

Capítulo 04 - Produto Misto.

01) a)	$\vec{u} = (3, -1, 1)$	$3 \quad -1 \quad 1$	$3 \quad -1$
	$\vec{v} = (1, 2, 2)$	$1 \quad 2 \quad 2$	$1 \quad 2$
	$\vec{w} = (2, 0, -3)$	$2 \quad 0 \quad -3$	$2 \quad 0$

$$= -18 - 4 - 4 + 3 = -29$$

b) $(\vec{w}, \vec{u}, \vec{v}) = -29$

02) $(\vec{u}, \vec{v}, \vec{w}) = -5$

a) $(\vec{w}, \vec{v}, \vec{u}) = 5$

b) $(\vec{v}, \vec{u}, \vec{w}) = 5$

c) $(\vec{w}, \vec{u}, \vec{v}) = -5$

d) $\vec{v} \cdot (\vec{w} \times \vec{u}) = (\vec{v}, \vec{w}, \vec{u}) = -5$

obs: Na ocorrência de permutação, ocorre a troca do sinal do determinante.

03) a) $\vec{u} \cdot (\vec{v} \times \vec{w}) = 2$
 $\vec{u} \cdot (\vec{w} \times \vec{v}) = -2$

b) $\vec{v} \cdot (\vec{w} \times \vec{u}) = -2$

c) $(\vec{v} \times \vec{w}) \cdot \vec{u} = 2$

d) $(\vec{u} \times \vec{w}) \cdot (3\vec{v}) = (\vec{u}, \vec{w}, 3\vec{v}) = -2 \cdot 3 = -6$

e) $\vec{u} \cdot (2\vec{w} \times \vec{v}) = (\vec{u}, 2\vec{w}, \vec{v}) = -2 \cdot 2 = -4$

f) $(\vec{u} + \vec{v}) \cdot (\vec{u} \times \vec{w}) = (2\vec{u}, \vec{w}) + (\vec{v}, \vec{u}, \vec{w}) = -2$

04) $(\vec{u}, \vec{w}, \vec{x}) = 2$
 $(\vec{v}, \vec{w}, \vec{x}) = 5$

a) $(\vec{u}, \vec{x}, -\vec{w}) = -2 \cdot (-1) = 2$

b) $(3\vec{u}, 3\vec{w}, -2\vec{x}) = 2 \cdot 3 \cdot 3 \cdot (-2) = -36$

c) $(2\vec{u} + 4\vec{v}, \vec{w}, \vec{x}) = (2\vec{u}, \vec{w}, \vec{x}) + (4\vec{v}, \vec{w}, \vec{x}) = 2 \cdot 2 + 5 \cdot 4 = 4 + 20 = 24$

d) $(5\vec{u} + (-3\vec{v}), 2\vec{w}, \vec{x}) = (5\vec{u}, 2\vec{w}, \vec{x}) + (-3\vec{v}, 2\vec{w}, \vec{x}) = 2 \cdot 5 \cdot 2 + 5 \cdot (-3) \cdot 2 = -10$

05) Os vetores são coplanares se $(\vec{u}, \vec{v}, \vec{w}) = 0$.

/ /

Seg	Ter	Qua	Qui	Sex	Sáb	Dom
-----	-----	-----	-----	-----	-----	-----

a) $\vec{u} = (1, -1, 2)$		1 -1 2	1 -1
$\vec{v} = (2, 2, 1)$	$(\vec{u}, \vec{v}, \vec{w}) =$	2 2 1	2 2
$\vec{w} = (-2, 0, 4)$		-2 0 4	-2 0

$$= 8 + 2 + 8 + 8 = 26 //$$

R: Os vetores não são coplanares.

b) $\vec{u} = (2, -1, 3)$		2 -1 3	2 -1
$\vec{v} = (3, 1, -2)$	$(\vec{u}, \vec{v}, \vec{w}) =$	3 1 -2	3 1
$\vec{w} = (7, -1, 4)$		7 -1 4	7 -1

$$= 8 + 14 - 9 - 21 - 4 + 12 = 34 - 34 = 0,$$

R: Os vetores são coplanares.

06) a) $\vec{u} = (2, -1, k)$		2 -1 k	2 -1
$\vec{v} = (1, 0, 2)$	$(\vec{u}, \vec{v}, \vec{w}) =$	1 0 2	1 0 = 0
$\vec{w} = (k, 3, k)$		k 3 k	k 3

$$-2k + 3k - 12 + k = 0$$

$$2k = 12 \Rightarrow k = 6 //$$

b) $\vec{u} = (2, k, 1)$		2 k 1	2 k
$\vec{v} = (1, 2, k)$	$(\vec{u}, \vec{v}, \vec{w}) =$	1 2 k	1 2 = 0
$\vec{w} = (3, 0, -3)$		3 0 -3	3 0

$$-12 + 3k^2 - 6 + 3k = 0$$

$$3k^2 + 3k - 18 = 0$$

$$k^2 + k - 6 = 0$$

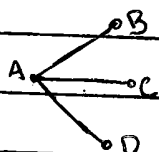
$$\Delta = 1 + 24 = 25$$

$$k = \frac{-1 \pm 5}{2} \rightarrow k' = -3$$

$$\rightarrow k'' = 2 //$$

07) a)	A(1,1,0)		-3	0	-6	-3	0
	B(-2,1,-6)	$(\vec{AB}, \vec{AC}, \vec{AD}) =$	-2	1	-1	-2	1
	C(-1,2,-1)		1	-2	-4	1	-2
	D(2,-1,-4)						

$$= 12 - 24 + 6 + 6 = 0$$



são coplanares.

b)	A(2,1,2)		-2	0	-4	-2	0
	B(0,1,-2)	$(\vec{AB}, \vec{AC}, \vec{AD}) =$	-1	-1	-5	-1	-1
	C(1,0,-3)		1	0	-4	1	0
	D(3,1,-2)						

$$= -8 - 4 = -12$$

Não são coplanares.

08) $(\vec{AB}, \vec{AC}, \vec{AD}) = 0$

A(m,1,2)		2-m	-3	-5	2-m	-3
B(2,-2,-3)	=>	5-m	-2	-1	5-m	-2
C(5,-1,1)		3-m	-3	-4	3-m	-3
D(3,-2,-2)						

$$8(2-m) + 3(3-m) + 15(5-m) - 10(3-m) - 3(2+m) - 12(6-m) = 0$$

$$16 - 8m + 9 - 3m + 75 - 5m - 30 + 10m - 6 + 3m - 60 + 12m = 0$$

$$16 + 3 - 15 - 5m + 4m = 0$$

$$m = 4$$

09) $(\vec{i}, \vec{j}, \vec{k}) = (1,1,1)$

$$V = |(\vec{u}, \vec{v}, \vec{w})|$$

$$\vec{u} = (1,0,0)$$

$$\vec{v} = (1,1,1)$$

$$V =$$

$$\vec{j} = (0,1,0)$$

$$\vec{k} = (0,0,1)$$

$$V = |1|$$

$$V = 1$$

/ /

Seg Ter Qua Qui Sex Sáb Dom

10) $\vec{u} = (3, -1, 4)$		3	-1	4	3	-1
$\vec{v} = (2, 0, 1)$	$= \text{I) } (\vec{u}, \vec{v}, \vec{w}) =$	2	0	1	2	0
$\vec{w} = (-2, 1, 5)$		-2	1	5	-2	1

$$= 2 + 8 - 3 + 10 = 17 //$$

$$\text{II) } V = |(\vec{u}, \vec{v}, \vec{w})| = |17| = 17 //$$

III) $V = (\text{Área}) h$		\hat{i}	\hat{j}	\hat{k}	\hat{i}	\hat{j}
$V = \vec{u} \times \vec{v} h$	$\vec{u} \times \vec{v} =$	3	-1	4	3	-1
$h = \frac{V}{ \vec{u} \times \vec{v} }$		2	0	1	2	0

$$-3\hat{i} + 8\hat{j} + 2\hat{k} - 3\hat{j} = -3\hat{i} + 5\hat{j} + 2\hat{k}$$

$$h = \frac{17}{|(-1, 5, 2)|} = \frac{17}{\sqrt{1+25+4}} = \frac{17}{\sqrt{30}} \text{ u.c. } (-1, 5, 2)$$

11) $\vec{v}_1 = (0, -1, 2)$	I)	0	-1	2	0	-1
$\vec{v}_2 = (-4, 2, -1)$	$(\vec{v}_1, \vec{v}_2, \vec{v}_3) =$	-4	2	-1	-4	2
$\vec{v}_3 = (3, m, -2)$		3	m	-2	3	m

$$V = 33 \quad = +3 - 8m - 12 + 8 = -8m - 1$$

$$\text{II) } V = |(\vec{v}_1, \vec{v}_2, \vec{v}_3)|$$

$$33 = |-8m - 1|$$

$$-8m - 1 = 33 \quad -(-8m - 1) = 33$$

$$-8m = 34 \quad \text{ou} \quad 8m + 1 = 33$$

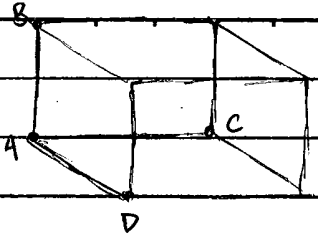
$$m = -34/8 = -17/4 //$$

$$8m = 32 \Rightarrow m = 4 //$$

III) $V = (\text{Área}) \cdot h$		\hat{i}	\hat{j}	\hat{k}	\hat{i}	\hat{j}
$h = \frac{33}{ \vec{v}_1 \times \vec{v}_2 }$	$\vec{v}_1 \times \vec{v}_2 =$	0	-1	2	0	-1
		-4	2	-1	-4	2

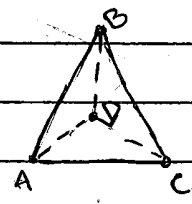
$$h = \frac{33}{\sqrt{9+64+16}} = \frac{33}{\sqrt{89}} \text{ u.c. } 1\hat{i} - 8\hat{j} - 4\hat{k} - 4\hat{i} = -3\hat{i} - 8\hat{j} - 4\hat{k}$$

$$= (-3, -8, -4)$$

12)		$A(1, -2, 3)$	$V = 20 \text{ u.v.}$
		$B(2, -1, 4)$	$m = ?$
		$C(0, 2, 0)$	
		$D(-1, m, 1)$	

I)	$\begin{vmatrix} 1 & 1 & -7 \\ -1 & 4 & -8 \\ -2 & m+2 & -2 \end{vmatrix}$	$\begin{vmatrix} 1 & 1 \\ -1 & 4 \end{vmatrix}$	II) $V = (\vec{AB}, \vec{AC}, \vec{AD}) $
$(\vec{AB}, \vec{AC}, \vec{AD}) =$	$-1 \cdot 4 \cdot -8$	$-1 \cdot 4$	i) $20 = 10m - 40$
	$-2 \cdot m+2 \cdot -2$	$-2 \cdot m+2$	$10m = 60 \Rightarrow m = 6$
	$= -8 + 16 + 7m + 4 + 16 + 8m + 6 - 2$		ii) $20 = -10m + 40$
	$= 10m - 40$		$-2 + 10m = -20 \Rightarrow m = 2$

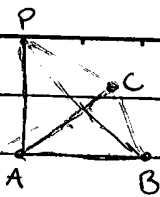
13)	$A(2, 1, 1)$	I)	$\begin{vmatrix} -3 & -1 & 0 \\ 1 & 1 & -3 \\ -2 & -1 & 2 \end{vmatrix}$	$\begin{vmatrix} -3 & -1 \\ 1 & 1 \end{vmatrix}$
	$B(1, 0, 1)$	$(\vec{AB}, \vec{AC}, \vec{AD}) =$	$\begin{vmatrix} 1 & 1 & -3 \\ -2 & -1 & 2 \end{vmatrix}$	$\begin{vmatrix} 1 & 1 \\ -2 & -1 \end{vmatrix}$
	$C(3, 2, -2)$		$-3z + 3 - 6 + 9 + z + 1 = -2z + 5$	
	$D(x, y, z)$			
	$D \rightarrow \text{eixo } z$	II) $V = (\vec{AB}, \vec{AC}, \vec{AD}) $	i) $25 = -2z + 5$	
	$x=0; y=0$	$25 = -2z + 5 $	$-2z = 20 \Rightarrow z = -10$	
			ii) $25 = 2z - 5$	
			$2z = 30 \Rightarrow z = 15$	
	$D(0, 0, -10)$ ou $D(0, 0, 15)$			

14)	$A(1, 1, 0)$		$V = \frac{1}{6} (\vec{AB}, \vec{AC}, \vec{AD}) $
	$B(6, 4, 1)$		
	$C(2, 5, 0)$		
	$D(0, 3, 3)$		II) $V = \frac{1}{6} 57 = \frac{57}{6}$

I)	$\begin{vmatrix} 5 & 3 & 1 \\ 1 & 4 & 0 \\ -1 & 2 & 3 \end{vmatrix}$	$\begin{vmatrix} 5 & 3 \\ 1 & 4 \end{vmatrix}$	$V = 19 \text{ u.v.}$
$(\vec{AB}, \vec{AC}, \vec{AD}) =$	$1 \cdot 4 \cdot 0$	$1 \cdot 4$	$\begin{vmatrix} 2 & 2 \end{vmatrix}$
	$-1 \cdot 2 \cdot 3$	$-1 \cdot 2$	
	$= 60 + 2 + 4 - 9$		
	$= 57$		

$$\frac{151}{3}$$

$$V = \frac{1}{6} |(\vec{AB}, \vec{AC}, \vec{AP})|$$



5)

A(2,0,0)

B(2, 4, 0)

$C(0, 3, 0)$

$$P(2, -2, 9)$$

$$\begin{pmatrix} \vec{AB} \\ \vec{AC} \\ \vec{AP} \end{pmatrix} = \begin{pmatrix} -2 & 3 & 0 \\ 0 & -2 & 9 \\ 0 & -2 & -2 \end{pmatrix}$$

$$= 72 //$$

$$\text{II) } V = \frac{11721}{6} = \frac{72}{6} = 12 \text{ u.v.}$$

$$\text{III) } V = \frac{1}{6} |\vec{AB} \times \vec{AC}| h \Rightarrow A = \frac{6 \cdot 12}{|\vec{AB} \times \vec{AC}|} = \frac{6 \cdot 12}{\sqrt{64}} = \frac{6 \cdot 12 \cdot 3}{8 \cdot 2} = \underline{\underline{9 \text{ u.c}}}$$

$$\vec{AB} \times \vec{AC} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & 4 & 0 \\ -2 & 3 & 0 \end{vmatrix} = \begin{vmatrix} \hat{i} & \hat{j} \\ 0 & 4 \\ -2 & 3 \end{vmatrix}$$

$$= +8^{\frac{1}{2}}K = (0, 0, 8)$$

16) $\vec{AB} = (2, 1, -4)$

$$\vec{AC} = (m, -1, 3)$$

$$\vec{AD} = (-3, 1, -2)$$

(Vielknedro = 3.

$$\underline{V = 6 \cdot 3 = 18 \text{ u.v.}}$$

$$\text{Vetnaedno} = \frac{V}{6}$$

5)

$$(\vec{AB}, \vec{AC}, \vec{AD}) =$$

$$\begin{array}{ccc|ccc} 2 & 1 & -4 & 2 & 1 & \\ m & -1 & 3 & m & -1 & \\ -3 & 1 & -2 & -3 & 1 & \end{array}$$

II) $V = |-2m + 1|$

i) $18 = -2m + 1$

$$-2m = 17 \Rightarrow m = -\frac{17}{2}$$

$$= 4 + 9 - 4m + 12 - 6 + 2m$$

ii) $18 = 2m - 1$

$$= -2m + 1$$

$$2m = 19 \Rightarrow m = 19/2 //$$

17) A(-2, 4, -1)

$$V_4 = \frac{1}{6} |(\vec{AB}, \vec{AC}, \vec{AD})| = 6 \quad \therefore J)$$

$$B(-3, 2, 3)$$

$$\frac{6}{6} \quad \frac{12y+12}{6} = 8 \Rightarrow 2y = 4$$

$$C(1, -2, -1)$$

$$y = 2x$$

$$D(0, Y, 0)$$

$$\text{iii) } \frac{-12y - 12}{6} = 6 \Rightarrow -2y = 8$$

$$y = -4$$

$$\begin{array}{ccc|cc} -3 & -2 & 4 & -1 & -2 \end{array}$$

$$\begin{bmatrix} 3 & -6 & 0 & | & 3 & -6 \end{bmatrix}$$

2	$y-4$	2	$y-9$
---	-------	---	-------

$$(\vec{AB}, \vec{AC}, \vec{AD}) =$$

$$D(0, -4, 0) = -6 + 12y - 48 + 48 + 6$$

$$= 12y + 12$$

18) $q = ?$

I) $(\vec{AB}, \vec{AC}, \vec{AD}) = \begin{vmatrix} -3 & -3 & 0 \\ -3 & 0 & 3 \\ -1 & 5 & 2 \end{vmatrix} = 9 + 45 - 18 = 36$

II) $V = |36| = 36 \text{ u.v.}$

III) $V = (A_{\text{base}}) \cdot h$

$A = \frac{36}{|\vec{AB} \times \vec{AC}|}$

$\vec{AB} \times \vec{AC} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -3 & -3 & 0 \\ -3 & 0 & 3 \end{vmatrix} = \begin{vmatrix} -3 & -3 \\ -3 & 0 \end{vmatrix} \hat{i} - \begin{vmatrix} -3 & 0 \\ -3 & 3 \end{vmatrix} \hat{j} + \begin{vmatrix} -3 & -3 \\ -1 & 5 \end{vmatrix} \hat{k}$

$= -9\hat{i} - 9\hat{j} + 9\hat{k}$

$= (-9, -9, 9)$

$h = \frac{36}{\sqrt{81+81+81}} = \frac{36}{\sqrt{243}} = \frac{4}{\sqrt{3}} \text{ u.c.}$

19) $|\vec{u}| = 3$

$|\vec{v}| = 4$

$\theta = 120^\circ$

a) $|\vec{u} + \vec{v}| = ?$

$|\vec{u} + \vec{v}|^2 = |\vec{u}|^2 + 2\vec{u} \cdot \vec{v} + |\vec{v}|^2$

$|\vec{u} + \vec{v}| = \sqrt{9 + 12 + 16} = \sqrt{27} = 3\sqrt{3}$

$\vec{u} \cdot \vec{v} = |\vec{u}| \cdot |\vec{v}| \cdot \cos 120^\circ = 3 \cdot 4 \cdot (-1/2) = -6$

b) $|\vec{u} \times (\vec{v} - \vec{u})| = ?$

$|\vec{u} \times \vec{v}| = |\vec{u}| |\vec{v}| \sin 120^\circ = 3 \cdot 4 \cdot \frac{\sqrt{3}}{2} = 6\sqrt{3}$

c) $V = |\vec{u} \cdot \vec{v}, \vec{u} \times \vec{v}| = (\vec{u} \cdot \vec{v}) \cdot |\vec{u} \times \vec{v}| = -6 \cdot 6\sqrt{3} = -36\sqrt{3}$

20) a) $(m, n, 2) \cdot (4, -1, 3) = -2$

$4m - n + 6 = -2$

$n = 4m + 8$

b) $(m, n, 2) \times (4, -1, 3) = (8, -1, -11)$

$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ m & n & 2 \\ 4 & -1 & 3 \end{vmatrix} = \begin{vmatrix} \hat{i} & \hat{j} \\ m & n \end{vmatrix} = 8\hat{i} - 1\hat{j} - 11\hat{k}$

$\Rightarrow 3n\hat{i} + 8\hat{j} - m\hat{k} - 4n\hat{k} + 2\hat{u} - 3m\hat{j}$

$3n\hat{i} - 3m\hat{j} - m\hat{k} - 4n\hat{k} = 8\hat{i} - 9\hat{j} - 11\hat{k}$

$(3n, -3m, -m-4n) = (8, -9, -11)$

$n = 8; m = 3$

/ /

Seg Ter Qua Qui Sex Sáb Dom

c) $(m, n, 2) \cdot ((3, 1, 2) \times (0, 1, -1)) = 9$

m	n	2	m	n	
3	1	2	3	1	$= 9$
0	1	-1	0	1	

$$-m + 6 - 2m + 3n = 9$$

$$-3m + 3n = 3$$

$$-m + n = 1 \Rightarrow n = \underline{m+1}$$