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Capítulo 07 - Distâncias.

01) S.

$P_1(-2,0,1)$

$P_2(1,-3,2)$

$\vec{P_1P_2}=(3,-3,1)$

$d(P_1,P_2)=|\vec{P_1P_2}|$

$=$

$d(P_1,P_2)=\sqrt{3^2+(-3)^2+1^2}$

$d(P_1,P_2)=\sqrt{19}$ u.c

02) S.

$P_1(1,0,1)$

$P_2(2,-1,0)$

$\vec{P_1P_2}=(1,-1,-1)$

$=$

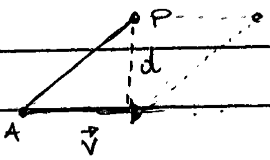
$d(P_1,P_2)=\sqrt{1^2+(-1)^2+(-1)^2}$

$d(P_1,P_2)=\sqrt{3}$ u.c

03) S.

$P(2,3,-1)$

$$d(P,r)=\frac{|\vec{AP}\times\vec{v}|}{|\vec{v}|}$$



$$r:\begin{cases} x=3+t \\ y=-2t \\ z=1-2t \end{cases}$$

$\vec{v}=(1,-2,-2)$

$A(3,0,1)$

$\vec{AP}=(1,-3,2)$

$=$ I)

$$\vec{AP}\times\vec{v}=\left|\begin{array}{ccc|cc} \hat{i} & \hat{j} & \hat{k} & \hat{i} & \hat{j} \\ 1 & -3 & 2 & 1 & -3 \\ 1 & -2 & -2 & 1 & -2 \end{array}\right|$$

$=10\hat{i}+4\hat{j}+\hat{k}=(10,4,1)$

II) $$d(P,r)=\frac{\sqrt{10^2+4^2+1^2}}{\sqrt{1^2+(-2)^2+(-2)^2}}=\frac{\sqrt{117}}{3}\ \text{u.c}$$

04) S.

$P(1,-1,0)$

$$r:\begin{cases} x=2-t \\ y=0 \\ z=t \end{cases}$$

$\vec{v}=(-1,0,1)$

$A(2,0,0)$

$\vec{AP}=(1,1,0)$

$=$

I)

$$\vec{AP}\times\vec{v}=\left|\begin{array}{ccc|cc} \hat{i} & \hat{j} & \hat{k} & \hat{i} & \hat{j} \\ 1 & 1 & 0 & 1 & 1 \\ -1 & 0 & 1 & -1 & 0 \end{array}\right|$$

$=\hat{i}-\hat{j}+\hat{k}=(1,-1,1)$

II) $$d(P,r)=\frac{\sqrt{3}}{\sqrt{2}}\cdot\frac{\sqrt{2}}{\sqrt{2}}=\frac{\sqrt{6}}{2}\ \text{u.c}$$

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05) $P(3,2,1)$

$r: \begin{cases} y = 2x \\ z = x+3 \end{cases}$

$\vec{v} = (1,2,1)$

$A(0,0,3)$

$\vec{AP} = (-3,-2,2)$

I)

$$\vec{AP} \times \vec{v} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -3 & -2 & 2 \\ 1 & 2 & 1 \end{vmatrix} \begin{matrix} \hat{i} & \hat{j} \\ -3 & -2 \\ 1 & 2 \end{matrix}$$

$= -6\hat{i} + 5\hat{j} - 4\hat{k} = (-6,5,-4)$

II) $d(P,r) = \dfrac{\sqrt{36+25+16}}{\sqrt{6}} = \dfrac{\sqrt{77}}{\sqrt{6}} = \dfrac{\sqrt{462}}{6}$ u.c.

06) $P(0,0,0)$

$r: \begin{cases} 2x - y + z - 3 = 0 \\ x + y - 2z + 1 = 0 \end{cases}$

I) Definindo: $x = 0$

$\begin{cases} -y + z = 3 \\ y - 2z = -1 \end{cases}$ $P'(0,-5,-2)$

$z = -2$ $y = -5$

II)

$$\vec{v} = (\vec{n_1} \times \vec{n_2}) = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & -1 & 1 \\ 1 & 1 & -2 \end{vmatrix} \begin{matrix} \hat{i} & \hat{j} \\ 2 & -1 \\ 1 & 1 \end{matrix}$$

$= \hat{i} + 5\hat{j} + 3\hat{k} = (1,5,3)$

III)

$$\vec{PP'} \times \vec{v} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & -5 & -2 \\ 1 & 5 & 3 \end{vmatrix} \begin{matrix} \hat{i} & \hat{j} \\ 0 & -5 \\ 1 & 5 \end{matrix}$$

$= -5\hat{i} - 2\hat{j} + 5\hat{k} = (-5,-2,5)$

IV) $d(P,r) = \dfrac{\sqrt{25+4+25}}{\sqrt{1+25+9}}$

$d(P,r) = \sqrt{\dfrac{54}{35}}$ u.c.

07) $P(3,-1,1)$

$\vec{v} = (1,-4,2)$

$A(2,3,-1)$

$\vec{AP} = (1,-4,2)$

I)

$$\vec{AP} \times \vec{v} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -4 & 2 \\ 1 & -4 & 2 \end{vmatrix} \begin{matrix} \hat{i} & \hat{j} \\ 1 & -4 \\ 1 & -4 \end{matrix}$$

$= 0$

II) $d(P,r) = 0$

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08) $P(1,2,3)$
r.
$\vec{v}=(1,0,0)$
$\vec{AP}=(1,2,3)$
$A(0,0,0)$

I) $\vec{AP}\times\vec{v}=\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & 3 \\ 1 & 0 & 0 \end{vmatrix}\begin{matrix} \hat{i} & \hat{j} \\ 1 & 2 \\ 1 & 0 \end{matrix}$

$= 3\hat{j}-2\hat{k}=(0,3,-2)$

II) $d(P,r)=\dfrac{\sqrt{9+4}}{1}=\sqrt{13}$ u.c //

09) $P(1,2,3)$
r.
$A(0,0,0)$
$\vec{v}(0,0,1)$
$\vec{AP}=(1,2,3)$

I) $\vec{AP}\times\vec{v}=\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & 3 \\ 0 & 0 & 1 \end{vmatrix}\begin{matrix} \hat{i} & \hat{j} \\ 1 & 2 \\ 0 & 0 \end{matrix}$

$= 2\hat{i}-\hat{j}=(2,-1,0)$

II) $d(P,r)=\sqrt{5}$ u.c //.

10) $P(1,2,3)$
r.
$\pi:\begin{cases} x=1 \\ z=-1 \end{cases}$
$\vec{v}=(0,1,0)$
$A(1,0,-1)$
$\vec{AP}=(0,2,4)$

I) $\vec{AP}\times\vec{v}=\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & 2 & 4 \\ 0 & 1 & 0 \end{vmatrix}\begin{matrix} \hat{i} & \hat{j} \\ 0 & 2 \\ 0 & 1 \end{matrix}$

$= -4\hat{i}=(-4,0,0)$

II) $d(P,r)=\dfrac{\sqrt{16}}{\sqrt{1}}=4$ u.c //.

11) $P(2,-1,2)$
r.
$\pi: 2x-2y-z+3=0$
$\vec{n}=(2,-2,-1)$

$d(P,\pi)=\dfrac{|2(2)-2(-1)-(2)+3|}{\sqrt{2^2+(-2)^2+(-1)^2}}=\dfrac{7}{3}$ u.c //.

12) $P(3,-1,4)$
r.
$\pi: x+y+z=0$
$\vec{n}=(1,1,1)$

$d(P,\pi)=\dfrac{|3-1+4|}{\sqrt{3}}=\dfrac{6}{\sqrt{3}}\cdot\dfrac{\sqrt{3}}{\sqrt{3}}=\dfrac{6\sqrt{3}}{3}=2\sqrt{3}$ u.c //.

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13) $P(1,3,-6)$

$\pi: 4x-y+z+5=0$

$\vec{n}=(4,-1,1)$

$$d(P,\pi)=\frac{|4-3-6+5|}{\sqrt{4^2+(-1)^2+1^2}}=\frac{|0|}{\sqrt{18}}=0$$

14) $P(0,0,0)$

$\pi: 3x-4y+20=0$

$\vec{n}=(3,-4,0)$

$$d(P,\pi)=\frac{|20|}{\sqrt{25}}=\frac{20}{5}=4\ u.c$$

15) $P(1,1,1)$

$$\pi: \begin{cases} x=2+2h+3t \\ y=-1+h+t \\ z=2-h \end{cases}$$

$\vec{v}_1'=(2,1,-1)$

$\vec{v}_1''=(3,1,0)$

I) $\vec{n}=(\vec{v}_1'\times\vec{v}_1'')=\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 1 & -1 \\ 3 & 1 & 0 \end{vmatrix}\begin{matrix} \hat{i} & \hat{j} \\ 2 & 1 \\ 3 & 1 \end{matrix}$

$= 1\hat{i}-3\hat{j}-\hat{k}=(1,-3,-1)$

II) $\pi\, x-3y-z+d=0$

$P(2,-1,2)$:

$2+3-2+d=0$

$d=-3$

III) Equação Geral do Plano:

$\pi: x-3y-z-3=0$

IV) $$d(P,\pi)=\frac{|\cancel{1}-3\cancel{-1}-3|}{\sqrt{1^2+(-3)^2+(-1)^2}}=\frac{|-6|}{\sqrt{11}}=\frac{6}{\sqrt{11}}\ u.c$$

16) $\pi_1: x+y+z-4=0$

$\pi_2: 2x+2y+2z-5=0$

$\vec{n}_2=(2,2,2)$

I) Atribuindo $x=1$ e $y=2$ para o plano π_1:

$1+2+z-4=0$ $\quad P(1,2,1)$

$z=1$

II) $$d(P,\pi_2)=\frac{|2+4+2-5|}{\sqrt{4+4+4}}=\frac{|3|}{\sqrt{12}}=\frac{3}{2\sqrt{3}}\cdot\frac{\sqrt{3}}{\sqrt{3}}=\frac{3\sqrt{3}}{6}=\frac{\sqrt{3}}{2}\ u.c$$

17) $$r: \begin{cases} x=4+3t \\ y=-1+t \\ z=t \end{cases}$$

$\pi: x-y-2z+4=0$

$\vec{n}=(1,-1,-2)$

$P(4,-1,0)$

$$d(P,\pi)=\frac{|4+1+4|}{\sqrt{6}}=\frac{9}{\sqrt{6}}\ u.c$$

Seg Ter Qua Qui Sex Sáb Dom   / /

18) $P(3,4,0)$ | $d(P,\pi) = \dfrac{|3+4-12|}{\sqrt{2}} = \dfrac{5}{\sqrt{2}}$ u.c
7

$\pi: x+y-12=0$

$\vec{n}=(1,1,0)$

19) $P(3,4,0)$ | $d(P,\pi) = \dfrac{|4|}{\sqrt{1}} = 4$ u.c
7.

$\pi: y=0$

$\vec{n}=(0,1,0)$

20) 7.

$r_1 \begin{cases} x=2-t \\ y=3+t \\ z=1-2t \end{cases}$ $r_2 \begin{cases} x=t \\ y=-1-3t \\ z=2t \end{cases}$ $d(r_1,r_2) = \dfrac{|(\vec{v_1},\vec{v_2},\vec{A_1A_2})|}{|\vec{v_1}\times\vec{v_2}|}$

$A_1(2,3,1)$ $A_2(0,-1,0)$

$\vec{A_1A_2}=(-2,-4,-1)$

$\vec{v_1}=(-1,1,-2)$ $\vec{v_2}=(1,-3,2)$

I) $(\vec{v_1},\vec{v_2},\vec{A_1A_2}) = \begin{vmatrix} -1 & 1 & -2 \\ 1 & -3 & 2 \\ -2 & -4 & -1 \end{vmatrix} \begin{matrix} -1 & 1 \\ 1 & -3 \\ -2 & -4 \end{matrix}$

$= -3-4+8+12-8+1 = 6$

II) $\vec{v_1}\times\vec{v_2} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -1 & 1 & -2 \\ 1 & -3 & 2 \end{vmatrix} \begin{matrix} i & j \\ -1 & 1 \\ 1 & -3 \end{matrix}$

$= -4\hat{i}+2\hat{k} = (-4,0,2)$

III) $d(r_1,r_2) = \dfrac{|6|}{\sqrt{20}} = \dfrac{6}{2\sqrt{5}} = \dfrac{3}{\sqrt{5}}$ u.c

21) 7.

$r_1: x=y=z$

$r_2: \begin{cases} y=x+1 \\ z=2x-1 \end{cases}$

$\vec{v_1}=(1,1,1)$ $\vec{v_2}=(1,1,2)$

$A_1(0,0,0)$ $A_2(0,1,-1)$

$\vec{A_1A_2}=(0,1,-1)$

I) $(\vec{v_1},\vec{v_2},\vec{A_1A_2}) = \begin{vmatrix} 1 & 1 & 1 \\ 1 & 1 & 2 \\ 0 & 1 & -1 \end{vmatrix} \begin{matrix} 1 & 1 \\ 1 & 1 \\ 0 & 1 \end{matrix} = -1+1-2+1 = -1$

II) $\vec{v_1}\times\vec{v_2} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 1 & 1 \\ 1 & 1 & 2 \end{vmatrix} \begin{matrix} \hat{i} & \hat{j} \\ 1 & 1 \\ 1 & 1 \end{matrix} = \hat{i}-\hat{j} = (1,-1,0)$

III) $d(r_1,r_2) = \dfrac{|-1|}{\sqrt{2}} = \dfrac{1}{\sqrt{2}}$ u.c

22) 7.

$r_1 \begin{cases} y=2x \\ z=3 \end{cases}$ $r_2 \begin{cases} x=2+t \\ y=-1-t \\ z=2+3t \end{cases}$ $\vec{A_1A_2}=(2,-1,-1)$

$\vec{v_1}=(1,2,0)$

$A_1(0,0,3)$ $\vec{v_2}=(1,-1,3)$

$A_2(2,-1,2)$

I) $(\vec{v_1},\vec{v_2},\vec{A_1A_2}) = \begin{vmatrix} 1 & 2 & 0 \\ 1 & -1 & 3 \\ 2 & -1 & -1 \end{vmatrix} \begin{matrix} 1 & 2 \\ 1 & -1 \\ 2 & -1 \end{matrix}$

$= 1+12+3+2 = 18$

I) $\vec{v}_1 \times \vec{v}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & 0 \\ 1 & -1 & 3 \end{vmatrix} \begin{matrix} \hat{i} & \hat{j} \\ 1 & 2 \\ 1 & -1 \end{matrix} = 6\hat{i} - 3\hat{j} - 3\hat{k}$ $(6,-3,-3)$

II) $d(r_1,r_2) = \dfrac{|18|}{\sqrt{54}} = \dfrac{18}{3\sqrt{6}} = \dfrac{6}{\sqrt{6}} \cdot \dfrac{\sqrt{6}}{\sqrt{6}}$

$= \sqrt{6}$ u.c.

23) $\vec{v}_1 = (1,1,-2)$
$\vec{v}_2 = (1,3,-4)$
$A_1(1,2,-2)$
$A_2(0,1,0)$
$\vec{A_1A_2} = (-1,-1,2)$

I) $(\vec{v}_1, \vec{v}_2, \vec{A_1A_2}) = \begin{vmatrix} 1 & 1 & -2 \\ 1 & 3 & -4 \\ -1 & -1 & 2 \end{vmatrix} \begin{matrix} 1 & 1 \\ 1 & 3 \\ -1 & -1 \end{matrix} = \cancel{6}+\cancel{4}+\cancel{2}-\cancel{6}-\cancel{4}-\cancel{2} = 0$

II) $d(r_1,r_2) = 0$

24) $r_1 \begin{cases} x = 3 \\ y = 2 \end{cases}$ $r_2 \begin{cases} x = 1 \\ y = 4 \end{cases}$

$\vec{v}_1 = (0,0,1)$ $\vec{v}_2 = (0,0,1)$
$A_1(3,2,0)$ $A_2(1,4,0)$
$\vec{A_1A_2} = (-2,2,0)$

I) $\vec{A_1A_2} \times \vec{v}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -2 & 2 & 0 \\ 0 & 0 & 1 \end{vmatrix} \begin{matrix} \hat{i} & \hat{j} \\ -2 & 2 \\ 0 & 0 \end{matrix} = 2\hat{i} - 2\hat{j} = (2,-2,0)$

II) $d(P,r) = \dfrac{\sqrt{4+4}}{1} = \sqrt{8} = 2\sqrt{2}$ u.c.

25) $r_1 \begin{cases} x = 3 \\ y = 4 \end{cases}$ $P(3,4,0)$ $A(0,0,0)$ $\vec{AP} = (3,4,0)$

r_2: eixo dos z
$\vec{v}_2 = (0,0,1)$

I) $\vec{AP} \times \vec{v}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & 4 & 0 \\ 0 & 0 & 1 \end{vmatrix} \begin{matrix} \hat{i} & \hat{j} \\ 3 & 4 \\ 0 & 0 \end{matrix}$

$= 4\hat{i} - 3\hat{j} = (4,-3,0)$

II) $d(P,r) = \dfrac{\sqrt{25}}{\sqrt{1}} = 5$ u.c.