1. $\vec{u} = 3\vec{i} - \vec{j} - 2\vec{k}$ $\qquad \vec{v} = 2\vec{i} + 4\vec{j} - \vec{k}$ $\qquad \vec{w} = -\vec{i} + \vec{k}$

a) $|\vec{u} \times \vec{u}| = 0$

$\vec{u} \times \vec{u} = 0$

b) $2\vec{v} \times 3\vec{v} = \vec{0}$

$$2\vec{v} \times 3\vec{v} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 4 & 8 & -2 \\ 6 & 12 & -3 \end{vmatrix} = \vec{i} \begin{vmatrix} 8 & -2 \\ 12 & -3 \end{vmatrix} - \vec{j} \begin{vmatrix} 4 & -2 \\ 6 & -3 \end{vmatrix} + \vec{k} \begin{vmatrix} 4 & 8 \\ 6 & 12 \end{vmatrix}$$

$$= i(-24 + 24) - \vec{j}(-12 + 12) + \vec{k}(48 - 48)$$

$$= \vec{0}$$

c) $(\vec{u} \times \vec{w}) + (\vec{w} \times \vec{u}) = (-1, -1, -1) + (1, 1, 1) = 0$

$$\vec{u} \times \vec{w} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 3 & -1 & -2 \\ -1 & 0 & 1 \end{vmatrix} = \vec{i} \begin{vmatrix} -1 & -2 \\ 0 & 1 \end{vmatrix} - \vec{j} \begin{vmatrix} 3 & -2 \\ -1 & 1 \end{vmatrix} + \vec{k} \begin{vmatrix} 3 & -1 \\ -1 & 0 \end{vmatrix}$$

$$= \vec{i}(-1 - 0) - \vec{j}(3 - 2) + \vec{k}(0 - 1)$$

$$= (-1, -1, -1)$$

$$\vec{u} \times \vec{w} = -(\vec{w} \times \vec{u}) = -(-1, -1, -1) = (1, 1, 1)$$

d) $(\vec{u} \times \vec{v}) \times (\vec{v} \times \vec{u}) = \vec{0}$

$$\vec{u} \times \vec{v} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 3 & -1 & -2 \\ 2 & 4 & -1 \end{vmatrix} = \vec{i} \begin{vmatrix} -1 & -2 \\ 4 & -1 \end{vmatrix} - \vec{j} \begin{vmatrix} 3 & -2 \\ 2 & -1 \end{vmatrix} + \vec{k} \begin{vmatrix} 3 & -1 \\ 2 & 4 \end{vmatrix}$$

$$= \vec{i}(1 + 8) - \vec{j}(-3 + 4) + \vec{k}(12 + 2)$$

$$= (9, 1, 14)$$

$$\vec{v} \times \vec{u} = (-9, -1, -14)$$

$$(\vec{u} \times \vec{v}) \times (\vec{v} \times \vec{u}) = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 9 & 1 & 14 \\ -9 & -1 & -14 \end{vmatrix} = \vec{i} \begin{vmatrix} 1 & 14 \\ -1 & -14 \end{vmatrix} - \vec{j} \begin{vmatrix} 9 & 14 \\ -9 & -14 \end{vmatrix} + \vec{k} \begin{vmatrix} 9 & 1 \\ -9 & -1 \end{vmatrix}$$

$$= \vec{i}(-14 + 14) - \vec{j}(-126 + 126) + \vec{k}(-9 + 9)$$

$$= \vec{0}$$

e) $(\vec{u} - \vec{v}) \times \vec{w} = (-5, 0, -5)$

$\vec{u} - \vec{v} = (1, 3, -3)$

$$(\vec{u} - \vec{v}) \times \vec{w} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & -5 & -1 \\ -1 & 0 & 1 \end{vmatrix} = \vec{i} \begin{vmatrix} -5 & -1 \\ 0 & 1 \end{vmatrix} - \vec{j} \begin{vmatrix} 1 & -1 \\ -1 & 1 \end{vmatrix} + \vec{k} \begin{vmatrix} 1 & -5 \\ -1 & 0 \end{vmatrix}$$

$$= \vec{i}(-5 - 0) - \vec{j}(1 - 1) + \vec{k}(0 - 5)$$

$$= (-5, 0, -5)$$

f) $(\vec{u} \times \vec{v}) \times \vec{w} = (-1, -23, -1)$

$\vec{u} \times \vec{v} = (9, -1, 14)$

$$(\vec{u} \times \vec{v}) \times \vec{w} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 9 & -1 & 14 \\ -1 & 0 & 1 \end{vmatrix} = \vec{i} \begin{vmatrix} -1 & 14 \\ 0 & 1 \end{vmatrix} - \vec{j} \begin{vmatrix} 9 & 14 \\ -1 & 1 \end{vmatrix} + \vec{k} \begin{vmatrix} 9 & -1 \\ -1 & 0 \end{vmatrix}$$

$$= \vec{i}(-1-0) - \vec{j}(9+14) + \vec{k}(0-1)$$

$$= (-1, -23, -1)$$

g) $\vec{u} \times (\vec{v} \times \vec{w}) = (-6, -20, 1)$

$$\vec{v} \times \vec{w} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 2 & 4 & -1 \\ -1 & 0 & 1 \end{vmatrix} = \vec{i} \begin{vmatrix} 4 & -1 \\ 0 & 1 \end{vmatrix} - \vec{j} \begin{vmatrix} 2 & -1 \\ -1 & 1 \end{vmatrix} + \vec{k} \begin{vmatrix} 2 & 4 \\ -1 & 0 \end{vmatrix}$$

$$= \vec{i}(4-0) - \vec{j}(2-1) + \vec{k}(0+4)$$

$$= (4, -1, 4)$$

$$\vec{u} \times (\vec{v} \times \vec{w}) = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 3 & -1 & -2 \\ 4 & -1 & 4 \end{vmatrix} = \vec{i} \begin{vmatrix} -1 & -2 \\ -1 & 4 \end{vmatrix} - \vec{j} \begin{vmatrix} 3 & -2 \\ 4 & 4 \end{vmatrix} + \vec{k} \begin{vmatrix} 3 & -1 \\ 4 & -1 \end{vmatrix}$$

$$= \vec{i}(-4-2) - \vec{j}(12+8) + \vec{k}(-3+4)$$

$$= (-6, 20, 1)$$

h) $\vec{u} \times (\vec{v} + \vec{w}) = (\vec{u} \times \vec{v}) + (\vec{u} \times \vec{w}) = (8, -2, 13)$

$\vec{u} \times \vec{v} = (9, -1, 14)$

$\vec{u} \times \vec{w} = (-1, -1, -1)$

i) $(\vec{u} \times \vec{v}) + (\vec{u} \times \vec{w}) = (8, -2, 13)$

j) $(\vec{u} \times \vec{v}) \cdot \vec{v} = \vec{u} \cdot (\vec{v} \times \vec{v}) = 0$

$\vec{v} \times \vec{v} = \vec{0}$

$\vec{u} \cdot \vec{0} = 0$

k) $(\vec{u} \times \vec{v}) \cdot \vec{w} = 5$

$\vec{u} \times \vec{v} = (9, -1, 14)$

$(\vec{u} \times \vec{v}) \cdot \vec{w} = (9, -1, 14) \cdot (-1, 0, 1) = -9 + 0 + 14 = 5$

l) $\vec{u} \cdot (\vec{v} \times \vec{w}) = (\vec{u} \times \vec{v}) \cdot \vec{w} = 5$

3. $A(2,1,-1)$ $\quad \vec{AD} = \vec{BC} \times \vec{AC}$

$B(3,0,1)$ $\quad \vec{AD} = (C-B) \times (C-A)$

$C(2,-1,-3)$ $\quad \vec{AD} = (-1,-1,-4) \times (0,-2,-2)$

$$\vec{AD} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ -1 & -1 & -4 \\ 0 & -2 & -2 \end{vmatrix} = \vec{i}\begin{vmatrix} -1 & -4 \\ -2 & -2 \end{vmatrix} - \vec{j}\begin{vmatrix} -1 & -4 \\ 0 & -2 \end{vmatrix} + \vec{k}\begin{vmatrix} -1 & -1 \\ 0 & -2 \end{vmatrix}$$

$$= \vec{i}(2-8) - \vec{j}(2-0) + \vec{k}(2-0)$$

$$= (-6,-2,2)$$

$\vec{AD} = (-6,-2,2)$

$D - A = (-6,-2,2)$

$D = (-6,-2,2) + (2,1,-1) = (-4,-1,1)$

4. $\vec{x} \cdot (1,4,-3) = -7$

$\vec{x} \times (4,-2,1) = (3,5,-2)$

$\vec{x} = (a,b,c)$ $\quad (a,b,c)\cdot(1,4,-3) = -7$

$\quad 3 \quad -1 \quad 2$ $\quad a + 4b - 3c = -7$

$$(a,b,c) \times (4,-2,1) = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ a & b & c \\ 4 & -2 & 1 \end{vmatrix} = \vec{i}\begin{vmatrix} b & c \\ -2 & 1 \end{vmatrix} - \vec{j}\begin{vmatrix} a & c \\ 4 & 1 \end{vmatrix} + \vec{k}\begin{vmatrix} a & b \\ 4 & -2 \end{vmatrix}$$

$$= \vec{i}(b+2c) - \vec{j}(a-4c) + \vec{k}(-2a-4b)$$

$(b+2c, -a+4c, -2a-4b) = (3,5,-2)$

$b + 2c = 3$ $\quad -a + 4c = 5$ $\quad -2a - 4b = -2 \quad (\div -2)$

$b = 3 - 2c$ $\quad a + 2b = 1$

$b = 3 - 2 \cdot 2$ $\quad a = 1 - 2b$

$b = -1$ $\quad a = 1 - 2(3-2c)$

$a + 4b - 3c = -7$ $\quad a = -5 + 4c$

$(-5+4c) + 4(3-2c) - 3c = -7$ $\quad a = -5 + 4 \cdot 2$

$-5 + 4c + 12 - 8c - 3c = -7$ $\quad a = 3$

$-7c = -7 - 7$

$-7c = -14$

$c = 2$ $\quad \vec{x} = (3,-1,2)$

5) a) $\begin{cases} \vec{x} \times \vec{j} = \vec{k} \\ \vec{x} \cdot (4\vec{i} - 2\vec{j} + \vec{k}) = 10 \end{cases}$ $x = (a, b, c)$

$$\vec{x} \times \vec{j} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ a & b & c \\ 0 & 1 & 0 \end{vmatrix} = \vec{i}(-c) - \vec{j}(0) + \vec{k}(a)$$

$\vec{x} \times \vec{j} = \vec{k}$

$(-c, 0, a) = (0, 0, 1)$

$c = 0 \qquad a = 1$

$\vec{x} = (1, -3, 0)$

$(1, b, 0) \cdot (4, -2, 1) = 10$

$4 - 2b + 0 = 10$

$-2b = 6$

$b = -3$

b) $\begin{cases} \vec{x} \times (2\vec{i} - \vec{j} + 3\vec{k}) = \vec{0} \\ \vec{x} \cdot (\vec{i} + 2\vec{j} - 2\vec{k}) = 12 \end{cases}$ $\vec{x} = (a, b, c)$

$$\vec{x} \times (2, -1, 3) = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ a & b & c \\ 2 & -1 & 3 \end{vmatrix} = \vec{i}(3b + c) - \vec{j}(3a - 2c) + \vec{k}(-a - 2b)$$

$(3b + c, -3a + 2c, -a - 2b) = (0, 0, 0)$

$3b + c = 0 \qquad -3a + 2c = 0 \qquad -a - 2b = 0$

$-3(-2b) + 2c = 0 \qquad a = -2b$

$6b = -2c \qquad a = -2 \cdot 2 = -4$

$c = -3b$

$c = -3 \cdot 2 = -6$

$(a, b, c) \cdot (1, 2, -2) = 12$

$a + 2b - 2c = 12$

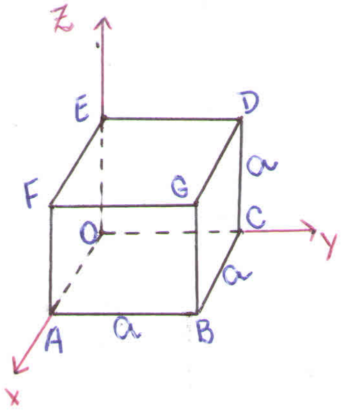
$-2b + 2b + 6b = 12$

$6b = 12$

$b = 2$

$\vec{x} = (-4, 2, -6)$

7.



$A(a, 0, 0)$

$B(a, a, 0)$

$C(0, a, 0)$

$D(0, a, a)$

$E(0, 0, a)$

$F(a, 0, a)$

$G(a, a, a)$

$O(0, 0, 0)$

a) $\vec{OF} \times \vec{OD} = (-a^2, -a^2, a^2)$

$(F-O) \times (D-O) = (a, 0, a) \times (0, a, 0)$

$$= \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ a & 0 & a \\ 0 & a & a \end{vmatrix} = \vec{i}(0 - a^2) - \vec{j}(a^2 - 0) + \vec{k}(a^2 - 0) = (-a^2, -a^2, a^2)$$

b) $\vec{AC} \times \vec{FA} = (-a^2, -a^2, 0)$

$(C-A) \times (A-F) = (-a, a, 0) \times (0, 0, -a)$

$$= \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ -a & a & 0 \\ 0 & 0 & -a \end{vmatrix} = \vec{i}(-a^2 - 0) - \vec{j}(a^2 - 0) + \vec{k}(0 - 0) = (-a^2, -a^2, 0)$$

c) $\vec{AB} \times \vec{AC} = (0, 0, a^2)$

$(B-A) \times (C-A) = (0, a, 0) \times (-a, a, 0)$

$$= \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 0 & a & 0 \\ -a & a & 0 \end{vmatrix} = \vec{i}(0-0) - \vec{j}(0-0) + \vec{k}(0 + a^2) = (0, 0, a^2)$$

d) $\vec{EC} \times \vec{EA} = (-a^2, -a^2, -a^2)$

$(C-E) \times (A-E) = (0, a, -a) \times (a, 0, -a)$

$$= \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 0 & a & -a \\ a & 0 & -a \end{vmatrix} = \vec{i}(-a^2 - 0) - \vec{j}(0 + a^2) + \vec{k}(0 - a^2) = (-a^2, -a^2, -a^2)$$

e) $\vec{OA} \cdot (\vec{OC} \times \vec{OE}) = a^3$

$(A-O) \cdot [(C-O) \times (E-O)] = (a, 0, 0) \cdot [(0, a, 0) \times (0, 0, a)] = (a, 0, 0) \cdot (a^2, 0, 0) = a^3$

$$\begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 0 & a & 0 \\ 0 & 0 & a \end{vmatrix} = \vec{i}(a^2 - 0) - \vec{j}(0-0) + \vec{k}(0-0) = (a^2, 0, 0)$$

f) $\vec{GB} \times \vec{AF} = \vec{0}$

$(B-G) \times (F-A) = (0, 0, -a) \times (0, 0, -a) = \vec{0}$

8. $\vec{u} = (1, -2, 1)$ $\quad \vec{v} = (1, 1, 1)$ $\quad \vec{w} = (1, 0, -1)$

a) $(\vec{u} \times \vec{v}) \cdot \vec{u} = 0$ e $(\vec{u} \times \vec{v}) \cdot \vec{v} = 0$

$$\begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & -2 & 1 \\ 1 & 1 & 1 \end{vmatrix} = \vec{i}(-2-1) - \vec{j}(1-1) + \vec{k}(1+2) = (-3, 0, 3)$$

$(\vec{u} \times \vec{v}) \cdot \vec{u} = (-3, 0, 3) \cdot (1, -2, 1) = -3 + 0 + 3 = 0$

$(\vec{u} \times \vec{v}) \cdot \vec{v} = (-3, 0, 3) \cdot (1, 1, 1) = -3 + 0 + 3 = 0$

$(\vec{u}\times\vec{w})\cdot\vec{u}=0$ e $(\vec{u}\times\vec{w})\cdot\vec{w}=0$

$$\begin{vmatrix}\vec{i} & \vec{j} & \vec{k}\\ 1 & -2 & 1\\ 1 & 0 & -1\end{vmatrix} = \vec{i}(2-0)-\vec{j}(-1-1)+\vec{k}(0+2)=(2,2,2)$$

$(2,2,2)\cdot(1,-2,1)=2-4+2=0$

$(2,2,2)\cdot(1,0,-1)=2+0-2=0$

$(\vec{v}\times\vec{w})\cdot\vec{v}=0$ e $(\vec{v}\times\vec{w})\cdot\vec{w}=0$

$$\begin{vmatrix}\vec{i} & \vec{j} & \vec{k}\\ 1 & 1 & 1\\ 1 & 0 & -1\end{vmatrix} = \vec{i}(-1-0)-\vec{j}(-1-1)+\vec{k}(0-1)=(-1,2,-1)$$

$(-1,2,-1)\cdot(1,1,1)=-1+2-1=0$

$(-1,2,-1)\cdot(1,0,-1)=-1+0+1=0$

b) $\vec{u}\times\vec{v}=(-3,0,3)$

$\vec{u}\times\vec{w}=(2,2,2)$

$\vec{v}\times\vec{w}=(-1,2,-1)$

$(\vec{u}\times\vec{v})\parallel\vec{w}$ $\quad \frac{-3}{1}=\frac{0}{0}=\frac{3}{-1}$

$(\vec{u}\times\vec{w})\parallel\vec{v}$ $\quad \frac{2}{1}=\frac{2}{1}=\frac{2}{1}$

$(\vec{v}\times\vec{w})\parallel\vec{u}$ $\quad -\frac{1}{1}=\frac{2}{-2}=-\frac{1}{1}$

c) $\vec{u}\times(\vec{v}\times\vec{w})=\vec{0}$

$$\vec{u}\times(-1,2,-1)=\begin{vmatrix}\vec{i} & \vec{j} & \vec{k}\\ 1 & -2 & 1\\ -1 & 2 & -1\end{vmatrix} = \vec{i}(0)-\vec{j}(0)+\vec{k}(0)=\vec{0}$$

13. $|\vec{x}|=2$ $\quad \vec{u}=(3,2,2)$ $\quad \vec{v}=(0,1,1)$

A partir de $\vec{u}\times\vec{v}$ obtém-se dois vetores unitários:

$$\vec{u}_1=\frac{\vec{u}\times\vec{v}}{|\vec{u}\times\vec{v}|}=\frac{(0,-3,3)}{3\sqrt{2}}=\left(0,-\frac{1}{\sqrt{2}},\frac{1}{\sqrt{2}}\right)$$

$\vec{u}_2=-\vec{u}_1=(0,\ 1/\sqrt{2},\ -1/\sqrt{2})$

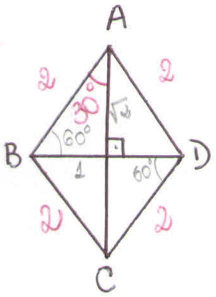
$$\vec{u}\times\vec{v}=\begin{vmatrix}\vec{i} & \vec{j} & \vec{k}\\ 3 & 2 & 2\\ 0 & 1 & 1\end{vmatrix} = \vec{i}(2-2)-\vec{j}(3-0)+\vec{k}(3-0)=(0,-3,3)$$

$|\vec{u}\times\vec{v}|=\sqrt{0^2+(-3)^2+3^2}=\sqrt{18}=3\sqrt{2}$

Para obter um vetor de módulo 2 que seja ortogonal a $\vec{u}$ e a $\vec{v}$ basta multiplicar por 2 o vetor unitário:

$(0,-\sqrt{2},\sqrt{2})$ e $(0,\sqrt{2},-\sqrt{2})$

14.



$\operatorname{sen} 30° = \frac{x}{2}$ $\qquad$ $\operatorname{sen} 60° = \frac{y}{2}$ $\qquad$ $|\vec{u} \times \vec{v}| = |\vec{u}| \cdot |\vec{v}| \cdot \operatorname{sen} \theta$

$\frac{1}{2} = \frac{x}{2}$ $\qquad$ $\frac{\sqrt{2}}{2} = \frac{y}{2}$

$x = 1$ $\qquad$ $y = \sqrt{2}$

a) $|\vec{AB} \times \vec{AD}| = 2\sqrt{3}$

$|\vec{AB} \times \vec{AD}| = |\vec{AB}| \cdot |\vec{AD}| \cdot \operatorname{sen}\theta$

$= 2 \cdot 2 \cdot \operatorname{sen} 60° = 4 \cdot \frac{\sqrt{3}}{2} = 2\sqrt{3}$

b) $|\vec{BA} \times \vec{BC}| = 2\sqrt{3}$

$|\vec{BA} \times \vec{BC}| = |\vec{BA}| \cdot |\vec{BC}| \cdot \operatorname{sen}\theta$

$= 2 \cdot 2 \cdot \operatorname{sen} 120° = 4 \cdot \frac{\sqrt{3}}{2} = 2\sqrt{3}$

c) $|\vec{AB} \times \vec{DC}| = 0$ $\qquad$ $\vec{AB} = \vec{DC}$

$|\vec{AB} \times \vec{DC}| = |\vec{AB}| \cdot |\vec{DC}| \cdot \operatorname{sen}\theta$

$= 2 \cdot 2 \cdot \operatorname{sen} 0° = 4 \cdot 0 = 0$

d) $|\vec{AB} \times \vec{CD}| = 0$

$|\vec{AB} \times \vec{CD}| = |\vec{AB}| \cdot |\vec{CD}| \cdot \operatorname{sen}\theta$

$= 2 \cdot 2 \cdot \operatorname{sen} 180° = 4 \cdot 0 = 0$

e) $|\vec{BD} \times \vec{AC}| = 4$

$|\vec{BD} \times \vec{AC}| = |\vec{BD}| \cdot |\vec{AC}| \cdot \operatorname{sen}\theta$

$= 2 \cdot 2 \cdot \operatorname{sen} 90° = 4 \cdot 1 = 4$

f) $|\vec{BD} \times \vec{CD}| = 2\sqrt{3}$

$|\vec{BD} \times \vec{CD}| = |\vec{BD}| \cdot |\vec{CD}| \cdot \operatorname{sen}\theta$

$= 2 \cdot 2 \cdot \operatorname{sen} 60° = 4 \cdot \frac{\sqrt{3}}{2} = 2\sqrt{3}$

15. $|\vec{u}| = 2\sqrt{2}$ $\qquad$ $|\vec{v}| = 4$ $\qquad$ $\vec{u}$ e $\Rightarrow 45°$

a) $|2\vec{u} \times \vec{v}| = |2\vec{u}| \cdot |\vec{v}| \cdot \operatorname{sen} 45°$

$= 2 \cdot 2\sqrt{2} \cdot 4 \cdot \frac{\sqrt{2}}{2}$

$= 16$

b) $\left|\frac{2}{5}\vec{u} \times \frac{1}{2}\vec{v}\right| = \left|\frac{2}{5}\vec{u}\right| \cdot \left|\frac{1}{2}\vec{v}\right| \cdot \operatorname{sen} 45°$

$= \frac{2}{5} \cdot 2\sqrt{2} \cdot \frac{1}{2} \cdot 4 \cdot \frac{\sqrt{2}}{2} = \frac{8}{5}$

16. $|\vec{u} \times \vec{v}| = 12$ $\quad |\vec{u}| = 13$ $\quad |\vec{v}| = 1$

$|\vec{u} \times \vec{v}| = |\vec{u}| \cdot |\vec{v}| \cdot \operatorname{sen}\theta$

$12 = 13 \cdot 1 \cdot \operatorname{sen}\theta$

$\operatorname{sen}\theta = \frac{12}{13}$

$\theta = \operatorname{sen}^{-1}\left(\frac{12}{13}\right) \cong 67{,}38°$

$\vec{u} \cdot \vec{v} = |\vec{u}| \cdot |\vec{v}| \cdot \cos\theta$

$\vec{u} \cdot \vec{v} = 13 \cdot 1 \cdot \cos 67{,}38°$

$= \pm 5$

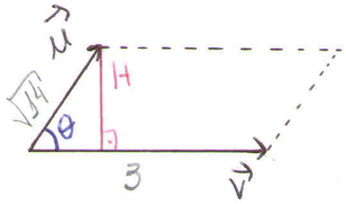
$\cos 67{,}38° = \cos -67{,}38°$

17. $\vec{u} = (3, -1, 2)$ $\quad \vec{v} = (-2, 2, 1)$

a) $A = |\vec{u} \times \vec{v}| = |(-5, -7, 4)| = \sqrt{25 + 49 + 16} = \sqrt{90} = \sqrt{9 \cdot 10} = 3\sqrt{10}$

$\vec{u} \times \vec{v} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 3 & -1 & 2 \\ -2 & 2 & 1 \end{vmatrix} = \vec{i}(-1-4) - \vec{j}(3+4) + \vec{k}(6-2) = (-5, -7, 4)$

b)



$|\vec{u}| = \sqrt{9+1+4} = \sqrt{14}$

$|\vec{v}| = \sqrt{4+4+1} = 3$

$|\vec{u} \times \vec{v}| = |\vec{u}| \cdot |\vec{v}| \cdot \operatorname{sen}\theta$

$\operatorname{sen}\theta = \frac{3\sqrt{10}}{\sqrt{14} \cdot 3} = \frac{\sqrt{10}}{\sqrt{14}}$

$\operatorname{sen}\theta = \frac{H}{\sqrt{14}} \Rightarrow \frac{\sqrt{10}}{\sqrt{14}} = \frac{H}{\sqrt{14}} \Rightarrow H = \sqrt{10}$

20) $\vec{u} = (m, -3, 1)$ $\quad \vec{v} = (1, -2, 2)$ $\quad A = \sqrt{26}$

$A = |\vec{u} \times \vec{v}| = \sqrt{26}$

$\vec{u} \times \vec{v} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ m & -3 & 1 \\ 1 & -2 & 2 \end{vmatrix} = \vec{i}(-6+2) - \vec{j}(2m-1) + \vec{k}(-2m+3)$

$= (-4, 1-2m, 3-2m)$

$|\vec{u} \times \vec{v}| = \sqrt{(-4)^2 + (1-2m)^2 + (3-2m)^2} = \sqrt{26}$

$16 + 1 - 4m + 4m^2 + 9 - 12m + 4m^2 = 26$

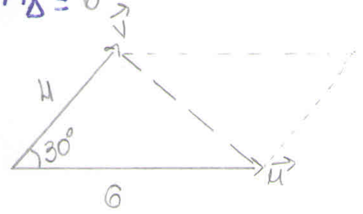
$8m^2 - 16m = 0$

$8m(m-2) = 0$

$m = 2$ ou $m = 0$

21. $|\vec{u}| = 6$ $\quad |\vec{v}| = 4$ $\quad \vec{u} \text{ e } \vec{v} \Rightarrow 30°$

a) $A_\Delta = 6$

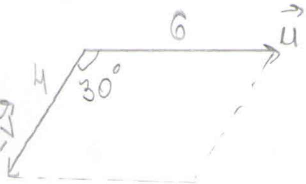


$$A_\Delta = \frac{1}{2} |\vec{u} \times \vec{v}|$$

$$|\vec{u} \times \vec{v}| = |\vec{u}| \cdot |\vec{v}| \cdot \operatorname{sen} \theta$$
$$= 6 \cdot 4 \cdot \operatorname{sen} 30°$$
$$= 24 \cdot \frac{1}{2}$$
$$= 12$$

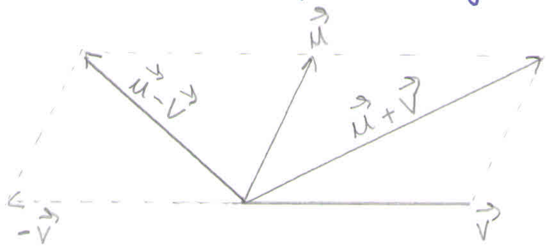
$$A_\Delta = \frac{1}{2} \cdot 12 = 6$$

b) área do paralelogramo determinado por $\vec{u}$ e $-\vec{v}$



$$|\vec{u} \times (-\vec{v})| = |\vec{u}| \cdot |-\vec{v}| \cdot \operatorname{sen} \theta$$
$$= 6 \cdot 4 \cdot \operatorname{sen} 30°$$
$$= 12$$

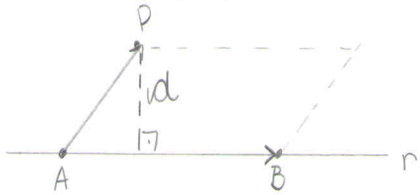
c) área do paralelogramo determinado por $\vec{u}+\vec{v}$ e $\vec{u}-\vec{v}$



$\vec{u}+\vec{v}$ e $\vec{u}-\vec{v}$ formam um paralelogramo com 4 triângulos:

$A = 4 \cdot 6 = 24$

23. $P(4,3,3)$ $\quad A(1,2,-1)$ $\quad B(3,1,1)$



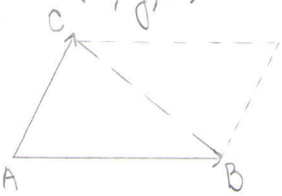
d é a distância do ponto P à reta r

$$d = \frac{|\vec{AB} \times \vec{AP}|}{|\vec{AB}|} = \frac{\sqrt{65}}{3}$$

$$\vec{AB} \times \vec{AP} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 2 & -1 & 2 \\ 3 & 1 & 4 \end{vmatrix} = \vec{i}(-4-2) - \vec{j}(8-6) + \vec{k}(2+3) = (-6, -2, 5)$$

$$|\vec{AB} \times \vec{AP}| = \sqrt{6^2+2^2+5^2} = \sqrt{65}$$
$$|\vec{AB}| = \sqrt{4+1+4} = 3$$

27. $A(2,1,-1)$
$B(0,2,1)$
$A_\Delta = 1{,}5$ u.a.
$C(0,y,0)$



$$A_\Delta = \frac{1}{2} |\vec{AB} \times \vec{AC}|$$
$$1{,}5 \cdot 2 = |\vec{AB} \times \vec{AC}|$$
$$3 = |\vec{AB} \times \vec{AC}|$$

$$\vec{AB} \times \vec{AC} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ -2 & 1 & 2 \\ -2 & y-1 & 1 \end{vmatrix}$$
$$= (1-2y+2,\ 2-4,\ -2y+2+2)$$
$$= (3-2y,\ -2,\ 4-2y)$$

$$|\vec{AB} \times \vec{AC}| = \sqrt{(3-2y)^2 + (-2)^2 + (4-2y)^2}$$

$$3^2 = 9 - 12y + 4y^2 + 4 + 16 - 16y + 4y^2$$

$$8y^2 - 28y + 20 = 0$$

$$y = \frac{28 \pm 12}{16} \qquad \begin{cases} y' = \frac{40}{16} = \frac{5}{2} \\ y'' = \frac{16}{16} = 1 \end{cases}$$

$C\left(0, \frac{5}{2}, 0\right)$ ou $C(0, 1, 0)$