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⇒ Capítulo 06 → O Plano.

01) $\pi: 3x + y - z - 4 = 0$

a) $P(1,3,z)$ | I) $3(1) + 3 - z - 4 = 0$; $-z = -2 \Rightarrow z = 2$ | II) $P(1,3,2)$

b) $A(0,y,2)$ | I) $3(0) + y - 2 - 4 = 0$; $y = 6$ | II) $A(0,6,2)$

c) $P(k,2,k-1)$ | $3k + 2 - k + 1 - 4 = 0$; $2k = 1 \therefore k = 1/2$

d) $B(2,2z,z)$; $y = 2z$ | I) $3(2) + 2z - z - 4 = 0$; $z = -2$ | II) $B(2,-4,-2)$

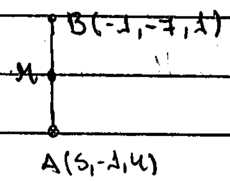
e) $\vec{n}_1 = (k,-4,4) \,//\, \vec{n} = (3,1,-1)$; $\pi_1: kx - 4y + 4z - 7 = 0$; $\pi_1 // \pi$ | $\dfrac{\vec{n}}{\vec{n}_1} = \dfrac{3}{k} = \dfrac{1}{-4} = \dfrac{-1}{4}$; $k = -12$

02) $\pi_1: ax + by + cz + d = 0$; $\pi: 2x - 3y - z + 5 = 0$; $A(4,-2,1)$; $A \in \pi_1$; $\pi_1 // \pi$ | I) $\vec{n} = (2,-3,-1)$; $a = 2$; $b = -3$; $c = -1$; $2x - 3y - z + d = 0$ | II) $A(4,-2,1)$; $2(4) - 3(-2) - 1 + d = 0$; $d = -13$.

III) $\pi_1: 2x - 3y - z - 13 = 0$

03) $r: \begin{cases} x = 2 + 2t \\ y = 1 - 3t \\ z = 4t \end{cases}$ | I) $\pi: 2x - 3y + 4z + d = 0$; $A \in \pi$; $2(-1) - 3(2) + 4(3) + d = 0$; $d = -4$

$r \perp \pi$; $\vec{n} = (2,-3,4)$; $A(-1,2,3)$ | II) $\pi: 2x - 3y + 4z - 4 = 0$

04) 

I) $M(2,-4,5/2)$

II) $\vec{MB} = (-3,-3,-3/2)$

III) $\pi: -3x - 3y - 3/2\,z + d = 0$; $M \in \pi$; $\pi: -3(2) - 3(-4) - 3/2(5/2) + d = 0$

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$\Rightarrow \pi: -6+12-15/4+d=0$

$\pi: 6+15/4=-d$

$d=-9/4$

IV) $\pi: -3x-3y-3/2z+9/4=0$

$\pi: -12x-12y-6z-9=0 \quad (\div -3)$

$\pi: 4x+4y+2z+3=0$ //.

05) $\pi: 3x-2y-z-6=0$ → Existem infinitos pontos.

Escolhido: $x=t$

$y=h$

$$\pi: \begin{cases} x=t \\ y=h \\ z=3t-2h-6 \end{cases}$$

06) $\begin{cases} x=1+h-2t \\ y=1-t \\ z=4+2h-2t \end{cases}$

I) $\vec{h}=(1,0,2)$

$\vec{v}=(-2,-1,-2)$

$\vec{n}=\vec{h}\times\vec{v}=\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 0 & 2 \\ -2 & -1 & -2 \end{vmatrix} \begin{matrix} \hat{i} & \hat{j} \\ 1 & 0 \\ -2 & -1 \end{matrix}$

$=2\hat{i}-2\hat{j}-\hat{k}=(2,-2,-1)$

II) $\pi: 2x-2y-z+d=0$

$P(1,1,4)$

$\pi: 2(1)-2(1)-4+d=0$

$d=4$

III) $\pi: 2x-2y-z+4=0$

07) $A(1,0,2)$ | I) $\vec{AB}=(-2,2,-3)$

$B(-1,2,-1)$ | $\vec{AC}=(0,1,-3)$

$C(1,1,-1)$ | $\vec{n}=\vec{AB}\times\vec{AC}=\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -2 & 2 & -3 \\ 0 & 1 & -3 \end{vmatrix} \begin{matrix} \hat{i} & \hat{j} \\ -2 & 2 \\ 0 & 1 \end{matrix}$

$=-3\hat{i}-6\hat{j}+2\hat{k}=(-3,-6,-2)$

II) $\pi: -3x-6y+2z+d=0$

Utilizando o ponto A:

$-3(1)-6\cdot0-2(2)+d=0$

$d=7$ //

IV) Equação Paramétrica (Ponto A):

$\vec{AB}=t$ e $\vec{AC}=h$

$$\pi: \begin{cases} x=1-2t \\ y=2t+h \\ z=2-3t-3h \end{cases}$$

III) Equação Geral:

$\pi: -3x-6y-2z+7=0 \Rightarrow \pi: 3x+6y+2z-7=0$ //

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I) $\vec{AB} = (1,1,5)$

$\vec{AC} = (-1,1,1)$

$\vec{n} = \vec{AB} \times \vec{AC} = \left|\begin{array}{ccc|cc} \hat{i} & \hat{j} & \hat{k} & \hat{i} & \hat{j} \\ 1 & 1 & 5 & 1 & 1 \\ -1 & 1 & 1 & -1 & 1 \end{array}\right|$

$= -4\hat{i} - 6\hat{j} + 2\hat{k} = (-4,-6,2)$

II. $\pi: -4x - 6y + 2z + d = 0 \; (x-1)$

$\Rightarrow \pi: 4x + 6y - 2z - d = 0$

Utilizando o ponto A:

$4 \cdot 0 + 6 \cdot 0 - 2 \cdot 0 - d = 0$

$d = 0$ //

III) Equação Geral:

$\pi: 4x + 6y - 2z = 0 \; (\div 2)$

$\pi: 2x + 3y - z = 0$ //

IV) Equação Paramétrica (Ponto A):

$\vec{AB} = t$ e $\vec{AC} = h$

$\pi: \begin{cases} x = t - h \\ y = t + h \\ z = 5t + h \end{cases}$

09) A(2,0,-1)

B(-2,6,3)

C(0,3,4)

I) $\vec{AB} = (-4,6,4)$

$\vec{AC} = (-2,3,5)$

$\vec{n} = \vec{AB} \times \vec{AC} = \left|\begin{array}{ccc|cc} \hat{i} & \hat{j} & \hat{k} & \hat{i} & \hat{j} \\ -4 & 6 & 4 & -4 & 6 \\ -2 & 3 & 5 & -2 & 3 \end{array}\right|$

$= 30\hat{i} - 12\hat{i} - 8\hat{j} + 20\hat{j} - 12\hat{k} + 12\hat{k} = (18,12,0)$

II) $\pi: 18x + 12y + d = 0$

Utilizando o ponto A:

$18 \cdot 2 + 12 \cdot 0 + d = 0$

$d = -36$

III) Equação Geral:

$\pi: 18x + 12y - 36 = 0 \; (\div 6)$

$\pi: 3x + 2y - 6 = 0$ //

IV) Equação Paramétrica (Ponto A):

$\vec{AB} = h$ e $\vec{AC} = t$

$\pi: \begin{cases} x = 2 - 4h - 2t \\ y = 6h + 3t \\ z = -1 + 4h + 5t \end{cases}$

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10) $A(2,1,0)$, $B(-4,-2,-1)$, $C(0,0,1)$

I) $\vec{AB} = (-6,-3,-1)$

$\vec{AC} = (-2,-1,1)$

$$\vec{n} = (\vec{AB} \times \vec{AC}) = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -6 & -3 & -1 \\ -2 & -1 & 1 \end{vmatrix} \begin{matrix} \hat{i} & \hat{j} \\ -6 & -3 \\ -2 & -1 \end{matrix}$$

$$= -3\hat{i} - 1\hat{i} + 2\hat{j} + 6\hat{j} + 6\hat{k} - 6\hat{k} = (-4,8,0)$$

II) $\pi: -4x + 8y + d = 0$

Utilizando o Ponto C:

$d = 0$ //

III) Equação Geral:

$\pi: -4x + 8y = 0 \quad (\div -4)$

$\pi: x - 2y = 0$ //

IV) Equação Paramétrica (Ponto A):

$\vec{AB} = h$ e $\vec{AC} = t$

$$\pi: \begin{cases} x = 2 - 6h - 2t \\ y = 1 - 3h - t \\ z = -h + t \end{cases}$$

11) $A(2,1,3)$, $B(-3,-1,3)$, $C(4,2,3)$

I) $\vec{AB} = (-5,-2,0)$

$\vec{AC} = (2,1,0)$

$$\vec{n} = (\vec{AB} \times \vec{AC}) = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -5 & 2 & 0 \\ 2 & 1 & 0 \end{vmatrix} \begin{matrix} \hat{i} & \hat{j} \\ -5 & 2 \\ 2 & 1 \end{matrix}$$

$$= -5\hat{k} - 4\hat{k} = (0,0,-9)$$

II) $\pi: -9z + d = 0$

Utilizando o Ponto A:

$-9(3) + d = 0$

$d = 27$ //

III) Equação Geral:

$\pi: -9z + 27 = 0 \quad (\div -9)$

$\pi: z - 3 = 0$ //

IV) Equação Paramétrica (Ponto A):

$\vec{AB} = h$ e $\vec{AC} = t$

$$\pi: \begin{cases} x = 2 - 5h + 2t \\ y = 1 - 2h + t \\ z = 3 \end{cases}$$

12) $A(x,1,9)$, $B(2,3,4)$, $C(-4,-1,6)$, $D(0,2,4)$

I) $\vec{BC} = (-6,-4,2)$

$\vec{BD} = (-2,-1,0)$

$$\vec{n} = (\vec{BC} \times \vec{BD}) = \begin{vmatrix} i & j & k \\ -6 & -4 & 2 \\ -2 & -1 & 0 \end{vmatrix} \begin{matrix} i & j \\ -6 & -4 \\ -2 & -1 \end{matrix}$$

$$= 2\hat{i} - 4\hat{j} - 2\hat{k} = (2,-4,-2)$$

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II) $\pi: 2x - 4y - 2z + d = 0$

Utilizando o ponto B:

$2\cdot 2 - 4\cdot 3 - 2\cdot 4 + d = 0$

$d = 16$ //

III) Equação Geral:

$\pi: 2x - 4y - 2z + 16 = 0 \quad (\div 2)$

$\pi: x - 2y - z + 8 = 0$

IV) Substituindo x, y e z.

$A(\alpha, 1, 9)$

$\alpha - 2\cdot 1 - 9 + 8 = 0$

$\alpha - 3 = 0 \Rightarrow \alpha = 3$

13) $\vec{u} = (1, -1, 1)$

$\vec{v} = (2, 3, 0)$

$A(2, 0, -2)$

I) $\vec{n} = \vec{u} \times \vec{v} = \left|\begin{array}{ccc|cc} \hat{i} & \hat{j} & \hat{k} & \hat{i} & \hat{j} \\ 1 & -1 & 1 & 1 & -1 \\ 2 & 3 & 0 & 2 & 3 \end{array}\right|$

$= -3\hat{i} + 2\hat{j} + 5\hat{k} = (-3, 2, 5)$

II) $\pi: -3x + 2y + 5z + d = 0$

Utilizando o ponto A:

$-3\cdot 2 + 2\cdot 0 + 5(-2) + d = 0$

$d = 16$ //

III) Equação Geral do Plano:

$\pi: -3x + 2y + 5z + 16 = 0 \quad x(-1)$

$\pi: 3x - 2y - 5z - 16 = 0$ //

14) $A(-3, 1, -2)$

$B(-1, 2, 1)$

$\vec{AB} = (2, 1, 3)$

$r: \frac{x}{2} = \frac{z}{-3}; y = 4$

$\vec{v} = (2, 0, -3)$

I) $\vec{n} = (\vec{AB} \times \vec{v}) = \left|\begin{array}{ccc|cc} \hat{i} & \hat{j} & \hat{k} & \hat{i} & \hat{j} \\ 2 & 1 & 3 & 2 & 1 \\ 2 & 0 & -3 & 2 & 0 \end{array}\right|$

$= -3\hat{i} + 6\hat{j} + 6\hat{j} - 2\hat{k} = (-3, 12, -2)$

II) $\pi: -3x + 12y - 2z + d = 0$

Utilizando o ponto B:

$-3(-1) + 12(2) - 2(1) + d = 0$

$d = -25$ //

III) Equação Geral do Plano:

$\pi: -3x + 12y - 2z - 25 = 0 \quad x(-1)$

$\pi: 3x - 12y + 2z + 25 = 0$ //

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15) $A(1,-2,2)$

$B(-3,1,-2)$

$\pi \perp \pi_1$

$\pi_1: 2x+y-z+8=0$

$\vec{n_1}=(2,1,-1)$

I) $\vec{AB}=(-4,3,-4)$

$\vec{n_1}=(2,1,-1)$

$$\vec{n}=(\vec{n_1}\times\vec{AB})=\left|\begin{array}{ccc|cc} \hat{i} & \hat{j} & \hat{k} & \hat{i} & \hat{j} \\ -4 & 3 & -4 & -4 & 3 \\ 2 & 1 & -1 & 2 & 1 \end{array}\right|$$

$= -3\hat{i}+4\hat{i}-8\hat{j}-4\hat{j}-4\hat{k}-6\hat{k}$

$= (1,-12,-10)$

II) $\pi: x-12y+10z+d=0$

Utilizando o ponto A:

$1-12(-2)+10\cdot 2+d=0$

$d=-5$

III) Equação geral do plano:

$\pi: x-12y+10z-5=0$

16) $A(2,1,2)$

$B(1,-1,4)$

$\vec{n_1}=(0,0,z)\to(0,0,1)$

$\vec{AB}=(-1,-2,2)$

I) $$\vec{n}=(\vec{n_1}\times\vec{AB})=\left|\begin{array}{ccc|cc} \hat{i} & \hat{j} & \hat{k} & \hat{i} & \hat{j} \\ 0 & 0 & 1 & 0 & 0 \\ -1 & -2 & 2 & -1 & 2 \end{array}\right|$$

$= +2\hat{i}-1\hat{j} = (+2,-1,0)$

II) $\pi: 2x-y+d=0$

Utilizando o ponto B:

$2(1)-1(-1)+d=0$

$d=-3$

III) Equação geral do Plano:

$\pi: 2x-y-3=0$

17) $r: \begin{cases} x=2+t \\ y=1-t \\ z=3+2t \end{cases}$

$\vec{v}=(1,-1,2)$

$P(2,1,3)$

$\pi_1: 2x+2y-3z=0$

$\vec{n_1}=(2,2,-3)$

I) $$\vec{n}=(\vec{n_1}\times\vec{v})=\left|\begin{array}{ccc|cc} \hat{i} & \hat{j} & \hat{k} & \hat{i} & \hat{j} \\ 2 & 2 & -3 & 2 & 2 \\ 1 & -1 & 2 & 1 & -1 \end{array}\right|$$

$= 4\hat{i}-3\hat{i}-3\hat{j}-4\hat{j}-2\hat{k}-2\hat{k}$

$= (1,-7,-4)$

II) $\pi: x-7y-4z+d=0$

Utilizando ponto P:

$1(2)-7(1)-4(3)+d=0$

$d=17$

III) Equação geral do Plano:

$\pi: x-7y-4z+17=0$

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18) $A(4,1,1)$

$\pi_1: 2x+y-3z=0$

$\vec{n_1}=(2,1,-3)$

$\pi_2: x+y-2z-3=0$

$\vec{n_2}=(1,1,-2)$

$P(x,y,z)$

I) $\vec{n}=(\vec{n_1}\times\vec{n_2})=\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 1 & -3 \\ 1 & 1 & -2 \end{vmatrix}\begin{matrix} \hat{i} & \hat{j} \\ 2 & 1 \\ 1 & 1 \end{matrix}$

$=(1,1,1)$

II) $\pi: x+y+z+d=0$

utilizando o ponto A:

$d=-6$

III) Equação Geral do Plano:

$\pi: x+y+z-6=0$

19) $P(0,-3,2)$

$r_1: \begin{cases} x=t \\ y=2t-3 \\ z=-t+2 \end{cases}$ $\quad r_2: \begin{cases} x=1+3t \\ y=-1 \\ z=1-t \end{cases}$

$\vec{v_1}=(1,2,-1)$ $\quad \vec{v_2}=(3,0,-1)$

I) $\vec{n}=(\vec{v_1}\times\vec{v_2})=\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & -1 \\ 3 & 0 & -1 \end{vmatrix}\begin{matrix} \hat{i} & \hat{j} \\ 1 & 2 \\ 3 & 0 \end{matrix}$

$=-2\hat{i}-2\hat{j}-6\hat{k}=(-2,-2,-6)$

II) $\pi: -2x-2y-6z+d=0$

utilizando o ponto P:

$-2(-3)-6(2)+d=0$

$d=6$

III) Equação geral do Plano:

$\pi: -2x-2y-6z+6=0 \quad (\div -2)$

$\pi: x+y+3z-3=0$

20) $\vec{v_1}=(2,3,-1)$

$\vec{v_2}=(-2,-1,2)$

$P(1,-2,3)$

I) $\vec{n}=(\vec{v_1}\times\vec{v_2})=\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 3 & -1 \\ -2 & -1 & 2 \end{vmatrix}\begin{matrix} \hat{i} & \hat{j} \\ 2 & 3 \\ -2 & -1 \end{matrix}$

$=5\hat{i}-2\hat{j}+4\hat{k}=(5,-2,4)$

II) $\pi: 5x-2y+4z+d=0$

utilizando o Ponto P:

$5(1)-2(-2)+4(3)+d=0$

$d=-21$

III) Equação geral do plano:

$\pi: 5x-2y+4z-21=0$

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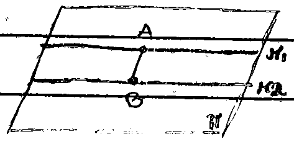
21) $\vec{v}_1 = (1,-1,0)$

$\vec{v}_2 = (1,-1,0)$



$A(-2,0,-3)$

$B(0,-1,3)$

$\vec{AB} = (2,-1,6)$

I) $\vec{n} = (\vec{AB} \times \vec{v}_1) = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & -1 & 6 \\ 1 & -1 & 0 \end{vmatrix} \begin{matrix} \hat{i} & \hat{j} \\ 2 & -1 \\ 1 & -1 \end{matrix}$

$= 6\hat{i} + 6\hat{j} + 2\hat{k} + 1\hat{k} = (6,6,-1)$

II) $\pi: 6x + 6y - z + d = 0$

Utilizando o ponto A

$-12 + 3 + d = 0$

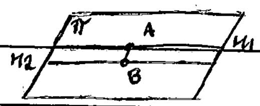
$d = 9$

III) Equação Geral do Plano

$\pi: 6x + 6y + z + 9 = 0$

22) $\vec{v}_1 = (1,0,1)$

$\vec{v}_2 = (-1,0,-1)$



$A(0,-3,0)$

$B(0,1,2)$

$\vec{AB} = (0,4,2)$

I) $\vec{n} = (\vec{AB} \times \vec{v}_1) = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & 4 & 2 \\ 1 & 0 & 1 \end{vmatrix} \begin{matrix} \hat{i} & \hat{j} \\ 0 & 4 \\ 1 & 0 \end{matrix}$

$= 4\hat{i} + 2\hat{j} - 4\hat{k} = (4,2,-4)$

II) $\pi: 4x + 2y - 4z + d = 0$

Utilizando o ponto A

$-6 + d = 0$

$d = 6$

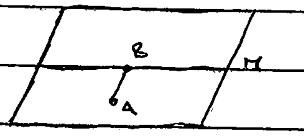
III) Equação Geral do Plano

$\pi: 4x + 2y - 4z + 6 = 0 \quad (:2)$

$\pi: 2x + y - 2z + 3 = 0$

23) $A(4,3,2)$

$B(0,2,3)$



$\vec{BA} = (4,1,-1)$

$\vec{v_r} = (1,-1,2)$

I) $\vec{n} = (\vec{BA} \times \vec{v_r}) = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 4 & 1 & -1 \\ 1 & -1 & 2 \end{vmatrix} \begin{matrix} \hat{i} & \hat{j} \\ 4 & 1 \\ 1 & -1 \end{matrix}$

$= 2\hat{i} - \hat{i} - \hat{j} - 8\hat{j} - 4\hat{k} - \hat{k}$

$= (1,-9,-5)$

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II) $\pi: x - 9y - 5z + d = 0$

Utilizando o ponto A

$4 - 27 - 10 + d = 0$

$d = 33$

III) Equação Geral do Plano

$\pi: x - 9y - 5z + 33 = 0$

24) $A(1,-1,2)$

$\vec{v} = (0,0,1)$

$P(0,0,0)$

$\vec{AP} = (1,-1,2)$

I) $\vec{n} = (\vec{AP} \times \vec{v}) = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -1 & 2 \\ 0 & 0 & 1 \end{vmatrix} \begin{matrix} \hat{i} & \hat{j} \\ 1 & -1 \\ 0 & 0 \end{matrix}$

$= -\hat{i} - \hat{j} = (-1,-1,0)$

II) $\pi: -x - y + d = 0$

Utilizando o ponto A

$-1 + 1 + d = 0$

$d = 0$ //

III) Equação Geral do Plano

$\pi: -x - y = 0 \quad \times(-1)$

$\pi: x + y = 0$ //

25) $A(0,3,4)$

$B(2,0,-2)$

$\vec{AB} = (2,-3,-6)$

$\vec{v} = (0,0,1)$

$\vec{n} = (\vec{AB} \times \vec{v}) = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & -3 & -6 \\ 0 & 0 & 1 \end{vmatrix} \begin{matrix} \hat{i} & \hat{j} \\ 2 & -3 \\ 0 & 0 \end{matrix}$

$= -3\hat{i} - 2\hat{j}$

II) $\pi: -3x - 2y + d = 0$

Utilizando o Ponto A:

$-6 + d = 0$

$d = 6$ //

III) Equação Geral do Plano:

$\pi: -3x - 2y + 6 = 0 \quad \times(-1)$

$\pi: 3x + 2y - 6 = 0$ //

26) $A(-2,0,2)$

$B(0,-2,1)$

$\vec{AB} = (2,-2,-1)$

$\vec{v} = (1,0,0)$

I) $\vec{n} = (\vec{AB} \times \vec{v}) = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & -2 & -1 \\ 1 & 0 & 0 \end{vmatrix} \begin{matrix} \hat{i} & \hat{j} \\ 2 & -2 \\ 1 & 0 \end{matrix}$

$= -\hat{j} + 2\hat{k}$

II) $\pi: -y + 2z + d = 0$

Utilizando o ponto A:

$4 + d = 0$

$d = -4$ //

III) Equação Geral do Plano:

$\pi: -y + 2z - 4 = 0 \quad \times(-1)$

$\pi: y - 2z + 4 = 0$ //

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27) $\vec{v}=(0,1,0)$

R. $A(2,3,0)$

$B(0,4,1)$

$\vec{AB}=(-2,1,1)$

I) $\vec{n}=(\vec{AB}\times\vec{v})=\begin{vmatrix}\hat{i}&\hat{j}&\hat{k}\\-2&1&1\\0&1&0\end{vmatrix}\begin{matrix}\hat{i}&\hat{j}\\-2&1\\0&1\end{matrix}$

$=-\hat{i}-2\hat{k}=(-1,0,-2)$

II) $\pi: -x+2z+d=0$

Utilizando o Ponto A:

$-2+d=0$

$d=2$

III) Equação Geral do Plano:

$\pi: -x-2z+2=0 \quad \times(-1)$

$\pi: x+2z-2=0$

28) $A(5,-2,3)$

R. $xOy \Rightarrow \vec{v}=(0,0,z)$

$\vec{n}=(0,0,1)$

$P(0,0,3)$

I) $\pi: z+d=0$

$3+d=0$

$d=-3$

II) $\pi: z-3=0$

$\pi: z=3$

R.

29) $\vec{n}=(0,1,0)$

R. $A(3,4,-1)$

I) $\pi: y+d=0$

Ponto A:

$d=-4$

II) $\pi: y-4=0$

$y=4$

30) $\vec{v}=(1,0,0)$

R. $A(1,-2,1)$

$P(0,0,0)$

$\vec{PA}=(1,-2,1)$

I) $\vec{n}=(\vec{PA}\times\vec{v})=\begin{vmatrix}\hat{i}&\hat{j}&\hat{k}\\1&-2&1\\1&0&0\end{vmatrix}\begin{matrix}\hat{i}&\hat{j}\\1&-2\\1&0\end{matrix}$

$=\hat{j}+2\hat{k}=(0,1,2)$

II) $\pi: y+2z+d=0$

Ponto A:

$-2+2+d=0$

$d=0$

III) Equação Geral do Plano.

$\pi: y+2z=0$

31)

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32) a) $\vec{n_1} = (1, -2, 1)$
$\vec{n_2} = (2, -1, -1)$

$$\cos\theta = \frac{|\vec{n_1} \cdot \vec{n_2}|}{|\vec{n_1}| \cdot |\vec{n_2}|}$$

$$\cos\theta = \frac{|2+2-1|}{\sqrt{6} \cdot \sqrt{6}} = \frac{3}{6} = \frac{1}{2} = 60° = \frac{\pi}{3}$$

b) $\vec{n_1} = (1, -1, 0)$
$\vec{n_2} = (2, -1, -1)$

$$\cos\theta = \frac{|2+1|}{\sqrt{2} \cdot \sqrt{6}} = \frac{3}{2\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \frac{\sqrt{3}}{2} = 30° = \frac{\pi}{6}$$

c) $\vec{n_1} = (1, 2, 0)$
$\vec{n_2} = (0, 1, 0)$

$$\cos\theta = \frac{|2|}{\sqrt{5} \cdot 1} = \frac{2}{\sqrt{5}} = \arccos \frac{2}{\sqrt{5}}$$

d) $v_1' = (1, 1, 1)$
$v_1'' = (-1, 2, 0)$

$$\vec{n_1} = (v_1' \times v_1'') = \left|\begin{array}{ccc|cc} \hat{i} & \hat{j} & \hat{k} & \hat{i} & \hat{j} \\ 1 & 1 & 1 & 1 & 1 \\ -1 & 2 & 0 & -1 & 2 \end{array}\right|$$

$$= -2\hat{i} - \hat{j} + 3\hat{k} = (-2, -1, 3)$$

$v_2' = (0, -2, 1)$
$v_2'' = (1, 0, 1)$

$$\vec{n_2} = (v_2' \times v_2'') = \left|\begin{array}{ccc|cc} \hat{i} & \hat{j} & \hat{k} & \hat{i} & \hat{j} \\ 0 & -2 & 1 & 0 & -2 \\ 1 & 0 & 1 & 1 & 0 \end{array}\right|$$

$$= -2\hat{i} + \hat{j} + 2\hat{k} = (-2, 1, 2)$$

$$\cos\theta = \frac{|4-1+6|}{\sqrt{14} \cdot \sqrt{9}} = \frac{9}{\sqrt{14} \cdot 3} = \frac{3}{\sqrt{14}} = \arccos \frac{3}{\sqrt{14}}$$

33) $\vec{n_1} = (1, m, 2)$
$\vec{n_2} = (4, 5, 3)$

$$\cos\theta = \frac{\sqrt{3}}{2} \Rightarrow \frac{\sqrt{3}}{2} = \frac{|4+5m+6|}{\sqrt{m^2+5} \cdot \sqrt{50}}$$

$$\Rightarrow (\sqrt{3} \cdot \sqrt{m^2+5} \cdot \sqrt{50})^2 = (20+10m)^2$$

$$3 \cdot (m^2+5) \cdot 50 = 400 + 400m + 100m^2$$

$$150m^2 + 750 = 400 + 400m + 100m^2$$

$$5\cancel{0}m^2 - 40\cancel{0}m + 35\cancel{0} = 0$$

$$m^2 - 8m + 7 = 0$$

$$\Delta = 36$$

$\hookrightarrow m = \frac{8 \pm 6}{2}$

$m' = 7$

$m'' = 1$

/ / 4+16

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34) a) $\vec{n_1} = (m, 1, -3)$ $\quad 0 = \dfrac{|2m - 3m - 12|}{\sqrt{4+m^2} \cdot \sqrt{9m^2+20}}$

$\vec{n_2} = (2, -3m, 4)$

$\cos 90° = \frac{\pi}{2} = 0$ $\quad |-m - 12| = 0$

$m + 12 = 0$

$m = -12$ //

b) $\vec{v_1} = (-1, 2, -2)$ $\quad$ I)

$\vec{v_2} = (2, 0, 1)$ $\quad \vec{n_1} = (\vec{v_1} \times \vec{v_2}) = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -1 & 2 & -2 \\ 2 & 0 & 1 \end{vmatrix} \begin{matrix} \hat{i} & \hat{j} \\ -1 & 2 \\ 2 & 0 \end{matrix}$

$\vec{n_2} = (2m, 4, -1)$

$= 2\hat{i} - 3\hat{i} - 4\hat{k} = (2, -3, -4)$

$\cos 90° = 0$ $\quad$ II) $\quad 0 = \dfrac{|4m - 12 + 4|}{\sqrt{29} \cdot \sqrt{4m^2+17}}$

$4m - 8 = 0$

$m = 2$ //

35) a) $r: \begin{cases} x = -3 + t \\ y = -1 + 2t \\ z = 4t \end{cases}$ $\quad \pi: mx - y - 2z - 3 = 0$

$\vec{n} = (m, -1, -2)$

$\vec{v} = (1, 2, 4)$

I) $r // \pi$ $\quad \vec{v} \cdot \vec{n} = 0 \Rightarrow (1, 2, 4) \cdot (m, -1, -2) = 0$

$\vec{v} \perp \vec{n}$ $\quad m - 2 - 8 = 0 \therefore m = 10$ //

II) $r \perp \pi$ $\quad \vec{v} = \alpha \vec{n}$ $\quad \alpha = \frac{2}{-1} = \frac{4}{-2} = -2$

$\vec{v} // \vec{n}$ $\quad (1, 2, 4) = -2(m, -1, -2)$

$-2m = 1$

$m = -1/2$ //

b) $\vec{v} = (2, m, -1)$ $\quad$ I) $r // \pi$ $\quad \vec{v} \cdot \vec{n} = 0$

$\vec{n} = (3, 2, m)$ $\quad \vec{v} \perp \vec{n}$ $\quad (2, m, -1) \cdot (3, 2, m) = 0$

$6 + 2m - m = 0 \therefore m = -6$ //

II) $r \perp \pi$ $\quad \alpha = \frac{2}{3}$ $\quad \vec{v} = \alpha \vec{n}$

$\vec{v} // \vec{n}$ $\quad (2, m, -1) = 2/3 (3, 2, m)$ $\quad$ Não existe um valor

$m = 4/3$ $\quad -3/2 = m$ $\quad$ para m.

Para r está contida em $\pi \Rightarrow \vec{v} \cdot \vec{n} = 0$

36) a) $\vec{v} = (1,4,2)$ $\qquad \vec{v} \cdot \vec{n} = 0$

$\vec{n} = (2,1,-3)$ $\qquad (1,4,2) \cdot (2,1,-3) = 0$ $\qquad$ logo,

$2 + 4 - 6 = 0$ $\qquad$ r está contida em π.

$0 = 0$

b) $\vec{v} = (1,2,1)$ $\qquad$ I)

$\vec{v}'_1 = (1,2,1)$ $\qquad \vec{n} = (\vec{v}'_1 \times \vec{v}''_1) = \left| \begin{array}{ccc|cc} \hat{i} & \hat{j} & \hat{k} & \hat{i} & \hat{j} \\ 1 & 2 & 1 & 1 & 2 \\ 1 & -3 & -1 & 1 & -3 \end{array} \right|$

$\vec{v}''_1 = (1,-3,-1)$

$= \hat{i} + 2\hat{j} - 5\hat{k} = (1,2,-5)$

II) $\vec{v} \cdot \vec{n} = 0$

$(1,2,1) \cdot (1,2,-5) = 0$ $\qquad$ logo,

$1 + 4 - 5 = 0$ $\qquad$ r está contida em π.

$0 = 0$

37) $\vec{v} = (1,-2,2)$ $\qquad$ I) $\vec{v} \cdot \vec{n} = 0$ $\qquad$ II) π: $10x + 2y - 3z + n = 0$

$A(-2,3,0)$ $\qquad (1,-2,2) \cdot (m,2,-3) = 0$ $\qquad$ Utilizando o ponto A:

$\vec{n} = (m,2,-3)$ $\qquad m - 4 - 6 = 0$ $\qquad -20 + 6 + n = 0$

$m = 10$ // $\qquad n = 14$ //

38) $\vec{v} = (1,2,-1)$ $\qquad$ I) $\vec{v} \cdot \vec{n} = 0$ $\qquad$ II) π: $5x - 2y + z + 2 = 0$

$P(0,-1,m)$ $\qquad (1,2,-1) \cdot (5,-n,1) = 0$ $\qquad$ Utilizando o ponto P:

$\vec{n} = (5,-n,1)$ $\qquad 5 - 2n - 1 = 0$ $\qquad 0 + 2 + m + 2 = 0$

$2n = 4$ $\qquad m = -4$ //

$n = 2$ //

39) $\vec{v} = (3,m,-4)$ $\qquad$ I) $\vec{v} \cdot \vec{n} = 0$ $\qquad$ II) π: $3x - 3y + z - 7 = 0$

$\vec{n} = (3,-3,1)$ $\qquad (3,m,-4) \cdot (3,-3,1) = 0$ $\qquad$ Ponto A:

$A(1,-2,n)$ $\qquad 9 - 3m - 4 = 0$ $\qquad 3 + 6 + n - 7 = 0$

$3m = 5$ $\qquad n = -2$ //

$m = 5/3$ //

40) ξ. $r: \begin{cases} 3x - y + 2z = 1 \\ x + 2y - 3z = 4 \end{cases}$

I) Definindo: $x = 0$

$\begin{cases} -y + 2z = 1 \\ 2y - 3z = 4 \end{cases}$

$z = 6$; $y = 11$

$P(0, 11, 6)$

II) $\vec{n_1} = (3, -1, 2)$

$\vec{n_2} = (1, 2, -3)$

$\vec{v} = (\vec{n_1} \times \vec{n_2}) = \left| \begin{array}{ccc|cc} \hat{i} & \hat{j} & \hat{k} & \hat{i} & \hat{j} \\ 3 & -1 & 2 & 3 & -1 \\ 1 & 2 & -3 & 1 & 2 \end{array} \right|$

$= -\hat{i} + 11\hat{j} + 7\hat{k} = (-1, 11, 7)$

III) $r: \begin{cases} x = -t \\ y = 11 + 11t \\ z = 6 + 7t \end{cases}$

IV) $r: \begin{cases} y = 11 - 11x \\ z = 6 - 7x \end{cases}$

41) ξ. $r: \begin{cases} 3x - 2y - z = 1 \\ x + 2y - z = 7 \end{cases}$

I) Definindo: $x = 0$

$\begin{cases} -2y - z = 1 \\ 2y - z = 7 \end{cases}$

$-2z = 8 \therefore z = -4$

$y = \frac{3}{2}$

$P(0, 3/2, -4)$

II) $\vec{n_1} = (3, -2, -1)$

$\vec{n_2} = (1, 2, -1)$

$\vec{v} = (\vec{n_1} \times \vec{n_2}) = \left| \begin{array}{ccc|cc} \hat{i} & \hat{j} & \hat{k} & \hat{i} & \hat{j} \\ 3 & -2 & -1 & 3 & -2 \\ 1 & 2 & -1 & 1 & 2 \end{array} \right|$

$= 4\hat{i} + 2\hat{j} + 8\hat{k} = (4, 2, 8)$

III) $r: \begin{cases} x = 4t \\ y = 3/2 + 2t \\ z = -4 + 8t \end{cases}$

IV) $t = x/4$

$r: \begin{cases} y = 3/2 + x/2 \\ z = -4 + 2x \end{cases}$

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42) $r: \begin{cases} x+y-z=-2 \\ x+y+2z=1 \end{cases}$

I) Definindo: $x=0$

$\begin{cases} y-z=-2 \\ y+2z=1 \end{cases}$ $P(0,-1,1)$

$3z=3$

$z=1$ // $y=-1$ //

II) $\vec{n_1}=(1,1,-1)$

$\vec{n_2}=(1,1,2)$

$\vec{v}=(\vec{n_1}\times\vec{n_2})=\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 1 & -1 \\ 1 & 1 & 2 \end{vmatrix}\begin{matrix} \hat{i} & \hat{j} \\ 1 & 1 \\ 1 & 1 \end{matrix}$

$= 3\hat{i}-3\hat{j} = (3,-3,0)$

III) $r: \begin{cases} x=3t \\ y=-1-3t \\ z=1 \end{cases}$

IV) $t=x/3$

$r: \begin{cases} y=-1-x \\ z=1 \end{cases}$

43) $r: \begin{cases} 3x+y-3z-5=0 \\ x-y-z-3=0 \end{cases}$

I) Definindo: $x=0$

$\begin{cases} y-3z=5 \\ -y-z=3 \end{cases}$ $P(0,-1,-2)$

$z=-2$ // $y=-1$ //

II) $\vec{n_1}=(3,1,-3)$

$\vec{n_2}=(1,-1,-1)$

$\vec{v}=(\vec{n_1}\times\vec{n_2})=\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & 1 & -3 \\ 1 & -1 & -1 \end{vmatrix}\begin{matrix} \hat{i} & \hat{j} \\ 3 & 1 \\ 1 & -1 \end{matrix}$

$= -4\hat{i}-4\hat{k} = (-4,0,-4)$

III) $r: \begin{cases} x=-4t \\ y=-1 \\ z=-2-4t \end{cases}$

44) $r: \begin{cases} 2x+y=4 \\ z=5 \end{cases}$

I) Definindo: $x=0$

$y=4$

$P(0,4,5)$

II) $\vec{n_1}=(2,1,0)$

$\vec{n_2}=(0,0,1)$

$\vec{v}=(\vec{n_1}\times\vec{n_2})=\begin{vmatrix} i & j & k \\ 2 & 1 & 0 \\ 0 & 0 & 1 \end{vmatrix}\begin{matrix} i & j \\ 2 & 1 \\ 0 & 0 \end{matrix}$

$= i-2j = (1,-2,0)$

III) $r: \begin{cases} x=t \\ y=4-2t \\ z=5 \end{cases}$

45) S.

$$r: \begin{cases} x = 3t \\ y = 1 - 2t \\ z = -t \end{cases} \qquad \pi: 2x + 3y - 2z - 7 = 0$$

I) Determinando o t:

$$2(3t) + 3(1-2t) - 2(-t) - 7 = 0$$

$$\cancel{6t} + 3 - \cancel{6t} + 2t - 7 = 0$$

$$2t = 4 \therefore t = 2$$

II) Determinando o ponto de interseção:

$x = 3 \cdot 2 = 6$

$y = 1 - 2 \cdot 2 = -3$ $\qquad P(6, -3, -2)$

$z = -2$

46) S.

$$r: \begin{cases} x = t \\ y = t - 10 \\ z = -t + 1 \end{cases} \qquad \pi: 2x - y + 3z - 9 = 0$$

I) Determinando o t:

$$2t - t + 10 - 3t + 3 - 9 = 0$$

$$-2t = -4 \therefore t = 2$$

II) Determinando o ponto de interseção:

$x = 2$

$y = -8$ $\qquad P(2, -8, -1)$

$z = -1$

47) S.

$$r: \begin{cases} x = 4 + K \\ y = 3 + 2K \\ z = -2 - 3K \end{cases}$$

I) $\vec{v}_1' = (1, -1, 3)$

$\vec{v}_1'' = (2, -1, -3)$

$$\vec{n} = (\vec{v}_1' \times \vec{v}_1'') = \left| \begin{array}{ccc|cc} \hat{i} & \hat{j} & \hat{k} & \hat{i} & \hat{j} \\ 1 & -1 & 3 & 1 & -1 \\ 2 & -1 & -3 & 2 & -1 \end{array} \right|$$

$$= 6\hat{i} + 9\hat{j} + \hat{k} = (6, 9, 1)$$

II) $\pi: 6x + 9y + z + d = 0$ $\qquad$ III) $\pi: 6x + 9y + z + 14 = 0$

$P(2, -3, 1)$

$12 - 27 + 1 + d = 0$

$d = 14$

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IV) Determinando o K:

$6(4+K)+9(3+2K)+1(-2-3K)+14=0$

$24+6K+27+18K-2-3K+14=0$

$21K+63=0$

$K=-63/21=-3$

V) Determinando o ponto de interseção:

$x=4-3=1$

$y=3+2\cdot(-3)=-3$ $P(1,-3,7)$

$z=-2-3(-3)=7$

48) a) Plano xOz I) $0=2x-3$ II) $z=-3/2+2$

$y=0$ $x=3/2$ $z=1/2$

$P(3/2, 0, 1/2)$

b) $r: \begin{cases} x=t \\ y=2t-3 \\ z=-t+2 \end{cases}$ I) Determinando o t:

$2(t)+4(2t-3)-(-t+2)-4=0$

$2t+8t-12+t-2-4=0$

$11t=18 \therefore t=18/11$

II) Determinando o ponto de interseção:

$x=18/11$ Logo,

$y=36/11-33/11=3/11$ $P(18/11, 3/11, 4/11)$

$z=-18/11+22/11=4/11$

c) Plano xOy $\Rightarrow z=0$ I) $4y=4+z-2x$ II) $z=0$

$\begin{cases} z=0 \\ 2x+4y-z-4=0 \end{cases}$ $y=\dfrac{4-2x}{4}$

$y=-1/2x+1$

$r: \begin{cases} y=-1/2x+1 \\ z=0 \end{cases}$

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49) a) $\vec{v} = \vec{n} = (2,1,1)$

$P(5,2,3)$

$r: \begin{cases} x = 5 + 2t \\ y = 2 + t \\ z = 3 + t \end{cases}$

b) $\pi: 2x + y + z - 3 = 0$

$2(5+2t) + 2 + t + \cancel{3} + t - \cancel{3} = 0$

$10 + 4t + 2 + t + t = 0$

$6t = -12$

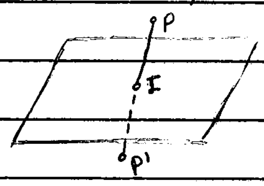
$t = -2$ //

$x = 1$

$y = 0$ $I(1,0,1)$ //

$z = 1$

c)



$\vec{PI} = \vec{IP'}$

$I - P = P' - I$

$P' = 2I - P$

$P' = (2,0,2) - (5,2,3) = (-3,-2,-1)$ //

d) $d(P,\pi) = \dfrac{|10 + 2 + 3 - 3|}{\sqrt{6}} = \dfrac{12\sqrt{6}}{6} = 2\sqrt{6}$ u.c //

50) $\vec{v} = \vec{n} = (1,-3,2)$

$A(3,-2,4)$

$r: \begin{cases} x = 3 + t \Rightarrow t = x - 3 \\ y = -2 - 3t \\ z = 4 + 2t \end{cases}$

$r: \begin{cases} y = -3x + 7 \\ z = 2x - 2 \end{cases}$ //

51) a) $\vec{n_1} = (2,1,1)$

$\vec{n_2} = (1,-3,-1)$

I) $\vec{v} = (\vec{n_1} \times \vec{n_2}) = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 1 & 1 \\ 1 & -3 & -1 \end{vmatrix} \begin{matrix} \hat{i} & \hat{j} \\ 2 & 1 \\ 1 & -3 \end{matrix} = 2\hat{i} + 3\hat{j} - 7\hat{k}$

$(2,3,-7)$

$A(-1,0,2)$

II) $\begin{cases} x = 2t - 1 \\ y = 3t \\ z = -7t + 2 \end{cases}$

b) $A(0,0,0)$ $\vec{n} = (1,-1,-1)$

$r: 2x = y = 3z$

$r: x = y/2 = 3z/2$

$r: x/3 = y/6 = z/2$

$\vec{v_r} = (3,6,2)$

I) $\vec{v} = (\vec{v_r} \times \vec{n}) = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & 6 & 2 \\ 1 & -1 & -1 \end{vmatrix} \begin{matrix} \hat{i} & \hat{j} \\ 3 & 6 \\ 1 & -1 \end{matrix} = -4\hat{i} + 5\hat{j} - 9\hat{k}$

II) $r': \begin{cases} x = -4t \\ y = 5t \\ z = -9t \end{cases}$ //

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52) $\vec{v_1}=(1,1,-3)$
$\vec{v_2}=(3,6,2)$
$A(-1,2,-1)$

I) $\vec{n}=(\vec{v_1}\times\vec{v_2})=\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 1 & -3 \\ 3 & 6 & 2 \end{vmatrix}\begin{matrix} \hat{i} & \hat{j} \\ 1 & 1 \\ 3 & 6 \end{matrix} = 20\hat{i}-11\hat{j}+3\hat{k}$

$(20,-11,3)$

II) $\pi: 20x-11y+3z+d=0$

Ponto A:

$-20-22-3+d=0$

$d=45$

III) $\pi: 20x-11y+3z+45=0$

53) a) $A(2,1,-4)$

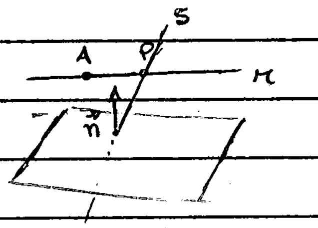
$\pi: x-y+3z-5=0$

$\vec{n}=(1,-1,3)$

$s: \begin{cases} x=1+3t \\ y=3-t \\ z=-2-2t \end{cases}$

$P(1+3t,3-t,-2-2t)$

$\vec{AP}=(3t-1,2-t,-2t+2)$



I) $\vec{AP}\cdot\vec{n}=0$

$(3t-1,2-t,-2t+2)\cdot(1,-1,3)=0$

$3t-1-2+t-6t+6=0$

$-2t=-3 \Rightarrow t=3/2$

II) $P(11/2,3/2,-5)$

III) $\vec{v}=\vec{AP}=(7/2,1/2,-1)=(7,1,-2)$

IV) $r: \begin{cases} x=2+7h \\ y=1+h \\ z=-4-2h \end{cases}$

b) $A(3,-2,-4)$

$\vec{n}=(3,-2,-3)$

$s: \begin{cases} x=2+t \\ y=-4-2t \\ z=1+3t \end{cases}$

$P(2+t,-4-2t,1+3t)$

$\vec{v}=\vec{AP}=(t-1,-2t-2,3t+5)$

I) $\vec{AP}\cdot\vec{n}=0$

$(t-1,-2t-2,3t+5)\cdot(3,-2,-3)=0$

$3t-3+4t+4-9t-15=0$

$-2t=14$

$t=-7$

II) $P(-5,10,-20)$

III) $\vec{v}=\vec{AP}=(-8,12,-16)$
$=(-2,3,-4)$

IV) $r: \begin{cases} x=3-2t \\ y=-2+3t \\ z=-4-4t \end{cases}$

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54)
$$r: \begin{cases} x = 3+t \\ y = 1-2t \\ z = -1+2t \end{cases}$$

I) Plano $xOy \rightarrow \pi: z=0$

$x=0$ e $y=0$

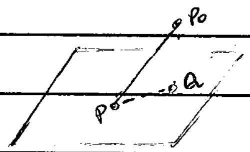
$\vec{n} = (0,0,1)$

i) Ponto P:

$-1+2t=0$

$t = 1/2$

$P(3+t, 1-2t, -1+2t)$



$$s: \begin{cases} x=3 \\ y=1 \\ z=-1+h \end{cases}$$

$P(7/2, 0, 0)$

ii) Ponto Q:

$-1+h=0$

$h=1$

$Q(3,1,0)$

iii) $\vec{PQ} = Q-P$

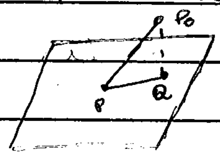
$\vec{PQ} = (-1/2, 1, 0)$

$= (1, -2, 0)$

iv)
$$r': \begin{cases} x = 3+t' \Rightarrow t' = x-3 \\ y = 1-2t' \\ z=0 \end{cases}$$

$$r' \begin{cases} y = -2x+7 \\ z=0 \end{cases}$$

II) Plano $xOz \rightarrow \pi: y=0$



$\vec{n} = (0,1,0)$

$P(3+t, 1-2t, -1+2t)$

i) Ponto P:

$1-2t=0$

$t = 1/2$

$P(7/2, 0, 0)$

ii)
$$s: \begin{cases} x=3 \\ y=1+h \\ z=-1 \end{cases}$$

Ponto Q:

$1+h=0$

$h=-1$

$Q(3,0,-1)$

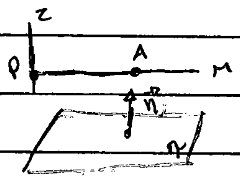
iii) $\vec{PQ} = (-1/2, 0, -1)$

$= (1, 0, 2)$

iv)
$$r' \begin{cases} x = 3+t' \Rightarrow t' = x-3 \\ y=0 \\ z = -1+2t' \end{cases}$$

$$r': \begin{cases} y=0 \\ z = 2x-7 \end{cases}$$

55)



$\pi: x-3y+5z-6=0$

$\vec{n} = (1,-3,5)$

$P(0,0,z)$

$A(3,6,4)$

I) $\vec{AP} \cdot \vec{n} = 0$

$(-3, -6, z-4) \cdot (1, -3, 5) = 0$

$-3+18+5z-20=0$

$5z=5$

$z=1$

$P(0,0,1)$

$\vec{AP} = (-3, -6, -3)$

$= (1, 2, 3)$

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II) utilizando o ponto A:

$$r \begin{cases} x = 3 + t \\ y = 6 + 2t \\ z = 4 + t \end{cases}$$ //

56) 7.

$\pi_1: x + z = 2$

$\pi_2: y - z = 0$

$\vec{n}_1 = (1, 0, 1)$

$\vec{n}_2 = (0, 1, -1)$

$A(-1, 2, -4)$

I) $\vec{n} = (\vec{n}_1 \times \vec{n}_2) = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 0 & 1 \\ 0 & 1 & -1 \end{vmatrix} \begin{matrix} \hat{i} & \hat{j} \\ 1 & 0 \\ 0 & 1 \end{matrix}$

$= -\hat{i} + \hat{j} + \hat{k} = (-1, 1, 1)$

II) $\pi: -x + y + z + d = 0$

utilizando o ponto A:

$+1 + 2 - 4 + d = 0$

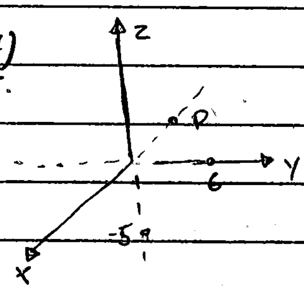
$d = 1$ //

III) Equação Geral do Plano π:

$\pi: -x + y + z + 1 = 0 \quad \times(-1)$

$\pi: x - y - z - 1 = 0$ //

57) 7.



$P(-3, 0, 0)$

$Q(0, 6, 0)$

$R(0, 0, -5)$

$-3 \cdot 6 \cdot -5 \neq 0$ //

Pela Equação segmentária do Plano:

$$\frac{x}{-3} + \frac{y}{6} + \frac{z}{-5} = 1$$

$$\frac{-10x + 5y + 6z + 30 = 0}{-30}$$

$\pi: 10x - 5y + 6z + 30 = 0$ //

58) 7.

$Q(p, 0, 0)$

$R(0, p, 0)$

$S(0, 0, p)$

$\vec{QR} = (-p, p, 0)$

$\vec{QS} = (-p, 0, p)$

$A(1, -3, 4)$

I) $\vec{n} = (\vec{QR} \times \vec{QS}) = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -p & p & 0 \\ -p & 0 & p \end{vmatrix} \begin{matrix} \hat{i} & \hat{j} \\ -p & p \\ -p & 0 \end{matrix}$

$= (p^2, p^2, p^2)$

Para $p = 1$

$\vec{n} = (1, 1, 1)$

II) $\pi: x+y+z+d=0$

utilizando o ponto A:

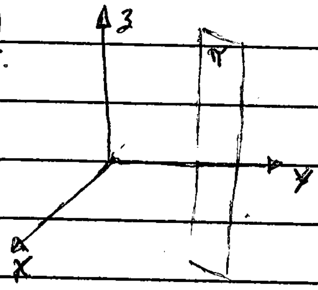
$1-3+4+d=0$

$d=-2$

III) Equação Geral do Plano π:

$\pi: x+y+z-2=0$

59) R.



$Q(-3,0,0)$

$R(0,4,0)$

$S(0,0,0)$

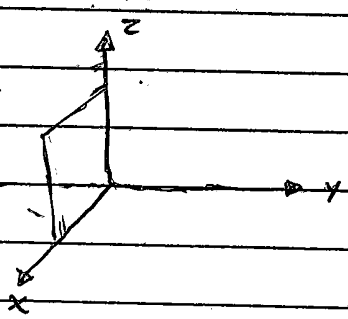
Pela equação segmentária do Plano π:

$$\frac{x}{-3}+\frac{y}{4}=1$$

$4x-3y+12=0$

$\pi: 4x-3y+12=0$

60) R.



$\pi_1: y=0$

$\pi // \pi_1$

Pela equação segmentária do Plano π:

$$\frac{y}{-7}=1 \therefore y=-7$$

$\pi: y=-7$

61) R. $\vec{v}_1=(1,-1,0)$

$\vec{v}_2=(1,3,-3)$

$Q(0,0,0)$

↳ origem

I) $\vec{n}=(\vec{v}_1\times\vec{v}_2)=\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -1 & 0 \\ 1 & 3 & -3 \end{vmatrix}\begin{matrix} \hat{i} & \hat{j} \\ 1 & -1 \\ 1 & 3 \end{matrix} = 3\hat{i}+3\hat{j}+4\hat{k}$

$(3,3,4)$

II) $\pi: 3x+3y+4z+d=0$

$d=0$

III) $\pi: 3x+3y+4z=0$

62) R. $\pi_1: 2x-y+3z-4=0$

$\vec{n}_1=(2,-1,3)$

$\pi_2: x+2y-4z+1=0$

$\vec{n}_2=(1,2,-4)$

$A(-1,2,5)$

I) $\vec{n}=(\vec{n}_1\times\vec{n}_2)=\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & -1 & 3 \\ 1 & 2 & -4 \end{vmatrix}\begin{matrix} \hat{i} & \hat{j} \\ 2 & -1 \\ 1 & 2 \end{matrix} = -2\hat{i}+11\hat{j}+5\hat{k}$

$(-2,11,5)$

II) $A(-1,2,5)$

$-2(-1)+11(2)+5(5)+d=0$

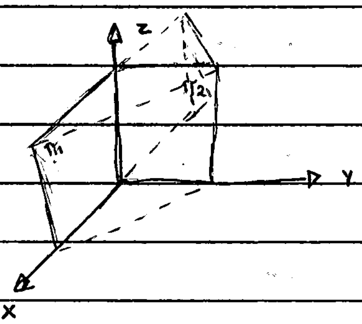
$2+22+25+d=0$

$d=-49_{//}$

III) $\pi: -2x+11y+5z-49=0 \quad x(-1)$

$\pi: 2x-11y-5z+49=0_{//}$

63) /5.



$\pi_1: y=0 \qquad \pi_2: x=0$

$\vec{n_1}=(0,1,0) \qquad \vec{n_2}=(1,0,0)$

i) $\vec{n}=(1,1,0)$

$\pi: x+y+d=0$

$Q(0,0,0)$

$d=0_{//}$

$\pi: x+y=0_{//}$

ii) $n=(-1,1,0)$

$\pi: -x+y=0 \quad x(-1)$

$\pi: x-y=0_{//}$

iii) $n(-1,-1,0)$

$\pi: -x-y=0$

$\pi: x+y=0_{//}$

iv) $n(1,-1,0)$

$\pi: x-y=0_{//}$

64) a) /5.

$$r: \begin{cases} x=2-2t \\ y=-1-t \\ z=3 \end{cases}$$

$P(2,-1,3)$

$Q(0,-2,3)$

i) $\pi: 2mx-ny-z+4=0$

$2m(2)-n(-1)-3+4=0$

$4m+n+1=0$

ii) $2n+1=0$

$n=-1/2_{//}$

iii) $4m+1/2+1=0$

$4m=-3/2$

$m=-1/8_{//}$

b)
$$r: \begin{cases} x=n+2t \\ y=2+mt \\ z=nt \end{cases}$$

$\pi: x-3y+z=1$

i) $P(n,2,0)$

$n-6=1$

$n=7_{//}$

ii) $t=1$; $Q(9;\ 2+m;\ 7)$

$9-3(2+m)+7=1$

$16-6-3m=1$

$-3m=-9 \Rightarrow m=3_{//}$

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65) $A(1,-1,0)$

q. $B(K,1,2)$

$\vec{AB} = (K-1, 2, 2)$

$r: \begin{cases} x = 1 + (K-1)t' \\ y = -1 + 2t' \\ z = 2t' \end{cases}$

$\begin{cases} r // \pi \\ \vec{v} \cdot \vec{n} = 0 \end{cases}$

$\pi: \begin{cases} x = 1 + 3h \\ y = 1 + 2h + t \\ z = 3 + 3t \end{cases}$

I) $\vec{n} = (\vec{v_1} \times \vec{v_2}) = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & 1 & 3 \\ 3 & 2 & 0 \end{vmatrix} \begin{matrix} \hat{i} & \hat{j} \\ 0 & 1 \\ 3 & 2 \end{matrix}$

$\vec{v_1} = (0,1,3)$ $Q(1,1,3)$

$\vec{v_2} = (3,2,0)$

$= -6\hat{i} + 9\hat{j} - 3\hat{k} = (-6, 9, -3)$

$= (2, -3, 1)$

II) $\pi: 2x - 3y + z + d = 0$

Ponto Q:

$2 - 3 + 3 + d = 0$

$d = -2$

III) $\pi: 2x - 3y + z - 2 = 0$

IV) $\vec{v} \cdot \vec{n} = 0$

$(K-1, 2, 2) \cdot (2, -3, 1) = 0$

$2K - 2 - 6 + 2 = 0$

$2K = 6 \Rightarrow K = 3$

66) $A(3,-2,-1)$

q. $r: \begin{cases} x + 2y + z - 1 = 0 & \text{I} \\ 2x + y - z + 7 = 0 & \text{II} \end{cases}$

I) $\underline{z = 1 - x - 2y}$

$z = 1 - x + 2x + 4$

$z = x + 5$

II) $2x + y - 1 + x + 2y + 7 = 0$

$3y + 3x = -6$

$y = -x - 2$

$r: \begin{cases} y = -x - 2 \\ z = x + 5 \end{cases}$

$\vec{n} = (\vec{P_0P_1} \times \vec{P_0A}) = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -1 & 1 \\ 3 & 0 & -6 \end{vmatrix} \begin{matrix} \hat{i} & \hat{j} \\ 1 & -1 \\ 3 & 0 \end{matrix}$

$P_0(0,-2,5)$

$P_1(1,-3,6)$

$= 6\hat{i} + 9\hat{j} + 3\hat{k} = (2,3,1)$

$\vec{P_0P_1} = (1,-1,1)$

$\vec{P_0A} = (3,0,-6)$

$\pi: 2x + 3y + z + 1 = 0$

$\pi: 2x + 3y + z + d = 0$

Ponto A:

$6 - 6 - 1 + d = 0 \Rightarrow d = 1$

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67) S. $r: \begin{cases} x-2y+z-3=0 \\ x=0 \end{cases}$ $r: \begin{cases} z=3+2y \\ x=0 \end{cases}$ $P_0(0,0,3)$
$P_1(0,1,5)$

$A(1,2,1)$ $\vec{P_0P_1}=(0,1,2)$

$\vec{P_0A}=(1,2,-2)$

$$\vec{n}=(\vec{P_0P_1}\times\vec{P_0A})=\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & 1 & 2 \\ 1 & 2 & -2 \end{vmatrix}\begin{matrix} \hat{i} & \hat{j} \\ 0 & 1 \\ 1 & 2 \end{matrix}$$

$$=-6\hat{i}+2\hat{j}-\hat{k}=(6,-2,1)$$

$\pi: 6x-2y+z+d=0$ $\pi: 6x-2y+z-3=0$ //

Ponto A

$6-4+1+d=0$

$d=-3$ //

68) S. $r_1: \begin{cases} 3x-y-z=0 & \text{I} \\ 8x-2y-3z+1=0 & \text{II} \end{cases}$ $r_2: \begin{cases} x-3y+z+3=0 & \text{I)} \\ 3x-y-z+5=0 & \text{II)} \end{cases}$

I) $z=3x-y$ II) $y=\frac{8x-3z+1}{2}$ I) $z=-3+3y-x$

$z=3x-x+1$ $2y=8x-9x+3y+1$ $z=-3+3x+6-x$

$z=2x+1$ // $y=x-1$ // $z=2x+3$ //

II) $y=3x-z+5$

$r_1: \begin{cases} y=x-1 \\ z=2x+1 \end{cases}$ $r_2: \begin{cases} y=x+2 \\ z=2x+3 \end{cases}$ $y=3x+3-3y+x+5$

$4y=4x+8 \Rightarrow y=x+2$ //

$\vec{v_1}=(1,1,2)$ $\vec{v_2}=(1,1,2)$

$P_0(0,-1,1)$ São Paralelos. $P_{02}(0,2,3)$

$\vec{P_0P_{02}}=(0,3,2)$

$$\vec{n}=(\vec{v_1}\times\vec{P_0P_{02}})=\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 1 & 2 \\ 0 & 3 & 2 \end{vmatrix}\begin{matrix} \hat{i} & \hat{j} \\ 1 & 1 \\ 0 & 3 \end{matrix}=-4\hat{i}-2\hat{j}+3\hat{k}$$
$(4,2,-3)$

$\pi: 4x+2y-3z+d=0$

Ponto P_0: $\pi: 4x+2y-3z+5=0$

$-2-3+d=0$

$d=5$

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69) ξ.

$$\begin{cases} 2x - y + z = 8 \\ x + 2y - 2z = -6 \\ 3x - z = 3 \end{cases}$$

$y = 5x - 11$

I) $z = 3x - 3$

$z = 3$

II) $\begin{cases} 5x - y = 11 \\ -5x + 2y = -12 \end{cases}$

$y = -1$

$x = 2$

$P(2, -1, 3)$

$P_0(0,0,0)$

$\vec{P_0P} = (2, -1, 3)$

$r: x = y;\ z = 2y$

$\vec{v} = (1, 1, 2)$

$$\vec{n} = (\vec{v} \times \vec{P_0P}) = \left|\begin{array}{ccc|cc} \hat{i} & \hat{j} & \hat{k} & \hat{i} & \hat{j} \\ 1 & 1 & 2 & 1 & 1 \\ 2 & -1 & 3 & 2 & -1 \end{array}\right|$$

$= 5\hat{i} + \hat{j} - 3\hat{k} = (5, 1, -3)$

$\pi: 5x + y - 3z + d = 0$

$\pi: 5x + y - 3z = 0$

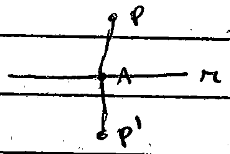
Ponto P:

$10 - 1 - 9 + d = 0$

$d = 0$

70.) a) ξ.

$r: \begin{cases} y = -2x \\ z = x \end{cases}$



$\vec{v} = (1, -2, 1)$

$P(1, 0, 5)$

$A(x, -2x, x)$

$\vec{PA} \cdot \vec{v} = 0$

$(x-1, -2x, x-5) \cdot (1, -2, 1) = 0$

$x - 1 + 4x + x - 5 = 0$

$6x = 6$

$x = 1$

$\vec{PA} = (0, -2, -4)$

$\vec{PA} = AP'$

$(0, -2, -4) = P' - (1, -2, 1)$

$(1, -4, -3) = P'$

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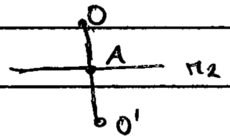
b)
7.

$$r_2 \begin{cases} x = 2 - t \\ y = -1 + t \\ z = 4 - 2t \end{cases}$$



$\vec{OA} \cdot \vec{v} = 0$

$(2-t, -1+t, 4-2t) \cdot (-1, 1, -2) = 0$

$-2+t-1+t-8+4t = 0$

$6t = 11$

$t = 11/6$

$\vec{v} = (-1, 1, -2)$

$O = (0,0,0)$

$A(2-t, -1+t, 4-2t)$

$\vec{OA} = (1/6, 5/6, 1/3)$

$\vec{OA} = \vec{AO'}$

$(1/6, 5/6, 1/3) + (1/6, 5/6, 1/3) = O'$

$O' = (1/3, 5/3, 2/3)$ //

7.1)
7.

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