Cap. 02 - Pgs. 66 a 70

1. $\vec{u} = (2, -3, -1)$ $\quad$ $\vec{v} = (1, -1, 4)$

a) $2\vec{u} \cdot (-\vec{v}) = (4, -6, -2) \cdot (-1, 1, -4) = (-4) + (-6) + 8 = -2$

b) $(\vec{u} + 3\vec{v}) \cdot (\vec{v} - 2\vec{u}) = [(2, -3, -1) + (3, -3, 12)] \cdot [(1, -1, 4) - (4, -6, -2)] =$
$(5, -6, 11) + (-3, 5, 6) = (-15) + (-30) + 66 = 21$

c) $(\vec{u} + \vec{v}) \cdot (\vec{u} - \vec{v}) = (3, -4, 3) \cdot (1, -2, -5) = 3 + 8 + (-15) = -4$

d) $(\vec{u} + \vec{v}) \cdot (\vec{v} - \vec{u}) = (3, -4, 3) \cdot (-1, 2, 5) = (-3) + (-8) + 15 = 4$

3. $A(4, 0, -1)$ $\quad$ $B(2, -2, 1)$ $\quad$ $C(1, 3, 2)$ $\quad$ $\vec{u} = (2, 1, 1)$ $\quad$ $\vec{v} = (-1, -2, 3)$

a) $3\vec{x} + 2\vec{v} = \vec{x} + (\vec{AB} \cdot \vec{u})\vec{v}$

$3\vec{x} + (-2, -4, 6) = \vec{x} + [(-2, -2, 2) \cdot (2, 1, 1)] \cdot (-1, -2, 3)$

$3\vec{x} - \vec{x} = -(-2, -4, 6) + [(-4) + (-2) + 2] \cdot (-1, -2, 3)$

$2\vec{x} = -(-2, -4, 6) + (4, 8, -12)$

$2\vec{x} = (6, 12, -18)$

$\vec{x} = (3, 6, -9)$

b) $(\vec{BC} \cdot \vec{v}) \cdot \vec{x} = (\vec{u} \cdot \vec{v}) \cdot \vec{v} - 3\vec{x}$

$[(-1, 5, 1) \cdot (-1, -2, 3)] \cdot \vec{x} = [(-2) + (-2) + 3] \cdot (-1, -2, 3) - 3\vec{x}$

$[1 + (-10) + 3] \cdot \vec{x} = (-1)(-1, -2, 3) - 3\vec{x}$

$-6\vec{x} = (1, 2, -3) - 3\vec{x}$

$-6\vec{x} + 3\vec{x} = (1, 2, -3)$

$-3\vec{x} = (1, 2, -3)$

$\vec{x} = \left(-\frac{1}{3}, -\frac{2}{3}, 1\right)$

4. $\vec{v} // \vec{u}$ $\quad$ $\vec{u} = (2, -1, 3)$ $\quad$ $\vec{v} \cdot \vec{u} = -42$

$\vec{v} // \vec{u} \Rightarrow \frac{x}{2} = \frac{y}{-1} = \frac{z}{3}$

$-x = 2y$ $\quad$ $3y = -z$

$x = -2y$ $\quad$ $z = -3y$

$x = -2 \cdot 13$ $\quad$ $z = -3 \cdot 13$

$x = -26$ $\quad$ $z = -39$

$\vec{v} \cdot \vec{u} = -42$

$(x, y, z) \cdot (2, -1, 3) = -42$

$(-2y, y, -3y) \cdot (2, -1, 3) = -42$

$(-4y) + (-y) + (-9y) = -42$

$-14y = -42$

$y = 13$

$\vec{v} = (-26, 13, -39) = (-2, 1, -3)$

5. $|\vec{v}| = 5$ $\quad$ $\vec{v} \perp Ox$ $\quad$ $\vec{v} \cdot \vec{\omega} = 6$ $\quad$ $\vec{\omega} = \vec{i} + 2\vec{j}$

$\vec{v} \perp Ox \Rightarrow \vec{v} = (0, y, z)$

$\vec{\omega} = \vec{i} + 2\vec{j} \Rightarrow \vec{\omega} = (1, 2, 0)$

$\vec{v} \cdot \vec{\omega} = 6$

$(0, y, z) \cdot (1, 2, 0) = 6$

$0 + 2y + 0 = 6$

$2y = 6$

$y = 3$

$|\vec{v}| = 5$

$\sqrt{0^2 + y^2 + z^2} = 5$

$3^2 + z^2 = 25$

$z^2 = 16$

$z = \pm 4$

$\vec{v} = (0, 3, \pm 4)$

6. $\vec{v} \perp O_y \qquad \vec{v}\cdot\vec{v_1} = 8 \qquad \vec{v}\cdot\vec{v_2} = -3 \qquad \vec{v_1} = (3, 1, -2) \qquad \vec{v_2} = (-1, 1, 1)$

$\vec{v} \perp O_y \Rightarrow \vec{v} = (x, 0, z)$

$\vec{v} = (2, 0, -1)$

$\vec{v}\cdot\vec{v_1} = 8$

$(x, 0, z)\cdot(3, 1, -2) = 8$

$3x - 2z = 8$

$3x - 2x + 6 = 8$

$x = 2$

$\vec{v}\cdot\vec{v_2} = -3$

$(x, 0, z)\cdot(-1, 1, 1) = -3$

$-x + z = -3$

$z = x - 3$

$z = 2 - 3$

$z = -1$

9. $\vec{u}\cdot\vec{v} + \vec{u}\cdot\vec{w} + \vec{v}\cdot\vec{w} \qquad \vec{u} + \vec{v} + \vec{w} = \vec{0} \qquad |\vec{u}| = 2 \qquad |\vec{v}| = 3 \qquad |\vec{w}| = 5$

$|\vec{u} + \vec{v} + \vec{w}|^2 = (\vec{u} + \vec{v} + \vec{w})\cdot(\vec{u} + \vec{v} + \vec{w})$

$|\vec{u} + \vec{v} + \vec{w}|^2 = \vec{u}\cdot\vec{u} + \vec{u}\cdot\vec{v} + \vec{u}\cdot\vec{w} + \vec{v}\cdot\vec{u} + \vec{v}\cdot\vec{v} + \vec{v}\cdot\vec{w} + \vec{w}\cdot\vec{u} + \vec{w}\cdot\vec{v} + \vec{w}\cdot\vec{w}$

$|\vec{u} + \vec{v} + \vec{w}|^2 = |\vec{u}|^2 + |\vec{v}|^2 + |\vec{w}|^2 + 2\vec{u}\cdot\vec{v} + 2\vec{u}\cdot\vec{w} + 2\vec{v}\cdot\vec{w}$

$|\vec{0}|^2 = 2^2 + 3^2 + 5^2 + 2(\vec{u}\vec{v} + \vec{u}\vec{w} + \vec{v}\vec{w})$

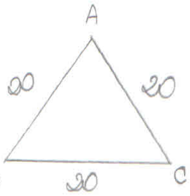
$\frac{-38}{2} = \vec{u}\vec{v} + \vec{u}\vec{w} + \vec{v}\vec{w}$

$\vec{u}\vec{v} + \vec{u}\vec{w} + \vec{v}\vec{w} = -19$

10. $\ell = 20\,cm$

$\vec{AB}\cdot\vec{AC}$

$\vec{AB}\cdot\vec{CA}$



soma dos ângulos internos de um triângulo equilátero = 180°

$180° \div 3 = 60°$

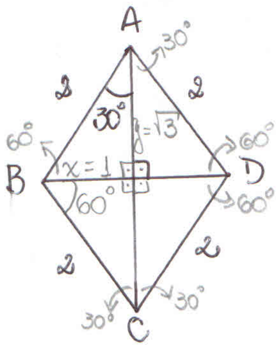
$d(\vec{AB}) = d(\vec{AC}) = d(\vec{BC}) = |\vec{AB}| = |\vec{AC}| = |\vec{BC}| = 20$

$\theta = 60°$

$\cos 60° = \frac{\vec{AB}\cdot\vec{AC}}{|\vec{AB}|\cdot|\vec{AC}|} \Rightarrow \frac{1}{2} = \frac{\vec{AB}\cdot\vec{AC}}{20\cdot 20} \Rightarrow \vec{AB}\cdot\vec{AC} = 200$

$\cos 60° = \frac{\vec{AB}\cdot\vec{CA}}{|\vec{AB}|\cdot|\vec{CA}|} \Rightarrow \frac{1}{2} = \frac{\vec{AB}\cdot(-\vec{AC})}{20\cdot 20} \Rightarrow \vec{AB}\cdot\vec{AC} = -200$

11.



$\text{sen } 30° = \frac{x}{2} \Rightarrow \frac{1}{2} = \frac{x}{2} \Rightarrow x = 1$

$\text{tg } 30° = \frac{1}{y} \Rightarrow \frac{\sqrt{3}}{3} = \frac{1}{y} \Rightarrow y = \frac{3}{\sqrt{3}}\cdot\frac{\sqrt{3}}{\sqrt{3}} = \sqrt{3}$

a) $\vec{AC}\cdot\vec{BD}$

$\cos 90° = \frac{\vec{AC}\cdot\vec{BD}}{|\vec{AC}|\cdot|\vec{BD}|} \Rightarrow 0 = \frac{\vec{AC}\cdot\vec{BD}}{2\sqrt{3}\cdot 2}$

$\vec{AC}\cdot\vec{BD} = 0$

b) $\vec{AB}\cdot\vec{AD}$

$\cos 60° = \frac{\vec{AB}\cdot\vec{AD}}{|\vec{AB}|\cdot|\vec{AD}|} \Rightarrow \frac{1}{2} = \frac{\vec{AB}\cdot\vec{AD}}{2\cdot 2}$

$\vec{AB}\cdot\vec{AD} = \frac{4}{2} = 2$

c) $\vec{BA}\cdot\vec{BC}$

$\cos 120° = \frac{\vec{BA}\cdot\vec{BC}}{|\vec{BA}|\cdot|\vec{BC}|}$

$-\frac{1}{2} = \frac{\vec{BA}\cdot\vec{BC}}{2\cdot 2} \Rightarrow \vec{BA}\cdot\vec{BC} = -\frac{4}{2} = -2$

d) $\vec{AB}\cdot\vec{BC}$

$$\cos 120° = \frac{\vec{AB}\cdot\vec{BC}}{|\vec{AB}|\cdot|\vec{BC}|} \Rightarrow -\frac{1}{2} = \frac{(-\vec{BA})\cdot\vec{BC}}{2\cdot 2}$$

$$(-\vec{BA})\cdot\vec{BC} = -\frac{4}{2} = -2 \Rightarrow \vec{AB}\cdot\vec{BC} = 2$$

e) $\vec{AB}\cdot\vec{DC}$

$\vec{AB} // \vec{DC}$ $\quad \theta = 0°$

sentido e direção =

$$\cos 0° = \frac{\vec{AB}\cdot\vec{DC}}{|\vec{AB}|\cdot|\vec{DC}|} \Rightarrow 1 = \frac{\vec{AB}\cdot\vec{DC}}{2\cdot 2}$$

$$\vec{AB}\cdot\vec{DC} = 4$$

f) $\vec{BC}\cdot\vec{DA}$

$\vec{BC} // \vec{DA}$ $\quad \theta = 180°$

sentido e direção ≠

$$\cos 180° = \frac{\vec{BC}\cdot\vec{DA}}{|\vec{BC}|\cdot|\vec{DA}|} \Rightarrow -1 = \frac{\vec{BC}\cdot\vec{DA}}{2\cdot 2}$$

$$\vec{BC}\cdot\vec{DA} = -4$$

12. $|\vec{u}| = 4$ $\quad |\vec{v}| = 3$ $\quad \vec{u}$ e $\vec{v} = 60°$

$|\vec{u}+\vec{v}| = \sqrt{37}$

$|\vec{u}-\vec{v}| = \sqrt{13}$

$(\vec{u}+\vec{v})\cdot(\vec{u}-\vec{v}) = 7$

$$\cos 60° = \frac{\vec{u}\cdot\vec{v}}{|\vec{u}|\cdot|\vec{v}|} \Rightarrow \frac{1}{2} = \frac{\vec{u}\cdot\vec{v}}{4\cdot 3} \Rightarrow \vec{u}\cdot\vec{v} = 6$$

$$|\vec{u}+\vec{v}|^2 = |\vec{u}|^2 + 2\vec{u}\vec{v} + |\vec{v}|^2$$

$$|\vec{u}+\vec{v}|^2 = 4^2 + 2\cdot 6 + 3^2 = 37$$

$$|\vec{u}+\vec{v}| = \sqrt{37}$$

$$|\vec{u}-\vec{v}|^2 = |\vec{u}|^2 - 2\vec{u}\vec{v} + |\vec{v}|^2$$

$$|\vec{u}-\vec{v}|^2 = 4^2 - 2\cdot 6 + 3^2 = 13$$

$$|\vec{u}-\vec{v}| = \sqrt{13}$$

$$(\vec{u}+\vec{v})\cdot(\vec{u}-\vec{v}) = |\vec{u}|^2 - |\vec{v}|^2 = 4^2 - 3^2 = 7$$

14. $\vec{u} = (4,-1,2)$ $\quad \vec{v} = (-3,2,-2)$

a) $|\vec{u}\cdot\vec{v}| \leq |\vec{u}|\,|\vec{v}|$ (desigualdade de Schwarz)

$$|4(-3)+(-1)\cdot 2 + 2(-2)| \leq \sqrt{4^2+(-1)^2+2^2}\cdot\sqrt{(-3)^2+2^2+(-2)^2}$$

$$|-18| \leq \sqrt{21}\cdot\sqrt{17}$$

$$18 \leq \sqrt{357}$$

$$18 \leq 18{,}89$$

b) $|\vec{u}+\vec{v}| \leq |\vec{u}| + |\vec{v}|$ (desigualdade Triangular)

$$|(4+(-3), (-1)+2, 2+(-2))| \leq \sqrt{21} + \sqrt{17}$$

$$|(1,1,0)| \leq 4{,}58 + 4{,}12$$

$$\sqrt{1^2+1^2+0^2} \leq 8{,}70$$

$$1{,}41 \leq 8{,}70$$

17. $A(-1,0,5)$

$B(2,-1,4)$

$C(1,1,1)$

$\vec{AC} \perp \vec{BP}$

$P(x,0,x-3)$

$$\vec{AC} \perp \vec{BP} \Rightarrow \vec{AC}\cdot\vec{BP} = 0$$

$$(C-A)\cdot(P-B) = 0$$

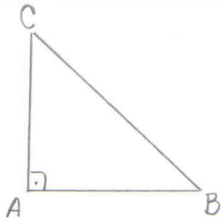
$$(2,1,-4)\cdot(x-2,1,x-7) = 0$$

$$2(x-2) + 1\cdot 1 + (-4)(x-7) = 0$$

$$2x - 4 + 1 - 4x + 28 = 0$$

$$-2x + 25 = 0 \Rightarrow x = \frac{25}{2}$$

19. $A(-m,1,0)$
$B(m-1,2m,2)$
$C(1,3,-1)$



$\vec{AB}\cdot\vec{AC} = |\vec{AB}|\cdot|\vec{AC}|\cdot\cos 90°$
$\vec{AB}\cdot\vec{AC} = 0$
$(B-A)\cdot(C-A) = 0$
$(-1, 2m-1, 2)\cdot(1-m, 2, -1) = 0$
$-1+m+4m-2-2 = 0$
$5m = 5$
$m = 1$

$A(1,1,0)$
$B(0,2,2)$
$C(1,3,-1)$

no triângulo retângulo: $\text{Área} = \dfrac{\text{cateto 1}\cdot\text{cateto 2}}{2}$

$$A_\Delta = \frac{|\vec{AB}|\cdot|\vec{AC}|}{2} = \frac{|(-1,1,2)|\cdot|(0,2,-1)|}{2} = \frac{\sqrt{1+1+4}\cdot\sqrt{0+4+1}}{2} = \frac{\sqrt{6}\cdot\sqrt{5}}{2} = \frac{\sqrt{30}}{2}$$

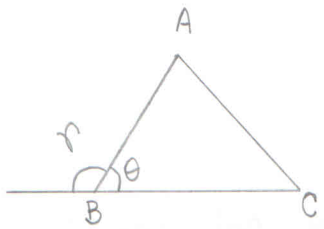
23. $\vec{a}\perp\vec{b}$ $\quad |\vec{a}| = 6$ $\quad |\vec{b}| = 8$
$|\vec{a}+\vec{b}| = 10$
$|\vec{a}-\vec{b}| = 10$

$\vec{a}\perp\vec{b} \Rightarrow \vec{a}\cdot\vec{b} = 0$

$|\vec{a}+\vec{b}|^2 = |\vec{a}|^2 + 2\vec{a}\vec{b} + |\vec{b}|^2$
$|\vec{a}+\vec{b}|^2 = 6^2 + 2\cdot 0 + 8^2$
$|\vec{a}+\vec{b}|^2 = 100$
$|\vec{a}+\vec{b}| = 10$

$|\vec{a}-\vec{b}|^2 = |\vec{a}|^2 - 2\vec{a}\vec{b} + |\vec{b}|^2$
$|\vec{a}-\vec{b}|^2 = 6^2 - 2\cdot 0 + 8^2$
$|\vec{a}-\vec{b}|^2 = 100$
$|\vec{a}-\vec{b}| = 10$

26. $A(3,4,4)$
$B(2,-3,4)$
$C(6,0,4)$



$\vec{BA} = A-B = (1,7,0)$
$\vec{BC} = C-B = (4,3,0)$
$|\vec{BA}| = \sqrt{1+49+0} = \sqrt{50}$
$|\vec{BC}| = \sqrt{16+9+0} = 5$
$\vec{BA}\cdot\vec{BC} = 4+21+0 = 25$

$\cos\theta = \dfrac{\vec{BA}\cdot\vec{BC}}{|\vec{BA}|\cdot|\vec{BC}|}$
$\cos\theta = \dfrac{25}{\sqrt{50}\cdot 5} = \dfrac{5}{\sqrt{50}}$
$\theta = \arccos\left(\dfrac{5}{\sqrt{50}}\right) = 45°$

θ = ângulo interno ao vértice B = 45°
γ = ângulo externo ao vértice B = 180° − 45° = 135°

30. $|\vec{u}| = 4$ $\quad |\vec{v}| = 2$ $\quad \vec{u}$ e $\vec{v} \Rightarrow 120°$
$\vec{u}+\vec{v}$ e $\vec{u}-\vec{v} \Rightarrow 49°$
$\vec{u}\cdot\vec{v} = |\vec{u}|\cdot|\vec{v}|\cdot\cos\theta$
$\vec{u}\cdot\vec{v} = 4\cdot 2\cdot\cos 120° = -4$

$\cos\theta = \dfrac{(\vec{u}+\vec{v})\cdot(\vec{u}-\vec{v})}{|\vec{u}+\vec{v}|\cdot|\vec{u}-\vec{v}|} = \dfrac{12}{\sqrt{12}\cdot\sqrt{28}} = \dfrac{12}{\sqrt{336}}$
$\theta = \arccos\left(\dfrac{12}{\sqrt{336}}\right) \cong 49°$

$|\vec{u}+\vec{v}|^2 = |\vec{u}|^2 + 2\vec{u}\vec{v} + |\vec{v}|^2$
$|\vec{u}+\vec{v}|^2 = 16 + 2(-4) + 4 = 12$
$|\vec{u}+\vec{v}| = \sqrt{12}$

$|\vec{u}-\vec{v}|^2 = |\vec{u}|^2 - 2\vec{u}\vec{v} + |\vec{v}|^2$
$|\vec{u}-\vec{v}|^2 = 16 - 2(-4) + 4 = 28$
$|\vec{u}-\vec{v}| = \sqrt{28}$

$(\vec{u}+\vec{v})\cdot(\vec{u}-\vec{v}) = |\vec{u}|^2 - |\vec{v}|^2 = 16 - 4 = 12$

32. $\vec{v} = (6,-2,3)$ $\qquad |\vec{v}| = \sqrt{36+4+9} = \sqrt{49} = 7$

$\cos\alpha = \dfrac{\vec{v}\cdot\vec{i}}{|\vec{v}|\cdot|\vec{i}|} = \dfrac{(6,-2,3)\cdot(1,0,0)}{7\cdot 1} = \dfrac{6}{7}$ $\qquad \alpha = \arccos\left(\dfrac{6}{7}\right) \cong 31^\circ$

$\cos\beta = \dfrac{\vec{v}\cdot\vec{j}}{|\vec{v}|\cdot|\vec{j}|} = \dfrac{(6,-2,3)\cdot(0,1,0)}{7\cdot 1} = -\dfrac{2}{7}$ $\qquad \beta = \arccos\left(-\dfrac{2}{7}\right) \cong 107^\circ$

$\cos\gamma = \dfrac{\vec{v}\cdot\vec{k}}{|\vec{v}|\cdot|\vec{k}|} = \dfrac{(6,-2,3)\cdot(0,0,1)}{7\cdot 1} = \dfrac{3}{7}$ $\qquad \gamma = \arccos\left(\dfrac{3}{7}\right) \cong 65^\circ$

36. $|\vec{v}| = 1$ $\qquad \sqrt{x^2+y^2+z^2} = 1$

$\vec{v} \perp O_z$ $\qquad \vec{v}\cdot\vec{k} = 0 \Rightarrow (x,y,z)\cdot(0,0,1) = 0 \Rightarrow z = 0$

$\alpha = 60^\circ$ $\qquad \cos 60^\circ = \dfrac{x}{|\vec{v}|} \Rightarrow \dfrac{1}{2} = \dfrac{x}{1} \Rightarrow x = \dfrac{1}{2}$

$\vec{v} = \left(\dfrac{1}{2}, y, 0\right)$ $\qquad$ mas $|\vec{v}| = 1$

$$\sqrt{\left(\frac{1}{2}\right)^2 + y^2 + 0^2} = 1$$

$$\frac{1}{4} + y^2 = 1$$

$$y^2 = \frac{3}{4} \Rightarrow y = \pm\frac{\sqrt{3}}{2}$$

$\vec{v} = \left(\dfrac{1}{2}, \pm\dfrac{\sqrt{3}}{2}, 0\right)$

40. $\vec{u} = (3,0,1)$ $\qquad \vec{v} = (-2,1,2)$

$\text{proj}_{\vec{v}}\,\vec{u} = \dfrac{\vec{u}\cdot\vec{v}}{\vec{v}\cdot\vec{v}}\cdot\vec{v} = \dfrac{-6+0+2}{4+1+4}\cdot(-2,1,2) = -\dfrac{4}{9}(-2,1,2) = \left(\dfrac{8}{9}, -\dfrac{4}{9}, -\dfrac{8}{9}\right)$

$\text{proj}_{\vec{u}}\,\vec{v} = \dfrac{\vec{v}\cdot\vec{u}}{\vec{u}\cdot\vec{u}}\cdot\vec{u} = \dfrac{-6+0+2}{9+0+1}\cdot(3,0,1) = -\dfrac{4}{10}\cdot(3,0,1) = \left(-\dfrac{6}{5}, 0, -\dfrac{2}{5}\right)$

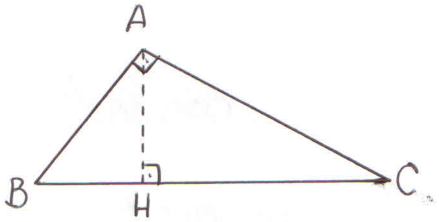
41. $\vec{v} = 4\vec{i} - 3\vec{j} + 2\vec{k} = (4,-3,2)$

$\text{proj}_{\vec{x}}\,\vec{v} = \dfrac{\vec{v}\cdot\vec{i}}{\vec{i}\cdot\vec{i}}\cdot\vec{i} = \dfrac{4+0+0}{1+0+0}\cdot(1,0,0) = (4,0,0)$

$\text{proj}_{\vec{y}}\,\vec{v} = \dfrac{\vec{v}\cdot\vec{j}}{\vec{j}\cdot\vec{j}}\cdot\vec{j} = \dfrac{0-3+0}{0+1+0}\cdot(0,1,0) = (0,-3,0)$

$\text{proj}_{\vec{z}}\,\vec{v} = \dfrac{\vec{v}\cdot\vec{k}}{\vec{k}\cdot\vec{k}}\cdot\vec{k} = \dfrac{0+0+2}{0+0+1}\cdot(0,0,1) = (0,0,2)$

43. $A(2,1,3)$ $B(m,3,5)$ $C(0,4,1)$



$B(3,3,5)$

a) $m = ?$ $\hat{A} = 90°$

$\vec{AB} \cdot \vec{AC} = |\vec{AB}| \cdot |\vec{AC}| \cdot \cos 90°$

$(B-A)\cdot(C-A) = 0$

$(m-2, 2, 2)\cdot(-2, 3, -2) = 0$

$-2m + 4 + 6 - 4 = 0$

$-2m = -6$

$m = 3$

b) $\text{proj}_{\vec{BC}} \vec{AB} = \dfrac{\vec{AB} \cdot \vec{BC}}{\vec{BC} \cdot \vec{BC}} \cdot \vec{BC} = \dfrac{(1,2,2)\cdot(-3,1,-4)}{(-3,1,-4)\cdot(-3,1,-4)} \cdot (-3,1,-4)$

$= \dfrac{-3+2-8}{9+1+16} \cdot (-3,1,-4) = -\dfrac{9}{26}(-3,1,-4) = \left(\dfrac{27}{26}, -\dfrac{9}{26}, \dfrac{36}{26}\right)$

medida $\text{proj}_{\vec{BC}} \vec{AB} = \left|\text{proj}_{\vec{BC}} \vec{AB}\right| = \sqrt{\left(\dfrac{27}{26}\right)^2 + \left(\dfrac{9}{26}\right)^2 + \left(\dfrac{36}{26}\right)^2} = \sqrt{\dfrac{2106}{676}} = \dfrac{9}{26}\sqrt{26}$

c) $H = ?$

$\vec{BH} = \text{proj}_{\vec{BC}} \vec{BA} = \dfrac{\vec{BA} \cdot \vec{BC}}{\vec{BC} \cdot \vec{BC}} \cdot \vec{BC} = \dfrac{(-1,-2,-2)\cdot(-3,1,-4)}{(-3,1,-4)\cdot(-3,1,-4)} \cdot (-3,1,-4)$

$= \dfrac{3-2+8}{9+1+16} \cdot (-3,1,-4) = \dfrac{9}{26}(-3,1,-4) = \left(-\dfrac{27}{26}, \dfrac{9}{26}, -\dfrac{36}{26}\right)$

$\vec{BH} = H - B$

$\left(-\dfrac{27}{26}, \dfrac{9}{26}, -\dfrac{36}{26}\right) = (x,y,z) - (3,3,5)$

$(x,y,z) = \left(-\dfrac{27}{26}, \dfrac{9}{26}, -\dfrac{36}{26}\right) + (3,3,5) = \left(\dfrac{51}{26}, \dfrac{87}{26}, \dfrac{94}{26}\right)$

d) $\vec{AH} \perp \vec{BC}$

$\vec{AH} \cdot \vec{BC} = 0$

$(H-A)\cdot(C-B) = 0$

$\left(-\dfrac{1}{26}, \dfrac{61}{26}, \dfrac{16}{26}\right)\cdot(-3,1,-4) = \dfrac{3}{26} + \dfrac{61}{26} - \dfrac{64}{26} = \dfrac{0}{26} = 0$

44. $\vec{u} = (-2,3)$ $\quad \vec{v} = (k,-4)$

a) $\vec{u} \parallel \vec{v}$ $\quad \frac{-2}{k} = \frac{3}{-4} \Rightarrow 3k = 8$

$k = \frac{8}{3}$

b) $\vec{u} \perp \vec{v}$ $\quad \vec{u} \cdot \vec{v} = 0$

$(-2,3) \cdot (k,-4) = 0$

$-2k - 12 = 0$

$-2k = 12$

$k = -6$

45. a) $4\vec{i} + 3\vec{j} = (4,3) = \vec{u}$

$\vec{v} = (a,b)$

$|\vec{v}| = 1$

$\vec{v} \cdot \vec{u} = 0$

$\vec{v} \cdot \vec{u} = 0$

$(a,b) \cdot (4,3) = 0$

$4a + 3b = 0$

$4a = -3b$

$a = -\frac{3b}{4}$

$|\vec{v}| = 1$

$\sqrt{a^2 + b^2} = 1$

$\sqrt{\frac{9b^2}{16} + b^2} = 1$

$\frac{25b^2}{16} = 1$

$b = \pm \frac{4}{5}$

$b = \frac{4}{5}$ $\quad a = -\frac{3}{5}$

$b = -\frac{4}{5}$ $\quad a = \frac{3}{5}$ $\quad \left(-\frac{3}{5}, \frac{4}{5}\right)$ ou $\left(\frac{3}{5}, -\frac{4}{5}\right)$

b) $(-2,3)$

$(-2,3) \cdot (a,b) = 0$

$-2a + 3b = 0$

$2a = 3b$

$a = \frac{3b}{2}$

$\sqrt{a^2 + b^2} = 1$

$\sqrt{\frac{9b^2}{4} + b^2} = 1$

$\frac{13b^2}{4} = 1$

$b = \pm \frac{2}{\sqrt{13}}$

$b = \frac{2}{\sqrt{13}}$ $\quad a = \frac{3}{\sqrt{13}}$

$b = -\frac{2}{\sqrt{13}}$ $\quad a = -\frac{3}{\sqrt{13}}$ $\quad \left(\frac{3}{\sqrt{13}}, \frac{2}{\sqrt{13}}\right)$ ou $\left(-\frac{3}{\sqrt{13}}, -\frac{2}{\sqrt{13}}\right)$

c) $(-1,-1)$

$(-1,-1) \cdot (a,b) = 0$

$-a - b = 0$

$a = -b$

$\sqrt{a^2 + b^2} = 1$

$\sqrt{b^2 + b^2} = 1$

$2b^2 = 1$

$b = \pm \frac{1}{\sqrt{2}}$

$b = \frac{1}{\sqrt{2}}$ $\quad a = -\frac{1}{\sqrt{2}}$

$b = -\frac{1}{\sqrt{2}}$ $\quad a = \frac{1}{\sqrt{2}}$ $\quad \left(-\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$ ou $\left(\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}\right)$

48. $\vec{u} = \vec{i} - \vec{j}$ $\qquad$ $\vec{v} = 2\vec{i} + \vec{j}$

a) $\vec{u}$ $\quad |\vec{u}| = \sqrt{1^2 + 1^2} = \sqrt{2}$

$$\cos\theta = \frac{\vec{u} \cdot \vec{i}}{|\vec{u}| \cdot |\vec{i}|} = \frac{x}{|\vec{u}|} = \frac{1}{\sqrt{2}}$$

$$\theta = \arccos \frac{1}{\sqrt{2}} = 45^\circ$$

b) $\vec{v}$ $\quad |\vec{v}| = \sqrt{4+1} = \sqrt{5}$

$$\cos\theta = \frac{\vec{v} \cdot \vec{i}}{|\vec{v}| \cdot |\vec{i}|} = \frac{x}{|\vec{v}|} = \frac{2}{\sqrt{5}}$$

$$\theta = \arccos \frac{2}{\sqrt{5}} \cong 26{,}56^\circ$$

c) $\vec{u} + \vec{v}$ $\quad \vec{u} + \vec{v} = (3, 0)$

$$|\vec{u} + \vec{v}| = \sqrt{9+0} = 3$$

$$\cos\theta = \frac{(\vec{u} + \vec{v}) \cdot \vec{i}}{|\vec{u} + \vec{v}| \cdot |\vec{i}|} = \frac{x}{|\vec{u} + \vec{v}|} = \frac{3}{3} = 1$$

$$\theta = \arccos 1 = 0^\circ$$

d) $\vec{u} - \vec{v}$ $\quad \vec{u} - \vec{v} = (-1, -2)$

$$|\vec{u} - \vec{v}| = \sqrt{1+4} = \sqrt{5}$$

$$\cos\theta = \frac{(\vec{u} - \vec{v}) \cdot \vec{i}}{|\vec{u} - \vec{v}| \cdot |\vec{i}|} = \frac{x}{|\vec{u} - \vec{v}|} = \frac{-1}{\sqrt{5}}$$

$$\theta = \arccos \frac{-1}{\sqrt{5}} \cong 116{,}56^\circ$$

e) $\vec{v} - \vec{u}$ $\quad \vec{v} - \vec{u} = (1, 2)$

$$|\vec{v} - \vec{u}| = \sqrt{1+4} = \sqrt{5}$$

$$\cos\theta = \frac{(\vec{v} - \vec{u}) \cdot \vec{i}}{|\vec{v} - \vec{u}| \cdot |\vec{i}|} = \frac{x}{|\vec{v} - \vec{u}|} = \frac{1}{\sqrt{5}}$$

$$\theta = \arccos \frac{1}{\sqrt{5}} \cong 63{,}43^\circ$$

49. $\vec{u}$ e $\vec{v} \Rightarrow 45^\circ$

$\vec{u} = (2, 1)$

$\vec{v} = (1, a)$

$|\vec{u}| = \sqrt{4+1} = \sqrt{5}$

$|\vec{v}| = \sqrt{1+a^2}$

$$\vec{u} \cdot \vec{v} = |\vec{u}| \cdot |\vec{v}| \cdot \cos\theta$$

$$2 + a = \sqrt{5} \cdot \sqrt{1+a^2} \cdot \cos 45^\circ$$

$$2 + a = \sqrt{5} \cdot \sqrt{1+a^2} \cdot \frac{\sqrt{2}}{2}$$

$$4 + 2a = \sqrt{10} \cdot \sqrt{1+a^2}$$

$$(4+2a)^2 = 10(1+a^2)$$

$$16 + 16a + 4a^2 = 10 + 10a^2$$

$$6a^2-16a-6=0$$

$$a=\frac{16\pm\sqrt{256+144}}{12}=\frac{16\pm 20}{12}\quad \begin{cases} a'=3 \\ a''=-\frac{1}{3}\end{cases}$$

50. $\text{proj}_{\vec{u}}\vec{v}$ $\quad \vec{v}=\vec{v}_1+\vec{v}_2 \quad \vec{v}_1 // \vec{u} \quad \vec{v}_2 \perp \vec{u}$

a) $\vec{u}=(1,0)$ $\quad \text{proj}_{\vec{u}}\vec{v}=\frac{\vec{v}\cdot\vec{u}}{\vec{u}\cdot\vec{u}}\cdot\vec{u}=\frac{4+0}{1+0}(1,0)=4(1,0)=(4,0)=\vec{v}_1$

$\vec{v}=(4,3)$

$\vec{v}=\vec{v}_1+\vec{v}_2$

$\vec{v}_2=\vec{v}-\vec{v}_1=(4,3)-(4,0)=(0,3)$

b) $\vec{u}=(1,1)$ $\quad \text{proj}_{\vec{u}}\vec{v}=\frac{2+5}{1+1}\cdot(1,1)=\frac{7}{2}(1,1)=\left(\frac{7}{2},\frac{7}{2}\right)=\vec{v}_1$

$\vec{v}=(2,5)$

$\vec{v}_2=\vec{v}-\vec{v}_1=(2,5)-\left(\frac{7}{2},\frac{7}{2}\right)$

$=\left(\frac{4}{2},\frac{10}{2}\right)-\left(\frac{7}{2},\frac{7}{2}\right)=\left(-\frac{3}{2},\frac{3}{2}\right)$

c) $\vec{u}=(4,3)$ $\quad \text{proj}_{\vec{u}}\vec{v}=\frac{4+6}{16+9}\cdot(4,3)=\frac{10}{25}(4,3)=\left(\frac{8}{5},\frac{6}{5}\right)=\vec{v}_1$

$\vec{v}=(1,2)$

$\vec{v}_2=\vec{v}-\vec{v}_1=(1,2)-\left(\frac{8}{5},\frac{6}{5}\right)$

$=\left(\frac{5}{5},\frac{10}{5}\right)-\left(\frac{8}{5},\frac{6}{5}\right)=\left(-\frac{3}{5},\frac{4}{5}\right)$