AMATH 455/655 # Lab 1: Cart Position **Winter 2021**

The report is due no later than 11:59pm Monday, February 1, and should be submitted through Crowdmark. Questions marked with a (*) are required for AMath655 students; optional bonus questions for Amath455. You may choose to collaborate with one classmate for the Simulation Part. However, the submitted work should reflect your own understanding and be in your own words. Indicate the name of your collaborator at the top of your report if applicable.

Consider a cart on a track, forced by a rotary motor. Let $x(t)$ indicate the position of the cart at time t, $V(t)$ the motor voltage at time t, η_g the gearbox efficiency, K_g the gearbox gear ratio, η_m the motor efficiency, K_t the motor torque constant, K_m the back-EMF constant, R_m the motor armature resistance, M is the mass of the cart, ν i the coefficient of viscous friction, and r_{mp} the motor pinion radius. (Except for x and V, all these quantities are constant.)

The governing equation is

$$M\ddot{x}(t) = \frac{\eta_g K_g \eta_m K_t (r_{mp} V(t) - K_g K_m \dot{x}(t))}{R_m r_{mp}^2} - \nu \dot{x}(t),$$

The voltage can be controlled by the operator. The object of the control is to drive the cart to a particular position.

1. Only the cart position is measured. Put this equation into state-space form, indicating the state, input and output.

2. What are the eigenvalues of the system matrix?

3. (*) For any initial conditions, and desired position r, the required voltage so that the steady-state value of x is r can be found by solving the differential equation.

   (a) (*) Calculate the required constant voltage V_{req} and time period T so that for the given parameter values and input voltage

   $$V(t) = \begin{cases} V_{req} & 0 \le t \le T, \\ 0 & t > T, \end{cases}$$

   we get

   $$\lim_{t\to\infty} x(t) = 1, \quad \lim_{t\to\infty} \dot{x}(t) = 0.$$

   Use zero initial conditions: $x(0) = 0, \dot{x}(0) = 0$. (There are different values of T and V_{req} that will give a correct answer, but the value of V_{req} will change with the choice of T. You may choose $T = 0.9$, for instance. Then, V_{req} can be obtained in terms of the system parameters only.)

   (b) (*) The steady-state value of $x(t)$ is $\lim_{t\to\infty} x(t)$, if it exists. What happens to the steady state position if there is an error in the system parameter values?

(c) (*) What happens to the steady state value if because of some disturbance d, the actual voltage to the motor is $V_{req} + d$?

4. The procedure described in the previous step for calculating the voltage to drive the cart to a particular position is complicated and has other issues; in particular it is affected by errors in the parameter values. Consider instead for some constant $K_p > 0$

$$V(t) = K_p(r - x(t)), \tag{1}$$

where r is the given desired position.

(a) What is the steady state (or final) position x by applying this voltage to the system?

(b) How does the final value of x change if there are errors in some parameter values?

(c) Is the final position dependent on the value of K_p? If so, how?

(d) How does the final position change if because of some disturbance d, the actual voltage to the cart is $K_p(r - x(t)) + d$ instead of that in (1)? How can the effect of the disturbance on the final position be reduced?

Simulation

In this part, you will need to include an screenshot of your Simulink diagrams and accompanied codes. If you are using subsystems, you should include their screenshots as well.

5. Using the parameter values in Table 1, implement the system in Simulink, and by using a "Signal Generator" block, simulate the effect of square-wave inputs of frequencies 0.5 and 0.1 Hertz and amplitude 1V. It should be noted that this square-wave signal is directly applied as input $V(t)$ to the system and there is no feedback in this part of simulations.

(a) Plot the position and velocity over a range of 30 seconds. Make sure your plots include enough information. For this purpose, plotting data from workspace is recommended.

6. In this step, replace the square-wave input in the previous step with the feedback loop, as in (1), in your simulink diagram. Then, in different scenarios, set K_p to 0.5, 1, 5. Use a square wave with frequency 0.1 Hz and amplitude 1 as the reference input r in (1).

(a) Discuss the behavior of the system with the closed feedback and the effect of different values of the feedback gain K_p.

(b) Repeat the simulation with feedback loop with a reference input r with frequency 0.5 Hertz.

(c) What happens if $K_p < 0$?

Cart Mass	M	0.455 kg
Coefficient of Friction	ν	10.682 kg/s
Gearbox Efficiency	η_g	1.00
Gearbox Gear Ratio	K_g	3.71
Motor Efficiency	η_m	.88
Motor Torque Constant	K_t	0.00767 Nm/A
Back-EMF Constant	K_m	0.00767 V.s/rad
Motor Armature Resistance	R_m	2.6 Ω
Motor Pinion Radius	r_{mp}	0.00635 m

Table 1: Parameter Values

7. Consider both configurations of the simulations, with and without feedback, with the square-wave input of frequency 0.1 Hertz and amplitude 1, and $K_p = 1$. For both cases, perturb M and ν values within ± 30% of their nominal values to consider the effect of model uncertainty.

   (a) How do the outputs of the system change compared to the results with no perturbation?

   (b) By comparing the results from (a) for both open-loop and closed-loop (with feedback) cases, discuss their robustness against perturbations in the model parameters. Support your conclusion with the simulation results.