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e, $T(n) = 9(T/3) + (n^3 \log n)$

$a = 9$

$b = 3$

$\log_a b = \log_3 9 = 2$

$f(n) = n^3 \log n$

1, $n^3 \log n \quad n^{2-\varepsilon}$

2, $n^3 \log n \quad n^2 \cdot \log^k n$

3, $n^3 \log n \quad n^{2+\varepsilon} \quad \varepsilon = 1$

$9T(n/b) \quad 9(n/3)^3 \cdot \log n/3 \leq \delta \; n^3 \cdot \log n$

$\frac{n^3}{3} \cdot \log n/3 < \delta \; n^3 \cdot \log n$

$\frac{n^3}{3} \cdot (\log n - \log 3) < \delta(n^3 \cdot \log n)$

$\frac{n^3}{3} (\log n - \log 3) < \frac{n^3}{3} \log n$

$\delta = \frac{1}{3} < 1 \quad T(n)$ is $\Theta(n^3 \log n)$

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d, $T(n) = 7T(n/3) + n.$

$a = 7$

$b = 3$

$\log_a b = \log_3 7 = 1.xxx.$

1, $f(n) = n < n^{1.xxx - \varepsilon}$

$\Rightarrow T(n)$ is $\Theta(n^{1.xxx})$

c, $T(n) = 16T(n/2) + (n\log n)^4.$

$a = 16$

$b = 2$

$\log_b a = \log_2 16 = 4.$

$f(n) = (n\log n)^4.$

1, $f(n) = (n\log n)^4 > n^{4-\varepsilon}$

2. $f(n)\ (n^4\log n)^4 = \Theta(n^4 \log^k n)$ $\quad k = 4.$

$\Rightarrow T(n)$ is $\Theta(n^4 \log^5 n)$

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b, $T(n) = 8T(n/2) + n^2$.

$a = 8$

$b = 2$

$\log_b a = \log_2 8 = 3$.

$f(n) = n^2$

1, $f(n) = n^2 < n^{3-\varepsilon}$

$\Rightarrow T(n)$ is $\Theta(n^3)$

a, $T(n) = 2T(n/2) + \log n$.

$a = 2$

$b = 2$.

$\log_b a = \log_2 2 = 1$.

1, $f(n) = \log n < n^{1-\varepsilon}$.

$\Rightarrow T(n)$ is $\Theta(n)$

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