Academic Integrity Checklist

Please read the checklist below. Once you have verified these points, sign the checklist and submit with your assignment or test.

- I understand that I am responsible for being honest and ethical in this assessment as per Policy 71
- I have included in-text citations or footnotes when referencing words, ideas, or other intellectual property from other sources in the completion of this assessment, if applicable
- I have included a proper bibliography or works cited, which includes acknowledgement of all sources used to complete this assessment, if applicable
- The assessment was completed by my own efforts and I did not collaborate with any other person for ideas or answers
- This is the first time I have submitted this assessment (either partially or entirely) for academic evaluation

_____
Student Name (by signing or typing my name here I affirm my agreement to the foregoing statements)

_____
Student I.D. Number

_____
Date

AMATH 342 Winter 2021: Assignment 2

Due Date: Friday 26 February, 2021. Total points: 49.

- Assignments below are either theory or computational. For the computational assignments you may use any programming language you want **except** Maple or Mathematica. Besides your written/typed solutions you must also submit your code.
- This assignment should be submitted to Crowdmark. Please make sure your upload has the correct orientation.
- Office hrs: Tuesday's and Thursday's 10:00–10:50am (local time Waterloo, ON). A link will be emailed to you before each office hour.
- If you want to use your one time 3-day extension, the due date is Wednesday 3 March, 2021. Please email the TA if you plan to use your extension.

---

1. **(0 points)** Please sign the Academic Integrity Checklist. If you do not sign the Academic Integrity Checklist you will receive a 0 for this assignment.

2. (a) **(5 points)** Using Theorem 2.1, derive the three-step Adams-Moulton method.
   (b) **(5 points)** Using Theorem 2.1, derive the three-step Adams-Bashforth method.

3. (a) **(5 points)** Using Theorem 2.1, derive the two-step Nystrom method.
   (b) **(5 points)** Implement the 2-step Nystrom method to solve the following scalar ODE:

$$y' + y = \sin(t^2), \quad t \geq 0, \quad y(0) = 0.$$

   Plot y as a function of t for $t \in [0, 8]$. Take $h = 0.04$.
   Regarding the implementation, use Euler's method to find y_1. Use the Nystrom method for all following time steps.

4. **(5 points)** Exercise 2.6 of book.

5. (a) **(5 points)** Construct the Gaussian quadrature formulae for the weight function $\omega(t) \equiv 1$, $0 \leq t \leq 1$, of order two.
   (b) **(5 points)** Construct the Gaussian quadrature formulae for the weight function $\omega(t) \equiv 1$, $0 \leq t \leq 1$, of order four.

(over)

6. Consider the linear ODE system

$$\mathbf{y}' = A\mathbf{y} \qquad \text{where} \qquad A = \begin{bmatrix} -20 & 10 & 0 & 0 \\ 10 & -20 & 10 & 0 \\ 0 & 10 & -20 & 10 \\ 0 & 0 & 10 & -20 \end{bmatrix}. \tag{1}$$

Let the initial condition be given by $\mathbf{y}(0) = [1\ 1\ 1\ 1]^T$.

(a) **(5 points)** Write down the 3-step Adams-Bashforth method for the given linear ODE. Implement this method to solve (1) over the time interval $t \in [0, 10]$. At each time step compute the Euclidean norm of the solution and plot this Euclidean norm as a function of time. In your implementation, define $h = 10/N$, with N a positive integer. Given that the norm of the true solution approaches zero as $t \to \infty$, find out for which values of $N \geq 50$ the numerical method gives you a good approximation.
Regarding the implementation of the 3-step Adams-Bashforth method, use Euler's method to find $\mathbf{y}_1$ and $\mathbf{y}_2$. Use the 3-step Adams-Bashforth method for all following time steps.

(b) **(3 points)** Repeat part (a) but use a 2-step BDF method instead of the 3-step Adams-Bashforth method.
Regarding the implementation of the 2-step BDF method, use the Backward Euler method to find $\mathbf{y}_1$. Use the 2-step BDF method for all following time steps.

(c) **(1 point)** According to you, which method is best for this problem, the 3-step Adams-Bashforth method or the 2-step BDF method? Explain your answer.

7. **(5 points)** The Rössler attractor arose from studying oscillations in chemical reactions. The equations describing this system are given by

$$\begin{aligned} \frac{dx}{dt} &= -(y + z) \\ \frac{dy}{dt} &= x + Ay \\ \frac{dz}{dt} &= B + xz - Cz \end{aligned}$$

where A, B and C are system parameters. Take $A = 0.2$, $B = 0.2$ and $C = 5.7$. Implement a 3-stage ERK method of order 3 to solve the Rössler system of equations (you can choose any 3-stage ERK of order 3 that you want, but write down which one you chose). Let $t \in [0, 500]$ and take $h = 0.02$. As initial condition take $(x_0, y_0, z_0) = (0, 0, 0)$. Plot a 3D figure (with xyz axes) of the solution.