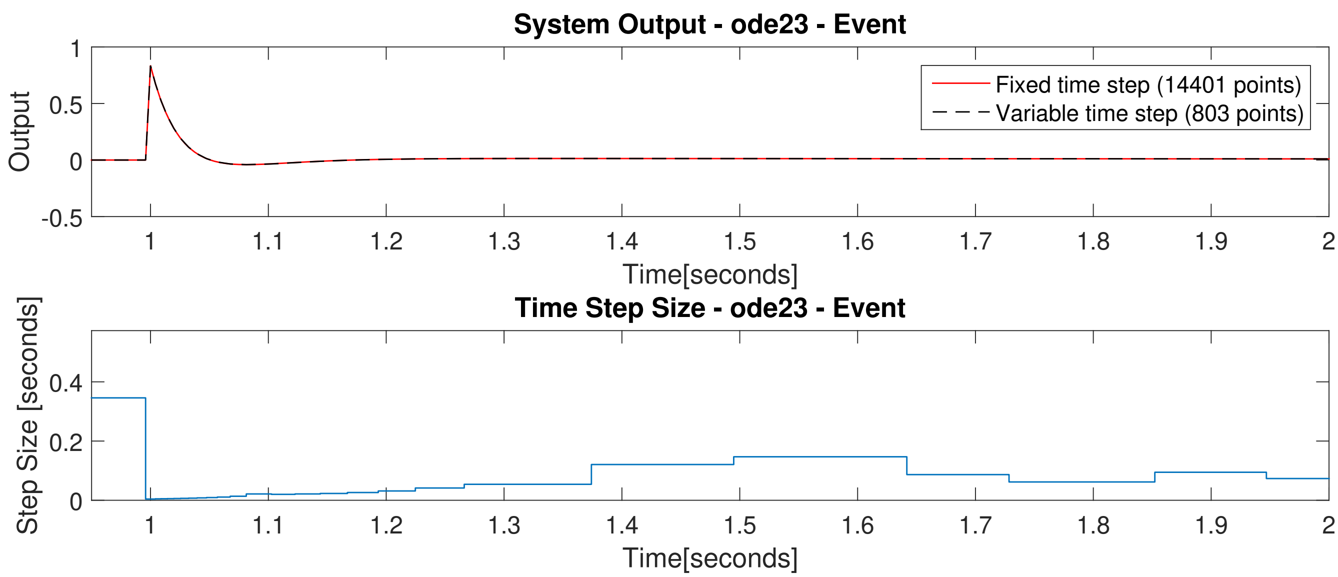

System Output - ode45
Output
Fixed time step (14401 points)
Variable time step (2426 points)
Time[seconds]
Variable Time Step Size - ode45

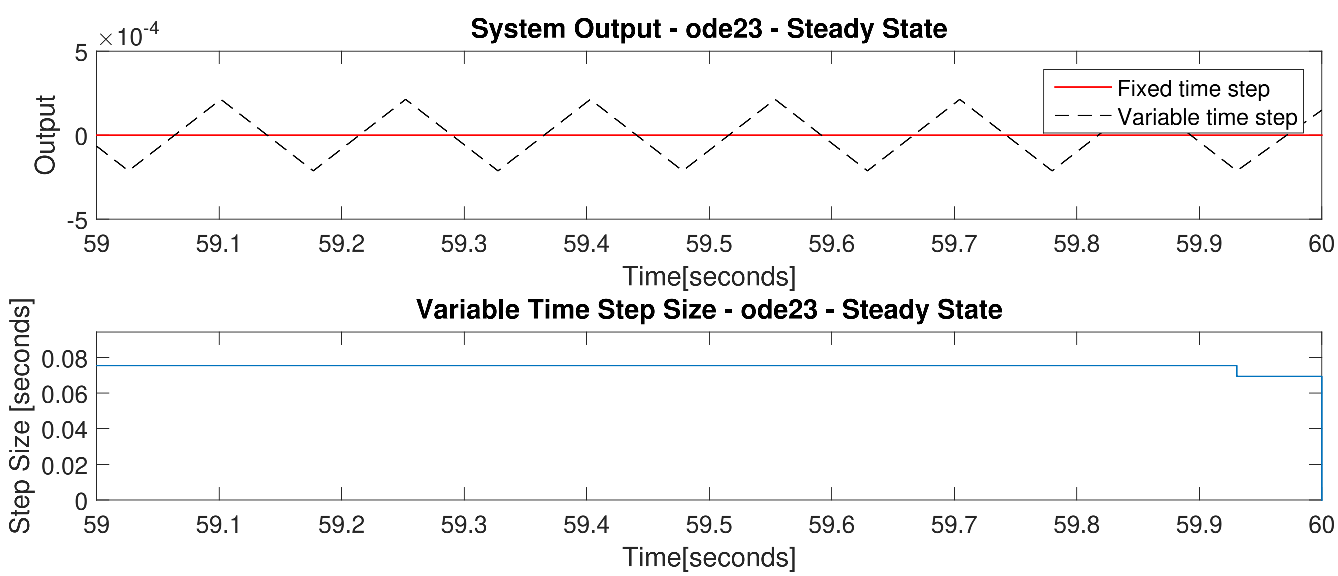

System Output - ode45 - Event
Output
Fixed time step (14401 points)
Variable time step (2426 points)
Time[seconds]
Time Step Size - ode45 - Event
Step Size [seconds]
Time[seconds]


System Output - ode45 - Steady State
×10^-4
Output
Fixed time step
Variable time step
Time[seconds]
Variable Time Step Size - ode45 - Steady State
Step Size [seconds]
Time[seconds]

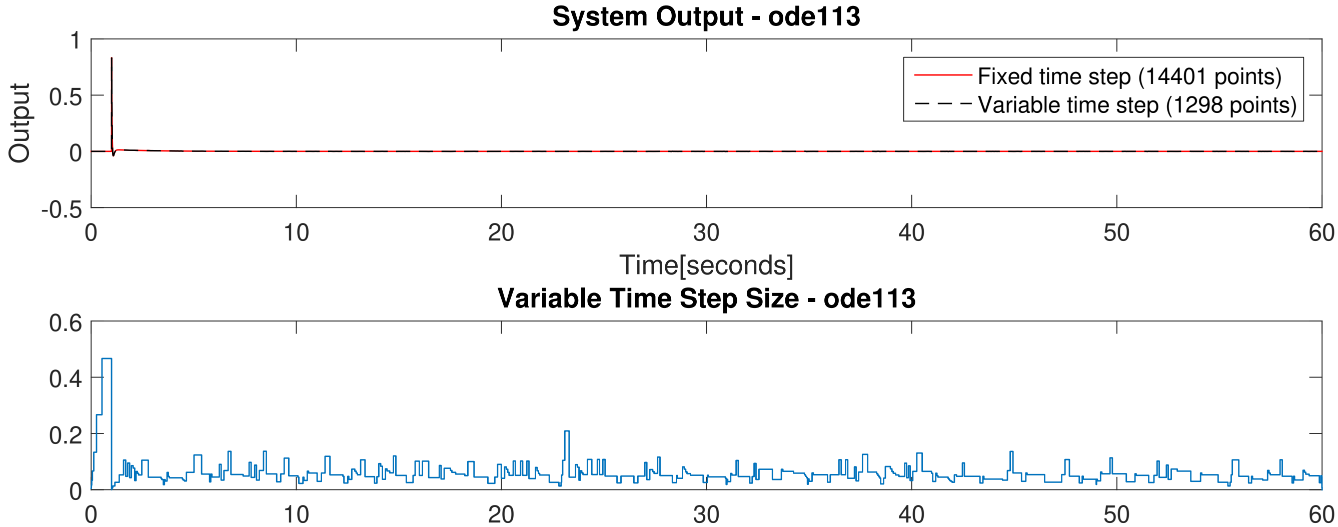
ode23

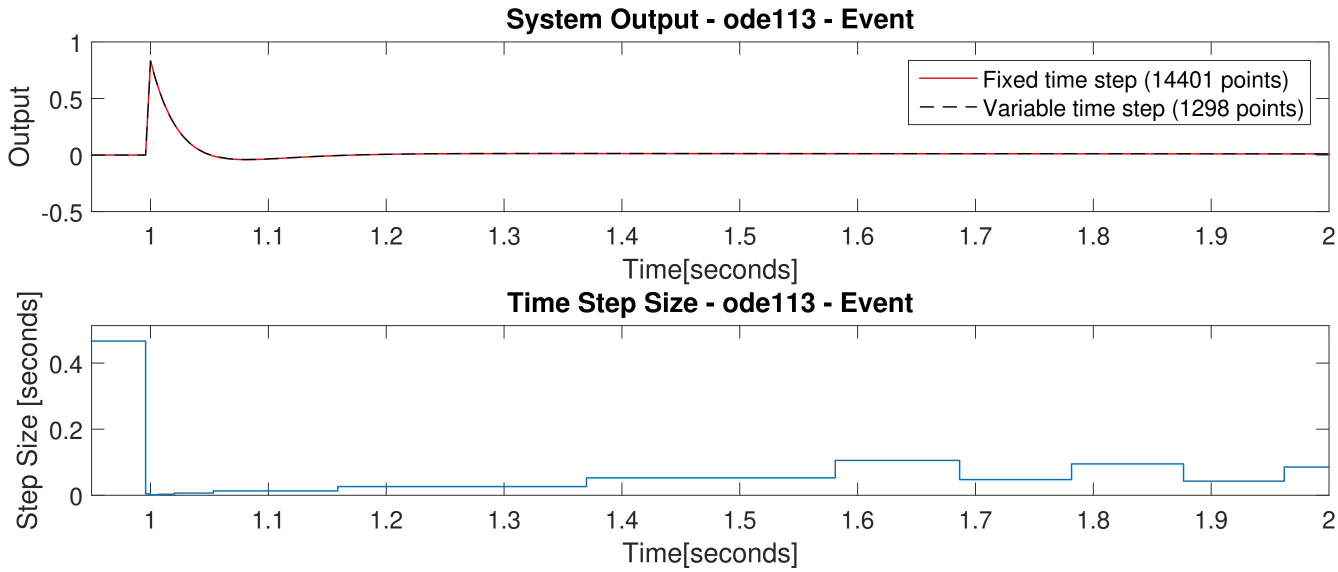

System Output - ode23
Output
Time[seconds]
Fixed time step (14401 points)
Variable time step (803 points)
Variable Time Step Size - ode23

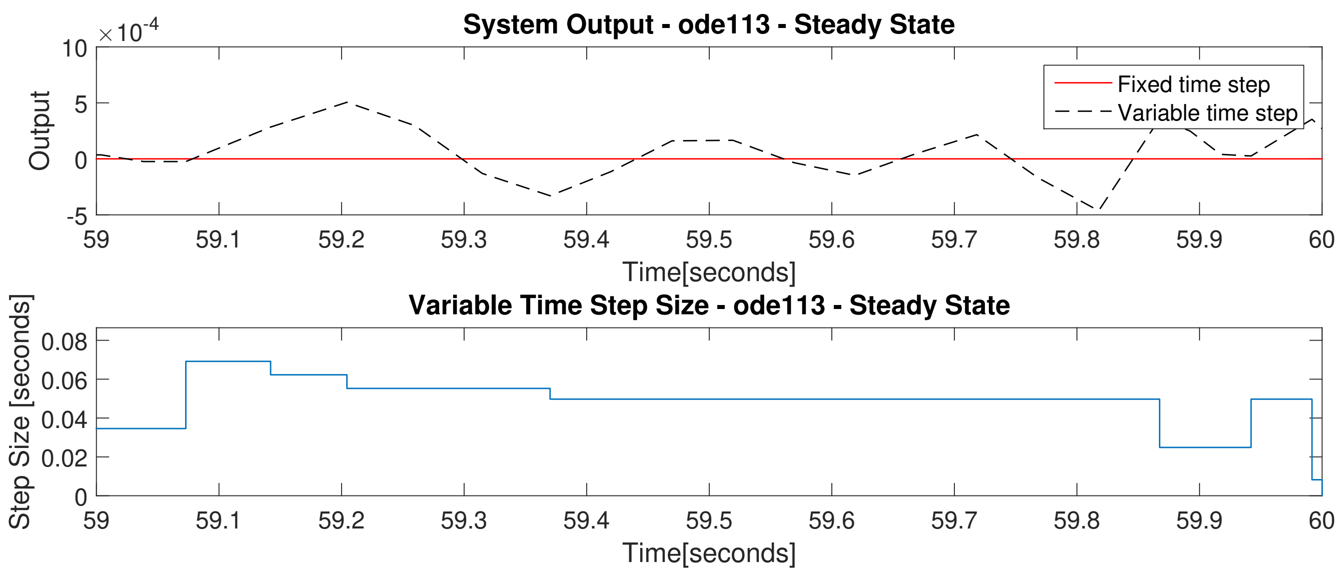

System Output - ode23 - Event
Output
Time[seconds]
Fixed time step (14401 points)
Variable time step (803 points)
Time Step Size - ode23 - Event
Step Size [seconds]
Time[seconds]


System Output - ode23 - Steady State
×10-4
Output
Time[seconds]
Fixed time step
Variable time step
Variable Time Step Size - ode23 - Steady State
Step Size [seconds]
Time[seconds]

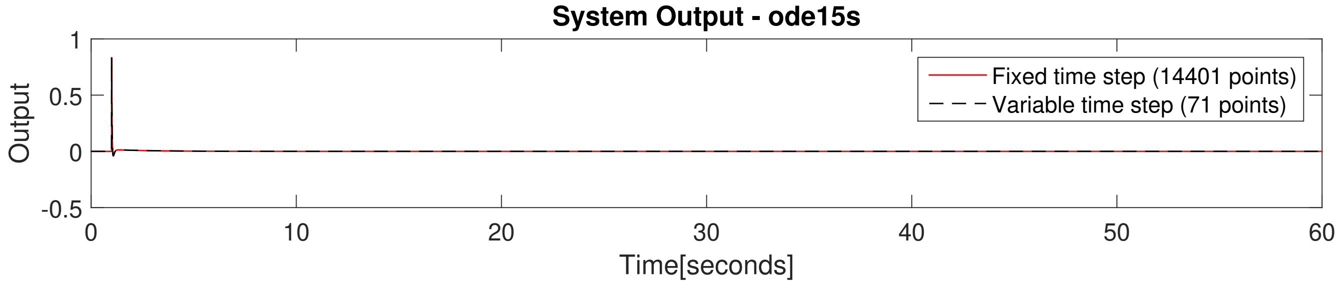
ode113

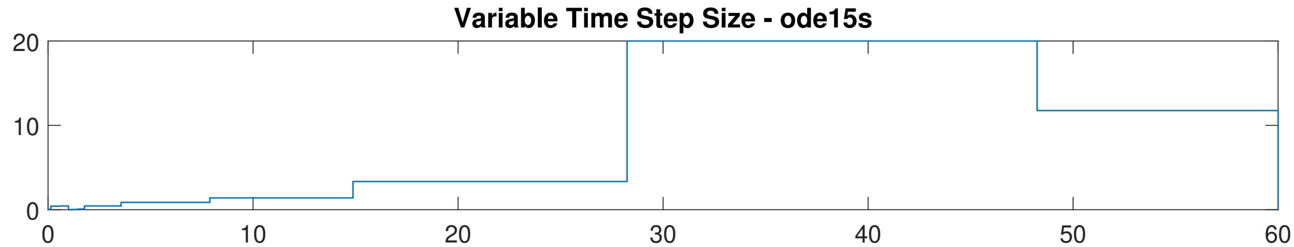

System Output - ode113
Fixed time step (14401 points)
Variable time step (1298 points)
Output
Time[seconds]
Variable Time Step Size - ode113

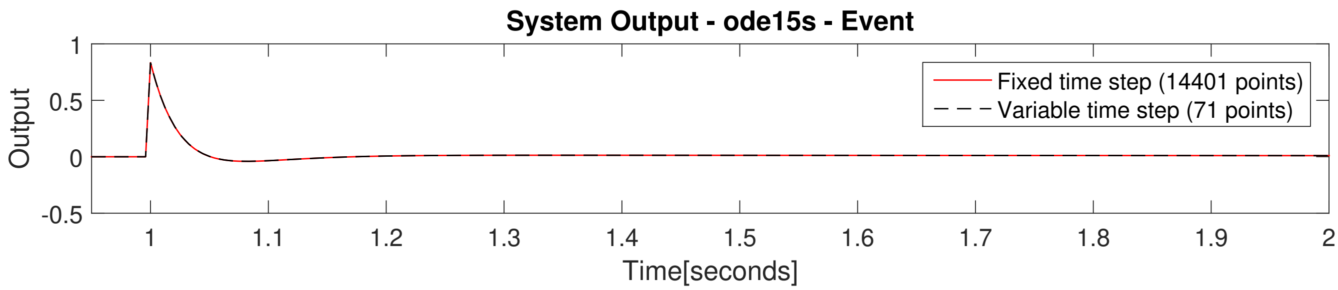

System Output - ode113 - Event
Fixed time step (14401 points)
Variable time step (1298 points)
Output
Time[seconds]
Time Step Size - ode113 - Event
Step Size [seconds]
Time[seconds]

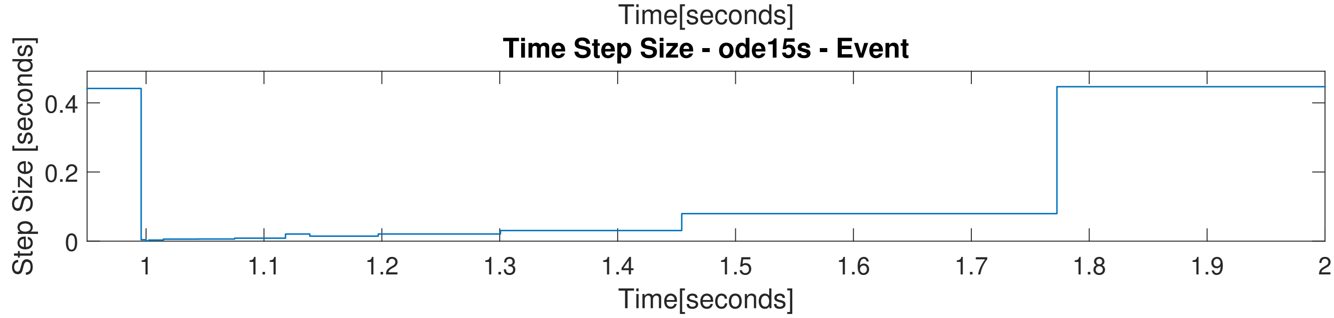

System Output - ode113 - Steady State
Fixed time step
Variable time step
Output
Time[seconds]
Variable Time Step Size - ode113 - Steady State
Step Size [seconds]
Time[seconds]

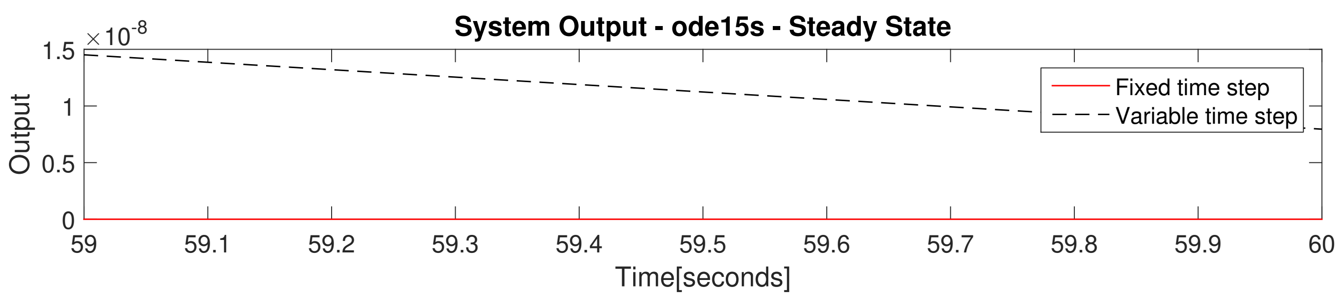
ode15s

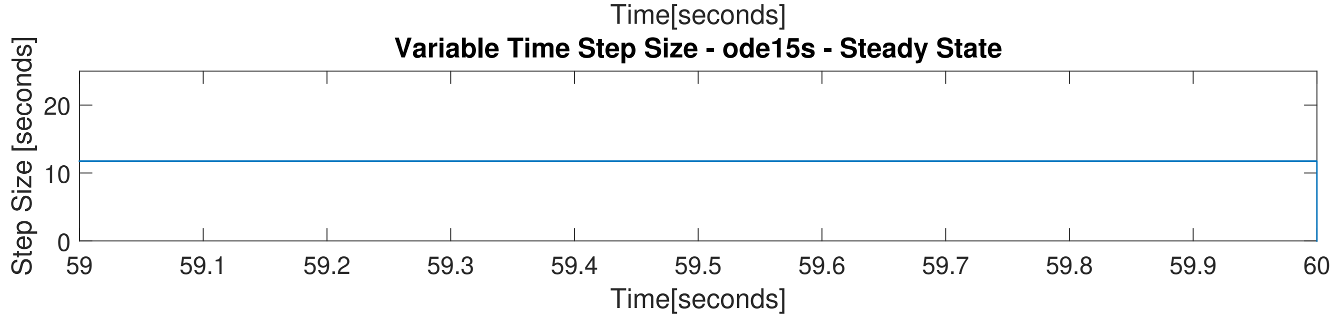

System Output - ode15s
Output
1
0.5
0
-0.5
0
10
20
30
40
50
60
Time[seconds]
Fixed time step (14401 points)
Variable time step (71 points)


Variable Time Step Size - ode15s
20
10
0
0
10
20
30
40
50
60

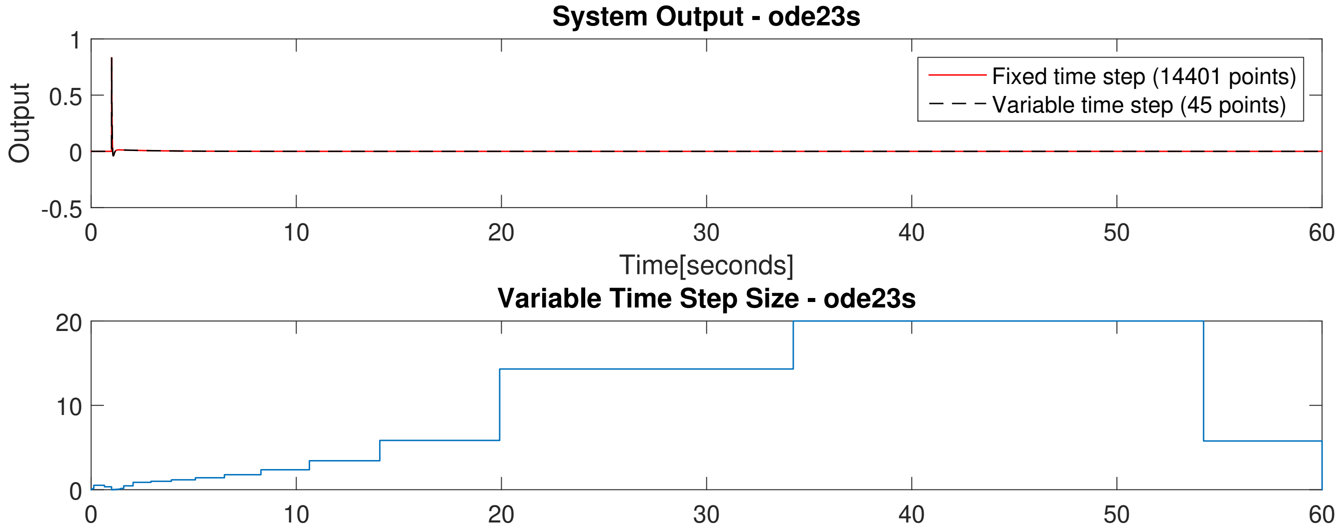

System Output - ode15s - Event
Output
1
0.5
0
-0.5
1
1.1
1.2
1.3
1.4
1.5
1.6
1.7
1.8
1.9
2
Time[seconds]
Fixed time step (14401 points)
Variable time step (71 points)

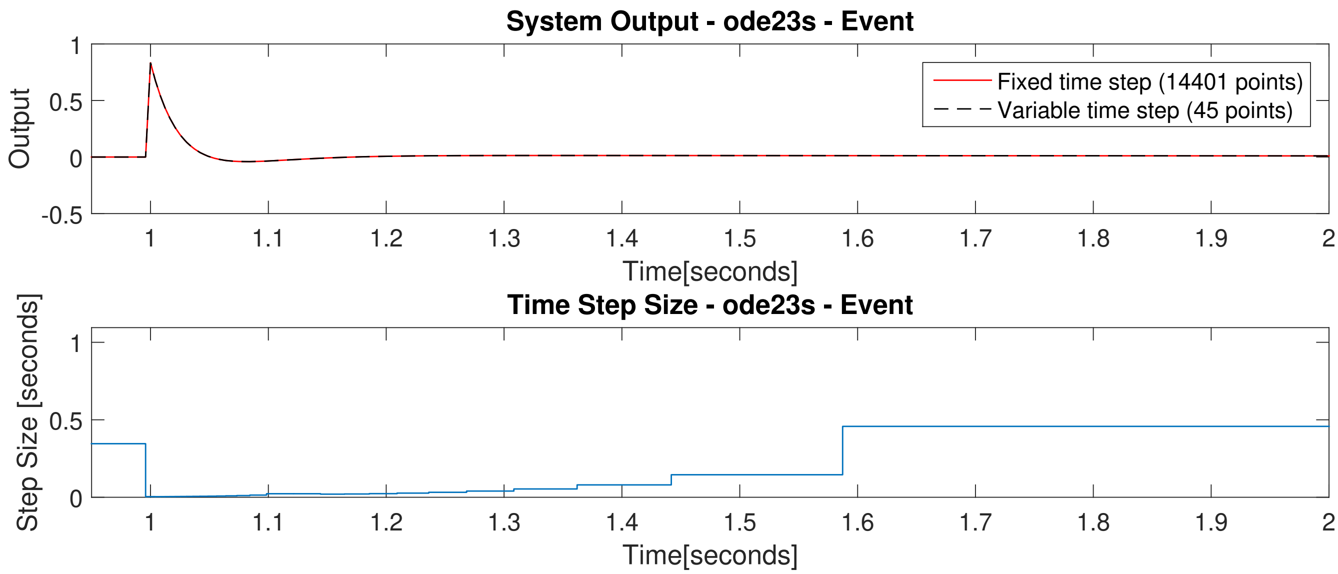

Time Step Size - ode15s - Event
Step Size [seconds]
0.4
0.2
0
1
1.1
1.2
1.3
1.4
1.5
1.6
1.7
1.8
1.9
2
Time[seconds]

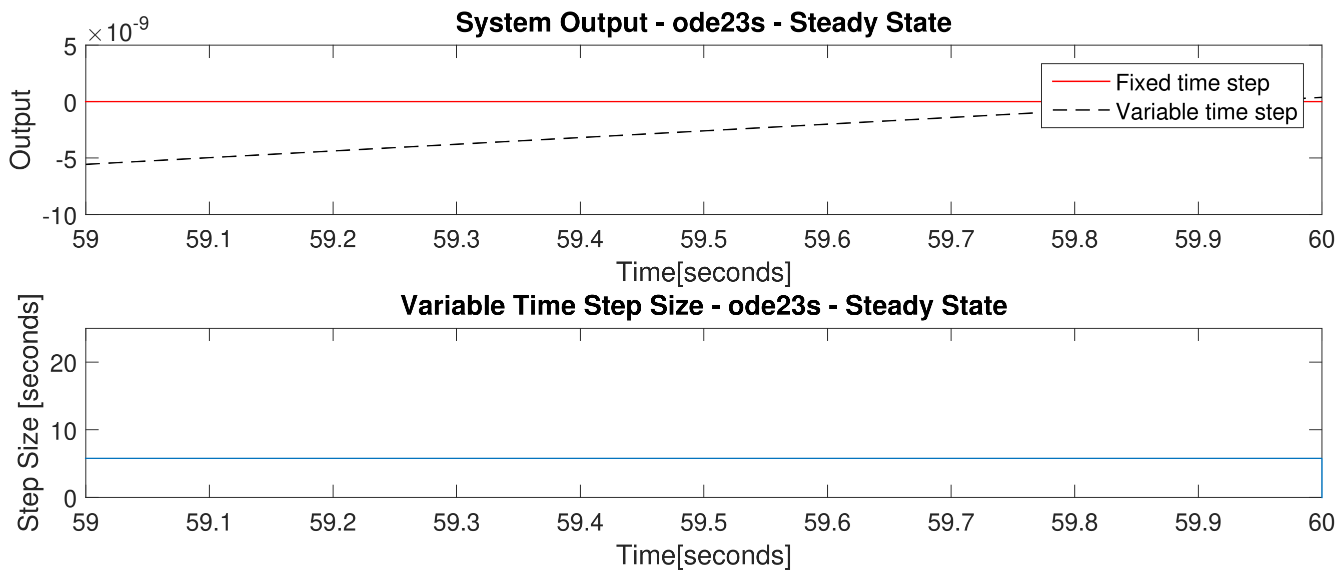

System Output - ode15s - Steady State
×10-8
Output
1.5
1
0.5
0
59
59.1
59.2
59.3
59.4
59.5
59.6
59.7
59.8
59.9
60
Time[seconds]
Fixed time step
Variable time step


Variable Time Step Size - ode15s - Steady State
Step Size [seconds]
20
10
0
59
59.1
59.2
59.3
59.4
59.5
59.6
59.7
59.8
59.9
60
Time[seconds]

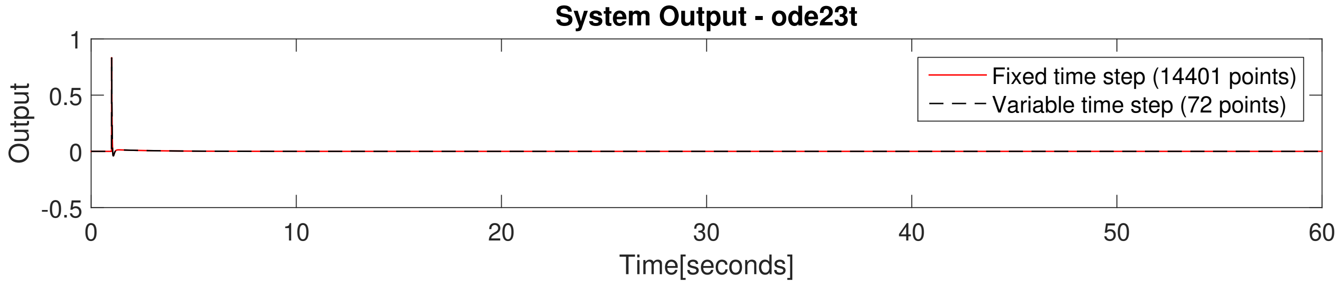
ode23s

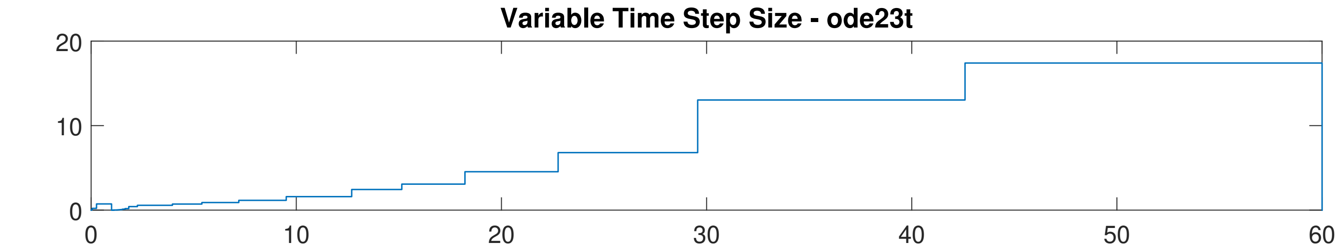

System Output - ode23s
Fixed time step (14401 points)
Variable time step (45 points)
Output
Time[seconds]
Variable Time Step Size - ode23s

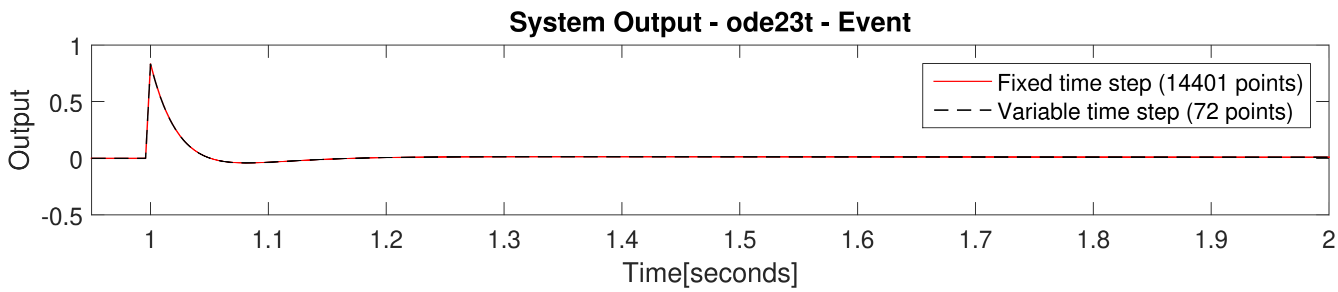

System Output - ode23s - Event
Fixed time step (14401 points)
Variable time step (45 points)
Output
Time[seconds]
Time Step Size - ode23s - Event
Step Size [seconds]
Time[seconds]

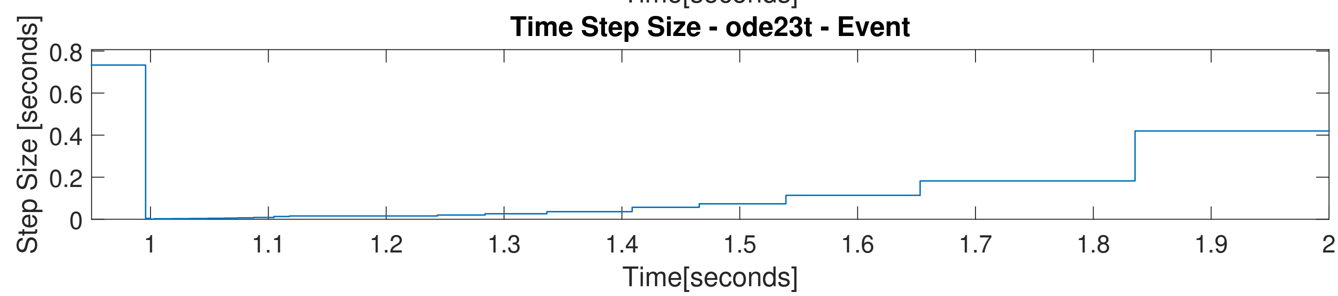

System Output - ode23s - Steady State
×10^-9
Fixed time step
Variable time step
Output
Time[seconds]
Variable Time Step Size - ode23s - Steady State
Step Size [seconds]
Time[seconds]

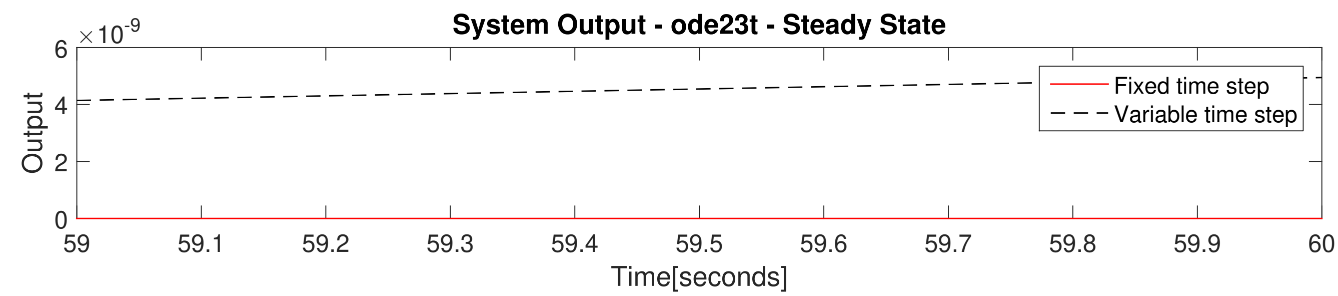
ode23t

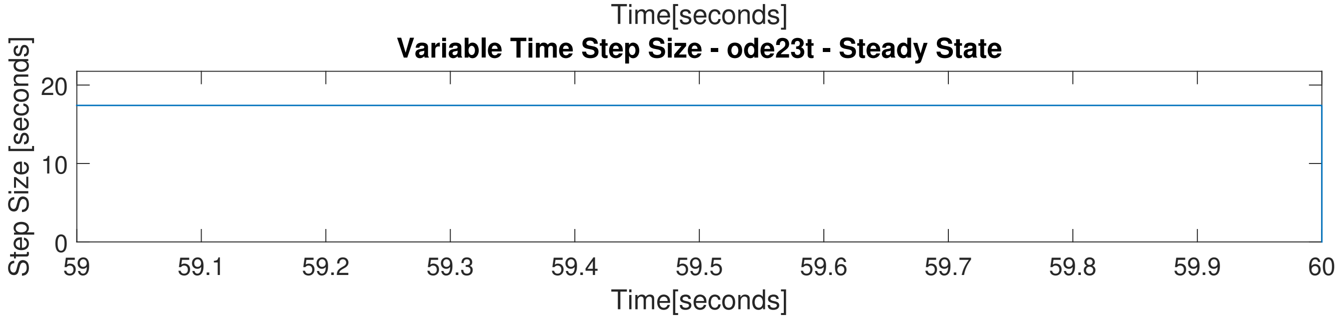

System Output - ode23t
Output
Time[seconds]
Fixed time step (14401 points)
Variable time step (72 points)


Variable Time Step Size - ode23t

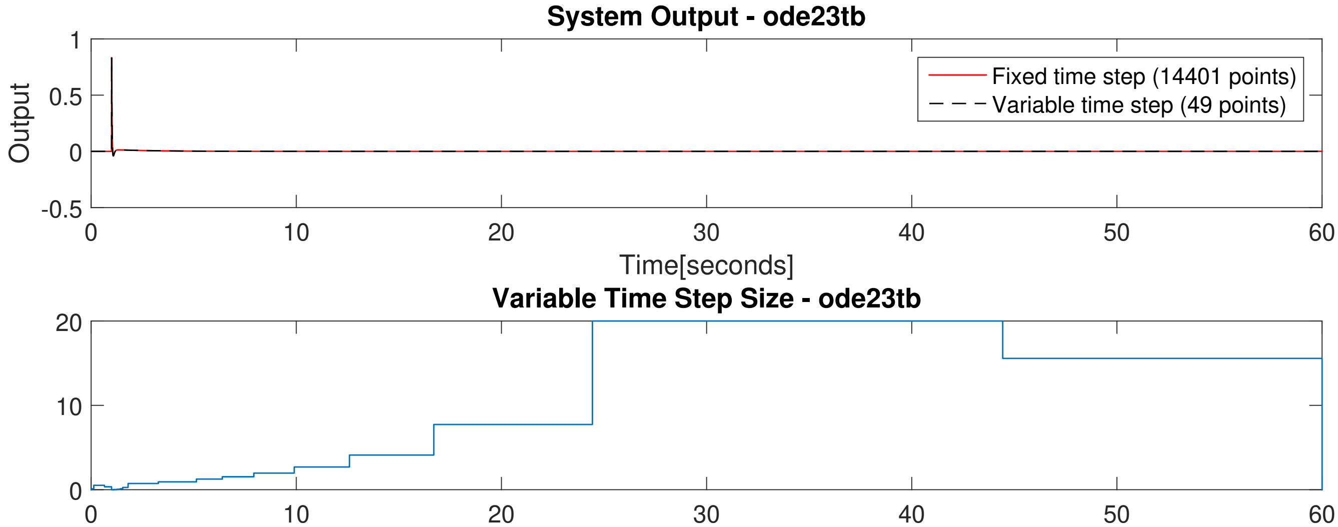

System Output - ode23t - Event
Output
Time[seconds]
Fixed time step (14401 points)
Variable time step (72 points)

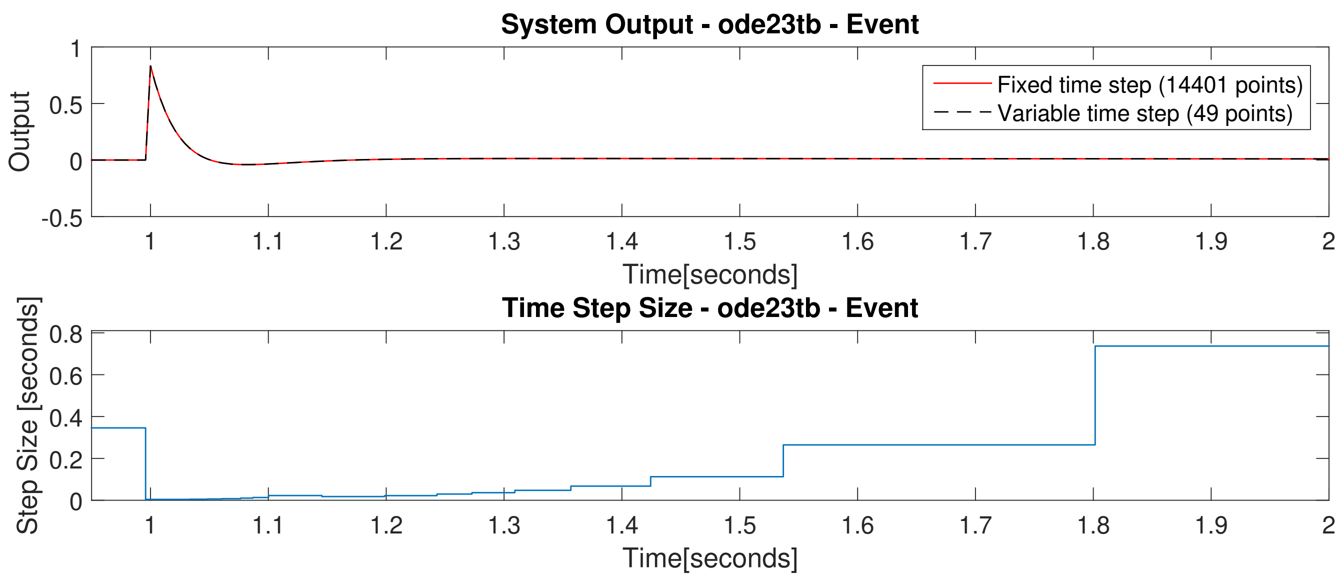

Time Step Size - ode23t - Event
Step Size [seconds]
Time[seconds]

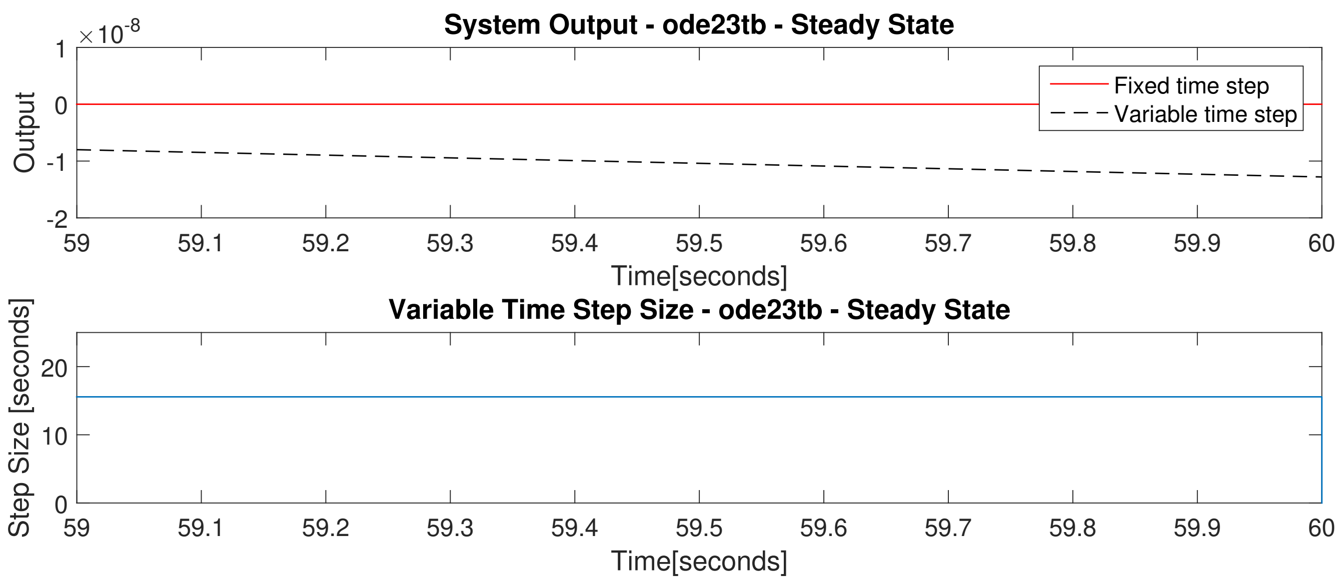

System Output - ode23t - Steady State
×10-9
Output
Time[seconds]
Fixed time step
Variable time step


Variable Time Step Size - ode23t - Steady State
Step Size [seconds]
Time[seconds]

ode23tb


System Output - ode23tb
Output
Fixed time step (14401 points)
Variable time step (49 points)
Time[seconds]
Variable Time Step Size - ode23tb


System Output - ode23tb - Event
Output
Fixed time step (14401 points)
Variable time step (49 points)
Time[seconds]
Time Step Size - ode23tb - Event
Step Size [seconds]
Time[seconds]


System Output - ode23tb - Steady State
×10-8
Output
Fixed time step
Variable time step
Time[seconds]
Variable Time Step Size - ode23tb - Steady State
Step Size [seconds]
Time[seconds]

MATLAB Code

A simple `getXdot` function is required to be passed into the ODE solver.

```
function [ xdot ] = getXdot( t, x)
%getXdot return xdot from statespace for ODE45 use
%   t = filler variable
%   A = A matrix from system
%   x = initial state vector
%   B = B matrix from system
%   U = Input to system
global A B U
    xdot = A*x + B*U;
end
```

```
%% test to use ode solvers to step PST-esq model
solverSelection = {'ode45', 'ode23', 'ode113','ode15s','ode23s','ode23t','ode23tb'}; % MATLAB
% 'ode15i' requires derivative at t=0... more thought required - availabe in octave aswell

%% pss model definition (miniWECC)
%           1   2   3   4       5   6   7       8       9       10
pss_con = [ 1           1   20  2     0.25 0.04 0.2   0.03      1.0     -1.0];

%% MATLAB model - fixed step using lsim
tend = 60;

% PSS model creation
block1 = tf([pss_con(3)*pss_con(4), 0],[pss_con(4), 1]);
block2 = tf([pss_con(5), 1],[pss_con(6), 1]);
block3 = tf([pss_con(7), 1],[pss_con(8), 1]);
G= block1*block2*block3;

% lsim input
tL = 0:1/60/4:tend; % quarter cycle steps
modSig = zeros(size(tL,1),1);
modSig(tL>=1) = .001; % very small input to avoid limiter

% fixed step solution
yL = lsim(G,modSig,tL);

%% stock solver with using statespace system
% manipulate test sytem to statespace
[num,den] = tfdata(G);
global A B U
[A,B,C,D] = tf2ss(num{1},den{1});
```

```
for slnNum = 1:length(solverSelection)
    clear t1 t2 y1 y2
    odeName =solverSelection{slnNum}; % select ode function name from cell

    % Configure ODE settings
    %options = odeset('RelTol',1e-3,'AbsTol',1e-6); % default settings
    options = odeset('RelTol',1e-5,'AbsTol',1e-8,'InitialStep', 1/60/4, 'MaxStep',20);

    % initial conditions
    x = zeros(size(A,1),1);
    y0 = x;
    U = 0;

    % Pre-perturbance time interval solution
    [t1,y1] = feval(odeName, @getXdot, [0,1-1/60/4],y0, options); % feval used for variable
    ↪  ode solver selection
    yOut1 = C*y1'+D*U; % could be handled using 'outputfunction'

    % Step input
    U = modSig(end); % magnitude from fixed step inputs
    [t2,y2] = feval(odeName, @getXdot, [1,tend],y1(end,:)', options); % second interval
    ↪  solution
    yOut2 = C*y2'+D*U;

    % combining output from variable step solution
    tCombined = [t1;t2];
    yCombined = [yOut1, yOut2];

    % calculate step size
    tStep = zeros(length(tCombined),1);
    for tNdx = 2:length(tCombined)
        tStep(tNdx-1) = tCombined(tNdx)-tCombined(tNdx-1);
    end

    % NOTE: Plotting code excluded
end
```