Placeholder

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1 Placeholder

Start from the Wiener-Hopf equations for **M=3**, we have three equations with three unknowns h_0, h_1, h_2 :

$$\begin{cases} h_0 r_0 + h_1 r_1 + h_2 r_2 = g_0, \\ h_0 r_1 + h_1 r_0 + h_2 r_1 = g_1, \\ h_0 r_2 + h_1 r_1 + h_2 r_0 = g_2. \end{cases}$$

Here:
$r_0, r_1, r_2 =$ autocorrelations $r_{xx}(k)$
$g_0, g_1, g_2 =$ cross-correlations $r_{dx}(k)$

Representing the equations in matrix form, we have

$$\begin{bmatrix} r_0 & r_1 & r_2 \\ r_1 & r_0 & r_1 \\ r_2 & r_1 & r_0 \end{bmatrix} \begin{bmatrix} h_0 \\ h_1 \\ h_2 \end{bmatrix} = \begin{bmatrix} g_0 \\ g_1 \\ g_2 \end{bmatrix}$$

This is $Rh = \gamma$.

In order to solve this set of linear equations, we will use the Gaussian elimination method.
(1) Original system

$$r_0 h_0 + r_1 h_1 + r_2 h_2 = g_0$$
$$r_1 h_0 + r_0 h_1 + r_1 h_2 = g_1$$
$$r_2 h_0 + r_1 h_1 + r_0 h_2 = g_2$$

(2) Eliminate h_0 from E_2 and E_3
From E_1 :

$$h_0 = \frac{g_0 - r_1 h_1 - r_2 h_2}{r_0}$$

Substitute into E_2 and E_3 :
Equation (2'):

$$r_1 \frac{g_0 - r_1 h_1 - r_2 h_2}{r_0} + r_0 h_1 + r_1 h_2 = g_1$$

Simplify:

$$\frac{r_1 g_0}{r_0} - \frac{r_1^2 h_1}{r_0} - \frac{r_1 r_2 h_2}{r_0} + r_0 h_1 + r_1 h_2 = g_1$$

Multiply both sides by r_0 to clear denominators:

$$r_1 g_0 - r_1^2 h_1 - r_1 r_2 h_2 + r_0^2 h_1 + r_0 r_1 h_2 = g_1 r_0$$

Group h_1, h_2 terms:

$$\left(r_0^2 - r_1^2\right) h_1 + \left(r_0 r_1 - r_1 r_2\right) h_2 = g_1 r_0 - r_1 g_0$$

Let's call this simplified equation (A).

Equation (3') (substitute h_0 into E_3):

$$r_2 \frac{g_0 - r_1 h_1 - r_2 h_2}{r_0} + r_1 h_1 + r_0 h_2 = g_2$$

Simplify:

$$\frac{r_2 g_0}{r_0} - \frac{r_1 r_2 h_1}{r_0} - \frac{r_2^2 h_2}{r_0} + r_1 h_1 + r_0 h_2 = g_2$$

Multiply both sides by r_0 :

$$r_2 g_0 - r_1 r_2 h_1 - r_2^2 h_2 + r_1 r_0 h_1 + r_0^2 h_2 = g_2 r_0$$

Group terms:

$$\left(r_0 r_1 - r_1 r_2\right) h_1 + \left(r_0^2 - r_2^2\right) h_2 = g_2 r_0 - r_2 g_0$$

Let's call this simplified equation (B).

(3) Solve equations (A) and (B) for h_1, h_2
We now have a 2×2 linear system:

$$\begin{cases} \left(r_0^2 - r_1^2\right) h_1 + \left(r_0 r_1 - r_1 r_2\right) h_2 = g_1 r_0 - r_1 g_0 \\ \left(r_0 r_1 - r_1 r_2\right) h_1 + \left(r_0^2 - r_2^2\right) h_2 = g_2 r_0 - r_2 g_0 \end{cases}$$

Let's define constants for readability:

$$\begin{aligned} A &= r_0^2 - r_1^2 \\ B &= r_0 r_1 - r_1 r_2 \\ C &= r_0^2 - r_2^2 \\ p &= g_1 r_0 - r_1 g_0 \\ q &= g_2 r_0 - r_2 g_0 \end{aligned}$$

Then:

$$\begin{cases} A h_1 + B h_2 = p \\ B h_1 + C h_2 = q \end{cases}$$

(4) Solve this 2×2 system
The determinant is:

$$D = AC - B^2$$

Then:

$$h_1 = \frac{pC - qB}{D}, \quad h_2 = \frac{qA - pB}{D}.$$

(5) Substitute back to find h_0
From earlier:

$$h_0 = \frac{g_0 - r_1 h_1 - r_2 h_2}{r_0}$$

We have now obtained all three coefficients.

Summary of explicit formulas for implementing the algorithm:

$$\begin{aligned}
A &= r_0^2 - r_1^2, \\
B &= r_0 r_1 - r_1 r_2, \\
C &= r_0^2 - r_2^2, \\
p &= g_1 r_0 - r_1 g_0, \\
q &= g_2 r_0 - r_2 g_0, \\
D &= AC - B^2, \\
h_1 &= \frac{pC - qB}{D}, \\
h_2 &= \frac{qA - pB}{D}, \\
h_0 &= \frac{g_0 - r_1 h_1 - r_2 h_2}{r_0}.
\end{aligned}$$