Assignments

Ch 5 Antiderivative

Submit in one single pdf file to thakur@prcampus.edu.np

filename: <FirstName_lastName.pdf>

Due: Baisakh 02, 2081

(1) Evaluate the upper and lower sums for $f(x) = 2 + \sin x$, $0 \le x \le \pi$, with $n = 2, 4 \,\&\, 8$.

(2) Estimate the area under the graph of $f(x) = \cos x$, $0 \le x \le \pi/2$ using four approximating rectangles and i) right end points, ii) left end points. Sketch the curve & rectangles as well. Give your comments on the differences on these two estimates.

(3) Determine the region whose area is equal to the given limit. Don't evaluate the limit.

a) $\lim\limits_{n\to\infty} \sum\limits_{i=1}^{n} \frac{2}{n}\left(5 + \frac{2}{n}i\right)^{10}$ (b) $\lim\limits_{n\to\infty} \sum\limits_{i=1}^{n} \frac{\pi}{4n} \tan\frac{i\pi}{4n}$

(4) Using max-min inequality estimate the value of the integral $\int_{\pi}^{2\pi} (x - \sin x)\, dx$.

(5) If $f(x) = e^x - 2$, $0 \le x \le 2$, find the Riemann sum with $n = 4$ correct to six decimal places, taking the sample points to be midpoints. What does the Riemann sum represent?

(6) Use the form of the definition of the integral and evaluate them.

a) $\int_1^4 (x^2 - 4x + 2)\, dx$ (b) $\int_0^2 (2x - x^3)\, dx$ (c) $\int_{-2}^{0} (x^2 + x)\, dx$

) Evaluate the integrals

a) $\int \frac{\sin 2x}{\sin x}\, dx$ (b) $\int_{-10}^{10} \frac{2e^x}{\sinh x + \cosh x}\, dx$ c) $\int_0^2 |2x - 1|\, dx$

) Find the area of region bounded by y-axis, $y = \sqrt[4]{x}$ and $y = 1$.

) Integrate the followings:

a) $\int t^2 e^t\, dt$ (b) $\int \sqrt{1 + x^2}\, x^5\, dx$ (c) $\int \sin^{-1} x\, dx$

d) $\int x \tan^{-1} x\, dx$ (e) $\int \sin 8x \cos 5x\, dx$ (f) $\int \frac{dx}{x^2\sqrt{4 - x^2}}$

Evaluate, i) $\int_0^{\infty} \frac{dv}{(1 + v^2)(1 + \tan^{-1} v)}$ ii) $\int_{-\infty}^{\infty} \frac{dx}{1 + x^2}$ iii) $\int_0^4 \frac{dx}{\sqrt{4 - x}}$

iv) $\int_0^3 \frac{dx}{(x-1)^{2/3}}$ v) $\int_0^{\infty} \frac{x \arctan x}{(1 + x^2)^2}\, dx$ (vi) $\int_{-\infty}^{\infty} \frac{x^2}{9 + x^6}\, dx$

) Use Comparison Theorem to determine whether the integral is convergent or divergent.

i) $\int_0^{\infty} \frac{x}{x^3 + 1}\, dx$ ii) $\int_1^{\infty} \frac{e^{-x} + 2}{x}\, dx$ iii) $\int_0^1 \frac{\sec^2 x}{x\sqrt{x}}\, dx$

iv) $\int_0^{\infty} \frac{\tan^{-1} x}{2 + e^x}\, dx$

Ch6 Application of antiderivatives

① Find the area of the region between x-axis and the graph of $f(x) = x^3 - x^2 - 2x$ for $-1 \le x \le 2$

② Find the area of the region enclosed by $y = 2 - x^2$ and the line $y = -x$.

③ Find the area of the region bounded by the curves $y = \sin x$, $y = \cos x$, $x = 0$ and $x = \pi/2$

4) Find the area of the region bounded by $y = e^x$, bounded below $y = x$, and bounded on the sides by $x = 0$ $x = 1$.

⑤ The region enclosed by x-axis and $y = 3x - x^2$ is revolved about the vertical line $x = -1$ to generate a solid. Find the volume of the solid thus formed.

⑥ Find the volume of the solid obtained by rotating about y-axis, the region between $y = x$ and $y = x^2$.

⑦ Use a) Trapezoidal rule b) mid-point rule, and c) Simpson's rule to approximate the given integral with specified n.
i) $\int_0^2 \frac{e^x}{1+x^2}\,dx$, $n = 10$ ii) $\int_1^4 \sqrt{\ln x}\,dx$, $n = 6$ iii) $\int_1^5 \frac{\cos x}{x}\,dx$, $n = 8$

⑧ Find the length of the graph
$$f(x) = \frac{x^3}{12} + \frac{1}{x}, \quad 1 \le x \le 4$$

9) Find the exact of the surface obtained by rotating the curve about the x-axis.
i) $y = x^3$, $0 \le x \le 2$ ii) $y = \sin \pi x$, $0 \le x \le 1$ iii) $x = \frac{1}{3}(y^2+2)^{3/2}$, $1 \le y \le 2$

Ch9 Plane and Space vectors

1) Find a unit vector in the direction of $\vec{u} = \vec{i} + 2\vec{j} - \vec{k}$
i) $(-4, 2, 4)$, (ii) $8\vec{i} - \vec{j} + 4\vec{k}$

2) Find the angle between the vectors:
1) $a = (3, -1, 5)$, $b = (-2, 4, 3)$ ii) $\vec{a} = \vec{i} + 2\vec{j} - 2\vec{k}$, $\vec{b} = 4\vec{i} - 3\vec{k}$

3) Determine whether the given vectors are orthogonal, parallel or neither.
i) $a = (-5, 3, 7)$, $b = (6, -8, 2)$ ii) $\vec{a} = 2\vec{i} + 6\vec{j} - 4\vec{k}$, $\vec{b} = -3\vec{i} - 9\vec{j} + 6\vec{k}$

4) Find the directional cosines and directional angles of the vectors
i) $(2, 1, 2)$ ii) $\vec{i} - 2\vec{j} - 3\vec{k}$ iii) $\frac{1}{2}\vec{i} + \vec{j} + \vec{k}$

⑤ Find the vector projection of b on a and scalar components.
i) $a = (3, 6, -2)$, $b = (1, 2, 3)$ ii) $a = 2\vec{i} - \vec{j} + 4\vec{k}$, $b = \vec{j} + \frac{1}{2}\vec{k}$

(6) Find two unit vectors orthogonal to both $\vec{j}-\vec{k}$ and $\vec{i}+\vec{j}$.

(7) Find a nonzero vector orthogonal to the plane through the points P, Q & R and also find the area of triangle PQR.
i) $P(1,0,1), Q(-2,1,3), R(4,2,5)$ ii) $P(-1,3,1), Q(0,5,2), R(4,3,-1)$

(8) Use the scalar triple product to verify that the vectors $\vec{u}=\vec{i}+5\vec{j}-2\vec{k}$, $\vec{v}=3\vec{i}-\vec{j}$ and $w=5\vec{i}+9\vec{j}-4\vec{k}$ are coplanar.

9) Find the equation of the plane passing through the points $(2,4,5)$, $(1,5,7)$ and $(-1,6,8)$

10) Find the distance from the point $(1,3,2)$ to the line $x=2+2t$, $y=1+6t$, $z=3$

11) Find the distance from the point $(2,1,3)$ to the plane $2x+y+2z=4$

12) Find the parametric equation for the line passing through $(2,4,6)$ that is perpendicular to the plane $x-y+3z=7$.

13) Find the distance between the given parallel planes
i) $2x-3y+z=4$, $4x-6y+2z=3$ ii) $6z=4y-2x$, $9z=1-3x+6y$.

14) Find the parametric equation for the tangent line to the helix with parametric equation, $x=2\cos t$, $y=\sin t$, $z=t$, at $(0,1,\pi/2)$

15) Find the length of one turn of the helix $\vec{r}(t)=\cos t\,\vec{i}+\sin t\,\vec{j}+t\,\vec{k}$.

16) Find the position, velocity and acceleration of a particle in space of the vector valued function $\vec{r}(t)=2\cos t\,\vec{i}+3\sin t\,\vec{j}+4t\,\vec{k}$, $t=\frac{\pi}{2}$

(17) Find $\vec{r}(t)$ if $\vec{r}(t)=2t\,\vec{i}+3t^2\vec{j}+\sqrt{t}\,\vec{k}$ and $\vec{r}(1)=\vec{i}+\vec{j}$.

18) Find the length of the curve
i) $\vec{r}(t)=(t, 3\cos t, 3\sin t)$, $-5\leq t\leq 5$
ii) $\vec{r}(t)=12t\,\vec{i}+8t^{1/2}\vec{j}+3t^2\vec{k}$, $0\leq t\leq 1$.

(18) Find unit tangent vector of $\vec{r}(t)=2\cos t\,\vec{i}+2\sin t\,\vec{j}+\sqrt{5}t\,\vec{k}$

(19) Find the curvature of the twisted cubic $\vec{r}(t)=(t,t^2,t^3)$ at a general point and at $(0,0,0)$.

(20) Show that the curvature of a circle of radius a is $\frac{1}{a}$.