FIRST ORDER PREDICATE LOGIC (FOPL)

Predicate logic (First Order Logic)

- It is an extension to propositional logic.
- Predicate Calculus deals with predicates, which are propositions containing variables.
- Predicate logic is a powerful language that develops information about the objects in a more easy way and can also express the relationship between those objects.
- Predicate logic assumes the world contains:
  - **Objects:** Mihir, numbers, house, cat, colors ….
  - **Relations:** has color, bigger than, comes between etc
  - **Facts :** (one value for a given input: is Father of, is best friend, can swim)

  Facts have truth value True or false
- As a natural language, first-order logic also has two main parts:
  - **Syntax**
  - **Semantics**

Predicate logic…

There are three kinds of symbols

- Constants: Objects
- Predicate: relations
- Functions: functions(i.e can return values other than truth and false)

Predicate logic statements can be divided into two parts:

- **Subject:** Subject is the main part of the statement.
- **Predicate:** A predicate can be defined as a relation, which binds two atoms together in a statement.

Subject

Intelligent is attribute, property, characteristic
Also called as predicate

- Sunil was intelligent

Intelligent(Sunil)

Predicate

- **Consider the statement: "x is an integer."**, it consists of two parts, the first part x is the subject of the statement and second part "is an integer," is known as a predicate. **Integer(x)**

Predicate logic ...

- Alka and Neelam are friends

  friends(Alka, Neelam)

We read it as Alka friends Neelam

- Shyam's brother is married to Ram's sister

Predicate are: brother, married, sister → Married() Brother(Shyam) Sister(Ram) → married(brother(Shyam),sister(Ram))

Syntax of First order Logic: Basic elements

Constant	**1, 2, A, Jagdish, Mumbai, cat,....**
Variables	**x, y, z, a, b,....**
Predicates	**Brother, Father, >,....**
Function	**sqrt, LeftLegOf, ...**
Connectives	$\wedge, \vee, \neg, \rightarrow, \Leftrightarrow$
Equality	**==**
Quantifier	$\forall, \exists$

Atomic sentences

- Are formed from a predicate symbol followed by a parenthesis with a sequence of terms.
- We can represent atomic sentences as:

  Predicate (term1, term2, ..., term n).
- **Example:** Ravi and Ajay are brothers:

  Brothers(Ravi, Ajay).

  Mini is a cat: cat (Mini).

Complex Sentences

- Complex sentences are made by combining atomic sentences using connectives.

  $\neg S$, $S_1 \wedge S_2$, $S_1 \vee S_2$, $S_1 \rightarrow S_2$, $S_1 \Leftrightarrow S_2$
- For example:

  "Ram *is a king or Ram is not a king*"

  $king(Ram) \vee \neg king(Ram)$

Quantifiers in Predicate logic

- A quantifier is a language element which generates quantification, and quantification specifies the quantity of specimen in the universe of discourse.
- The variable of predicates is quantified by quantifiers.
- These are the symbols that permit to determine or identify the range and scope of the variable in the logical expression.
- There are two types of quantifier:
  - **Universal Quantifier,** (for all, everyone, everything)
  - **Existential quantifier,** (for some, at least one)

Universal Quantifier

- Universal quantifier is a symbol of logical representation, which specifies that the statement within its range is true for everything or every instance of a particular thing.
- The Universal quantifier is represented by a symbol ∀, which resembles an inverted A.

  ∀ <variable> <sentence>
- In universal quantifier implication "→" is used.
- If x is a variable, then ∀x is read as:
  - **For all x**
  - **For each x**
  - **For every x**
- Example:
  - **All man drink coffee.**
  - **∀x: man(x) → drink (x, coffee).**

  (There are all x where x is a man who drink coffee)

Existential Quantifier

- Existential quantifiers express that the statement within its scope is true for at least one instance of something.
- It is denoted by the logical operator ∃, which resembles as inverted E. When it is used with a predicate variable then it is called as an existential quantifier.
- In Existential quantifier we always use AND or Conjunction symbol (∧).
- If x is a variable, then existential quantifier will be ∃x or ∃(x). And it will be read as:
  - **There exists a 'x.'**
  - **For some 'x.'**
  - **For at least one 'x.'**
- Example:
  - **Some boys are intelligent.**
  - **∃x: boys(x) ∧ intelligent(x)**

  (There are some x where x is a boy who is intelligent)

Points to remember

- The main connective for universal quantifier ∀ is implication →.
- The main connective for existential quantifier ∃ is and ∧.

Properties of Quantifiers:

- In universal quantifier, ∀x∀y is similar to ∀y∀x.
- In Existential quantifier, ∃x∃y is similar to ∃y∃x.
- ∃x ∀y is not similar to ∀y∃x.

∃x ∀y Loves(x,y)

-There is a person who loves everyone in the world

∀y ∃x Loves(y,x)

Everyone in the world is loved by at least one person.

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Examples using quantifier

- All birds fly.
  Here the predicate is "**fly(bird)**."
  And since there are all birds who fly so it will be represented as follows.
  ∀x : bird(x) →fly(x).
- All graduates are unemployed
  ∀x: graduatex(x)→unemployed(x)
- Someone is crying
  ∃ x:crying(x)
- Everyone likes everyone
  ∀x ∀y:Likes(x,y)

Example...

- **Not all students like both Mathematics and Science.**
  In this question, the predicate is "**like(x, y),**" **where x= student, and y= subject**.
  Since there are not all students, so we will use ∀ **with negation, so** following representation for this:

  ¬∀ (x) [student(x) → like(x, Mathematics) ∧ like(x, Science)].

Free and Bound Variables

- The quantifiers interact with variables which appear in a suitable way.
- There are two types of variables in First-order logic:
  - **Free Variable:** A variable is said to be a free variable in a formula if it occurs outside the scope of the quantifier.

Example: **∀x ∃(y)[P (x, y, z)], where z is a free variable.**

- **Bound Variable:** A variable is said to be a bound variable in a formula if it occurs within the scope of the quantifier.

Example: ∀x [A (x)→ B(y)], here x and y are the bound variables.

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Predicate Formulas

- Consider a Predicate P with n variables as $P(x_1, x_2, x_3, \ldots, x_n)$. Here P is n-place predicate and $x_1, x_2, x_3, \ldots, x_n$ are n individuals variables. This n-place predicate is known as atomic formula of predicate calculus. For Example: P(), Q(x, y), R(x,y,z)
- **Well Formed Formula (wff)**

  Well Formed Formula (wff) is a predicate holding any of the following –
  - All propositional constants and propositional variables are wffs
  - If x is a variable and Y is a wff, $\forall x \, Y$ and $\forall x \, Y$ are also wff
  - Truth value and false values are wffs
  - Each atomic formula is a wff
  - All connectives connecting wffs are wffs

Well Formed Formula (wff)…

- If a formula contains a variable *x*, then for the formula to be a wff, all occurrences of *x* must be bounded by the scope of either an *universal* quantifier "∀" or an *existential* quantifier "∃"
- $(\forall x)P(x)$ means that all objects in the domain have attribute *P*, while $(\exists x)P(x)$ means that at least one object in the domain has attribute *P*. Each quantifier has its own *scope*, which is the wff that directly follows it.
- When there are multiple variable in a sentence, the order of quantifiers may matter. For example:
  - "Every key can open every lock."
    $(\forall x)(\forall y)((Key(x) \wedge Lock(y) \rightarrow Open(x, y))$
  - "Every key can open some lock."
    $(\forall x)((Key(x) \rightarrow ((\exists y)\ Lock(y) \wedge Open(x, y)))$
  - "Every key can open the same lock."
    $(\exists y)\ (Lock(y) \wedge ((\forall x)(Key(x) \rightarrow Open(x, y)))$

Operators in predicate logic

- Greater than **(gt)**>
- Less than**(lt)** <
- Less than or Equal **(le)** <=
- Greater than or Equal**(ge)**>=
- Equal**(eq)**=
- Not Equal**(ne)**≠

Note: **( )** used as functions and < ,=,…..are used as operators

Examples of Predicate logic

- The ball color is red

  Color(Ball,Red)
- Rishi likes Mango

  Likes(Rishi,Mango)
- Everybody loves somebody

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  $\forall x \exists y$:loves(x,y)

Example…

- Mihir is a man

  Man(Mihir)
- Mihir is a Mumbaikar

  Mumbaikar(Mihir)
- All Mumbaikars are Indian.

  ∀x: Mumbaikars(x)→ Indian(x)

Here left hand side is information and right hand side is inference

Example…

- Every Mumbaikar likes Mumbai

  ∀x: Mumbaikar(x)→likes(x,Mumbai)
- All red flowers are beautiful

  ∀x: flowers(x)Λred(x)→beautiful(x)
- Everyone is loyal to someone

  ∀x ∃y: loyal(x,y)
- Everyone loves everyone

  ∀x ∀y: loves(x,y)

Example…

- Everyone loves everyone except himself

  $\forall x\ \forall y$: loves(x,y)$\wedge$ ¬loves(x,x)

- All Romans were either loyal to Caesar or hated him.

  $\forall x$:Romans(x)$\rightarrow$loyal(x,caesar) $\vee$ hated(x,ceasar)

Example…

- People only try to assassinate rulers they aren't loyal to.

  $\forall x\ \forall y$:Person(x)$\wedge$ ruler(y) $\wedge$ ¬ loyal(x,y) $\rightarrow$tryassasinate(x,y)

- Right solution is:

  $\forall x\ \forall y$:Person(x)$\wedge$ruler(y)$\wedge$ tryassasinate(x,y) $\rightarrow$ ¬ loyal(x,y)

Example...

- Anyone who is mumbaikar and owns more than one house is a millionaire

  $\forall x$: mumbaikar(x) $\wedge$

  (ownshouse(x)>1)$\rightarrow$millionaire(x)

- Right solution:

  $\forall x$: mumbaikar(x) $\wedge$

  gt(ownshouse(x),1)$\rightarrow$millionaire(x)

Example...

- You can fool all the people some of the time

$\forall x \exists t$:Person(x)$\rightarrow$time(t)$\wedge$canfool(x,t)

- You can fool some of the people all the time

$\exists x \forall t$: Person(x)$\wedge$time(t)$\rightarrow$canfool(x,t)

Example...

- Every city has a Museum which has been visited by every person in the city

$\forall x\ \exists y\ \forall z : city(x) \rightarrow Museum(y) \land person(z) \land lives\ (z,x) \land visits(z,y)$

Example...

- Parents and children are inverse relation

  $\forall\ p \forall c: Parent(p,c) \rightarrow child(c,p)$

- The best score in Maths is always higher than the best score in English

  $\forall x\ \forall y: bestmathscore(x) \land bestenglishscore(y) \rightarrow x>y$