

**Escola de Economia de São Paulo - Fundação Getulio Vargas**

**Course:** Econometria 2

**Instructor:** Vitor Possebom

**Problem Set 4 - Deadline: June 15 at 9:00 am - Total = 300 points**

**Question 1 (Testing for Cointegration when the Cointegrating Vector is Known - 150 points)**

*The dataset `brazil_data.csv` contains monthly information from January 1953 to January 2022 about the exchange rate between the Real and the Dollar and about IPCA index.*

*The dataset `usa_data.csv` contains monthly information from January 1960 to December 2021 about the American CPI index.*

*We will only use data from January 1995 to December 2019 in our analysis.*

- 1. Subset your data to cover only the analyzed period. (10 points)*
- 2. For each variable  $X_{k,t}$ ,  $k \in \{1, 2, 3\}$  in your dataset, define  $Y_{k,t} := 100 \cdot [\log(X_{k,t}) - \log(X_{k, \text{January}1995})]$ . (10 points)*
- 3. According to the purchasing power parity, what is the value of the cointegrating vector  $a$ ? To answer this question, you must be clear about the ordering of your variables and careful about measurement units. (10 points)*
- 4. Define  $Z_t = a'Y_t$ , where  $Y_t = (Y_{1,t}, Y_{2,t}, Y_{3,t})'$ . (10 points)*
- 5. Plot the data for  $Y_{k,t}$ ,  $k \in \{1, 2, 3\}$ . (10 points)*
- 6. Using the Augmented Dickey-Fuller test, test whether your  $Y_{k,t}$  variables are each individually  $I(1)$ . Be clear about the specification of your Augmented Dickey-Fuller test and about your null hypothesis, explaining how you choose the number of lags and your null hypothesis. (60 points)*
- 7. Plot the data for  $Z_t$ . (10 points)*

8. Using the Augmented Dickey-Fuller test, test whether  $Z_t$  is  $I(1)$ . Be clear about the specification of your Augmented Dickey-Fuller test and about your null hypothesis, explaining how you choose the number of lags and your null hypothesis. (20 points)
9. Based on your analysis, do you believe that the purchasing power parity holds in this context? Explain. (10 points)

**Question 2 (Testing for Cointegration when the Cointegrating Vector is Unknown - 120 points)**

The dataset `brazil_data.csv` contains monthly information from January 1953 to January 2022 about the exchange rate between the Real and the Dollar and about IPCA index.

The dataset `usa_data.csv` contains monthly information from January 1960 to December 2021 about the American CPI index.

We will only use data from January 1995 to December 2019 in our analysis. Therefore, you will use the same subset of the sample from Question 1.

In the previous question, we investigated whether a particular vector  $a$  was a cointegrating vector.

Now, we investigate whether there is any cointegrating relation between our three variables. To do so, we use the Phillips-Ouliaris-Hansen test.

1. Choose a order for your variables and justify your choice. (10 points)
2. Estimate  $Y_{1,t} = \alpha + \gamma_2 \cdot Y_{2,t} + \gamma_3 \cdot Y_{3,t} + U_t$ , save the estimated residuals and report the estimated cointegrating vector. (30 points)
3. Using the Phillips-Ouliaris-Hansen test, test whether  $\hat{U}_t$  is  $I(1)$ . Be clear about the specification of your test and your null hypothesis, explaining how you choose the critical values of your test. (30 points)
4. Based on your analysis, do you believe that the three original variable are cointegrated? Explain. (10 points)

5. Repeat the entire analysis in items 1-4 for the two remaining variable orderings. Are your conclusions robust to changing the order of your variables? (40 points)

**Question 3 (Testing for Cointegration: Johansen's Approach - 30 points)**

The dataset `brazil_data.csv` contains monthly information from January 1953 to January 2022 about the exchange rate between the Real and the Dollar and about IPCA index.

The dataset `usa_data.csv` contains monthly information from January 1960 to December 2021 about the American CPI index.

We will only use data from January 1995 to December 2019 in our analysis. Therefore, you will use the same subset of the sample from Question 1.

We investigate whether there is any cointegrating relation between our three variables. To do so, we use the Johansen's Approach.

1. Using Johansen's Eigenvalue test, find the number of cointegration relations among our variables of interest. Be clear about and justify your choice of lags and your choice for the option `ecdet`. (20 points)
2. Based on your results, do you believe that the purchasing power parity holds in this context. Explain. (10 points)