NUMERICAL SIMULATION OF SHALLOW WATER WAVES

ANDREW CANNON, KATHLEEN DUPRE

OVERVIEW

- Numerically model shallow water waves
- Governed by an extension of the Navier-Stokes equations
- Objectives:
  - Make a working code
  - Parallelize
  - Obtain outputs for a variety of cases

GOVERNING EQUATIONS

- $\frac{\partial \eta}{\partial t} + \frac{\partial (u\eta)}{\partial x} + \frac{\partial (v\eta)}{\partial y} = 0$
- $\frac{\partial (u\eta)}{\partial t} + \frac{\partial}{\partial x}\left(\eta u^2 + \frac{1}{2} g\eta^2\right) + \frac{\partial}{\partial y}(\eta uv) = 0$
- $\frac{\partial (u\eta)}{\partial t} + \frac{\partial}{\partial x}(\eta uv) + \frac{\partial}{\partial y}\left(\eta v^2 + \frac{1}{2} g\eta^2\right) = 0$

- η – fluid height
- u – x velocity
- v - y velocity
- g – gravitational acceleration

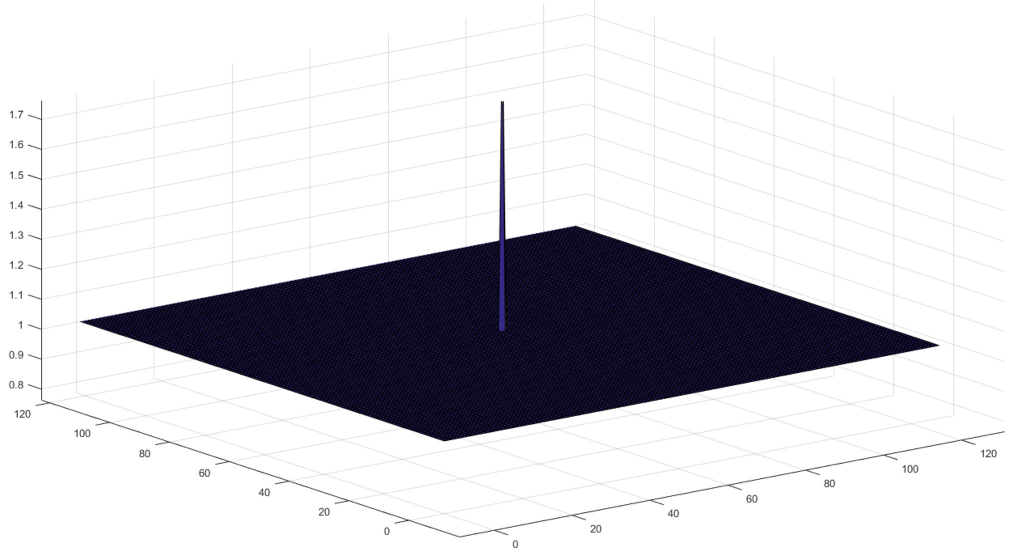
DIFFERENTIAL ALGORITHMS

- Forward difference in time:
  - $\frac{\partial \varphi}{\partial t} \rightarrow \frac{\varphi_{n+1} - \varphi_n}{\Delta t}$
- Central difference in space:
  - $\frac{\partial \varphi}{\partial x} \rightarrow \frac{\varphi_{i+1} - \varphi_{i-1}}{2 * \Delta x}$
- CFL number:
  - $C = U \frac{\Delta t}{\Delta x}$

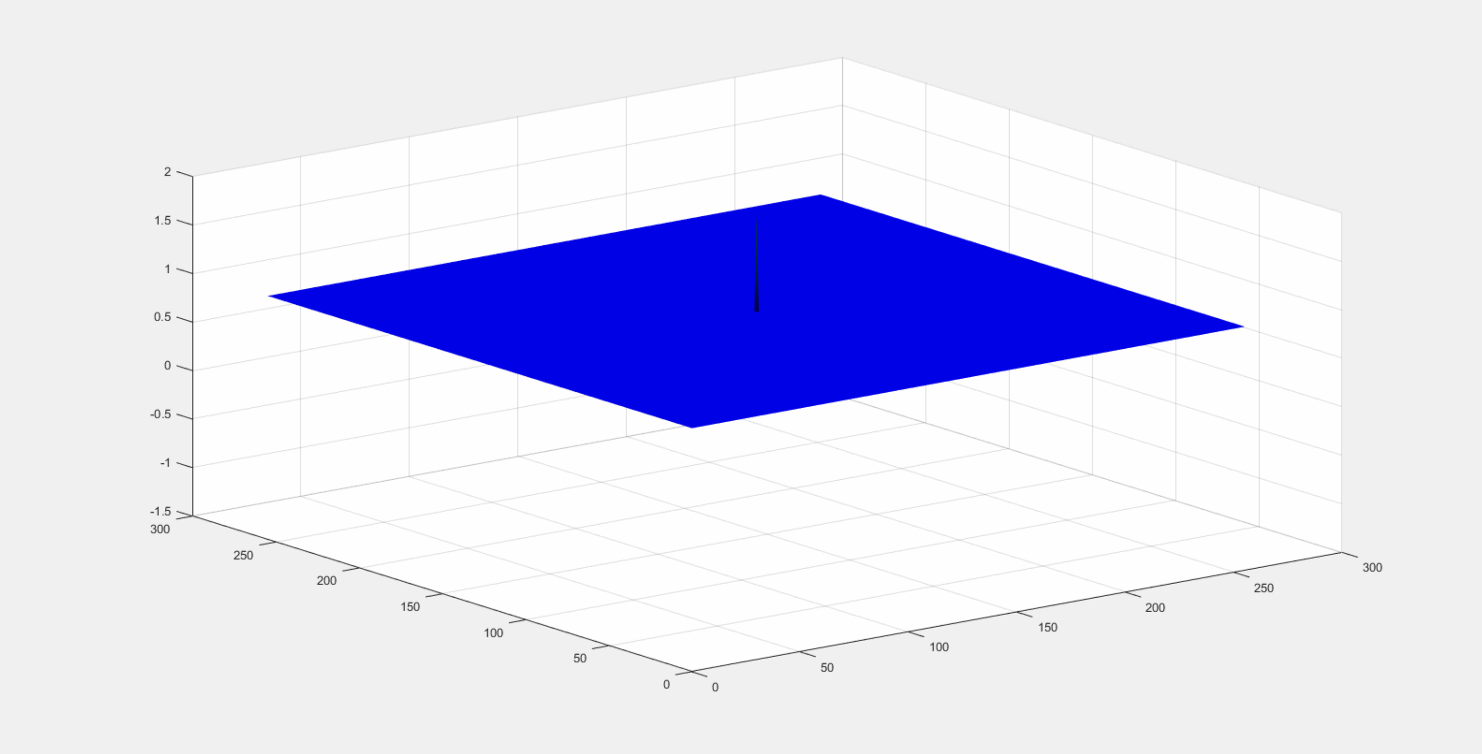
BOUNDARY AND INITIAL CONDITIONS

- Boundaries: Periodic
- Grid: 256 X 256
- Initials: Normal depth of 1

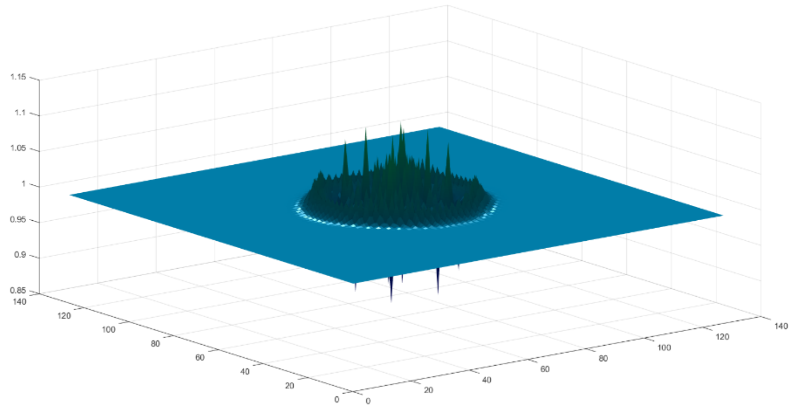
  Spike with depth 1.5 in center

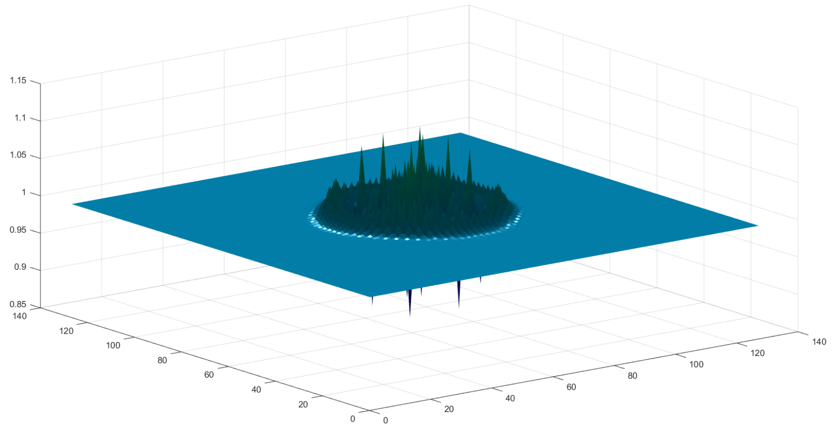


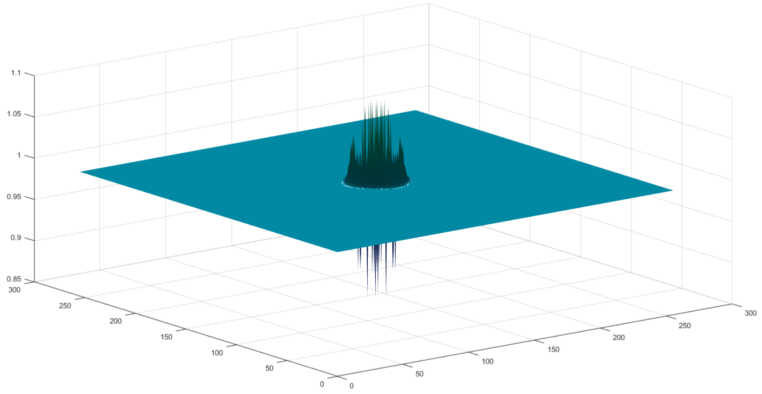
SIMPLE CASE



GRID REFINEMENTS

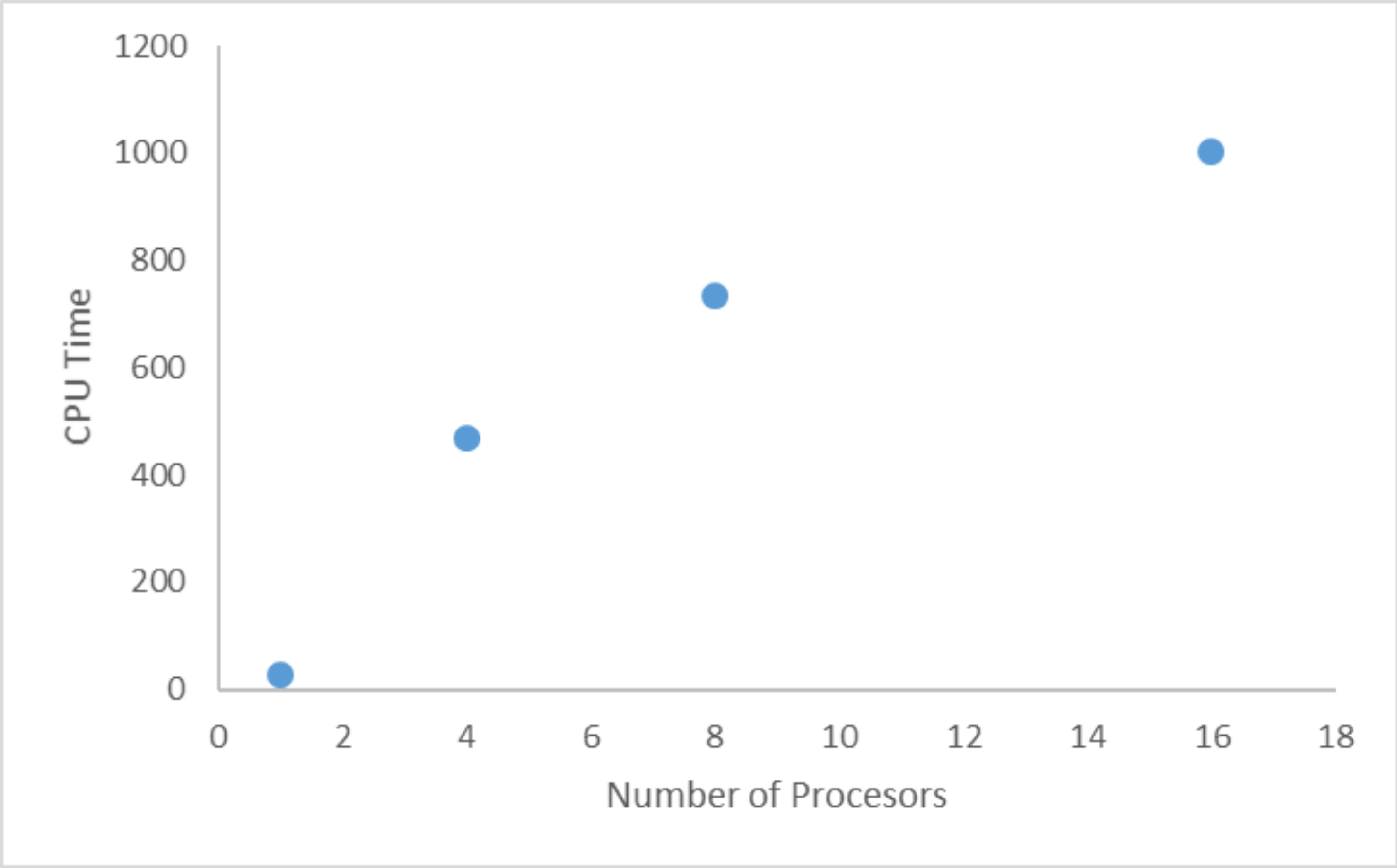




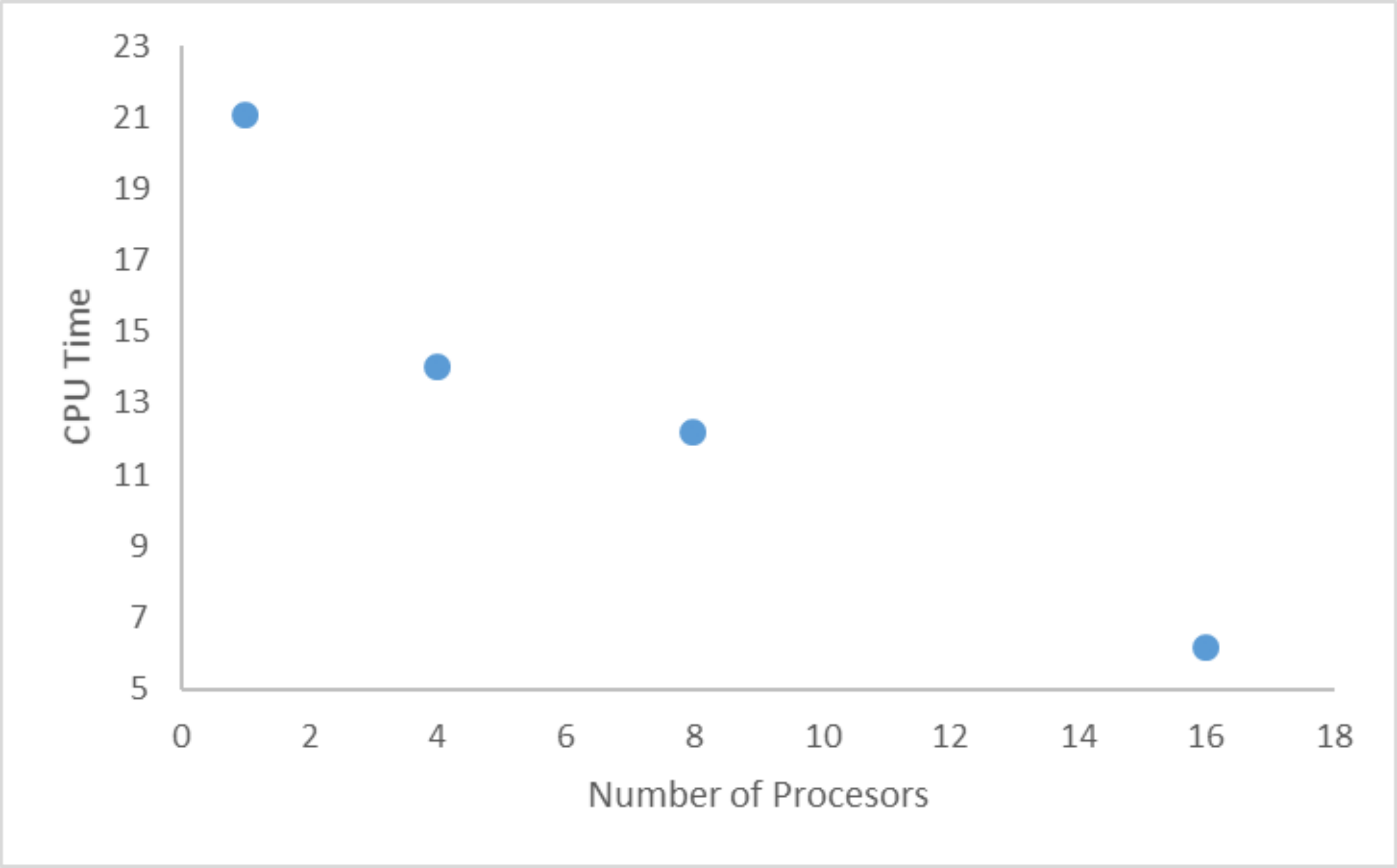


Root Mean Square Difference:
2.7E-4

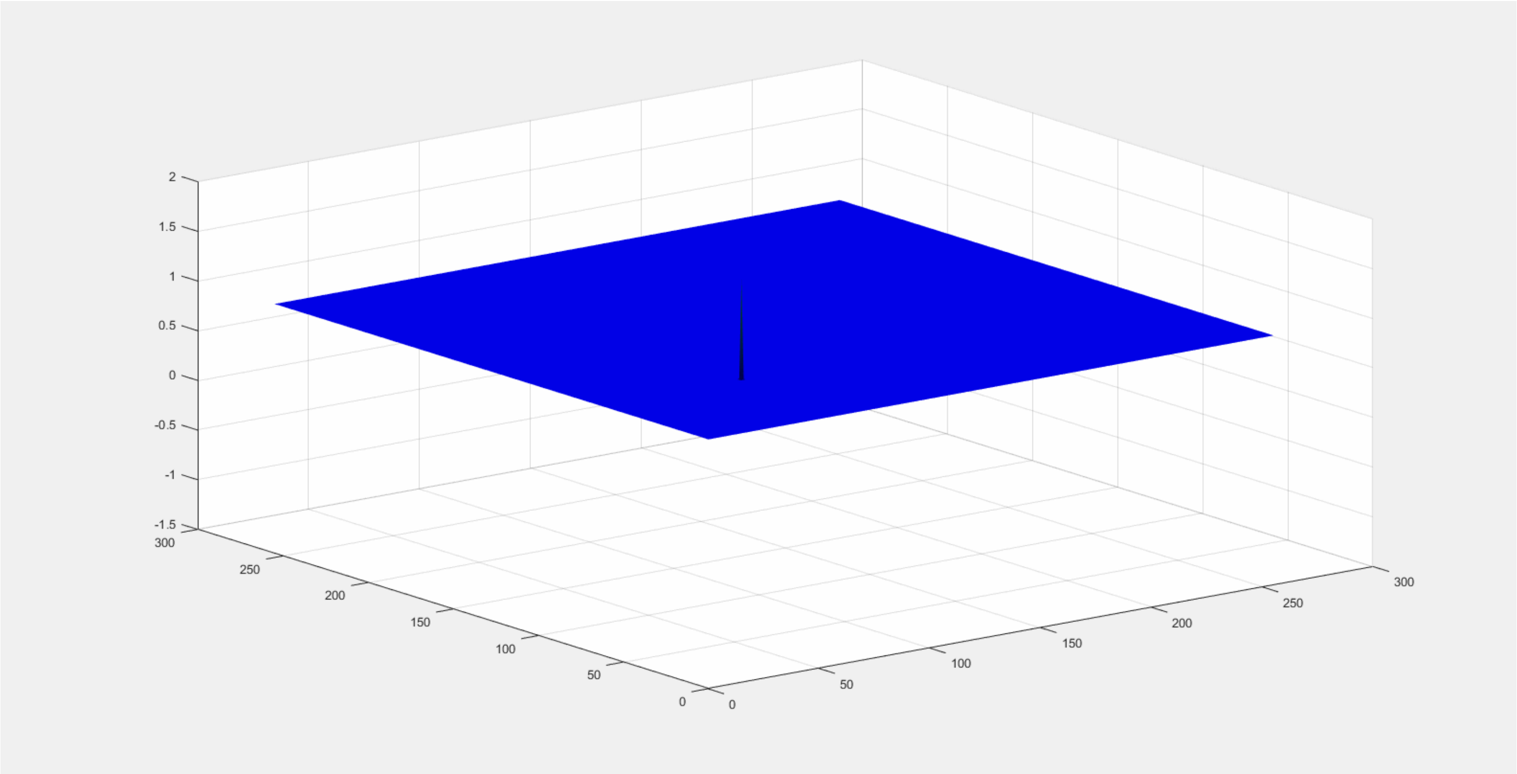
PARALLELIZING MPI



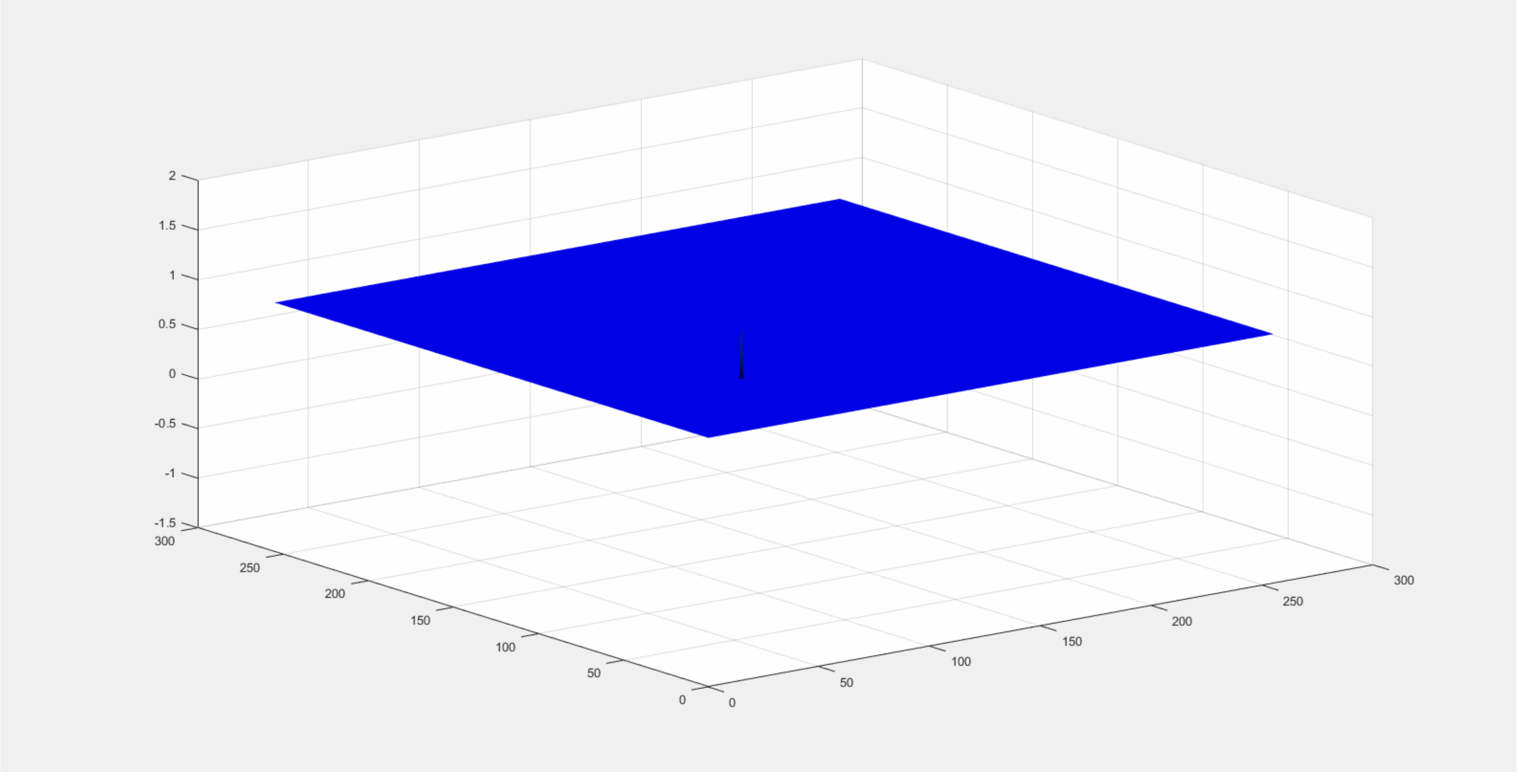
PARALLELIZING OMP



MORE COMPLEX CASES: OFF-CENTER DROP



MORE COMPLEX CASES: TIME DELAYED DROPS



CONCLUSIONS AND FUTURE WORK

- The code is effective in modeling shallow waves
- Could be improved using a smoothing function
- Nine-point stencil
- Different types boundary conditions
- Adaptive meshing