

Capítulo 01 - Pgs. 40 a 45

$$1. \vec{u} = 2\vec{i} - 3\vec{j} \quad \vec{v} = \vec{i} - \vec{j} \quad \vec{w} = -2\vec{i} + \vec{j}$$

$$a) 2\vec{u} - \vec{v} = 2(2, -3) - (1, -1) = (4, -6) - (1, -1) = (3, -5)$$

$$b) \vec{v} - \vec{u} + 2\vec{w} = (1, -1) - (2, -3) + 2(-2, 1) = (-1, 2) + (-4, 2) = (-5, 4)$$

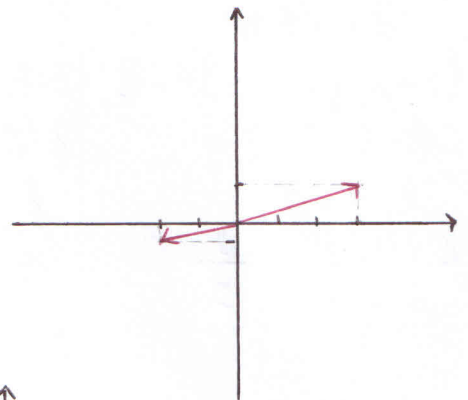
$$c) \frac{1}{2}\vec{u} - 2\vec{v} - \vec{w} = \frac{1}{2}(2, -3) - 2(1, -1) - (-2, 1) = \left(1, -\frac{3}{2}\right) - (2, -2) - (-2, 1) = (1, -\frac{1}{2})$$

$$d) 3\vec{u} - \frac{1}{2}\vec{v} - \frac{1}{2}\vec{w} = 3(2, -3) - \frac{1}{2}(1, -1) - \frac{1}{2}(-2, 1) = (6, -9) - \left(\frac{1}{2}, -\frac{1}{2}\right) - \left(-1, \frac{1}{2}\right) = \left(\frac{13}{2}, -9\right)$$

$$6. A(-5, 1) \quad B(1, 3) \quad \vec{v} = (a, b)$$

$$a) B = A + 2\vec{v} \\ B - A = 2\vec{v} \\ (6, 2) = 2(a, b) \\ (3, 1) = (a, b) = \vec{v}$$

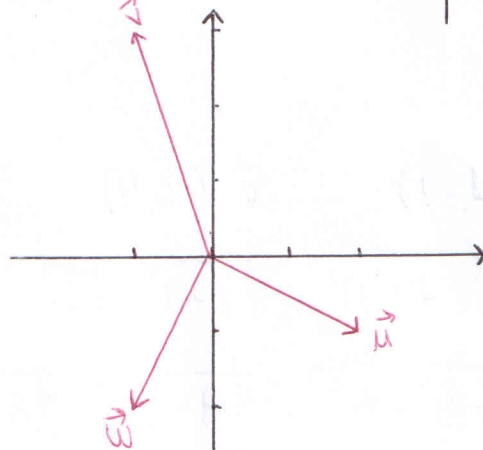
$$b) A = B + 3\vec{v} \\ A - B = 3\vec{v} \\ (-6, -2) = 3(a, b) \\ (-2, -\frac{2}{3}) = (a, b) = \vec{v}$$



$$10. P(2, 3) \quad Q(4, 2) \quad R(3, 5)$$

$$a) Q = P + \vec{u} \quad \vec{u} = Q - P = (2, -1) \\ R = Q + \vec{v} \quad \vec{v} = R - Q = (-1, 3) \\ P = R + \vec{w} \quad \vec{w} = P - R = (-1, -2)$$

$$b) \vec{u} + \vec{v} + \vec{w} = (2, -1) + (-1, 3) + (-1, -2) = (0, 0) = \vec{0}$$



14. $A(-2, 3)$ $B(6, -3)$

a) $\overline{A \quad C \quad D \quad E \quad B}$

$C(a, b) = (0, 3/2)$

$D(c, d) = (2, 0)$

$E(e, f) = (4, -3/2)$

D é ponto médio de AB:

$AD = DB$

$(-2+2, 3-d) = (6-c, -3-d)$

$-2+2 = 6-c$ $3-d = -3-d$

$2c = 4$

$c = 2$

$2d = 0$

$d = 0$

$AC = CD$

$(-2+2, b-3) = (2-a, 0-b)$

$-2+2 = 2-a$

$2a = 0$

$a = 0$

$b-3 = 0-b$

$2b = 3$

$b = \frac{3}{2}$

$DE = EB$

$(-2-e, 3-f) = (6-e, -3-f)$

$-2-e = 6-e$

$2e = 8$

$e = 4$

$3-f = -3-f$

$2f = -3$

$f = -\frac{3}{2}$

b) $\overline{A \quad F \quad G \quad B}$

$AF = GB$

$(-2+2, 3-f) = (6-g, -3-h)$

$-2+2 = 6-g$

$g = 4$

$g = 4 - \frac{10}{3} = \frac{2}{3}$

$3-f = -3-h$

$f = -h$

$f = 1$

$AF = FG$

$(-2+2, 3-f) = (g-e, h-f)$

$-2+2 = g-e$

$2e = g-2$

$8-2g = g-2$

$10 = 3g$

$g = \frac{10}{3}$

$3-f = h-f$

$2f = h+3$

$-2h = h+3$

$-3h = 3$

$h = -1$

16. $\vec{u} = (1, -1)$ $\vec{v} = (-3, 4)$ $\vec{w} = (8, -6)$

a) $|\vec{u}| = \sqrt{1^2 + (-1)^2} = \sqrt{1+1} = \sqrt{2}$

b) $|\vec{v}| = \sqrt{(-3)^2 + 4^2} = \sqrt{9+16} = \sqrt{25} = 5$

c) $|\vec{w}| = \sqrt{8^2 + (-6)^2} = \sqrt{64+36} = \sqrt{100} = 10$

d) $|\vec{u} + \vec{v}| = |(-2, 3)| = \sqrt{(-2)^2 + 3^2} = \sqrt{4+9} = \sqrt{13}$

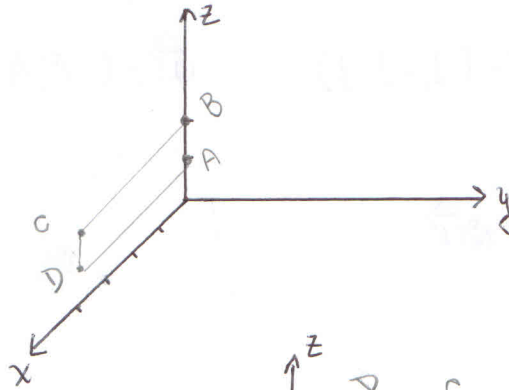
e) $|2\vec{u} - \vec{w}| = |(2, -2) - (8, -6)| = |(-6, 4)| = \sqrt{(-6)^2 + 4^2} = \sqrt{36+16} = \sqrt{52} = \sqrt{2^2 \cdot 13} = 2\sqrt{13}$

$$f) |\vec{w} - 3\vec{u}| = |(8, -6) - (3, -3)| = |(5, -3)| = \sqrt{5^2 + (-3)^2} = \sqrt{25 + 9} = \sqrt{34}$$

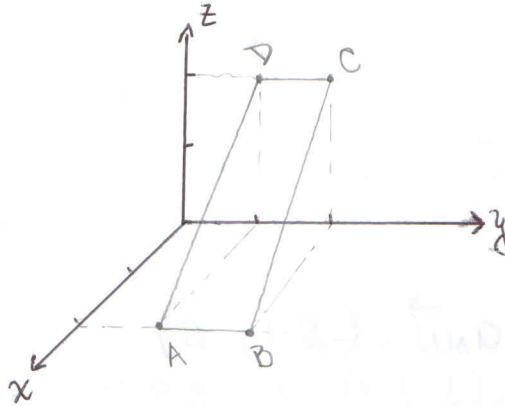
$$g) \frac{\vec{v}}{|\vec{v}|} = \frac{(-3, 4)}{5} = \left(-\frac{3}{5}, \frac{4}{5}\right)$$

$$h) \left| \frac{\vec{u}}{|\vec{u}|} \right| = \left| \frac{(1, -1)}{\sqrt{2}} \right| = \sqrt{\left(\frac{1}{\sqrt{2}}\right)^2 + \left(\frac{-1}{\sqrt{2}}\right)^2} = \sqrt{\frac{1}{2} + \frac{1}{2}} = \sqrt{\frac{2}{2}} = \sqrt{1} = 1$$

24. a) $A(0, 0, 1)$
 $B(0, 0, 2)$
 $C(4, 0, 2)$
 $D(4, 0, 1)$



b) $A(2, 1, 0)$
 $B(2, 2, 0)$
 $C(0, 2, 2)$
 $D(0, 1, 2)$



28. $A(3, 4, -2)$

a) in plane xy $|z| = 2$

b) in plane xz $|y| = 4$

c) in plane yz $|x| = 3$

d) in line AB $x=0$ $\sqrt{0^2 + 4^2 + (-2)^2} = \sqrt{16 + 4} = \sqrt{20} = 2\sqrt{5}$

e) in line AC $y=0$ $\sqrt{3^2 + 0^2 + (-2)^2} = \sqrt{9 + 4} = \sqrt{13}$

f) in line AD $z=0$ $\sqrt{3^2 + 4^2 + 0^2} = \sqrt{9 + 16} = \sqrt{25} = 5$

31. $A(2, -2, 3)$ $B(1, 1, 5)$ $\vec{v} = (1, 3, -4)$

a) $A + 3\vec{v} = (2, -2, 3) + 3(1, 3, -4) = (2, -2, 3) + (3, 9, -12) = (5, 7, -9)$

b) $(A - B) - \vec{v} = (1, -3, -2) - (1, 3, -4) = (0, -6, 2)$

c) $B + 2(B - A) = (1, 1, 5) + 2(-1, 3, 2) = (-1, 7, 9)$

d) $2\vec{v} - 3(B - A) = (2, 6, -8) - 3(-1, 3, 2) = (5, -3, -14)$

35. $\vec{u} = (2, 3, -1)$ $\vec{v} = (1, -1, 1)$ $\vec{w} = (-3, 4, 0)$

a) \vec{x}

$$3\vec{u} - \vec{v} + \vec{x} = 4\vec{x} + 2\vec{w}$$

$$3\vec{u} - \vec{v} - 2\vec{w} = 3\vec{x}$$

$$(6, 9, -3) - (1, -1, 1) - (-6, 8, 0) = 3\vec{x}$$

$$(11, 2, -4) = 3\vec{x}$$

$$\vec{x} = \left(\frac{11}{3}, \frac{2}{3}, -\frac{4}{3}\right)$$

b) a_1, a_2 и a_3

$$a_1\vec{u} + a_2\vec{v} + a_3\vec{w} = (-2, 13, -5)$$

$$a_1(2, 3, -1) + a_2(1, -1, 1) + a_3(-3, 4, 0) = (-2, 13, -5)$$

$$(2a_1 + a_2 - 3a_3, 3a_1 - a_2 + 4a_3, -a_1 + a_2) = (-2, 13, -5)$$

$$\begin{cases} 2a_1 + a_2 - 3a_3 = -2 \\ 3a_1 - a_2 + 4a_3 = 13 \\ -a_1 + a_2 = -5 \end{cases}$$

$$3a_1 - a_2 + 4a_3 = 13$$

$$-a_1 + a_2 = -5$$

$$3 + a_3 - a_3 + 4 + 4a_3 = 13$$

$$6a_3 = 6$$

$$a_3 = 1$$

$$a_2 = a_1 - 5$$

$$a_2 = a_3 + 1 - 5$$

$$a_2 = a_3 - 4$$

$$a_2 = 1 - 4$$

$$a_2 = -3$$

$$2a_1 + a_1 - 5 - 3a_3 = -2$$

$$3a_1 - 3a_3 = 3$$

$$a_1 - a_3 = 1$$

$$a_1 = 1 + a_3$$

$$a_1 = 1 + 1$$

$$a_1 = 2$$

37. $A(2, -5, 3)$

$B(7, 3, -1)$

$M(4, -3, 3)$

$AM = MC$

$M - A = C - M$

$(2, 2, 0) = (a - 4, b + 3, c - 3)$

$2 = a - 4$

$2 = b + 3$

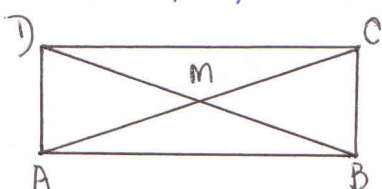
$0 = c - 3$

$a = 6$

$b = -1$

$c = 3$

$C(6, -1, 3)$



$$BM = MD$$

$$m - B = D - m$$

$$(-3, -6, 4) = (d-4, e+3, f-3)$$

$$-3 = d-4$$

$$d = 1$$

$$-6 = e+3$$

$$e = -9$$

$$4 = f-3$$

$$f = 7$$

$$D(1, -9, 7)$$

$$40. a) A(-2, 1, 3)$$

$$B(6, -7, 1)$$

$$C(a, b, c)$$

$$D(d, e, f)$$

$$E(g, h, i)$$



$$AC = EB$$

$$C - A = B - E$$

$$(a+2, b-1, c-3) = (6-g, -7-h, 1-i)$$

$$a+2 = 6-g$$

$$a = 4-g$$

$$b-1 = -7-h$$

$$b = -6-h$$

$$c-3 = 1-i$$

$$c = 4-i$$

$$AC = CD = DE = EB$$

$$AC = CD$$

$$C - A = D - C$$

$$(a+2, b-1, c-3) = (d-a, e-b, f-c)$$

$$a+2 = d-a$$

$$2a = d-2$$

$$2a = d-2$$

$$a = 0$$

$$b-1 = e-b$$

$$2b = e+1$$

$$2b = -3+1$$

$$b = -1$$

$$2c = f-c$$

$$2c = 3+f$$

$$2c = 3+2$$

$$c = \frac{5}{2}$$

$$C\left(0, -1, \frac{5}{2}\right)$$

$$CD = DE$$

$$D - C = E - D$$

$$(d-a, e-b, f-c) = (g-d, h-e, i-f)$$

$$d-a = g-d$$

$$d-4+g = g-d$$

$$2d = 4$$

$$d = 2$$

$$e-b = h-e$$

$$2e = h+b$$

$$2e = h-6-h$$

$$e = -3$$

$$f-c = i-f$$

$$2f = i+c$$

$$2f = i+4-i$$

$$f = 2$$

$$D(2, -3, 2)$$

$$DE = EB$$

$$E - D = B - E$$

$$(g-d, h-e, i-f) = (6-g, -7-h, 1-i)$$

$$g-d = 6-g$$

$$2g = 6+d$$

$$2g = 6+2$$

$$g = 4$$

$$h-e = -7-h$$

$$2h = -7-e$$

$$2h = -7-3$$

$$h = -5$$

$$i-f = 1-i$$

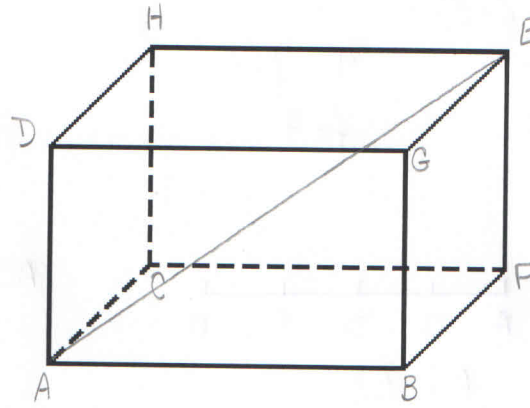
$$2i = 1+f$$

$$2i = 1+2$$

$$i = \frac{3}{2}$$

$$E\left(4, -5, \frac{3}{2}\right)$$

41.



\vec{AE} é uma diagonal do paralelepípedo.

$$\vec{AE} = \vec{AB} + \vec{AC} + \vec{AD}$$

$$E - A = \vec{AB} + \vec{AC} + \vec{AD}$$

$$E = A + (\vec{AB} + \vec{AC} + \vec{AD})$$

a) $A(3, 5, 0)$

$B(1, 5, 0)$

$C(3, 5, 4)$

$D(3, 2, 0)$

$$A' = (3, 5, 0) + [(-2, 0, 0) + (0, 0, 4) + (0, -3, 0)]$$

$$A' = (3, 5, 0) + (-2, -3, 4)$$

$$A' = (1, 2, 4)$$

b) $A(-1, 2, 1)$

$B(3, -1, 2)$

$C(4, 1, -3)$

$D(0, -3, -1)$

$$A' = (-1, 2, 1) + [(4, -3, 1) + (5, -1, -4) + (1, -5, -2)]$$

$$A' = (-1, 2, 1) + (10, -9, -5)$$

$$A' = (9, -7, -4)$$

c) $A(-1, 2, 3)$

$B(2, -1, 0)$

$C(3, 1, 4)$

$D(-2, 0, 5)$

$$A' = (-1, 2, 3) + [(3, -3, -3) + (4, -1, 1) + (-1, -2, 2)]$$

$$A' = (-1, 2, 3) + (6, -6, 0)$$

$$A' = (5, -4, 3)$$

44. $\vec{w} = (3, 2, 5)$ $\vec{u} = (3, 2, -1)$ $\vec{v} = (a, 6, b) + 2\vec{w}$

$\vec{u} \parallel \vec{v}$

$\vec{v} = (a, 6, b) + 2(3, 2, 5)$

$\vec{v} = (a+6, 6+4, b+10)$

$\vec{u} \parallel \vec{v}$

$$\frac{3}{a+6} = \frac{2}{10} = \frac{-1}{b+10}$$

$$\frac{3}{a+6} = \frac{2}{10}$$

$$30 = 2a + 12$$

$$18 = 2a$$

$$a = 9$$

$$\frac{2}{10} = \frac{-1}{b+10}$$

$$2b + 20 = -10$$

$$2b = -30$$

$$b = -15$$

45. $A(-2, 5, 1)$ $B(1, 3, 0)$
 $C(3, -1, -1)$ $D(0, m, n)$

$AB \parallel CD$

$B-A \parallel D-C$

$B-A = (3, -2, -1)$

$D-C = (-3, m+1, n+1)$

$$\frac{3}{-3} = \frac{-2}{m+1} = \frac{-1}{n+1}$$

$$3m+3=6$$

$$3m=3$$

$$m=1$$

$$3n+3=3$$

$$3n=0$$

$$n=0$$

$D(0, 1, 0)$

47. $P(m, 4, n)$ $A(-1, -2, 3)$ $B(2, 1, -5)$

$AP \parallel PB$

$P-A = (m+1, 6, n-3)$

$P-A \parallel B-P$

$B-P = (2-m, -3, -5-n)$

$$\frac{m+1}{2-m} = \frac{-6}{3} = \frac{n-3}{-5-n}$$

$$3m+3 = -12+6m$$

$$15 = 3m$$

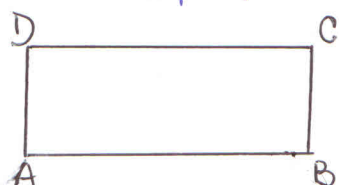
$$m=5$$

$$30+6n = 3n-9$$

$$3n = -39$$

$$n = -13$$

48. a) $A(-1, 0, 3)$
 $B(1, 1, 2)$
 $C(3, -2, 5)$



$\vec{AD} = \vec{BC}$

$D-A = C-B$

$(a+1, b, c-3) = (2, -3, 3)$

$a+1=2$

$a=1$

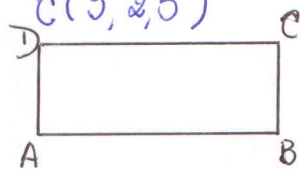
$b=-3$

$c-3=3$

$c=6$

$D(1, -3, 6)$

b) $A(4, 0, 1)$
 $B(5, 1, 3)$
 $C(3, 2, 5)$



$$\vec{AD} = \vec{BC}$$

$$D - A = C - B$$

$$(x-4, y, z-1) = (-2, 1, 2)$$

$$x-4 = -2 \quad y = 1 \quad z-1 = 2$$

$$x = 2 \quad \quad \quad z = 3$$

$$D(2, 1, 3)$$

49. $\vec{u} = (1, 1, 1)$

$$\vec{v} = \left(\frac{1}{\sqrt{6}}, -\frac{2}{\sqrt{6}}, \frac{1}{\sqrt{6}} \right)$$

vetor unitário: $|\vec{v}| = 1$

$$|\vec{u}| = \sqrt{1^2 + 1^2 + 1^2} = \sqrt{3}$$

mas é unitário

$$|\vec{v}| = \sqrt{\left(\frac{1}{\sqrt{6}}\right)^2 + \left(-\frac{2}{\sqrt{6}}\right)^2 + \left(\frac{1}{\sqrt{6}}\right)^2} = \sqrt{\frac{1}{6} + \frac{4}{6} + \frac{1}{6}} = \sqrt{\frac{6}{6}} = \sqrt{1} = 1 \quad \text{é unitário}$$

50. $\vec{v} = \left(n, -\frac{1}{2}, \frac{3}{4} \right)$

$$|\vec{v}| = 1$$

$$\sqrt{n^2 + \left(-\frac{1}{2}\right)^2 + \left(\frac{3}{4}\right)^2} = 1$$

$$n^2 + \frac{1}{4} + \frac{9}{16} = 1$$

$$n^2 = 1 - \frac{13}{16} = \frac{3}{16}$$

$$n = \pm \frac{\sqrt{3}}{4}$$

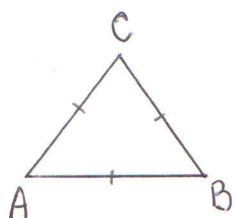
51. $\vec{u} = (a, -2a, 2a)$

$$\frac{\vec{u}}{|\vec{u}|} = \frac{(a, -2a, 2a)}{\sqrt{a^2 + (-2a)^2 + (2a)^2}} = \frac{(a, -2a, 2a)}{3a} = \frac{(1, -2, 2)}{3} = \left(\frac{1}{3}, -\frac{2}{3}, \frac{2}{3} \right)$$

mas como estamos trabalhando com módulo:

$$a = \pm \frac{1}{3}$$

53. $A(4, 4, 4)$
 $B(10, 4, -2)$
 $C(2, 0, -4)$



$$d(AB) = d(BC) = d(CA)$$

$$\begin{aligned}
 d(6,0,-6) &= d(-2,-y,-8) = d(-8,-y,-2) \\
 \sqrt{6^2+0^2+(-6)^2} &= \sqrt{(-2)^2+(-y)^2+(-8)^2} = \sqrt{(-8)^2+(-y)^2+(-2)^2} \\
 72 &= 68+y^2 = 68+y^2 \\
 72 &= 68+y^2 \\
 y^2 &= 4 \\
 y &= \pm 2
 \end{aligned}$$

54. A(3,-1,4)
B(1,-2,-3)
P(x,0,0)

$$\begin{aligned}
 d(PA) &= d(PB) \\
 d(3-x,-1,4) &= d(1-x,-2,-3) \\
 \sqrt{(3-x)^2+(-1)^2+4^2} &= \sqrt{(1-x)^2+(-2)^2+(-3)^2} \\
 9-6x+x^2+17 &= 1-2x+x^2+13 \\
 26-6x+x^2 &= 14-2x+x^2 \\
 12 &= 4x \\
 x &= 3
 \end{aligned}$$

P(3,0,0)

55. A(-1,2,-2)
d(PA)=3
P(0,0,z)

$$\begin{aligned}
 |A-P| &= 3 \\
 |(-1,2,-2-z)| &= 3 \\
 \sqrt{(-1)^2+2^2+(-2-z)^2} &= 3 \\
 1+4+4+4z+z^2 &= 9 \\
 4z+z^2 &= 0 \\
 z(4+z) &= 0 \\
 z &= 0 \text{ ou } z = -4
 \end{aligned}$$

P(0,0,±4)

56. $\vec{v} = (2,-1,-3)$

a) sentido contrário e $3|\vec{v}|$
 $-3|\vec{v}| = (-6,3,9)$

b) mesmo sentido e módulo 4

um vetor unitário obtido a partir de \vec{v} é o seu versor:

$$\frac{\vec{v}}{|\vec{v}|} = \frac{(2,-1,-3)}{\sqrt{4+1+9}} = \left(\frac{2}{\sqrt{14}}, \frac{-1}{\sqrt{14}}, \frac{-3}{\sqrt{14}} \right)$$

como o vetor deve ter módulo 4:

$$4 \cdot \frac{\vec{v}}{|\vec{v}|} = \left(\frac{8}{\sqrt{14}}, \frac{-4}{\sqrt{14}}, \frac{-12}{\sqrt{14}} \right)$$

c) sentido contrário e módulo 5

$$5 \cdot \frac{\vec{v}}{|\vec{v}|} = \left(\frac{10}{\sqrt{14}}, \frac{-5}{\sqrt{14}}, \frac{-15}{\sqrt{14}} \right)$$