

IEEE PES Task Force on Benchmark Systems for Stability Controls

Ian Hiskens

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Abstract

This report summarizes a study of an IEEE 10-generator, 39-bus system. Three types of analysis were performed: load flow, small disturbance analysis, and dynamic simulation. All analysis was carried out using **MATLAB**, and this report's objective is to demonstrate how to use **MATLAB** to obtain results that are comparable to benchmark results from other analysis methods. Data from other methods may be found on the website www.sel.eesc.usp.br/ieee.

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The IEEE 39-bus system analyzed in this report is commonly known as "the 10-machine New-England Power System." This system's parameters are specified in a paper by T. Athay et al[1] and are published in a book titled 'Energy Function Analysis for Power System Stability'[2]. A diagram of the system is shown in Figure 1, and system models and parameters are introduced in the following section.

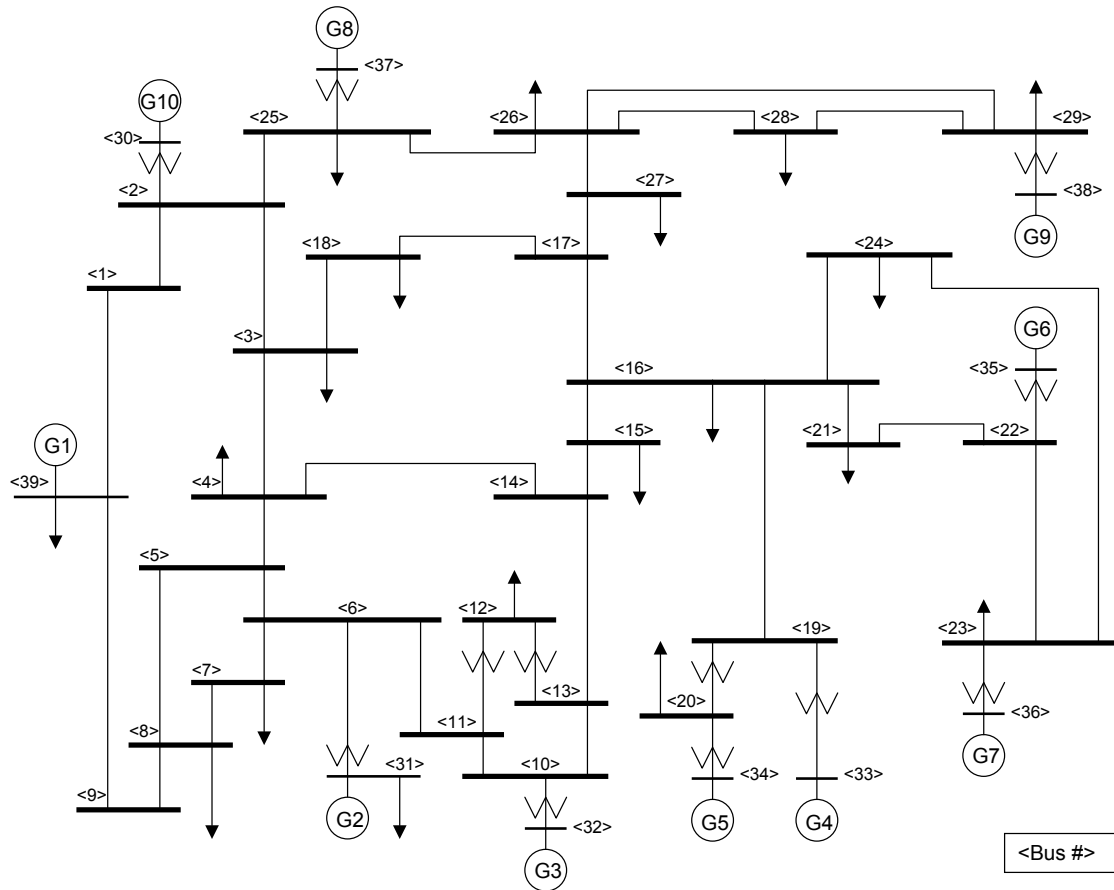


Figure 1: IEEE 39-bus network

1 System Model

1.1 Generators

1.1.1 State Variable Model

Generator analysis was carried out using a fourth-order model, as defined in the equations below. Equations 1 through 4 model the generator, while the remaining equations relate various parameters. Parameter values for the model are shown in Table 2 on the system base MVA. For more information on this generator model and its parameters, refer to Sauer and Pai pages 101-103 [3].

$$\dot{E}'_q = \frac{1}{T'_{do}}(-E'_q - (x_d - x'_d)I_d + E_{fd}) \quad (1)$$

$$\dot{E}'_d = \frac{1}{T'_{qo}}(-E'_d + (x_q - x'_q)I_q) \quad (2)$$

$$\dot{\delta} = \omega \quad (3)$$

$$\dot{\omega} = \frac{1}{M}(T_{mech} - (\phi_d I_q - \phi_q I_d) - D\omega) \quad (4)$$

$$0 = r_a I_d + \phi_q + V_d \quad (5)$$

$$0 = r_a I_q - \phi_d + V_q \quad (6)$$

$$0 = -\phi_d - x'_d I_d + E'_q \quad (7)$$

$$0 = -\phi_q - x'_q I_q - E'_d \quad (8)$$

$$0 = V_d \sin \delta + V_q \cos \delta - V_r \quad (9)$$

$$0 = V_q \sin \delta - V_d \cos \delta - V_i \quad (10)$$

$$0 = I_d \sin \delta + I_q \cos \delta - I_r \quad (11)$$

$$0 = I_q \sin \delta - I_d \cos \delta - I_i \quad (12)$$

1.1.2 Model Parameters

Generator inertia data is given in Table 1. Note that the per unit conversion for M associated with ω is in radians per second. Also, we changed the value of T'_{qo} for Unit 10 from 0.0 to 0.10.

Table 1: Generator inertia data

Unit No.	M=2*H
1	$2 \cdot 500.0/(120\pi)$
2	$2 \cdot 30.3/(120\pi)$
3	$2 \cdot 35.8/(120\pi)$
4	$2 \cdot 28.6/(120\pi)$
5	$2 \cdot 26.0/(120\pi)$
6	$2 \cdot 34.8/(120\pi)$
7	$2 \cdot 26.4/(120\pi)$
8	$2 \cdot 24.3/(120\pi)$
9	$2 \cdot 34.5/(120\pi)$
10	$2 \cdot 42.0/(120\pi)$

All other generator parameters are set according to Table 2.

Table 2: Generator data

Unit No.	H	R_a	x'_d	x'_q	x_d	x_q	T'_{do}	T'_{qo}	x_l
1	500	0	0.006	0.008	0.02	0.019	7	0.7	0.003
2	30.3	0	0.0697	0.17	0.295	0.282	6.56	1.5	0.035
3	35.8	0	0.0531	0.0876	0.2495	0.237	5.7	1.5	0.0304
4	28.6	0	0.0436	0.166	0.262	0.258	5.69	1.5	0.0295
5	26	0	0.132	0.166	0.67	0.62	5.4	0.44	0.054
6	34.8	0	0.05	0.0814	0.254	0.241	7.3	0.4	0.0224
7	26.4	0	0.049	0.186	0.295	0.292	5.66	1.5	0.0322
8	24.3	0	0.057	0.0911	0.29	0.28	6.7	0.41	0.028
9	34.5	0	0.057	0.0587	0.2106	0.205	4.79	1.96	0.0298
10	42	0	0.031	0.008	0.1	0.069	10.2	0	0.0125

1.1.3 AVR Model

All generators in the system are equipped with automatic voltage regulators (AVRs). We chose to use static AVRs with E_{fd} limiters. The model for this controller is shown in Figure 2 below.

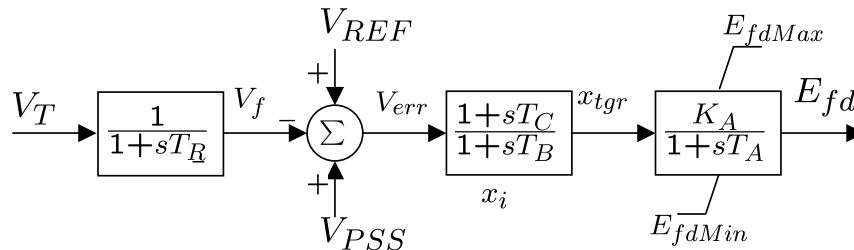


Figure 2: AVR block diagram

AVR parameters are related according to equations 13 to 24.

$$\dot{V}_f = \frac{1}{T_R}(V_T - V_f) \quad (13)$$

$$\dot{E}_{fd} = \frac{1}{T_A}(K_A x_{tgr} - E_{fd}) \quad (14)$$

$$\dot{x}_i = x_{err} - x_{tgr} \quad (15)$$

$$\dot{V}_{REF} = \begin{cases} 0 & \text{when } t > 0 \\ V_T - V_{\text{setpoint}} & \text{when } t < 0 \end{cases} \quad (16)$$

$$0 = V_T^2 - (V_r^2 + V_i^2) \quad (17)$$

$$0 = T_B x_{tgr} - T_C x_{err} - x_i \quad (18)$$

$$0 = x_{err} - (V_{REF} + V_{PSS} - V_f) \quad (19)$$

$$0 = E_{fd} - E_{fd} \quad (20)$$

$$\text{Upper Limit Detector} < 0 \begin{cases} 0 = \text{Upper Limit Switch} - 1 \\ 0 = \text{Upper Limit Detector} - E_{fd} - E_{fd,\text{Max}} \end{cases} \quad (21)$$

$$\text{Upper Limit Detector} > 0 \begin{cases} 0 = \text{Upper Limit Switch} \\ 0 = \text{Upper Limit Detector} - (K_A x_{tgr} - E_{fd}) \end{cases} \quad (22)$$

$$\text{Lower Limit Detector} > 0 \begin{cases} 0 = \text{Lower Limit Switch} - 1 \\ 0 = \text{Lower Limit Detector} - E_{fd} - E_{fd,\text{Min}} \end{cases} \quad (23)$$

$$\text{Lower Limit Detector} < 0 \begin{cases} 0 = \text{Lower Limit Switch} \\ 0 = \text{Lower Limit Detector} - (K_A x_{tgr} - E_{fd}) \end{cases} \quad (24)$$

1.1.4 AVR Parameters

AVR parameters, as defined in the previous section, are specified in Table 3.

Table 3: Generator AVR parameters

Unit No.	T_R	K_A	T_A	T_B	T_C	V_{setpoint}	$E_{fd,\text{Max}}$	$E_{fd,\text{Min}}$
1	0.01	200.0	0.015	10.0	1.0	1.0300	5.0	-5.0
2	0.01	200.0	0.015	10.0	1.0	0.9820	5.0	-5.0
3	0.01	200.0	0.015	10.0	1.0	0.9831	5.0	-5.0
4	0.01	200.0	0.015	10.0	1.0	0.9972	5.0	-5.0
5	0.01	200.0	0.015	10.0	1.0	1.0123	5.0	-5.0
6	0.01	200.0	0.015	10.0	1.0	1.0493	5.0	-5.0
7	0.01	200.0	0.015	10.0	1.0	1.0635	5.0	-5.0
8	0.01	200.0	0.015	10.0	1.0	1.0278	5.0	-5.0
9	0.01	200.0	0.015	10.0	1.0	1.0265	5.0	-5.0
10	0.01	200.0	0.015	10.0	1.0	1.0475	5.0	-5.0

1.1.5 PSS Model

Each generator in the system is equipped with a δ and ω type PSS with two phase shift blocks. The model for this PSS is shown in Figure 3 below.

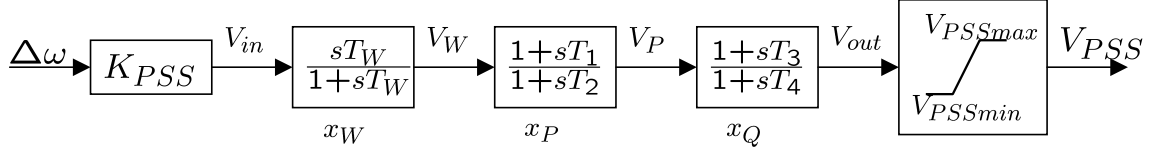


Figure 3: PSS block diagram

PSS parameters are related according to equations 25 to 33.

$$\dot{x}_W = \frac{V_W}{T_W} \quad (25)$$

$$\dot{x}_P = V_W - V_P \quad (26)$$

$$\dot{x}_Q = V_P - V_{out} \quad (27)$$

$$0 = \omega K_{PSS} - V_W - x_W \quad (28)$$

$$0 = V_P T_2 - V_W T_1 - x_P \quad (29)$$

$$0 = V_{out} T_4 - V_P T_3 - x_Q \quad (30)$$

$$0 = x_{PSS,Max} - V_{out} - (V_{PSS,Max} - V_{out}) \quad (31)$$

$$0 = V_{out} - x_{PSS,Min} - (V_{out} - V_{PSS,Min}) \quad (32)$$

$$0 = \begin{cases} V_{PSS} - x_{PSS,Max} & \text{when } (V_{PSS,Max} - V_{out}) < 0 \\ V_{PSS} - x_{PSS,Min} & \text{when } (V_{out} - V_{PSS,Min}) < 0 \\ V_{PSS} - V_{out} & \text{otherwise} \end{cases} \quad (33)$$

1.1.6 PSS Parameters

PSS parameters defined in the previous section are specified according to Table 4.

Table 4: Generator PSS parameters

Unit No.	K	T_W	T_1	T_2	T_3	T_4	$V_{PSS,Max}$	$V_{PSS,Min}$
1	$1.0/(120\pi)$	10.0	5.0	0.60	3.0	0.50	0.2	-0.2
2	$0.5/(120\pi)$	10.0	5.0	0.40	1.0	0.10	0.2	-0.2
3	$0.5/(120\pi)$	10.0	3.0	0.20	2.0	0.20	0.2	-0.2
4	$2.0/(120\pi)$	10.0	1.0	0.10	1.0	0.30	0.2	-0.2
5	$1.0/(120\pi)$	10.0	1.5	0.20	1.0	0.10	0.2	-0.2
6	$4.0/(120\pi)$	10.0	0.5	0.10	0.5	0.05	0.2	-0.2
7	$7.5/(120\pi)$	10.0	0.2	0.02	0.5	0.10	0.2	-0.2
8	$2.0/(120\pi)$	10.0	1.0	0.20	1.0	0.10	0.2	-0.2
9	$2.0/(120\pi)$	10.0	1.0	0.50	2.0	0.10	0.2	-0.2
10	$1.0/(120\pi)$	10.0	1.0	0.05	3.0	0.50	0.2	-0.2

1.1.7 Governor Model

No governor dynamics are included in our analysis, and each generator's mechanical torque is assumed to be constant. Since Unit 2 is the angle reference and resides at the swing node, $P_{set \text{ point}}$ is determined by the

power flow initialization. The value of $P_{\text{set point}}$ is given for each generator in Table 5 on the system base of 100 MVA.

Table 5: Generator setpoint data

Unit No.	$P_{\text{set point}}$
1	10.00
2	-
3	6.50
4	6.32
5	5.08
6	6.50
7	5.60
8	5.40
9	8.30
10	2.50

1.2 Loads

1.2.1 Load Model

We use a constant impedance load model in our analysis. Loads modeled this way are voltage dependent and behave according to the following equations.

$$\text{if } t < 0 \begin{cases} 0 = P_0 + V_r I_r + V_i I_i \\ 0 = Q_0 + V_i I_r - V_r I_i \\ V = (V_r^2 + V_i^2)^2 \end{cases} \quad (34)$$

$$\text{if } t > 0 \begin{cases} 0 = P_0 \left(\frac{V}{V_0} \right)^\alpha + V_r I_r + V_i I_i \\ 0 = Q_0 \left(\frac{V}{V_0} \right)^\beta + V_i I_r - V_r I_i \\ V = (V_r^2 + V_i^2)^2 \end{cases} \quad (35)$$

1.2.2 Load Parameters

Table 6 contains load behavior at initial voltage. Due to the voltage dependence discussed above, these values may not be accurate after voltages change.

Table 6: Active and reactive power draws for all loads at initial voltage

Bus	Load	
	P [PU]	Q [pu]
1	0.000	0.000
2	0.000	0.000
3	3.220	0.024
4	5.000	1.840
5	0.000	0.000
6	0.000	0.000
7	2.338	0.840
8	5.220	1.760
9	0.000	0.000
10	0.000	0.000
11	0.000	0.000
12	0.075	0.880
13	0.000	0.000
14	0.000	0.000
15	3.200	1.530
16	3.290	0.323
17	0.000	0.000
18	1.580	0.300
19	0.000	0.000
20	6.280	1.030
21	2.740	1.150
22	0.000	0.000
23	2.475	0.846
24	3.086	-0.920
25	2.240	0.472
26	1.390	0.170
27	2.810	0.755
28	2.060	0.276
29	2.835	0.269
31	0.092	0.046
39	11.040	2.500

1.3 Lines and Transformers

Network data for the system is shown in Table 7. As with generator data, all values are given on the system base MVA at 60 Hz.

Table 7: Network data

Line Data					Transformer Tap	
From Bus	To Bus	R	X	B	Magnitude	Angle
1	2	0.0035	0.0411	0.6987	-	-
1	39	0.001	0.025	0.75	-	-
2	3	0.0013	0.0151	0.2572	-	-
2	25	0.007	0.0086	0.146	-	-
3	4	0.0013	0.0213	0.2214	-	-
3	18	0.0011	0.0133	0.2138	-	-
4	5	0.0008	0.0128	0.1342	-	-
4	14	0.0008	0.0129	0.1382	-	-
5	6	0.0002	0.0026	0.0434	-	-
5	8	0.0008	0.0112	0.1476	-	-
6	7	0.0006	0.0092	0.113	-	-
6	11	0.0007	0.0082	0.1389	-	-
7	8	0.0004	0.0046	0.078	-	-
8	9	0.0023	0.0363	0.3804	-	-
9	39	0.001	0.025	1.2	-	-
10	11	0.0004	0.0043	0.0729	-	-
10	13	0.0004	0.0043	0.0729	-	-
13	14	0.0009	0.0101	0.1723	-	-
14	15	0.0018	0.0217	0.366	-	-
15	16	0.0009	0.0094	0.171	-	-
16	17	0.0007	0.0089	0.1342	-	-
16	19	0.0016	0.0195	0.304	-	-
16	21	0.0008	0.0135	0.2548	-	-
16	24	0.0003	0.0059	0.068	-	-
17	18	0.0007	0.0082	0.1319	-	-
17	27	0.0013	0.0173	0.3216	-	-
21	22	0.0008	0.014	0.2565	-	-
22	23	0.0006	0.0096	0.1846	-	-
23	24	0.0022	0.035	0.361	-	-
25	26	0.0032	0.0323	0.513	-	-
26	27	0.0014	0.0147	0.2396	-	-
26	28	0.0043	0.0474	0.7802	-	-
26	29	0.0057	0.0625	1.029	-	-
28	29	0.0014	0.0151	0.249	-	-
12	11	0.0016	0.0435	0	1.006	0
12	13	0.0016	0.0435	0	1.006	0
6	31	0	0.025	0	1.07	0
10	32	0	0.02	0	1.07	0
19	33	0.0007	0.0142	0	1.07	0
20	34	0.0009	0.018	0	1.009	0
22	35	0	0.0143	0	1.025	0
23	36	0.0005	0.0272	0	1	0
25	37	0.0006	0.0232	0	1.025	0
2	30	0	0.0181	0	1.025	0
29	38	0.0008	0.0156	0	1.025	0
19	20	0.0007	0.0138	0	1.06	0

1.4 Power and Voltage Setpoints

Table 8 contains power and voltage setpoint data, specified on the system MVA base. Note that Generator 2 is the swing node, and Generator 1 represents the aggregation of a large number of generators.

Table 8: Power and voltage setpoint data

Bus	Type	Voltage	Load		Generator		Unit No.
		[PU]	MW	MVar	MW	MVar	
1	PQ	-	0	0	0	0	
2	PQ	-	0	0	0	0	
3	PQ	-	322	2.4	0	0	
4	PQ	-	500	184	0	0	
5	PQ	-	0	0	0	0	
6	PQ	-	0	0	0	0	
7	PQ	-	233.8	84	0	0	
8	PQ	-	522	176	0	0	
9	PQ	-	0	0	0	0	
10	PQ	-	0	0	0	0	
11	PQ	-	0	0	0	0	
12	PQ	-	7.5	88	0	0	
13	PQ	-	0	0	0	0	
14	PQ	-	0	0	0	0	
15	PQ	-	320	153	0	0	
16	PQ	-	329	32.3	0	0	
17	PQ	-	0	0	0	0	
18	PQ	-	158	30	0	0	
19	PQ	-	0	0	0	0	
20	PQ	-	628	103	0	0	
21	PQ	-	274	115	0	0	
22	PQ	-	0	0	0	0	
23	PQ	-	247.5	84.6	0	0	
24	PQ	-	308.6	-92	0	0	
25	PQ	-	224	47.2	0	0	
26	PQ	-	139	17	0	0	
27	PQ	-	281	75.5	0	0	
28	PQ	-	206	27.6	0	0	
29	PQ	-	283.5	26.9	0	0	
30	PV	1.0475	0	0	250	-	Gen10
31	PV	0.982	9.2	4.6	-	-	Gen2
32	PV	0.9831	0	0	650	-	Gen3
33	PV	0.9972	0	0	632	-	Gen4
34	PV	1.0123	0	0	508	-	Gen5
35	PV	1.0493	0	0	650	-	Gen6
36	PV	1.0635	0	0	560	-	Gen7
37	PV	1.0278	0	0	540	-	Gen8
38	PV	1.0265	0	0	830	-	Gen9
39	PV	1.03	1104	250	1000	-	Gen1

This completes the description of the system and its model. The remainder of this report focuses on the three analyses performed on the system: load flow, small signal stability assessment via Eigenvalue calculation, and numerical simulation. The results of load flow analysis are presented in the next section.

2 Load Flow Results

Load flow for the system was calculated using **MATLAB**. The results are in Table 9. Note that all voltages, active power values, and reactive power values are given in per unit on the system MVA base.

For a more complete description of power flow, including flow on each line, see Section 5.2 in the Appendix.

Table 9: Power flow results

Bus	V	Angle [deg]	Bus Total		Load		Generator		Unit No.
			P	Q	P	Q	P	Q	
1	1.0474	-8.44	0	0	0	0			
2	1.0487	-5.75	0	0	0	0			
3	1.0302	-8.6	-322	-2.4	-322	-2.4			
4	1.0039	-9.61	-500	-184	-500	-184			
5	1.0053	-8.61	0	0	0	0			
6	1.0077	-7.95	0	0	0	0			
7	0.997	-10.12	-233.8	-84	-233.8	-84			
8	0.996	-10.62	-522	-176	-522	-176			
9	1.0282	-10.32	0	0	0	0			
10	1.0172	-5.43	0	0	0	0			
11	1.0127	-6.28	0	0	0	0			
12	1.0002	-6.24	-7.5	-88	-7.5	-88			
13	1.0143	-6.1	0	0	0	0			
14	1.0117	-7.66	0	0	0	0			
15	1.0154	-7.74	-320	-153	-320	-153			
16	1.0318	-6.19	-329	-32.3	-329	-32.3			
17	1.0336	-7.3	0	0	0	0			
18	1.0309	-8.22	-158	-30	-158	-30			
19	1.0499	-1.02	0	0	0	0			
20	0.9912	-2.01	-628	-103	-628	-103			
21	1.0318	-3.78	-274	-115	-274	-115			
22	1.0498	0.67	0	0	0	0			
23	1.0448	0.47	-247.5	-84.6	-247.5	-84.6			
24	1.0373	-6.07	-308.6	92.2	-308.6	92.2			
25	1.0576	-4.36	-224	-47.2	-224	-47.2			
26	1.0521	-5.53	-139	-17	-139	-17			
27	1.0377	-7.5	-281	-75.5	-281	-75.5			
28	1.0501	-2.01	-206	-27.6	-206	-27.6			
29	1.0499	0.74	-283.5	-26.9	-283.5	-26.9			
30	1.0475	-3.33	250	146.16			250	146.16	10
31	0.982	0	511.61	193.65	-9.2	-4.6	520.81	198.25	2
32	0.9831	2.57	650	205.14			650	205.14	3
33	0.9972	4.19	632	109.91			632	109.91	4
34	1.0123	3.17	508	165.76			508	165.76	5
35	1.0493	5.63	650	212.41			650	212.41	6
36	1.0635	8.32	560	101.17			560	101.17	7
37	1.0278	2.42	540	0.44			540	0.44	8
38	1.0265	7.81	830	22.84			830	22.84	9
39	1.03	-10.05	-104	-161.7	-1104	-250	1000	88.28	1

3 Small Disturbance Analysis

Small signal stability was assessed by calculating Eigenvalues of the system. To find Eigenvalues, we first calculated reduced A matrices as follows:

$$\begin{aligned} \dot{x} &= f(x, y) \mid 0 = g(x, y) \\ \text{Differential eq.} \quad & \text{Algebraic eq.} \\ \text{Linearize:} \quad & \Delta \dot{x} = f_x \Delta x + f_y \Delta y \mid 0 = g_x \Delta x + g_y \Delta y \\ \text{Eliminate algebraic equations:} \quad & \Delta \dot{x} = (f_x - f_y g_y^{-1} g_x) \Delta x \\ & \text{Reduced } A \text{ matrices} \end{aligned}$$

Eigenvalues, which correspond to machine oscillatory modes, are calculated from the reduced A matrices. Some Eigenvalues of our system are given in Table 10 below. To access all Eigenvalues calculated in our analysis, see Section 5.1 in the Appendix.

Table 10: Eigenvalues calculated through small disturbance analysis

Index	Real part	Imaginary part
27	-2.553	j10.566
29	-1.8494	j10.028
31	-1.5817	j8.5503
33	-2.5633	j8.6706
35	-1.8626	j7.4388
37	-1.3118	j7.1081
39	-1.8437	j7.0812
41	-1.523	j6.3180
57	-2.9563	j2.5076

The Eigenvalues in Table 10 are plotted in Figure 4.

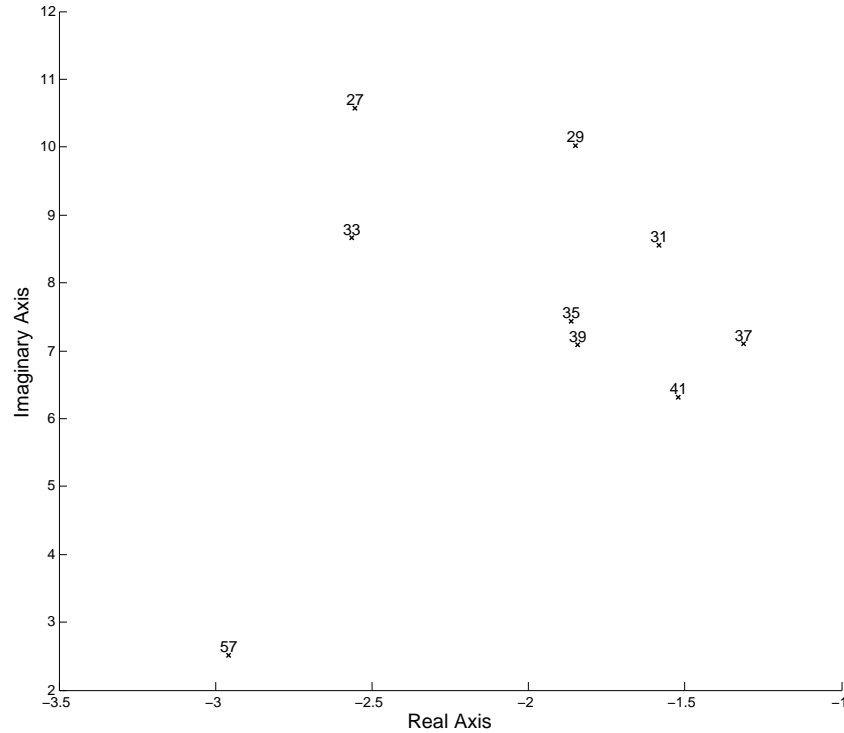


Figure 4: Graphical depiction of Eigenvalues

4 Dynamic Simulation

Dynamic simulation was performed for a three-phase fault at Bus 16 occurring at $t = 0.5\text{s}$. The fault impedance is 0.001 PU, and it is cleared at $t = 0.7\text{s}$. The simulation method used was numerical trapezoidal integration with a time step of 20 ms.

Figure 5 shows the generator state δ as a function of time for the ten generators, with Unit 1 as the angle reference. Figure 6 on the following page shows the generator state ω as a function of time for the ten generators.

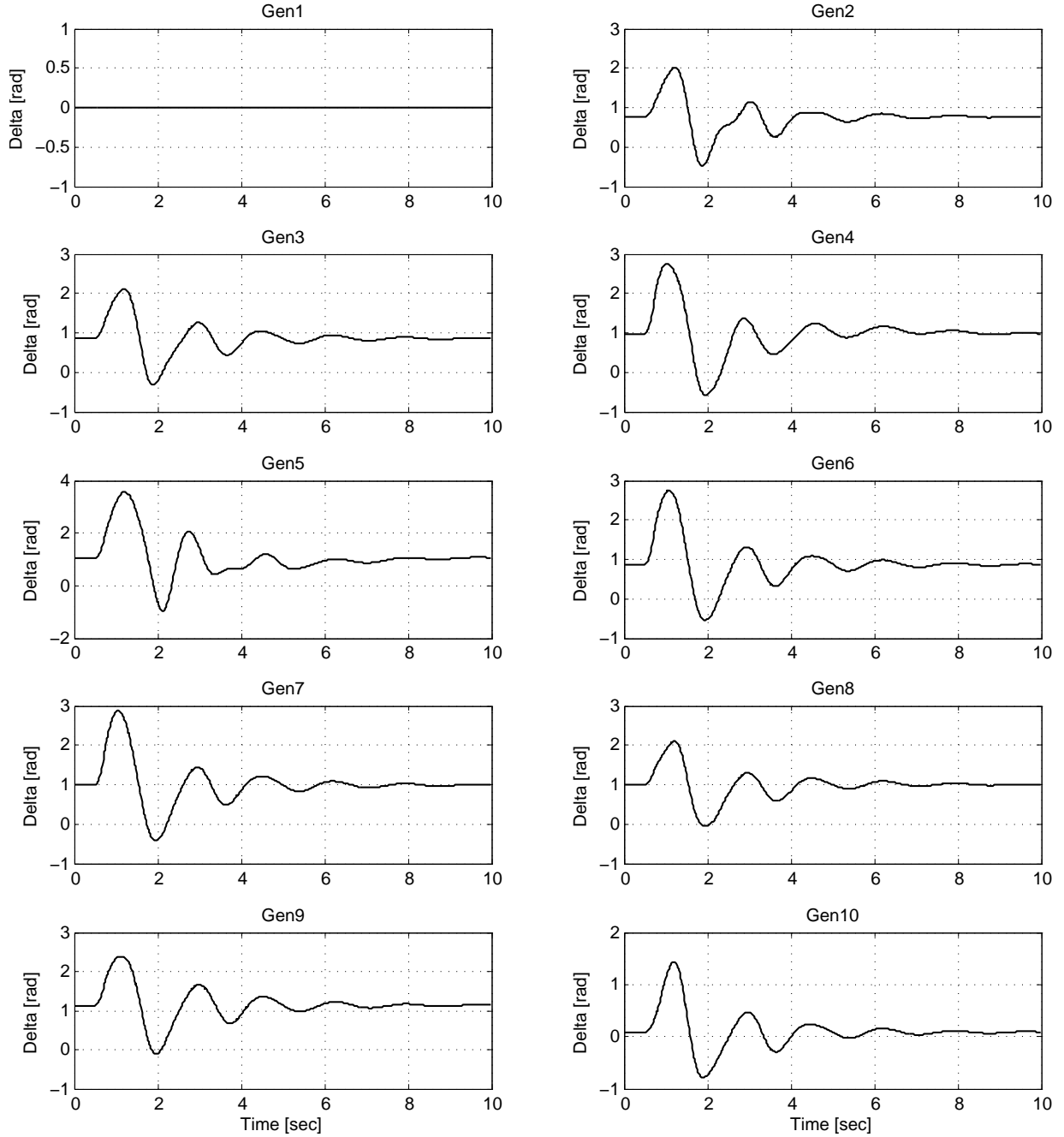


Figure 5: δ (in radians) versus time for generators

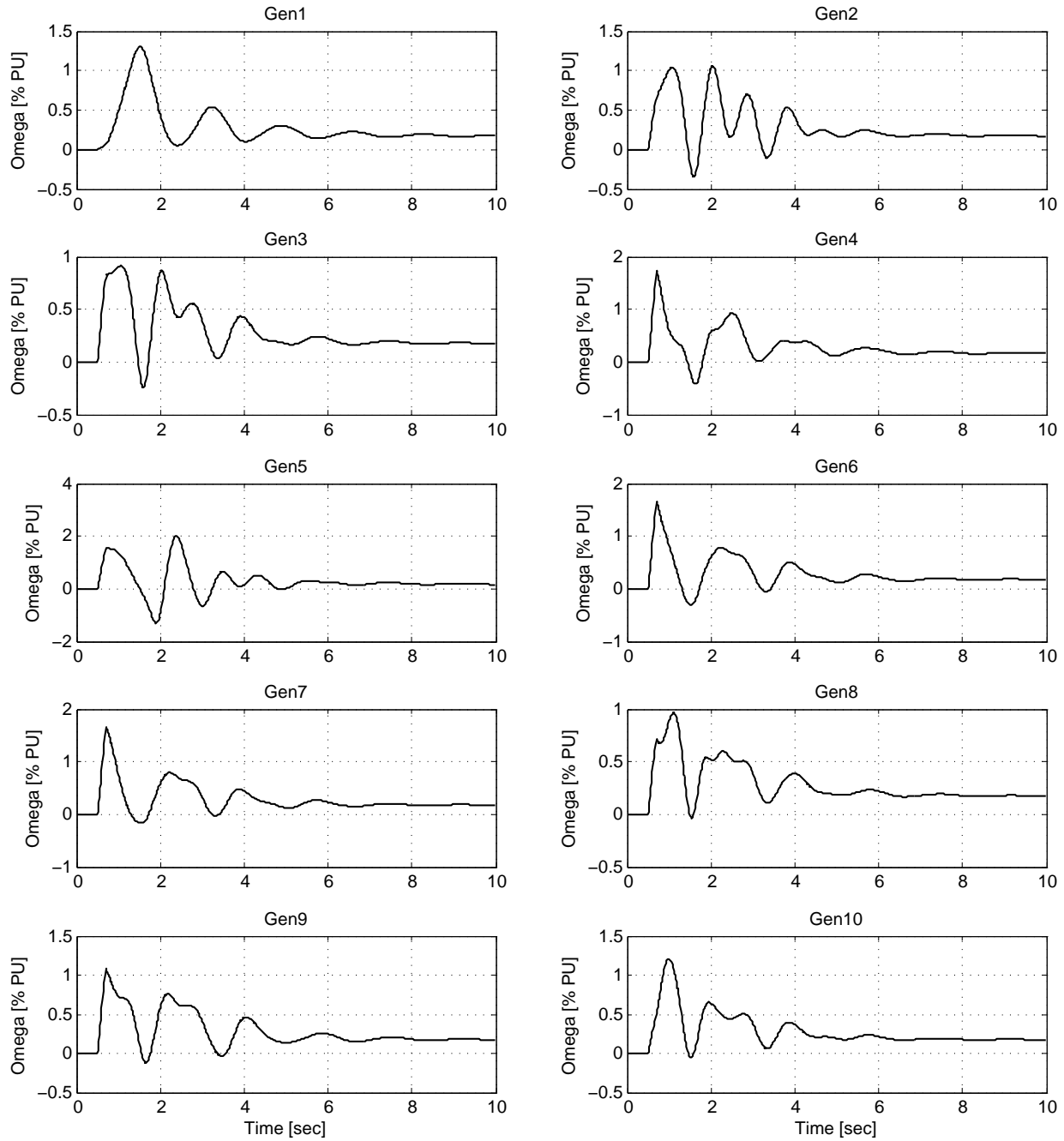


Figure 6: ω (in percent per-unit) for generators

To generate these and other figures from the MATLAB files available on the website, see "Accessing Simulation Data with MATLAB" in the appendix.

5 Appendix

5.1 Accessing Simulation Data with MATLAB®

There are seven MATLAB® `.mat` files available for download. Each `.mat` file has a `csv` counterpart for those who prefer to work with Excel or another CSV-friendly program (the procedure described here may be adapted for such programs). Each file is described below.

- `time.mat` is a row vector with 677 elements. This vector represents the simulation time period with increments of 20ms. To observe the variation of any model variable with respect to time, simply plot the corresponding row of `x.mat` or `y.mat` versus `time.mat`.
- `x.mat` contains 209 row vectors, each corresponding to a model state variable. See Table 11.
- `x0.mat` contains 209 elements corresponding to initial values of model state variables. It is indexed the same as `x.mat`.
- `y.mat` contains 617 row vectors, each corresponding to a non-state variable in the model. See Table 11.
- `y0.mat` contains 617 elements corresponding to initial values of non-state variables. It is indexed the same as `y.mat`.
- `Aeig.mat` contains 209 complex elements corresponding to system Eigenvalues. To obtain any entry of Table 10, one need only extract the element of `Aeig` corresponding to that entry's Index. Of course, there are many more Eigenvalues stored in `Aeig` than those listed in Table 10, and all Eigenvalues may be easily plotted so long as care is taken in isolating the real and imaginary parts.
- `Ared.mat` is a 209-by-209 variable containing reduced A matrix data for the system. (Reduced A matrices are discussed in Section 3.)

Model state variables are stored in `x.mat` while non-state variables are located in `y.mat`. Table 11 maps from these files to specific model variables by specifying the range of variables corresponding to each component of the model. The order of variables within a component's range is discussed in the next five subsections.

Table 11: Mapping from MATLAB® files to simulation data

Model	Model ID	x range		y range		Description
		from	to	from	to	
Generator	G1	1	4	1	13	4 x states and 13 y variables
	G10	37	49	118	130	
AVR	G1	110	114	438	448	5 x states and 11 y variables
	G10	155	159	537	547	
PSS	G1	160	164	548	554	5 x states and 7 y variables
	G10	205	209	611	617	
Load	Bus1	47	48	303	306	2 x states and 4 y variables (x states are real and reactive power)
	Bus39	107	108	423	426	
Network	Bus1			131	134	4 y variables
	Bus39			283	286	

5.1.1 Variable Indexing

This section defines all variables contained in the MATLAB files. When writing code to extract specific variables, refer to this section to determine which indices to use.

Generator Variables (See Section 1.1.1)

	y_1	:	V_r
	y_2	:	V_i
	y_3	:	I_r
	y_4	:	I_i
	y_5	:	V_d
	y_6	:	V_q
	y_7	:	I_d
	y_8	:	I_q
	y_9	:	ψ_d
	y_{10}	:	ψ_q
	y_{11}	:	E_{fd}
	y_{12}	:	ω
	y_{13}	:	T_{mech}
x_1	:	E'_q	
x_2	:	E'_d	
x_3	:	δ	
x_4	:	ω	

AVR Variables (See Section 1.1.3)

	y_1	:	V_r
	y_2	:	V_i
	y_3	:	V_T
	y_4	:	V_{PSS}
	y_5	:	x_{err}
	y_6	:	x_{tgr}
	y_7	:	E_{fd}
	y_8	:	Upper Limit Detector
	y_9	:	Lower Limit Detector
	y_{10}	:	Upper Limit Switch
	y_{11}	:	Lower Limit Switch
x_1	:	V_f	
x_2	:	E_{fd}	
x_3	:	x_i	
x_4	:	V_{REF}	

PSS Variables (See Section 1.1.5)

	y_1	:	ω (input)
	y_2	:	V_W
	y_3	:	V_P
	y_4	:	V_{out}
	y_5	:	V_{PSS}
	y_6	:	$V_{PSS,Max} - V_{out}$
	y_7	:	$V_{out} - V_{PSS,Min}$
x_1	:	x_W	
x_2	:	x_P	
x_3	:	x_Q	

Load Variables (See Section 1.2.1)

	y_1	:	V_r
	y_2	:	V_i
	y_3	:	I_r
	y_4	:	I_i
x_1	:	P	
x_2	:	Q	

Network Variables (See ??) Network variables are indexed in the MATLAB files as follows:

5.1.2 Example Application

The code below will generate Figures 5 and 6. By modifying or adapting this code, the reader may plot or manipulate any variables in `x.mat` and `y.mat`.

(Note: The placeholders "TIMEPATH", "XPATH", and "YPATH" should be replaced with paths to the respective files on your computer.)


```

%% Import Data
% Import time.mat
newData = load('-mat', 'TIMEPATH\time.mat');

% Create new variables in the base workspace from those fields.
vars = fieldnames(newData);
for i = 1:length(vars)
    assignin('base', vars{i}, newData.(vars{i}));
end

% Import x.mat
newData = load('-mat', 'XPATH\x.mat');

% Create new variables in the base workspace from those fields.
vars = fieldnames(newData);
for i = 1:length(vars)
    assignin('base', vars{i}, newData.(vars{i}));
end

% Import y.mat
newData = load('-mat', 'YPATH\y.mat');

% Create new variables in the base workspace from those fields.
vars = fieldnames(newData);
for i = 1:length(vars)
    assignin('base', vars{i}, newData.(vars{i}));
end

clear newData, clear vars

%% Plot x or y vs. time
% Plot omega vs time
figure('name','Generator Omega vs Time')
for i = 1:10
    subplot(5,2,i);
    p = plot(time,x(i*4 - 0,:)/4,'k'); % Note factor of 1/4
    set(p,'LineWidth',1);
    figtitle = strcat('Gen',num2str(i));
    title(figtitle)
    if or(i==9,i==10)
        xlabel('Time [sec]')
    end
    ylabel('Omega [% PU]')
    grid on
end

% Plot delta vs time
figure('name','Generator Delta vs Time')
for i = 1:10
    subplot(5,2,i);
    p = plot(time,x(i*4 - 1,:)-x(3,:), 'k'); % Subtract Unit 1 (angle reference)
    set(p,'LineWidth',1);

```

```
    figtitle = strcat('Gen',num2str(i));  
    title(figtitle)  
    if or(i==9,i==10)  
        xlabel('Time [sec]')  
    end  
    ylabel('Delta [rad]')  
    grid on  
end
```

5.2 Contents of file 39bus.out

The next four pages show the contents of the file 39bus.out. This file contains power flow data with enough granularity to observe flows on all lines.

IEEE 39 bus test system													39bus.out	
BUS Bus1	100.0	1.0474PU 104.7KV	-8.44		CKT	MW	MVAR	MVA	MW(NOM)	MVR(NOM)	MW(LOSS)	MVAR(LOSS)	TAP	
				TO Bus2	100.0	1	-124.34	-28.32	127.52		0.50	-70.92		
				TO Gen39	100.0	1	124.34	28.32	127.52		0.18	-76.30		
BUS Bus10	100.0	1.0172PU 101.7KV	-5.43		CKT	MW	MVAR	MVA	MW(NOM)	MVR(NOM)	MW(LOSS)	MVAR(LOSS)	TAP	
				TO Bus11	100.0	1	365.25	70.36	371.96		0.54	-1.74		
				TO Bus13	100.0	1	284.75	38.65	287.37		0.32	-4.07		
				TO Gen32	100.0	1	-650.00	-109.01	659.08		0.00	96.14	1.0700FX	
BUS Bus11	100.0	1.0127PU 101.3KV	-6.28		CKT	MW	MVAR	MVA	MW(NOM)	MVR(NOM)	MW(LOSS)	MVAR(LOSS)	TAP	
				TO Bus6	100.0	1	364.76	29.01	365.92		0.92	-3.43		
				TO Bus10	100.0	1	-364.71	-72.10	371.77		0.54	-1.74		
				TO Bus12	100.0	1	-0.06	43.09	43.09		0.03	0.79	1.0060UN	
BUS Bus12	100.0	1.0002PU 100.0KV	-6.24		CKT	MW	MVAR	MVA	MW(NOM)	MVR(NOM)	MW(LOSS)	MVAR(LOSS)	TAP	
				LOAD			7.50	88.00	88.32					
				TO Bus11	100.0	1	0.09	-42.30	42.30		0.03	0.79	1.0060FX	
				TO Bus13	100.0	1	-7.59	-45.70	46.32		0.03	0.94	1.0060FX	
BUS Bus13	100.0	1.0143PU 101.4KV	-6.10		CKT	MW	MVAR	MVA	MW(NOM)	MVR(NOM)	MW(LOSS)	MVAR(LOSS)	TAP	
				TO Bus10	100.0	1	-284.43	-42.72	287.62		0.32	-4.07		
				TO Bus14	100.0	1	276.81	-3.92	276.84		0.67	-10.16		
				TO Bus12	100.0	1	7.62	46.64	47.26		0.03	0.94	1.0060UN	
BUS Bus14	100.0	1.0117PU 101.2KV	-7.66		CKT	MW	MVAR	MVA	MW(NOM)	MVR(NOM)	MW(LOSS)	MVAR(LOSS)	TAP	
				TO Bus4	100.0	1	271.01	42.41	274.31		0.59	-4.47		
				TO Bus13	100.0	1	-276.14	-6.23	276.21		0.67	-10.16		
				TO Bus15	100.0	1	5.14	-36.17	36.54		0.01	-37.53		
BUS Bus15	100.0	1.0154PU 101.5KV	-7.74		CKT	MW	MVAR	MVA	MW(NOM)	MVR(NOM)	MW(LOSS)	MVAR(LOSS)	TAP	
				LOAD			320.00	153.00	354.70					
				TO Bus14	100.0	1	-5.13	-1.36	5.31		0.01	-37.53		
				TO Bus16	100.0	1	-314.87	-151.64	349.48		1.04	-7.02		
BUS Bus16	100.0	1.0318PU 103.2KV	-6.19		CKT	MW	MVAR	MVA	MW(NOM)	MVR(NOM)	MW(LOSS)	MVAR(LOSS)	TAP	
				LOAD			329.00	32.30	330.58					
				TO Bus15	100.0	1	315.91	144.63	347.45		1.04	-7.02		
				TO Bus17	100.0	1	230.04	-43.63	234.14		0.36	-9.78		
				TO Bus19	100.0	1	-502.68	-48.08	504.97		3.81	13.54		
				TO Bus21	100.0	1	-329.60	13.03	329.85		0.82	-13.26		
				TO Bus24	100.0	1	-42.68	-98.24	107.11		0.03	-6.68		
BUS Bus17	100.0	1.0336PU 103.4KV	-7.30		CKT	MW	MVAR	MVA	MW(NOM)	MVR(NOM)	MW(LOSS)	MVAR(LOSS)	TAP	
				TO Bus16	100.0	1	-229.68	33.85	232.16		0.36	-9.78		
				TO Bus18	100.0	1	210.66	9.73	210.88		0.29	-10.63		
				TO Bus27	100.0	1	19.02	-43.58	47.55		0.01	-34.32		
BUS Bus18	100.0	1.0309PU 103.1KV	-8.22		CKT	MW	MVAR	MVA	MW(NOM)	MVR(NOM)	MW(LOSS)	MVAR(LOSS)	TAP	
				LOAD			158.00	30.00	160.82					
				TO Bus3	100.0	1	52.36	-9.64	53.24		0.03	-22.36		
				TO Bus17	100.0	1	-210.36	-20.36	211.35		0.29	-10.63		
BUS Bus19	100.0	1.0499PU 105.0KV	-1.02		CKT	MW	MVAR	MVA	MW(NOM)	MVR(NOM)	MW(LOSS)	MVAR(LOSS)	TAP	
				TO Bus16	100.0	1	506.49	61.62	510.22		3.81	13.54		
				TO Gen33	100.0	1	-629.10	-51.14	631.18		2.90	58.76	1.0700FX	
				TO Bus20	100.0	1	122.62	-10.48	123.06		0.11	2.13	1.0600FX	
BUS Bus2	100.0	1.0487PU 104.9KV	-5.75		CKT	MW	MVAR	MVA	MW(NOM)	MVR(NOM)	MW(LOSS)	MVAR(LOSS)	TAP	
				TO Bus1	100.0	1	124.83	-42.60	131.90		0.50	-70.92		
				TO Bus3	100.0	1	364.26	92.24	375.76		1.70	-8.02		
				TO Bus25	100.0	1	-239.09	82.68	252.98		4.16	-11.08		
				TO Gen30	100.0	1	-250.00	-132.32	282.86		0.00	13.83	1.0250FX	
BUS Bus20	100.0	0.9912PU 99.1KV	-2.01		CKT	MW	MVAR	MVA	MW(NOM)	MVR(NOM)	MW(LOSS)	MVAR(LOSS)	TAP	
				LOAD			628.00	103.00	636.39					

Page 1

Figure 7: Contents of file 39bus.out (Page 1 of 4)

Page 2

39bus.out												
BUS Bus6	100.0	1.0077PU 100.8KV	-7.95	CTK	MW	MVAR	MVA	MW(NOM)	MVR(NOM)	MW(LOSS)	MVAR(LOSS)	TAP
				TO Bus5	100.0	1	454.84	56.94	458.39	0.41	0.99	
				TO Bus7	100.0	1	420.62	91.57	430.47	1.10	5.53	
				TO Bus11	100.0	1	-363.85	-32.44	365.29	0.92	-3.43	
				TO Gen31	100.0	1	-511.61	-116.07	524.61	0.00	77.58	1.0700FX
BUS Bus7	100.0	0.9970PU 99.7KV	-10.12	CTK	MW	MVAR	MVA	MW(NOM)	MVR(NOM)	MW(LOSS)	MVAR(LOSS)	TAP
				LOAD			233.80	84.00	248.43			
				TO Bus6	100.0	1	-419.52	-86.04	428.25	1.10	5.53	
				TO Bus8	100.0	1	185.72	2.04	185.73	0.14	-6.15	
BUS Bus8	100.0	0.9960PU 99.6KV	-10.62	CTK	MW	MVAR	MVA	MW(NOM)	MVR(NOM)	MW(LOSS)	MVAR(LOSS)	TAP
				LOAD			522.00	176.00	550.87			
				TO Bus5	100.0	1	-316.46	-61.88	322.45	0.83	-3.14	
				TO Bus7	100.0	1	-185.58	-8.19	185.76	0.14	-6.15	
				TO Bus9	100.0	1	-19.96	-105.94	107.80	0.18	-36.06	
BUS Bus9	100.0	1.0282PU 102.8KV	-10.32	CTK	MW	MVAR	MVA	MW(NOM)	MVR(NOM)	MW(LOSS)	MVAR(LOSS)	TAP
				TO Bus8	100.0	1	20.15	69.88	72.73	0.18	-36.06	
				TO Gen39	100.0	1	-20.15	-69.88	72.73	0.00	-126.98	
BUS Gen30	100.0	1.0475PU 104.8KV	-3.33	CTK	MW	MVAR	MVA	MW(NOM)	MVR(NOM)	MW(LOSS)	MVAR(LOSS)	TAP
				GENERATION			250.00	146.16	289.59	(250.00	0.00)	
				TO Bus2	100.0	1	250.00	146.16	289.59	0.00	13.83	1.0250UN
BUS Gen31	100.0	0.9820PU 98.2KV	0.00	CTK	MW	MVAR	MVA	MW(NOM)	MVR(NOM)	MW(LOSS)	MVAR(LOSS)	TAP
				LOAD			9.20	4.60	10.29			
				GENERATION			520.81	198.25	557.27	(0.00	0.00)	
				TO Bus6	100.0	1	511.61	193.65	547.03	0.00	77.58	1.0700UN
BUS Gen32	100.0	0.9831PU 98.3KV	2.57	CTK	MW	MVAR	MVA	MW(NOM)	MVR(NOM)	MW(LOSS)	MVAR(LOSS)	TAP
				GENERATION			650.00	205.15	681.60	(650.00	0.00)	
				TO Bus10	100.0	1	650.00	205.15	681.60	0.00	96.14	1.0700UN
BUS Gen33	100.0	0.9972PU 99.7KV	4.19	CTK	MW	MVAR	MVA	MW(NOM)	MVR(NOM)	MW(LOSS)	MVAR(LOSS)	TAP
				GENERATION			632.00	109.91	641.49	(632.00	0.00)	
				TO Bus19	100.0	1	632.00	109.91	641.49	2.90	58.76	1.0700UN
BUS Gen34	100.0	1.0123PU 101.2KV	3.18	CTK	MW	MVAR	MVA	MW(NOM)	MVR(NOM)	MW(LOSS)	MVAR(LOSS)	TAP
				GENERATION			508.00	165.76	534.36	(508.00	0.00)	
				TO Bus20	100.0	1	508.00	165.76	534.36	2.51	50.16	1.0090UN
BUS Gen35	100.0	1.0493PU 104.9KV	5.63	CTK	MW	MVAR	MVA	MW(NOM)	MVR(NOM)	MW(LOSS)	MVAR(LOSS)	TAP
				GENERATION			650.00	212.41	683.83	(650.00	0.00)	
				TO Bus22	100.0	1	650.00	212.41	683.83	0.00	60.73	1.0250UN
BUS Gen36	100.0	1.0635PU 106.4KV	8.32	CTK	MW	MVAR	MVA	MW(NOM)	MVR(NOM)	MW(LOSS)	MVAR(LOSS)	TAP
				GENERATION			560.00	101.17	569.07	(560.00	0.00)	
				TO Bus23	100.0	1	560.00	101.17	569.07	1.43	77.88	1.0000UN
BUS Gen37	100.0	1.0278PU 102.8KV	2.42	CTK	MW	MVAR	MVA	MW(NOM)	MVR(NOM)	MW(LOSS)	MVAR(LOSS)	TAP
				GENERATION			540.00	0.44	540.00	(540.00	0.00)	
				TO Bus25	100.0	1	540.00	0.44	540.00	1.66	64.04	1.0250UN
BUS Gen38	100.0	1.0265PU 102.6KV	7.81	CTK	MW	MVAR	MVA	MW(NOM)	MVR(NOM)	MW(LOSS)	MVAR(LOSS)	TAP
				GENERATION			830.00	22.84	830.31	(830.00	0.00)	
				TO Bus29	100.0	1	830.00	22.84	830.31	5.23	102.07	1.0250UN
BUS Gen39	100.0	1.0300PU 103.0KV	-10.05	CTK	MW	MVAR	MVA	MW(NOM)	MVR(NOM)	MW(LOSS)	MVAR(LOSS)	TAP
				LOAD			1104.00	250.00	1131.95			
				GENERATION			1000.00	88.28	1003.89	(1000.00	0.00)	
				TO Bus1	100.0	1	-124.15	-104.61	162.35	0.18	-76.30	
				TO Bus9	100.0	1	20.15	-57.10	60.56	0.00	-126.98	
SYSTEM TOTALS												

39bus.out

	MW	MVAR
GENERATION	6140.81	1250.37
LOAD	6097.10	1408.90
LOSSES	43.71	-158.52
BUS SHUNTS	0.00	0.00
LINE SHUNTS	0.00	0.00
SWITCHED SHUNTS		0.00
MI SMATCH	0.00	0.01

BASE MVA : 100.0
TOLERANCE: 0.000100 PU

SLACK BUSES

Gen31 100.0

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