Assignment 3 & 4 — Due: 8 Chaitra, 2080

Ch 3

(1) Show that the line $y = mx + b$ is its own tangent at any point (x_0, y_0) of itself.

(2) Find the equation of the tangent line to the curve at the given point.
i) $y = \sqrt{x}$ at $(1, 1)$ ii) $y = \frac{2x+1}{x+2}$ at $(1, 1)$

(3) Find the velocity at $t = 2$ s, if a ball is thrown into the air with a velocity of 40 ft/sec and its height (in feet) after t seconds is given by $y = 40t - 16t^2$.

(4) Find the average velocities over the intervals $[3.5, 4]$ and $[4, 4.5]$ for a particle moving in a straight line with displacement function as $s(t) = t^2 - 8t + 18$.

(5) Find the first derivatives of the followings:
i) $y = x^{\sqrt{x}}$ ii) $y = \ln\left(\frac{x+1}{x-2}\right)$ iii) $y = \tan^{-1}(\sqrt{x})$
iv) $\sin(x+y) = y^2 \cos x$ v) $xe^y = x - y$ vi) $y = \ln|1 + t - t^3|$

(6) If $z = x^2y + 3xy^4$, where $x = \sin 2t$, and $y = \cos t$, find $\frac{dz}{dt}$ at $t = 0$.

(7) Find the derivative of the function, simplify where possible.
i) $y = \tan^{-1}(x^3)$ ii) $y = x \sin^{-1} x + \sqrt{1 - x^2}$ iii) $y = \arctan\sqrt{\frac{1-x}{1+x}}$

(8) Find the points on the curve $y = 2x^3 + 3x^2 - 12x + 1$ where the tangent is horizontal.

(9) Show that the curve $y = 2e^x + 3x + 5x^3$ has no tangent line with slope 2.

(10) Find an equation of the normal line to the parabola $y = x^2 - 5x + 4$ that is parallel to the line $x - 3y = 5$.

(11) Verify that the following functions satisfy the three hypotheses of Rolle's theorem on the given interval, then find the value of c.
i) $f(x) = 5 - 12x + 3x^2$, $[1, 3]$ ii) $f(x) = 5x - \frac{x}{3}$, $[0, 9]$
iii) $f(x) = \cos 2x$, $[\pi/8, \frac{7\pi}{8}]$

12) Show that the equation $x^3 - 15x + c = 0$ has at most one root in the interval $[-2, 2]$.

Ch4

1) Sketch the graph of $f(x) = \frac{x^2}{\sqrt{x+1}}$ following the guidelines.

2) Sketch the graph of $f(x) = x^3 - 3x + 3$.

3) Find the dimensions of a rectangle with perimeter 100m whose area is as large as possible.

4) Find the point on the curve $y = \sqrt{x}$ that is closest to the point $(1, 0)$

5) Find the area of the largest rectangle that can be inscribed in a semicircle with radius r.

6) Approximate the roots of the following functions correct to four decimal places using N-R method.

i) $\frac{x^3}{3} + \frac{1}{2}x^2 + 3 = 0$, $x_1 = -3$ ii) $x^7 + 4 = 0$, $x_1 = -1$

iii) $\sqrt[100]{100}$ iv) $3\cos x = x + 1$ $(x_1 = -3.7)$.