Problem Set 3

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This problem set was based on the code by Matheus Junqueira and the code used in the classroom.

Problem 1

1.1

Model 1: Run an ADL(2,1) using GDP growth as your dependent variable and Exchange Rate as your predictor

a)

Report the estimated coefficient and their standard errors.

Table 1

	Dependent variable:
	$\mathbf{Y_t}$
GDP Growth$_{\mathbf{t-1}}$	0.323***
	(0.101)
GDP Growth$_{\mathbf{t-2}}$	0.230**
	(0.111)
Exchange Rate$_{\mathbf{t-1}}$	−0.591*
	(0.340)
Constant	2.490***
	(0.865)
Note:	*p<0.1; **p<0.05; ***p<0.01

b)

Predict GDP growth in 2020 using model 1.

Table 2: **Predict GDP growth in 2020 using model 1**

Model	GDP growth (2020)	Forecast for GDP growth	Forecast error
ADL(2,1)	-3.88	0.97	-4.84

1.2

Model 2: Run an ADL(2,2) using GDP growth as your dependent variable and inflation as your predictor.

a)

Report the estimated coefficient and their standard errors.

Table 3

	Dependent variable:
	$\mathbf{Y_t}$
GDP Growth$_{\mathbf{t-1}}$	0.368***
	(0.107)
GDP Growth$_{\mathbf{t-2}}$	0.264***
	(0.098)
Inflation$_{\mathbf{t-1}}$)	0.0003
	(0.001)
Inflation$_{\mathbf{t-1}}$	−0.001
	(0.001)
Constant	1.777***
	(0.640)
Note:	*p<0.1; **p<0.05; ***p<0.01

b)

Predict GDP growth in 2020 using model 2.

Table 4: **Predict GDP Growth in 2020 using model 2**

Model	GDP growth (2020)	Forecast for GDP growth	Forecast error
ADL(2,2)	-3.88	2.67	-6.56

1.3

Model 3: Run an general time series regression model using GDP growth as your dependent variable and two lags of GDP growth, Exchange Rate and Inflation as your predictors

a)

Report the estimated coefficient and their standard errors.

Table 5

	Dependent variable:
	$\mathbf{Y_t}$
GDP Growth$_{\mathbf{t-1}}$	0.282**
	(0.111)
GDP Growth$_{\mathbf{t-2}}$	0.195*
	(0.112)
Exchange Rate$_{\mathbf{t-1}}$	−1.497
	(1.166)
Exchange Rate$_{\mathbf{t-1}}$	0.751
	(1.178)
Inflation$_{\mathbf{t-1}}$	−0.0004
	(0.002)
Inflation$_{\mathbf{t-1}}$	−0.001
	(0.001)
Constant	3.200***
	(1.027)
Note:	*p<0.1; **p<0.05; ***p<0.01

b)

Predict GDP growth in 2020 using model 3.

Table 6: **Predict GDP Growth in 2020 using model 3**

Model	GDP growth (2020)	Forecast for GDP growth	Forecast error
General Regression	-3.88	0.72	-4.60

1.4

Model 4: Run an ARMA(2,0) using GDP growth as your dependent variable.

a)

Report the estimated coefficient and their standard errors.

Table 7

	Dependent variable:
	$\mathbf{Y_t}$
GDP Growth$_{\mathbf{t-1}}$	0.373***
	(0.104)
GDP Growth$_{\mathbf{t-2}}$	0.273***
	(0.101)
Constant	1.625***
	(0.593)
Note:	*p<0.1; **p<0.05; ***p<0.01

b)

Predict GDP growth in 2020 using model 4.

Table 8: **Predict GDP Growth in 2020 using model 4**

Model	GDP growth (2020)	Forecast for GDP growth	Forecast error
General Regression	-3.88	2.57	-6.45

1.5

Which model generate the prediction that is closest to the realized value?

Based on the provided information, out of the four models analyzed (ADL(2,1), ADL(2,2), General Time Series Regression, and ARMA(2,0)), Model 3, which is the General Time Series Regression model, generated the prediction that was closest to the realized value of GDP growth in 2020.

Problem 2

A p-th order vector autoregression process (VAR(p)) is given by[1]:

$$Y_t = c + \Phi_1 Y_{t-1} + \Phi_2 Y_{t-2} + \ldots + \Phi_p Y_{t-p} + \epsilon_t \tag{1}$$

where

- Y_t : $(n \times 1)$-vector of variables
- c : $(n \times 1)$-vector of constants
- Φ_j : $(n \times n)$-matrix of coefficients for $j \in \{1, 2, \ldots, p\}$
- ϵ_t : $(n \times 1)$-vector generalization of white noise
  - $\mathbb{E}[\epsilon_t] = 0$ - $\{\epsilon_t\}$ is white noise
  - $\underbrace{\mathbb{E}[\epsilon_t \epsilon_\tau']}_{n\times n} = \begin{cases} \Omega & \text{for } t = \tau \\ 0 & \text{otherwise} \end{cases}$ with Ω an $(n \times n)$ symmetric positive definite matrix.

Formally, we impose that ϵ_t is uncorrelated with $Y_{t-p-1}, Y_{t-p-2}, \cdots$, i.e., we impose that the model is well specified. Then, a vector process $\{Y_t\}$ is said to be covariance-stationary if it's first and second moments ($\mathbb{E}[Y_t]$ and $\mathbb{E}\left[Y_t Y_{t-j}'\right]$) are independent of time t.

A VAR(p) is a covariance-stationary if all values of z satisfying

$$|I_n - \Phi_1 \cdot z - \Phi_2 \cdot z^2 - \cdots - \Phi_p \cdot z^p| = 0$$

lie outside the unit circle.

Then:

- $\mathbb{E}[\epsilon_t] = \mu$
- $\mathbb{E}\left[(Y_t - \mu)(Y_{t-j} - \mu)'\right] = \Gamma_j$

To find $\mu =: E[Y_t]$ we can take the expected value on both sides of (1) given by:

$$Y_t = c + \Phi_1 Y_{t-1} + \Phi_2 Y_{t-2} + \ldots + \Phi_p Y_{t-p} + \epsilon_t$$

Then:

$$\begin{gathered}
\mathbb{E}[Y_t] = \mathbb{E}[c + \Phi_1 Y_{t-1} + \Phi_2 Y_{t-2} + \ldots + \Phi_p Y_{t-p} + \epsilon_t] \\
\mu = c + \Phi_1 \mu + \Phi_2 \mu + \ldots + \Phi_p \mu \\
\mu - \Phi_1 \mu + \Phi_2 \mu + \ldots + \Phi_p \mu = c \\
(I - \Phi_1 - \Phi_2 - \ldots - \Phi_q)\, \mu = c \\
\mu = (I - \Phi_1 - \Phi_2 - \ldots - \Phi_q)^{-1} c
\end{gathered}$$

Problem 3

To prove that $\Gamma_j' = \Gamma_{-j}$ we can **replace** t **with** $t + j$ in the jth autocovariance which is a (n x n) matrix defined by[2]: $\mathbb{E}\left[(Y_t - \mu)(Y_{t-j} - \mu)'\right] = \Gamma_j$.

Then, we have:

$$\begin{gathered}
\mathbb{E}\left[(Y_t - \mu)(Y_{t-j} - \mu)'\right] = \Gamma_j \\
\mathbb{E}\left[(Y_{t+j} - \mu)(Y_{t+j-j} - \mu)'\right] = \Gamma_j
\end{gathered}$$

[1] Hamilton's Time Series Analysis page 258

[2] Hamilton's Time Series Analysis page 262

$$\mathbb{E}\left[(Y_{t+j}-\mu)(Y_t-\mu)^{'}\right]=\Gamma_j$$

Taking transposes we have the desired result:

$$\Gamma_j^{'}=\mathbb{E}\left[(Y_t-\mu)(Y_{t+j}-\mu)^{'}\right]=\Gamma_{-j}$$

$$\Gamma_j^{'}=\Gamma_{-j}$$

Problem 4

An **MA (q) process** is given by [3]:

$$Y_t=\mu+\varepsilon_t+\Theta_1\varepsilon_{t-1}+\Theta_2\varepsilon_{t-2}+\ldots+\Theta_q\epsilon_{t-q}$$

where ε_t **is a white noise process** such that:

- $\mathbb{E}\left[\varepsilon\right]=0$
- $\underbrace{\mathbb{E}\left[\varepsilon_t\varepsilon_\tau'\right]}_{n\times n}=\begin{cases}\Omega & \text{for } t=\tau\\ 0 & \text{otherwise}\end{cases}$ with Ω an $(n\times n)$ symmetric positive definite matrix.

Worth mentioning that the **mean** of Y_t is:

$$\begin{aligned}\mathbb{E}\left[Y_t\right]&=\mathbb{E}\left[\mu+\varepsilon_t+\Theta_1\varepsilon_{t-1}+\Theta_2\varepsilon_{t-2}+\ldots+\Theta_q\epsilon_{t-q}\right]\\&=\mu+\mathbb{E}\left[\varepsilon_t\right]+\Theta_1\mathbb{E}\left[\varepsilon_{t-1}\right]+\Theta_2\mathbb{E}\left[\varepsilon_{t-2}\right]+\ldots+\Theta_q\mathbb{E}\left[\varepsilon_{t-q}\right]\\&=\mu\end{aligned}$$

and the **variance** is, i.e., when $j=0$:

$$\begin{aligned}\mathbb{E}\left[\left(Y_t-\mu\right)\left(Y_t-\mu\right)'\right]&=\mathbb{E}\left[\left(\varepsilon_t+\Theta_1\varepsilon_{t-1}+\Theta_2\varepsilon_{t-2}\ldots+\Theta_q\varepsilon_{t-q}\right)\left(\varepsilon_t+\Theta_1\varepsilon_{t-1}+\Theta_2\varepsilon_{t-2}+\ldots+\Theta_q\varepsilon_{t-q}\right)'\right]\\&=\mathbb{E}\left[\varepsilon_t\varepsilon_t'\right]+\Theta_1\mathbb{E}\left[\varepsilon_{t-1}\varepsilon_{t-1}'\right]\Theta_1'+\Theta_2\mathbb{E}\left[\varepsilon_{t-2}\varepsilon_{t-2}'\right]\Theta_2'+\ldots+\Theta_q\mathbb{E}\left[\varepsilon_{t-j}\varepsilon_{t-j}'\right]\Theta_q'\\&=\Omega+\Theta_1\Omega\Theta_1'+\Theta_2\Omega\Theta_2'+\ldots+\Theta_q\Omega\Theta_q'\\&=\Gamma_0\end{aligned}$$

Moreover, the **autocovariances** are separated in **3 situations**:

- When $j=\{1,2,\cdots,q\}$

  $$\mathbb{E}\left[\left(Y_t-\mu\right)\left(Y_{t-j}-\mu\right)'\right]=\mathbb{E}\left[\left(\varepsilon_t+\Theta_1\varepsilon_{t-1}+\Theta_2\varepsilon_{t-2}\ldots+\Theta_q\varepsilon_{t-q}\right)\left(\varepsilon_{t-j}+\Theta_1\varepsilon_{t-j-1}+\Theta_2\varepsilon_{t-j-2}\ldots+\Theta_q\varepsilon_{t-j-q}\right)'\right]$$

  such that:

  $$\mathbb{E}\left[\left(\varepsilon_t+\Theta_1\varepsilon_{t-1}+\Theta_2\varepsilon_{t-2}\ldots+\Theta_q\varepsilon_{t-q}\right)\left(\varepsilon_{t-j}+\Theta_1\varepsilon_{t-j-1}+\Theta_2\varepsilon_{t-j-2}\ldots+\Theta_q\varepsilon_{t-j-q}\right)'\right]\tag{2}$$

  can be written as:

  $$\Theta_j\mathbb{E}\left[\varepsilon_{t-j}\varepsilon_{t-j}'\right]+\Theta_{j+1}\mathbb{E}\left[\varepsilon_{t-j-1}\varepsilon_{t-j-1}'\right]\Theta_1'+\Theta_{j+2}\mathbb{E}\left[\varepsilon_{t-j-2}\varepsilon_{t-j-2}'\right]\Theta_2'+\ldots+\Theta_q\mathbb{E}\left[\varepsilon_{t-j-q}\varepsilon_{t-j-q}'\right]\Theta_{q-j}'\tag{3}$$

  Hence, (3) is equal to:

  $$\Theta_j\Omega+\Theta_{j+1}\Omega\Theta_1'+\Theta_{j+2}\Omega\Theta_2'+\ldots+\Theta_q\Omega\Theta_{q-j}'\tag{4}$$

[3] Hamilton's time series analysis, page 262

- When $j = \{-1, -2, \cdots, -q\}$

  $\mathbb{E}\left[(Y_t - \mu)(Y_{t-j} - \mu)'\right] = \mathbb{E}\left[(\varepsilon_t + \Theta_1\varepsilon_{t+1} + \Theta_2\varepsilon_{t+2} \ldots + \Theta_q\varepsilon_{t+q})(\varepsilon_{t+j} + \Theta_1\varepsilon_{t+j-1} + \Theta_2\varepsilon_{t+j-2} \ldots + \Theta_q\varepsilon_{t+j-q})'\right]$

  such that:

$$\mathbb{E}\left[(\varepsilon_t + \Theta_1\varepsilon_{t+1} + \Theta_2\varepsilon_{t+2} \ldots + \Theta_q\varepsilon_{t+q})(\varepsilon_{t+j} + \Theta_1\varepsilon_{t+j-1} + \Theta_2\varepsilon_{t+j-2} \ldots + \Theta_q\varepsilon_{t+j-q})'\right] \tag{5}$$

  can be written as:

$$\mathbb{E}\left[\varepsilon_t\varepsilon'_{t+j-j}\right]\Theta'_{-j} + \Theta_1\mathbb{E}\left[\varepsilon_{t-1}\varepsilon'_{t-1}\right]\Theta'_{-j+1} + \Theta_2\mathbb{E}\left[\varepsilon_{t-2}\varepsilon'_{t-2}\right]\Theta'_{-j+2} + \ldots + \Theta_{q+j}\mathbb{E}\left[\varepsilon_{t-q-j}\varepsilon'_{t-q-j}\right]\Theta'_q \tag{6}$$

  Hence, (6) is equal to:

$$\Omega\Theta'_{-j} + \Theta_1\Omega\Theta'_{-j+1} + \Theta_2\Omega\Theta'_{-j+2} + \ldots + \Theta_{q+j}\Omega\Theta'_q \tag{7}$$

- When $|j| > q$

$$\mathbb{E}\left[(Y_t - \mu)(Y_{t-j} - \mu)'\right] = 0$$

Therefore, the mean, the variance and all the autocovariances above do not depend on time, i.e., any vector MA(q) process is covariance-stationary.

Problem 5

Let's prove Proposition 1 from the Lecture Notes[4]

Proposition 1: Let $\{Y_t\}$ be a covariance-stationary process with moments given by

$$E(\mathbf{Y}_t) = \boldsymbol{\mu} \tag{8}$$

$$E\left[(\mathbf{Y}_t - \boldsymbol{\mu})(\mathbf{Y}_{t-j} - \boldsymbol{\mu})'\right] = \boldsymbol{\Gamma}_j \tag{9}$$

and with absolutely summable autocovariances (i.e., $\left(\sum_{v=\infty}^{+\infty}\boldsymbol{\Gamma}_{\mathbf{v}}\right) \in \mathbb{R}$). Assume we have a sample of size T drawn from $\{Y_t\}$. Then, the sample mean $\bar{\mathbf{Y}}_T := \frac{\sum_{t=1}^{T} Y_t}{T}$ satisfies

1. $\bar{\mathbf{Y}}_T \xrightarrow{p} \mu$
2. $\lim_{T\to+\infty}\left\{T \cdot \mathbb{E}\left[\left(\bar{\mathbf{Y}}_T - \mu\right)\left(\bar{\mathbf{Y}}_T - \mu\right)'\right]\right\} = \sum_{v=-\infty}^{+\infty}\boldsymbol{\Gamma}_{\mathbf{v}}$

First[5], to prove **2** recall that

$$\overline{\mathbf{Y}}_T = (1/T)\sum_{t=1}^{T}\mathbf{Y}_t$$

and

$$\mathbb{E}[\overline{\mathbf{Y}}_T] = (1/T)\sum_{t=1}^{T}\mathbb{E}[\mathbf{Y}_t] = \boldsymbol{\mu}$$

We can see what's inside equation 9 doing the following:

$$\begin{aligned}
&\mathbb{E}\left[\left(\overline{\mathbf{Y}}_T - \boldsymbol{\mu}\right)\left(\overline{\mathbf{Y}}_T - \boldsymbol{\mu}\right)'\right] \\
&= \left(1/T^2\right)\mathbb{E}\left\{(\mathbf{Y}_1 - \boldsymbol{\mu})\left[(\mathbf{Y}_1 - \boldsymbol{\mu})' + (\mathbf{Y}_2 - \boldsymbol{\mu})' + \cdots + (\mathbf{Y}_T - \boldsymbol{\mu})'\right]\right. \\
&+ (\mathbf{Y}_2 - \boldsymbol{\mu})\left[(\mathbf{Y}_1 - \boldsymbol{\mu})' + (\mathbf{Y}_2 - \boldsymbol{\mu})' + \cdots + (\mathbf{Y}_T - \boldsymbol{\mu})'\right] \\
&+ \cdots + \left.(\mathbf{Y}_T - \boldsymbol{\mu})\left[(\mathbf{Y}_1 - \boldsymbol{\mu})' + (\mathbf{Y}_2 - \boldsymbol{\mu})' + \cdots + (\mathbf{Y}_T - \boldsymbol{\mu})'\right]\right\} \\
&= \left(1/T^2\right)\left\{\left[\boldsymbol{\Gamma}_0 + \boldsymbol{\Gamma}_{-1} + \cdots + \boldsymbol{\Gamma}_{-(T-1)}\right]\right. \\
&+ \left[\boldsymbol{\Gamma}_1 + \boldsymbol{\Gamma}_0 + \boldsymbol{\Gamma}_{-1} + \cdots + \boldsymbol{\Gamma}_{-(T-2)}\right] \\
&+ \left[\boldsymbol{\Gamma}_2 + \boldsymbol{\Gamma}_1 + \boldsymbol{\Gamma}_0 + \boldsymbol{\Gamma}_{-1} + \cdots + \boldsymbol{\Gamma}_{-(T-3)}\right] \\
&= \left(1/T^2\right)\left\{T\boldsymbol{\Gamma}_0 + (T-1)\boldsymbol{\Gamma}_1 + (T-2)\boldsymbol{\Gamma}_2 + \cdots + \boldsymbol{\Gamma}_{T-1}\right. \\
&\left.+(T-1)\boldsymbol{\Gamma}_{-1} + (T-2)\boldsymbol{\Gamma}_{-2} + \cdots + \boldsymbol{\Gamma}_{-(T-1)}\right\}.
\end{aligned} \tag{10}$$

[4] Lecture 04A - VAR theory, slide 10

[5] Hamilton's Time Series Analysis, page 279

If we multiply both sides by T in equation 9 we have

$$\begin{aligned} T \cdot \mathbb{E}\Big[\left(\overline{\mathbf{Y}}_T - \boldsymbol{\mu}\right)\left(\overline{\mathbf{Y}}_T - \boldsymbol{\mu}\right)'\Big] & \\ =& \boldsymbol{\Gamma}_0 + [(T-1)/T]\boldsymbol{\Gamma}_1 + [(T-2)/T]\boldsymbol{\Gamma}_2 + \cdots \\ & + [1/T]\boldsymbol{\Gamma}_{T-1} + [(T-1)/T]\boldsymbol{\Gamma}_{-1} + [(T-2)/T]\boldsymbol{\Gamma}_{-2} \\ & + \cdots + [1/T]\boldsymbol{\Gamma}_{-(T-1)}. \end{aligned} \tag{11}$$

To prove that $\lim_{T\to+\infty}\left\{T \cdot \mathbb{E}\left[\left(\bar{\mathbf{Y}}_T - \mu\right)\left(\bar{\mathbf{Y}}_T - \mu\right)'\right]\right\} = \sum_{v=\infty}^{+\infty} \boldsymbol{\Gamma}_{\mathbf{v}}$ we have to consider that:

$$\sum_{v=-\infty}^{+\infty} \boldsymbol{\Gamma}_v - T \cdot \mathbb{E}\left[\left(\bar{\mathbf{Y}}_T - \mu\right)\left(\bar{\mathbf{Y}}_T - \mu\right)'\right] = \sum_{|v|\geq T} \boldsymbol{\Gamma}_v + \sum_{v=-(T-1)}^{T-1} \frac{|v|}{T}\boldsymbol{\Gamma}_v \tag{12}$$

Where 12 follows from 10, such that, $\gamma_{ij}^{(i)}$ denote the row i, column j element of $\boldsymbol{\Gamma}_v$. The row i, column j element of the matrix in 12 can be written

$$\sum_{|v|\geq T} \gamma_{ij}^{(v)} + \sum_{v=-(T-1)}^{T-1} (|v|/T)\gamma_{ij}^{(v)}$$

Since $\{\Gamma_v\}_{v=-\infty}^{\infty}$ are absolutely summable by assumption, for every $\varepsilon > 0$, there exists a q such that

$$\sum_{|v|\geq q} \left|\gamma_{ij}^{(v)}\right| < \varepsilon/2$$

Hence

$$\left|\sum_{|v|\geq T} \gamma_{ij}^{(v)} + \sum_{v=-(T-1)}^{T-1} (|v|/T)\gamma_{ij}^{(v)}\right| < \varepsilon/2 + \sum_{v=-q}^{q} (|v|/T)\left|\gamma_{ij}^{(v)}\right|$$

When T is sufficiently large:

$$\left|\sum_{|v|\geq T} \gamma_{ij}^{(v)} + \sum_{v=-(T-1)}^{T-1} \left(\frac{|v|}{T}\gamma_{ij}^{(v)}\right)\right| < \varepsilon$$

Therefore, with T sufficiently large and $\forall\varepsilon > 0$:

$$|\sum_{v=-\infty}^{+\infty} \boldsymbol{\Gamma}_v - T \cdot \mathbb{E}\left[\left(\bar{\mathbf{Y}}_T - \mu\right)\left(\bar{\mathbf{Y}}_T - \mu\right)'\right]| < 0 \tag{13}$$

So,

$$\lim_{T\to+\infty}\left\{T \cdot \mathbb{E}\left[\left(\bar{\mathbf{Y}}_T - \mu\right)\left(\bar{\mathbf{Y}}_T - \mu\right)'\right]\right\} = \sum_{v=-\infty}^{\infty} \boldsymbol{\Gamma}_{\mathbf{v}} \tag{14}$$

Second[6], to prove **1** we can use

$$\lim_{T\to+\infty}\left\{T \cdot \mathbb{E}\left[\left(\bar{\mathbf{Y}}_T - \mu\right)\left(\bar{\mathbf{Y}}_T - \mu\right)'\right]\right\} = \sum_{v=-\infty}^{\infty} \boldsymbol{\Gamma}_{\mathbf{v}}$$

and

$$\mathbb{E}[\overline{\mathbf{Y}}_T] = (1/T)\sum_{t=1}^{T} \mathbb{E}[\mathbf{Y}_t] = \boldsymbol{\mu}$$

In terms of each element of the matrix, we have that:

$$\lim_{t\to\infty} \mathbb{E}\left[\left(\bar{\mathbf{Y}}_T - \mu\right)\left(\bar{\mathbf{Y}}_T - \mu\right)'\right] = 0 \tag{15}$$

[6] Hamilton's Time Series Analysis, page 280

which implies that:

$$\lim_{t\to\infty} \mathbb{E}\left[\left(\bar{Y}_{i,T} - \mu_i\right)^2\right] = 0$$

such that,

$$\lim_{t\to\infty} \mathbb{E}\left[\left(\bar{Y}_{i,T} - \mu_i\right)^2\right]$$

is the i-th entry on the diagonal of

$$\lim_{t\to\infty} \mathbb{E}\left[\left(\bar{\mathbf{Y}}_T - \mu\right)\left(\bar{\mathbf{Y}}_T - \mu\right)'\right]$$

therefore, $\bar{\mathbf{Y}}_T \xrightarrow{p} \mu$

Problem 6

[7]

6.1.a

First, we will use a VAR (p) to forecast GDP growth in 2020.
Model A: Set p = 1 and estimate a reduced-form VAR (1) model.

i)

Report the estimated coefficient and their standard errors.

Table 9: **Estimated Model - Reduced-Form** $VAR(1)$

	Dependent variable:		
	$\mathbf{Y_t}$	$\mathbf{e_t}$	$\pi_\mathbf{t}$
$\mathbf{Y_{t-1}}$	0.264**	−0.001	−15.701*
	(0.109)	(0.007)	(9.381)
$\mathbf{e_{t-1}}$	0.066	1.010***	−150.974**
	(0.667)	(0.041)	(57.447)
$\pi_\mathbf{t-1}$	−0.001	0.0001	0.550***
	(0.001)	(0.0001)	(0.096)
Constant	6.567***	−0.041	30.594
	(1.243)	(0.076)	(106.994)
Trend	−0.075**	0.002	5.433*
	(0.032)	(0.002)	(2.794)
Observations	77	77	77
R^2	0.349	0.971	0.546
Adjusted R^2	0.313	0.970	0.521
Residual Std. Error (df = 72)	3.359	0.204	289.208
F Statistic (df = 4; 72)	9.660***	607.613***	21.672***
Note:			*p<0.1; **p<0.05; ***p<0.01

[7] Y_t is the GDP Growth, π_t is the inflation rate and e_t is the exchange rate

ii)

Predict GDP growth in 2020 using model A.

Table 10: Forecast Results

Observed Y_t in 2020	Forecast	Lower Bound	Upper Bound	CI
-3.880	1.245	-5.339	7.829	6.584

6.1.b

Model B: Set $p = 2$ and estimate a reduced-form VAR (2) model.

i)

Report the estimated coefficient and their standard errors.

Table 11: **Estimated Model - Reduced-Form** $VAR(2)$

	Dependent variable:		
	$\mathbf{Y_t}$	$\mathbf{e_t}$	$\pi_\mathbf{t}$
$\mathbf{Y_{t-1}}$	0.235*	0.003	−14.424
	(0.121)	(0.007)	(10.303)
$\mathbf{e_{t-1}}$	−1.069	1.315***	−233.080
	(2.025)	(0.121)	(172.454)
$\mathbf{e_{t-1}}$	0.0001	0.0001	0.655***
	(0.001)	(0.0001)	(0.120)
$\mathbf{Y_{t-2}}$	0.197*	−0.0003	−13.342
	(0.115)	(0.007)	(9.768)
$\mathbf{e_{t-2}}$	1.403	−0.338***	54.167
	(2.110)	(0.126)	(179.746)
$\pi_\mathbf{t-2}$	−0.0004	−0.0001	−0.219*
	(0.001)	(0.0001)	(0.117)
Constant	5.203***	−0.088	108.590
	(1.492)	(0.089)	(127.094)
Trend	−0.066*	0.003	5.933**
	(0.035)	(0.002)	(2.945)
Observations	76	76	76
R^2	0.382	0.974	0.582
Adjusted R^2	0.318	0.971	0.539
Residual Std. Error (df = 68)	3.350	0.200	285.334
F Statistic (df = 7; 68)	6.002***	362.063***	13.543***
Note:			*p<0.1; **p<0.05; ***p<0.01

ii)

Predict GDP growth in 2020 using model B.

Table 12: Forecast Results

Observed Y_t in 2020	Forecast	Lower Bound	Upper Bound	CI
−3.880	1.519	−5.047	8.084	6.565

6.1.c

Model C: Set $p = 3$ and estimate a reduced-form VAR (3) model.

i)

Report the estimated coefficient and their standard errors.

Table 13: **Estimated Model - Reduced-Form** $VAR(3)$

	Dependent variable:		
	$\mathbf{Y_t}$	$\mathbf{e_t}$	$\pi_{\mathbf{t}}$
$\mathbf{Y_{t-1}}$	0.235*	0.005	−10.426
	(0.128)	(0.007)	(9.677)
$\mathbf{e_{t-1}}$	−0.913	1.306***	−127.587
	(2.183)	(0.127)	(164.868)
$\pi_{\mathbf{t-1}}$	0.0001	0.0001	0.770***
	(0.002)	(0.0001)	(0.114)
$\mathbf{Y_{t-2}}$	0.211	0.003	−14.082
	(0.130)	(0.008)	(9.803)
$\mathbf{e_{t-2}}$	0.645	−0.324	−185.928
	(3.497)	(0.204)	(264.099)
$\pi_{\mathbf{t-2}}$	−0.001	−0.00004	−0.569***
	(0.002)	(0.0001)	(0.136)
$\mathbf{Y_{t-3}}$	−0.026	−0.013*	1.166
	(0.122)	(0.007)	(9.196)
$\mathbf{e_{t-3}}$	0.596	−0.033	204.081
	(2.303)	(0.134)	(173.872)
$\pi_{\mathbf{t-3}}$	0.0002	−0.0001	0.466***
	(0.001)	(0.0001)	(0.111)
Constant	5.202***	−0.050	125.939
	(1.707)	(0.099)	(128.872)
Trend	−0.065*	0.004	3.074
	(0.038)	(0.002)	(2.875)
Observations	75	75	75
R^2	0.380	0.975	0.673
Adjusted R^2	0.283	0.971	0.622
Residual Std. Error (df = 64)	3.446	0.201	260.204
F Statistic (df = 10; 64)	3.926***	250.384***	13.157***
Note:	*p<0.1; **p<0.05; ***p<0.01		

ii)

Predict GDP growth in 2020 using model C.

Table 14: **Forecast Results - Reduced-Form** $VAR(3)$

Observed Y_t in 2020	Forecast	Lower Bound	Upper Bound	CI
-3.880	1.370	-5.384	8.123	6.754

6.1.d

Which model generate the prediction that is closest to the realized value?

In our analysis, we considered the unique characteristics of the series and addressed them by incorporating a constant and trend in all the VAR(p) models we estimated. This allowed us to account for the deterministic and constant trend observed in the inflation series, as well as handle the presence of a unit root in the exchange rate series.

When forecasting GDP growth in 2020, we found that the VAR(1) model had the lowest prediction error, indicating its superior performance as the best model. The VAR(3) model performed slightly worse but still ranked as the second best model.

6.2.a

Now, we will focus on a VAR (2) model and focus on structural IRFs. Choose the order of your variables and justify your exclusion restrictions.

The selected period from 1942 to 2019 poses challenges for both the exchange rate and inflation series. Brazil predominantly operated under a fixed exchange rate regime, indicating relative exogeneity compared to other series. However, even under a flexible exchange rate regime, the exchange rate's pass-through effect on domestic prices, particularly for imported goods, significantly impacts inflation. This effect is supported by studies on the Brazilian case. Recent crises, such as the Covid-19 pandemic and the Ukraine conflict, have shown the exchange rate's impact on prices, leading to currency devaluations and price increases. As the exchange rate is highly speculative and influenced by exogenous factors, it emerges as the most exogenous variable in the model.

GDP is the second most exogenous variable in the model, experiencing slower shock transmission compared to prices. Then, GDP is more exogenous than domestic prices. Moreover, while acknowledging that prices are sticky and require time to respond to demand shocks based on empirical evidence, the exchange rate's pass-through effect remains a significant factor contributing to the greater endogeneity of prices. Recent supply shocks, have demonstrated that domestic price levels react more swiftly than other variables.

In conclusion, our exclusion restriction will be the exchange rate as the first variable (more exogenous), followed by GDP growth (in the middle), and finally inflation (more endogenous).

6.2.b

Estimate and plot all nine structural impulse response functions and their 90%-confidence intervals based on 1,000 bootstrap repetitions.

Figure 1: SIRF of a real GDP growth shock on GDP growth itself

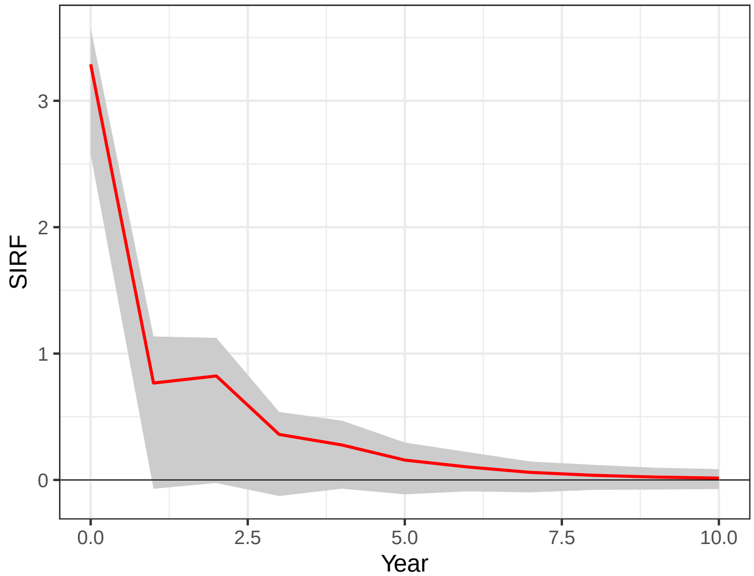


Figure 2: SIRF of a real GDP growth shock on inflation

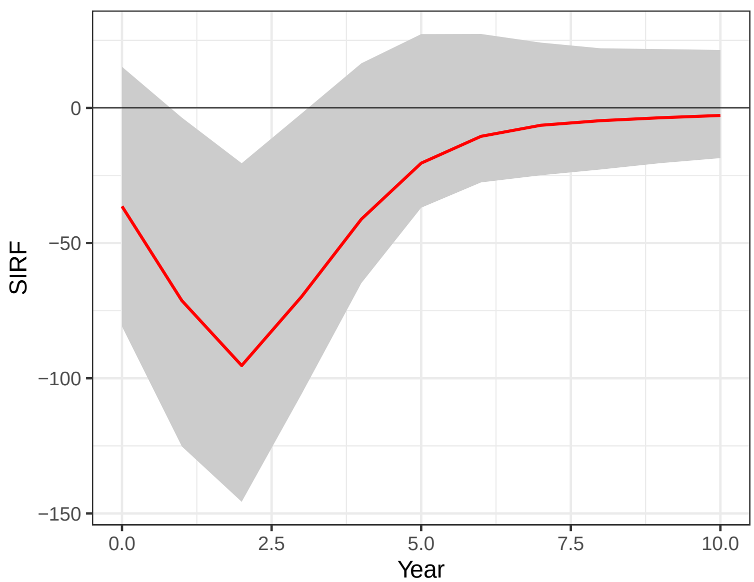


Figure 3: SIRF of a real GDP growth shock on exchange rate

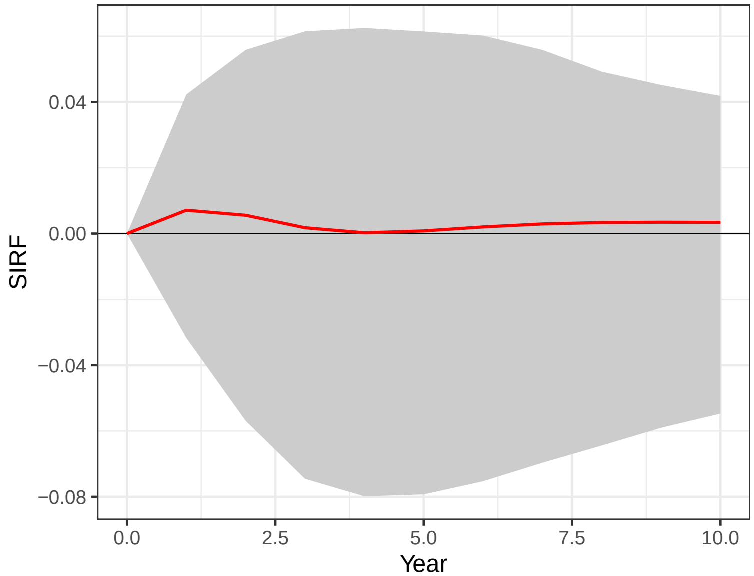


Figure 4: SIRF of an inflation shock on GDP growth

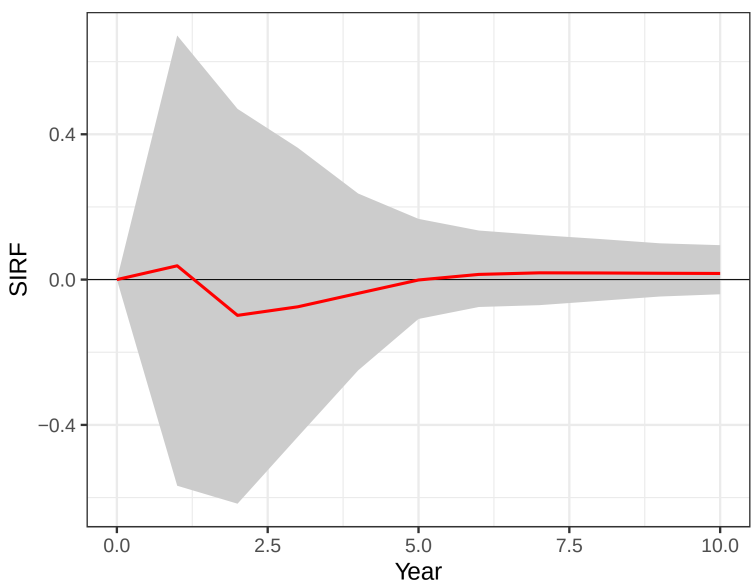


Figure 5: SIRF of an inflation shock on inflation itself

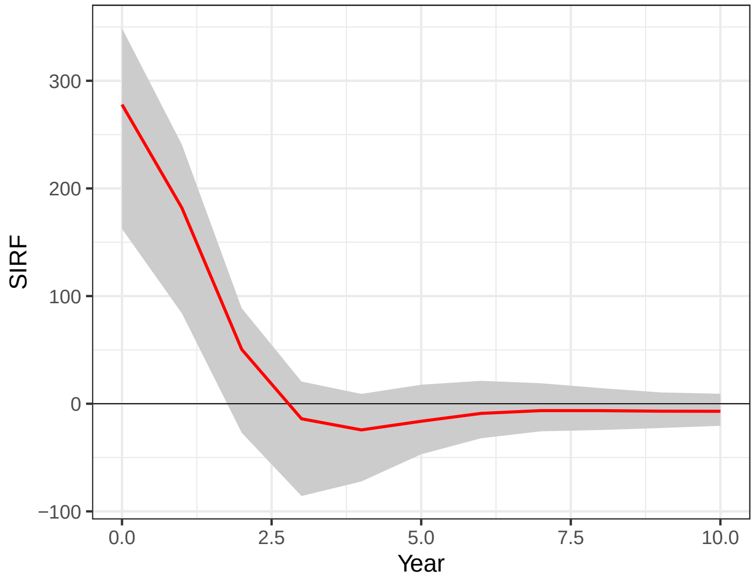


Figure 6: SIRF of an inflation shock on exchange rate

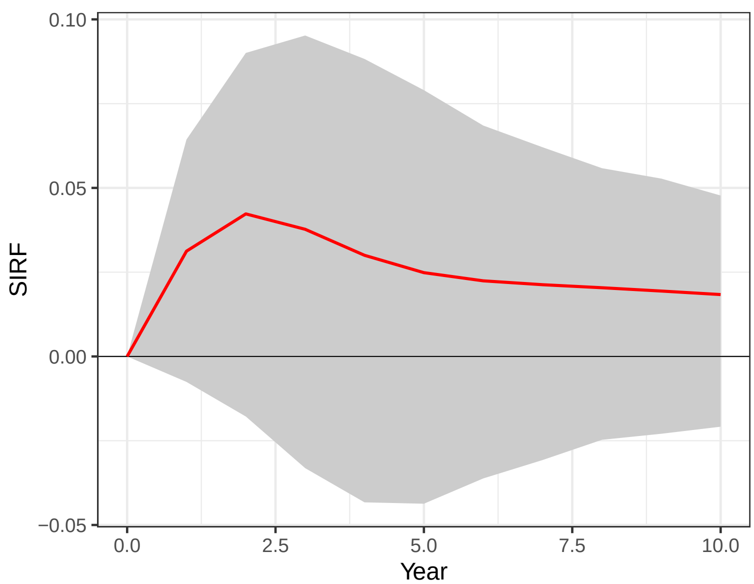


Figure 7: SIRF of an exchange rate on exchange rate itself

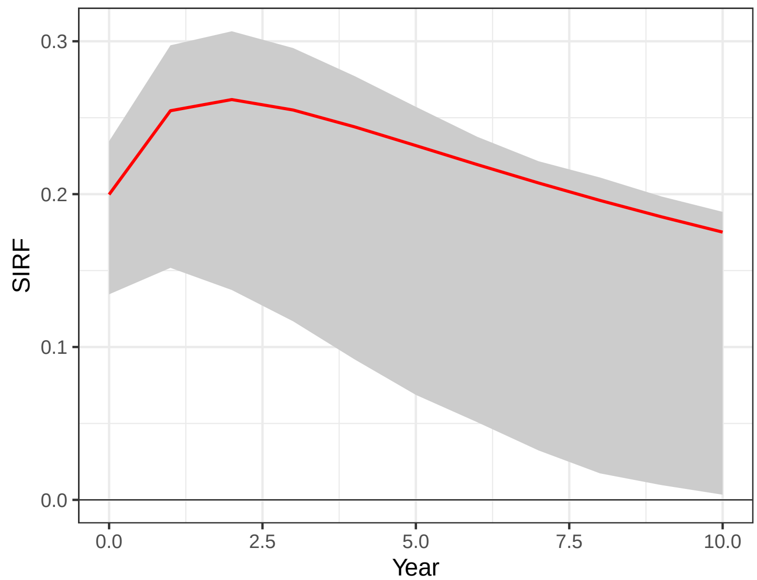


Figure 8: SIRF of an exchange rate on GDP growth

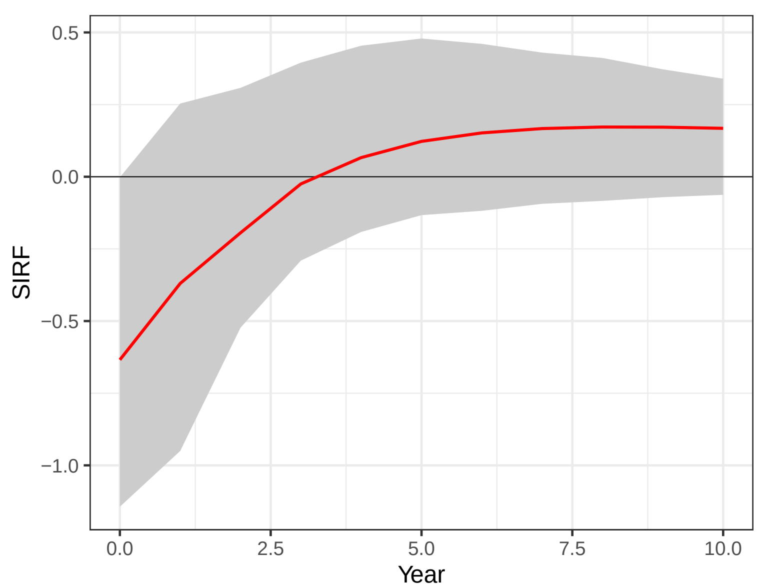
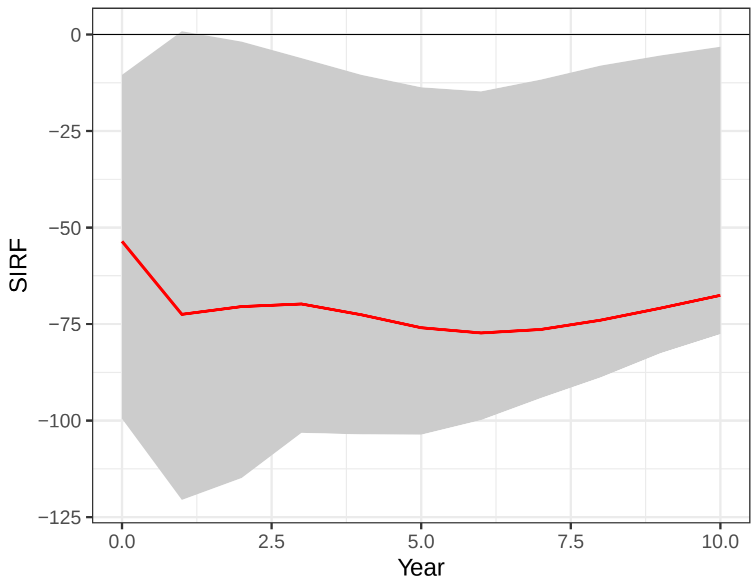


Figure 9: SIRF of an exchange rate on inflation



6.2.c

Do you believe your results are credible? Justify your answer.

Given the non-stationarity and potential presence of unit roots in the series, it would be prudent to explore whether cointegration exists among the variables. The presence of cointegration would imply a significant long-term relationship that the model fails to capture. This further highlights the imprecision and inadequacy of the model's specification.

In the absence of cointegration, it would be advisable to difference the series to ensure that all variables in the VAR(2) model are stationary. By doing so, the predictive capability of the model can be enhanced.

Additionally, it is important to note specific issues within the series themselves. First, the exchange rate series displays distinct regime shifts throughout the analyzed period,which indicates structural breaks in the Exchange Rate series. In a similar way, the inflation series exhibits periods of hyperinflation and it is impacted by several events throughout history. Hence, utilizing these series without appropriate treatment and considering the relevant time periods would introduce errors and compromise the model's ability to generate accurate predictions.

Moreover, the omission of important variables, such as interest rates and commodity prices in the models shows a huge limitation, which undermines the credibility of the estimated models' results.

The models' estimations are compromised, and the ability to make accurate predictions is compromised as a result.