Assignment 4 CG4

1. What is a determinant? Compute the det(A) using co-factor expansion.

$$A = \begin{bmatrix} 1 & -2 & 5 & 2 \\ 0 & 0 & 3 & 0 \\ 2 & -6 & -7 & 5 \\ 5 & 0 & 4 & 4 \end{bmatrix}$$

(2) Let $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ and k be a scalar. Find a formula that relates det(kA) to k and det(A).

(3) Compute det(A_i) without co-factor expansion.

$$A_1 = \begin{bmatrix} 5 & -7 & 2 & 2 \\ 0 & 3 & 0 & -4 \\ -5 & -8 & 0 & 3 \\ 0 & 5 & 0 & -6 \end{bmatrix}, \quad A_2 = \begin{bmatrix} 1 & 3 & -1 & 0 & -2 \\ 0 & 2 & -4 & -1 & -6 \\ -2 & -6 & 2 & 3 & 9 \\ 3 & 7 & -3 & 8 & -7 \\ 3 & 5 & 5 & 2 & 7 \end{bmatrix}$$

(4) Find the determinants of the following if $\begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix} = 7$

(a) $\begin{bmatrix} a & b & c \\ 3d & 3e & 3f \\ g & h & i \end{bmatrix}$ (b) $\begin{bmatrix} a & b & c \\ 2d+a & 2e+b & 2f+c \\ g & h & i \end{bmatrix}$ (c) $\begin{bmatrix} a & b & c \\ g & h & i \\ d & e & f \end{bmatrix}$

(5) Determine if the set of vectors is linearly independent.

(a) $\begin{bmatrix} 4 \\ 6 \\ -7 \end{bmatrix}, \begin{bmatrix} -7 \\ 0 \\ 2 \end{bmatrix}, \begin{bmatrix} -3 \\ -5 \\ 6 \end{bmatrix}$ (b) $\{(3,5,-6,4), (2,-6,0,7), (-2,-1,3,0), (0,0,0,-3)\}$

(6) Use Cramer's rule to compute the soln.

(a) $2x_1 + x_2 = 7$
$-3x_1 + x_3 = -8$
$x_2 + 2x_3 = -3$

(b) $8sx_1 + 4x_2 = 5$
$7x_1 + 2sx_2 = -2$
(what values of s give the unique soln?)

(7) Find the area of the parallelograms whose vertices are

(a) $(-1,0), (0,5), (1,-4), (2,1)$ (b) $(0,-2), (6,-1), (-3,1), (3,2)$

(8) Find the volume of parallelopiped with one vertex at origin and adjacent vertices are: $(1,0,-2), (1,2,4)$ & $(7,1,4)$