

ANLY 500 Laboratory #3 – Prescriptive Statistics

Evans Chapter 13 through and including Chapter 16

*“Performance Lawn Equipment Case Study” from Evans, **Business Analytics***

Contents

Introduction.....	1
Chapter 13.....	2
Step 1	2
Step 2	3
Step 3	4
Chapter 14.....	6

Introduction

In the documentation before on LP problems we generated solutions using both graphical and computer-based means to obtain a solution. In the first part of the lab you’ll see yet another way to solve LP problems using a computer program. You can use whatever method you prefer for your Laboratory #3 solutions, as long as you are using R/RStudio.

This laboratory follows the exercises in the book for the Performance Lawn Equipment Case Study, specifically Chapters 13 and 14, except this laboratory requires that you use R to complete the exercises. That is, you should answer all questions in the textbook exercises and complete all computations using R. Each laboratory in ANLY 500 will build on the laboratories you have completed before. So, you will want to continue to keep your work in the folder you set-up for Laboratories #1 and #2 so that you can refer back to previous laboratories if necessary. You will also continue to use the data files for ANLY 500 on Moodle.

Chapter 13

One of PLE's manufacturing facilities produces metal engine housings from sheet metal for both mowers and tractors. Production of each product consists of five steps: stamping, drilling, assembly, painting, and packaging to ship to its final assembly plant. The production rates in hours per unit and the number of production hours available in each department are given in the following table:

Department	Mower Housings	Tractor Housings	Production Hours Available
Stamping	0.03	0.07	200
Drilling	0.09	0.06	300
Assembly	0.15	0.10	300
Painting	0.04	0.06	220
Packaging	0.02	0.04	100

In addition, mower housings require 1.2 square feet of sheet metal per unit and tractor housings require 1.8 square feet per unit, and 2,500 square feet of sheet metal is available. The company would like to maximize the total number of housings they can produce during the planning period.

Formulate and solve a linear optimization model and recommend a production plan. Illustrate the results visually to help explain them in a presentation to Ms. Burke. In addition, conduct whatever what-if analyses (e.g., run different scenarios and apply parameter analysis) you feel are appropriate to include in your presentation. Summarize your results in a well-written report.

Step 1

The first step is to identify the decision variables. PLE wants to maximize the total number of housings they can produce during the planning period. So, the variables involved are:

- Production rates (time in hours)
- Production hours available (time in hours)
- Metal required for mower housings (square feet)
- Metal required for tractor housings (square feet)
- Amount of sheet metal available (square feet)

Following the chapter's discussion from the beginning, at the top level we'll define the following:

- MH (mower housings) = the number of mower housings produced/hour
- TH (tractor housings) = the number of tractor housings produced/hour

Step 2

Next we set up the constraints and the objective function. From the table we are given we can write the following equations:

Production Rates/Time Available				
Department	Mower Housings Hours per unit	Tractor Housings Hours per unit	Production Hours Available	Mathematical model
Stamping	0.03	0.07	200	$.03MH + .07TH \leq 200$
Drilling	0.09	0.06	300	$.09MH + .06TH \leq 300$
Assembly	0.15	0.1	300	$.15MH + .1TH \leq 300$
Painting	0.04	0.06	220	$.04MH + .06TH \leq 220$
Packaging	0.02	0.04	100	$.02MH + .06TH = 100$
Material Requirements/Availability				
	Sq. ft. per unit	Sq. ft. per unit	Sq. ft. Available	$1.2MH + 1.8TH \leq 2500$
Sheet Metal	1.2	1.8	MAvail	$MH, TH \geq 0$

Or,

$$\begin{aligned}
 &.03MH + .07TH \leq 200 \\
 &.09MH + .06TH \leq 300 \\
 &.15MH + .10TH \leq 300 \\
 &.04MH + .06TH \leq 220 \\
 &.02MH + .06TH \leq 100 \\
 &1.2MH + 1.8TH \leq 2500 \\
 &MH, TH \geq 0
 \end{aligned}$$

So, what we wind up with is a set of linear equations for which we want to maximize the number of mower and tractor housings (MH + TH) produced. Note that many of these are constraint functions because they limit the time to the number of hours available or the amount of sheet metal required to the total material available. So, our objective function is:

Maximize MH + TH

You will want to make sure that in all the equations you set-up that the units follow through and are consistent so that the objective equation yields the number of housings produced.

Step 3

Next, we set-up the linear optimization (or linear programming) problem in R. If we want to use the `lpSolve` package in R to solve this problem it is actually more helpful to use the table format of the problem than the equations. However, either can be used. I'll show the solution using the table format. That is, we have two variables; MH and TH, and six constraints with the following values in columns:

RowName	MH	TH	Constraint Type	Constraint (or RHS)
Stamping	.03	.07	<=	200
Drilling	.09	.06	<=	300
Assembly	.15	.10	<=	300
Painting	.04	.06	<=	220
Packaging	.02	.06	<=	110
Sheet Metal	1.2	1.8	<=	2500

To get this into R you need to have the `lpSolveAPI` package installed and attached with the `library(lpSolveAPI)` command. Then:

1. Initialize the linear programming object, i.e. the LPMO, with the number of variables and the number of constraints. I'll leave it up to you to verify I have the correct numbers for these. You should say something about these numbers in your lab report.

```
> lprec <- make.lp(6, 2)
```

2. Next use the columns to set up the data in the LMPO ([set_column](#) command).
3. Now enter the information for the objective function ([set.objfn](#) command).
4. And, enter the information for the constraint types, e.g. \leq , $=$, or \geq ([set.constr.type](#) command).
5. Enter the constraint (or RHS – right-hand-side) values ([set.rhs](#) command).
6. Set-up the linear programming control to maximize the objective function. I believe the default is to minimize so you need to be sure to include this. (Note that I got a lot of stuff back when I did this that I just ignored.) ([lp.control](#) command).
7. What remains is to set-up the column names, rownames and dimension using [colnames](#), [rownames](#), and [dimnames](#) respectively.
8. Let's look at the problem as we have set it up in R by just calling the data object name, lprec:

```
> lprec
```

Model name:

	MH	TH	
Maximize	1	1	
Stamping	0.03	0.07	\leq 200
Drilling	0.09	0.06	\leq 300
Assembly	0.15	0.1	\leq 300
Painting	0.04	0.06	\leq 220
Packaging	0.02	0.06	\leq 100
Sheet Metal	1.2	1.8	\leq 2500
Kind	Std	Std	
Type	Real	Real	
Upper	Inf	Inf	
Lower	0	0	

This looks like the problem we want to solve.

9. To solve is easy, just use [solve](#) command. You should see following statement in console:

```
[1] 0
```

The return of “0” indicates that the linear programming problem successfully ran and the solution is available.

10. To get the solution view the total for the objective function and for each of the variables; MH and TH using [get.objective](#) and [get.variables](#).

11. Get the values of the constraints that were using [get.constraints](#).

So, you've successfully solved your first linear programming problem.

Chapter 14

Completion of Chapter 14's exercises for the Performance Lawn Equipment Case Study earns you **extra credit!** That is, you are not required to finish the exercises in Chapter 14 unless you want to earn the extra credit. You will find examples, e.g. the multi-period production planning models beginning on page 481 in the textbook that are very helpful in completing this work. If you complete this work, in your lab report include an executive summary of your conclusions. Then, in an appendix include the R commands you used to generate your solution to the LP problem and its sensitivity.

Elizabeth Burke wants to develop a model to more effectively plan production for the next year. Currently, PLE has a planned capacity of producing 9,100 mowers each month, which is approximately the average monthly demand over the previous year. However, looking at the unit sales figures for the previous year, she observed that the demand for mowers has a seasonal fluctuation, so with this "level" production strategy, there is overproduction in some months, resulting in excess inventory buildup and underproduction in others, which may result in lost sales during peak demand periods. In discussing this with her, she explained that she could change the production rate by using planned overtime or undertime (producing more or less than the average monthly demand), but this incurs additional costs, although it may offset the cost of lost sales or of maintaining excess inventory. Consequently, she believes that the company can save a significant amount of money by optimizing the production plan. Ms. Burke saw a presentation at a conference about a similar model that another company used but didn't fully understand the approach. The PowerPoint notes didn't have all the details, but they did explain the variables and the types of constraints used in the model. She thought they would be helpful to you in implementing an optimization model. Here are the highlights from the presentation: Variables:

Variables:

- X_t = planned production in period t
- I_t = inventory held at the end of period t

- L_t = number of lost sales incurred in period t
- O_t = amount of overtime scheduled in period t
- U_t = amount of undertime scheduled in period t
- R_t = increase in production rate from period $t-1$ to period t
- D_t = decrease in production rate from period $t-1$ to period t

Material balance constraint:

$$X_t + I_{t-1} - I_t + L_t = \text{demand in month } t$$

Overtime/undertime constraint:

$$O_t - U_t = X_t - \text{normal production capacity}$$

Production rate-change constraint:

$$X_t - X_{t-1} = R_t - D_t$$

Ms. Burke also provided the following data and estimates for the next year:

- unit production cost=\$70.00;
- inventory-holding cost=\$1.40per unit per month;
- lost sales cost=\$200per unit;
- overtime cost=\$6.50per unit;
- undertime cost=\$3.00per unit; and
- production-rate-change cost=\$5.00per unit ,

which applies to any increase or decrease in the production rate from the previous month. Initially, 900 units are expected to be in inventory at the beginning of January, and the production rate for December 2012 was 9,100 units. She believes that monthly demand will not change substantially from last year, so the sales figures for last year in the PLE database should be used for the monthly demand forecasts.

Your task is to design a spreadsheet that provides detailed information on monthly production, inventory, lost sales, and the different cost categories and solve a linear optimization model for minimizing the total cost of meeting demand over the next year. Compare your solution with the level production strategy of producing 9,100 units each month. Interpret the Sensitivity report, and conduct an appropriate study of how the solution will be affected by changing the assumption of the lost sales costs. Summarize all your results in a report to Ms. Burke.

