Assignment 2 ch2

① Let $A=\begin{bmatrix}0.5 & 0 & 0\\ 0 & 0.5 & 0\\ 0 & 0 & 0.5\end{bmatrix}$, $u=\begin{bmatrix}1\\ 0\\ -4\end{bmatrix}$ and $v=\begin{bmatrix}a\\ b\\ c\end{bmatrix}$. Define $T:\mathbb{R}^3\to\mathbb{R}^3$ by $T(x)=Ax$. Find $T(u)$ & $T(v)$.

② Let $A=\begin{bmatrix}1 & -5 & -7\\ -3 & 7 & 5\end{bmatrix}$, $b=\begin{bmatrix}-2\\ -2\end{bmatrix}$. Define T by $T(x)=Ax$. find a vector x whose image under T is b.

③ Let $b=\begin{bmatrix}-1\\ 1\\ 0\end{bmatrix}$ and $A=\begin{bmatrix}1 & -4 & 7 & -5\\ 0 & 1 & -4 & 3\\ 2 & -6 & 6 & -4\end{bmatrix}$. Is b in the range of LT $x\to Ax$? Why or why not?

④ Is the transformation $T(x_1,x_2,x_3,x_4)=(0, x_1+x_2, x_2+x_3, x_3+x_4)$ Linear? Also, find the matrix A for this mapping.

⑤ If LT $T:\mathbb{R}^3\to\mathbb{R}^2$ defined by $T(x_1,x_2,x_3)=(x_1-5x_2+4x_3, x_2-6x_3)$ Check whether T is i) one-to-one or ii) onto

⑥ Let $T:\mathbb{R}^5\to\mathbb{R}^5$ be a LT whose standard matrix is given. Describe if T is a onto map. Justify your answer.

$$\begin{bmatrix}4 & -7 & 3 & 7 & 5\\ 6 & -8 & 5 & 12 & -8\\ -7 & 10 & -8 & -9 & 14\\ 3 & -5 & 4 & 2 & -6\\ -5 & 6 & -6 & -7 & 3\end{bmatrix}$$