Theoritical Derivation:

1. Economic interaction model Equation.

Discrete - Time From (most Common in economics)

$$x_{t+1} = Ax_t$$

where: x_t = state vector of the economy

A = economic interaction matrix.

Continious - time Form

$$\dot{x}(t) = Ax(t)$$

2. Eigenvalue Equation (main Equation)

$$\det(A - \lambda I) = 0$$

=) This is the called characteristic equation.

where: λ = eigen value

I = identify matrix.

3. Eigenvector Relation (Definition)

Eigenvalues come From:

$$Av = \lambda v$$

where: v is eigen vector

λ tells decay along that direction.

4. Stability conditions (core Result)

if the system is:

$$x_{t+1} = Ax_t$$

The equilibrium is stable if:

$$|\lambda_i| < 1 \quad \forall i$$

So, $\rho(A) = \max |\lambda_i| < 1$

where $\rho(A)$ is the spectral Radius.

=) continious = Time stability condition.

$\dot{x}(t) = Ax(t)$

5. Solution Behavior (What Happens over time).

Discrete - Time Evalution:

$x_t = A^t x_0$

So if eigenvalues are < 1, then:

$x_t \to 0$ (stable)

Continious - Time Evalution:

$x(t) = e^{At} x(0)$

6. Final checklist of Equations:

(1) Economic system model:

$x_{t+1} = A x_t$

(2) Characteristic Equation:

$\det(A - \lambda I) = 0$

3) Eigenvalue defination:

$AV = \lambda V$

4) stability conditions:

$|\lambda_i| < 1$

5) stability conditions (continious).

$(\lambda_i) < 0$

6) spectral radius:

$\rho(A) = \max |\lambda_i|$

Matlab Code:

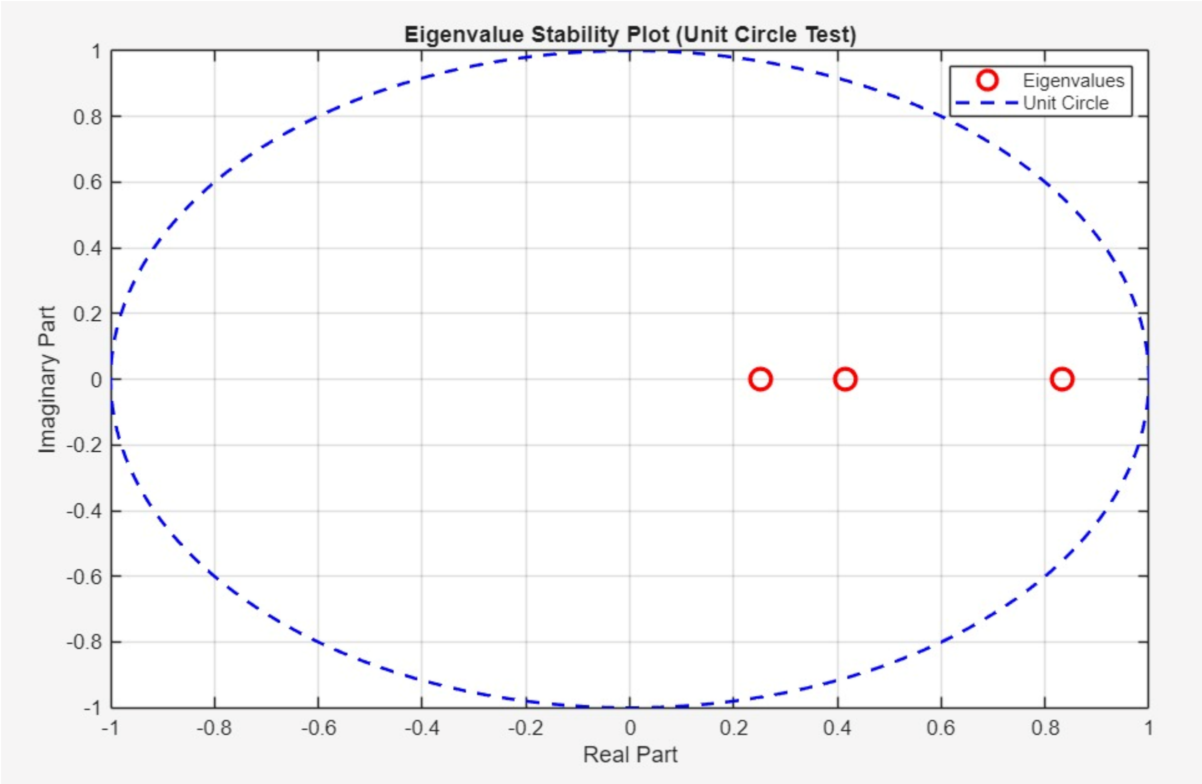
```
% EXPERIMENT: Eigenvalue Based Stability Analysis
% Economic Interaction Matrix
clc;
clear;
close all;
disp("  Eigenvalue Based Stability of Economic System ");
% Example Matrix (You can replace with your own)
A = [0.4  0.2  0.1;
     0.1  0.5  0.2;
     0.0  0.3  0.6];
disp("Economic Interaction Matrix A:");
disp(A);
% Step 2: Compute Eigenvalues
eigvals = eig(A);
disp("Eigenvalues of A:");
disp(eigvals);
% Step 3: Spectral Radius
rho = max(abs(eigvals));
disp("Spectral Radius (max |lambda|): ");
disp(rho);
% Step 4: Discrete-Time Stability Check
% Condition: |lambda| < 1
if all(abs(eigvals) < 1)
```

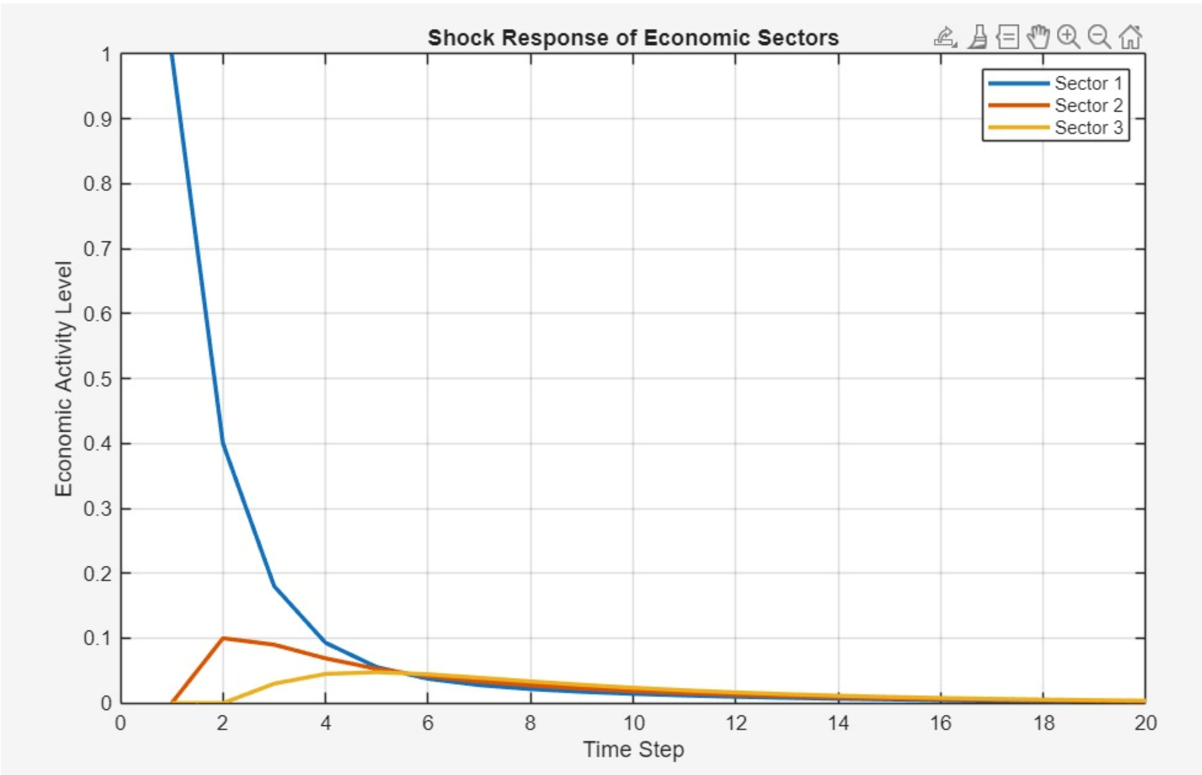
```
if all(abs(eigvals) < 1)
    disp("System is DISCRETE-TIME STABLE (Shock dies out).");
else
    disp("System is DISCRETE-TIME UNSTABLE (Shock grows).");
end
% Step 5: Plot Eigenvalues in Complex Plane

figure;
plot(real(eigvals), imag(eigvals), 'ro', ...
    'MarkerSize', 10, 'LineWidth', 2);

hold on;
grid on;

% Draw Unit Circle
theta = linspace(0, 2*pi, 300);
plot(cos(theta), sin(theta), 'b--', 'LineWidth', 1.5);

xlabel("Real Part");
ylabel("Imaginary Part");
title("Eigenvalue Stability Plot (Unit Circle Test)");
legend("Eigenvalues", "Unit Circle");
```

```
% Step 6: Simulate Economic Shock Propagation
% x(t+1) = A*x(t)

T = 20; % number of time steps
% Initial shock in Sector 1
x0 = [1; 0; 0];

% Store results
x = zeros(length(x0), T);
x(:,1) = x0;

% Time evolution
for t = 2:T
    x(:,t) = A * x(:,t-1);
end
% Step 7: Plot Sector Response Over Time

figure;
plot(1:T, x', 'LineWidth', 2);

figure;
plot(1:T, x', 'LineWidth', 2);

xlabel("Time Step");
ylabel("Economic Activity Level");
title("Shock Response of Economic Sectors");
legend("Sector 1", "Sector 2", "Sector 3");
grid on;

% Final Report
disp("FINAL STABILITY REPORT");
disp("Eigenvalues:");
disp(eigvals);

disp("Spectral Radius:");
disp(rho);

if rho < 1
    disp("Conclusion: Stable economy (interactions dampen shocks).");
elseif rho == 1
    disp("Conclusion: Marginal stability (system is fragile).");
else
    disp("Conclusion: Unstable economy (feedback amplifies shocks).");
end
```

Results:

```
  Eigenvalue Based Stability of Economic System
Economic Interaction Matrix A:
    0.4000    0.2000    0.1000
    0.1000    0.5000    0.2000
         0    0.3000    0.6000

Eigenvalues of A:
    0.2513
    0.4155
    0.8332

Spectral Radius (max |lambda|):
    0.8332

System is DISCRETE-TIME STABLE (Shock dies out).
FINAL STABILITY REPORT
Eigenvalues:
    0.2513
    0.4155
    0.8332

Spectral Radius:
    0.8332
```


Eigenvalue Stability Plot (Unit Circle Test)
Eigenvalues
Unit Circle
Imaginary Part
Real Part
1
0.8
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
-0.8
-1
-1
-0.8
-0.6
-0.4
-0.2
0
0.2
0.4
0.6
0.8
1


Shock Response of Economic Sectors
Sector 1
Sector 2
Sector 3
Economic Activity Level
Time Step
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
0
2
4
6
8
10
12
14
16
18
20