(1) Important questions to be taken care of

Ch 1 Function of one variable (we talk only real valued fn)

$f: \mathbb{R} \to \mathbb{R}$ ← Range, domain

(function definition)

Study carefully Example 1 of the book

Find domain and range of:

i) $y = x^2$ ii) $y = \frac{1}{x}$ iii) $y = \sqrt{4-1}$, iv) $y = \sqrt{x-1}$

See all the solutions in book.

Here are some more from exercise:

Here are some domain and range of

(3) Find domain and range of

i) $h(x) = \sqrt{4 - x^2}$

For domain (all the valid values of x that when plugged into $h(x) = \sqrt{4-x^2}$ should give a real number).

$\Rightarrow (4 - x^2) \geq 0$ ($\because$ if $(4-x^2) < 0$, then the square root of $-ve$ number is not real number)

changing sign, we get

$x^2 - 4 \leq 0$

$\Rightarrow (x-2)(x+2) \leq 0$

$\Rightarrow$ Domain $-2 \leq x \leq 2$

$x - 2 \leq 0$ & $(x+2) \leq 0$

$\Rightarrow x \leq 2$ $\Rightarrow x \leq -2$

i.e. $x \geq 2$

domain

$-2 \; -1 \; 0 \; 2$

To find range, we have

$h(x) = y = \sqrt{4 - x^2}$ (Squaring both sides)

$y^2 = 4 - x^2 \Rightarrow x^2 = 4 - y^2$

$\Rightarrow x = \pm\sqrt{4 - y^2}$, Again, for all values of $(4 - y^2) \geq 0$, we get corresponding real values for x.

$\Rightarrow y^2 - 4 \leq 0 \Rightarrow (y+2)(y-2) \leq 0 \Rightarrow -2 \leq y \leq 2$

(Imp. to check with question) But in our question, the only possible $x \in [-2, 2]$, is positive values of y given because square root of $-ve$ number not allowed $\Rightarrow$ range $= 0 \leq y \leq 2$

(2)

③ (iii) given $g(x) = \frac{2x+1}{x-3}$, the valid domain in this case is $x-3 \neq 0$, this is to avoid division by zero issue!

$\Rightarrow x \neq 3$ not included

So, domain $= (-\infty, 3) \cup (3, \infty)$

For range, let's inverse the function:

$g(x) = y = \frac{2x+1}{x-3} \Rightarrow xy - 3y = 2x+1$

$\Rightarrow xy - 2x = 3y+1$

$x(y-2) = 3y+1$

$\Rightarrow x = \frac{3y+1}{y-2}$

$\Rightarrow y-2 \neq 0$

The possible value of $y = (-\infty, 2) \cup (2, \infty)$

Now, checking back to the question, there is no restriction imposed by the question for this range! $\Rightarrow$ range $= (-\infty, 2) \cup (2, \infty)$

(Let's see, if we say if 2 is also in the range, then, we can say that $y = 2 \Rightarrow 2 = \frac{2x+1}{x-3}$

$\Rightarrow 2x - 6 = 2x+1$, $-6 = 1$, does not make sense.)

(Hence, 2 should be excluded from the range)

$\Rightarrow$ See the Vertical Line test to check for a function. solve the examples from book. like $x^2+y^2=9$

$\Rightarrow$ Also study carefully the examples in the book about Even and odd function: Symmetry

one more example from exercise:

5) (iv) Check for odd or even function

$f(x) = 2|x| + 1$

$\Rightarrow f(-x) = 2|-x| + 1 = 2|x| + 1 = f(x) \Rightarrow$ even f^n

even f^n

v) $f(x) = 3 \Rightarrow g(-x) = 3 = g(x) \Rightarrow$ even f^n

(iii) $h(x) = 1 + x^3 - x^5 \Rightarrow h(-x) = 1 + (-x)^3 - (-x)^5$

$= 1 - x^3 + x^5 \neq h(x)$

$\Rightarrow$ Neither odd nor even | take -1 common $\Rightarrow -1(x^3 - 1 - x^5) \neq -h(x)$

Study carefully. example 8 of the book

$f(x) = \begin{cases} 1-x & \text{if } x \le -1 \\ x^2 & \text{if } x > -1 \end{cases}$ } we discussed this in detail in the class!

Imp

Exercise 1.1 (1) Evaluation:

i) $f(x) = 4 - 3x$, $\frac{f(3+h) - f(3)}{h}$ [Just put $3+h$ and 3 in place of x]

$\Rightarrow \frac{4 - 3(3+h) - (4 - 3\times 3)}{h} = \frac{\cancel{4} - \cancel{9} - 3\cancel{h} - \cancel{4} + \cancel{9}}{\cancel{h}} = -3$

1.2 Linear mathematical model

Study carefully the example 1 in the book (we also discussed this in the class).

Ex. 1.2 (2) we have: 100 chairs cost \$2200

$\Rightarrow (100, 2200)$ one point in x, y coordinate

Imp

Also given in 300 chairs cost \$4800

$\Rightarrow (300, 4800)$. Here the x variable is number of chairs and y is the cost. So, we can build a linear model in the form of $y = mx + b$ with those two data points given.

We know, $m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{4800 - 2200}{300 - 100} = \frac{\cancel{2600}^{13}}{\cancel{200}} = 13$

So, we can take any one point and this slope to ~~make a linear model.~~ find the intercept or b!

$2200 = 13 \times 100 + b \Rightarrow b = 2200 - 1300 = 900$

$(y = mx + b)$

So, our final model is $\boxed{y = 13x + 900}$

$\Rightarrow$ If we want to build one more chair, the cost would go up by \$13, $\frac{dy}{dx} = 13$

The value of b indicates even if we don't produce a single chair ($x = 0$), we incur the cost (fixed cost) of \$900!

(1.3) Combination of function Composite f^n (4)

Study carefully Example (2-5) in the book (Imp) to composite functions and finding their domain.

Exercise 1.3

② (vi) $f(x) = x^2$, $g(x) = 1-\sqrt{x}$

$\Rightarrow f\circ g = f(g(x)) = f(1-\sqrt{x}) = (1-\sqrt{x})^2$

$\left[\because f(x)=x^2 \text{ we plug } (1-\sqrt{x}) \text{ as } x\right]$

$= 1-2\sqrt{x}+x = 1+x-2\sqrt{x} = f\circ g$

Here, domain of $f\circ g$ is that we can't take $x<0$ values as we have $\sqrt{x}$ term in the f^n definition

And hence the domain of $f\circ g$ is $= [0,\infty)$, 0 included

(try yourself to find the range)

$\Rightarrow g\circ g = g(g(x)) = g(1-\sqrt{x}) = (1-\sqrt{1-\sqrt{x}}) = g\circ g.$

Domain is, we can't have $(\sqrt{x}) < 0$

First, need to find domain of $g(x)$ $\Rightarrow (\sqrt{x}) \geq 0 \Rightarrow x \geq 0$

So, domain of $g(x) = [0,\infty] = A$

Then, we need to find the domain of $g(g(x)) = 1-\sqrt{1-\sqrt{x}}$,

Here, we must have $1-\sqrt{x} \geq 0$

$\Rightarrow \sqrt{x} \leq 1 \Rightarrow x \leq 1$

Again, $\sqrt{x}$ can't be negative,

i.e. $x \geq 0 \Rightarrow 0 \leq x \leq 1$, i.e. domain $= [0,1] = B$

$\therefore$ Domain of $g\circ g = A \cap B = [0,\infty] \cap [0,1] = [0,1]$

Note: We need to find the domain of inner f^n first & give it a name say A and then the whole composite f^n, $g(g(x))$ and give it a name, say B. AND the domain of the composite f^n is $A \cap B$.

But, in the case of $f\circ g$ just above, we had inner f^n $g(x)$ whose domain is only restricted by $x \geq 0$ i.e. $A = [0,\infty]$ and for B we had $f\circ g = f(g(x)) = 1+x-2\sqrt{x}$, again the same condition of $x \geq 0$

so $B = [0,\infty] \Rightarrow A\cap B = [0,\infty]$

Continuing with previous question

$g \circ f = g(f(x)) = g(x^2) = 1 - \sqrt{x^2} = 1 - x$

in this case, domain of $g \circ f \supset A \cap B$, inner f^n

A = domain of $f(x) = x^2 = (-\infty, \infty)$

B = domain of $g \circ f = 1 - x = (-\infty, \infty)$

$\Rightarrow \underline{A \cap B = (-\infty, \infty)}$

You can try for $f \circ f$, yourself!

Ch 2 Limit

1) Find the limit: $\lim_{x \to \infty} \frac{\sqrt{2x^2+1}}{3x-5}$, if we directly apply the limit $x \to \infty$ in our f^n, then we get $\frac{\infty}{\infty}$ = Indeterminate

So, to solve this type of problem, we need to bring the variable in the form of $\frac{c}{x}$, c is a constant

We can manipulate the f^n as

$$\lim_{x \to \infty} \frac{\sqrt{\frac{2x^2}{x^2} + \frac{1}{x^2}}}{\frac{3x}{x} - \frac{5}{x}}$$, multiplying both denominator & numerator by $\frac{1}{x}$.

$$\Rightarrow \lim_{x \to \infty} \frac{\sqrt{2 + \frac{1}{x^2}}}{3 - 5/x}$$; now we can apply limit

$$\Rightarrow \frac{\sqrt{2+0}}{3-0} = \sqrt{2}/3.$$ Since $\frac{1}{\infty} = 0$

1) $\lim_{x \to 2} \frac{x^4 - 16}{x^2 - 4}$, here also, we can't apply limit directly as we get $\frac{0}{0}$ if done so!

$$\Rightarrow \lim_{x \to 2} \frac{\cancel{(x^2-4)}(x^2+4)}{\cancel{(x^2-4)}} = \lim_{x \to 2} x^2 + 4 = 4 + 4 = \underline{\underline{8}}$$

We will review limit & continuity in class next week

ch 3 Derivatives

3.1 Tangent (Imp)

Study carefully Examples to find the equation of tangent to $y = x^2$ at the point $P(1,1)$.

(Imp) We discussed this in the class!

Exercise 3.1 Find eq^n of tangent to $y = f(x)$ at $P(x,y)$

1 (d) $y = \frac{2x+1}{x+2}$, $P = (1,1)$ of tangent,

First, we need to find the slope, m, at $x = 1$

$$m = \lim_{x\to1} \frac{f(x)-f(1)}{x-1} = \lim_{x\to1} \frac{1}{x-1}\left[\frac{2x+1}{x+2} - \frac{2+1}{1+2}\right]$$

$$\Rightarrow \lim_{x\to1} \frac{1}{(x-1)}\left[\frac{2x+1}{x+2} - 1\right) = \lim_{x\to1} \frac{2x+1-x-2}{(x+2)(x-1)}$$

$$\Rightarrow \lim_{x\to1} \frac{\cancel{(x-1)}}{\cancel{(x-1)}(x+2)} = \lim_{x\to1} \frac{1}{x+2} = \frac{1}{1+2} = \frac{1}{3}$$

Now, we have m and a given point $P(1,1)$ for a tangent line.

So, eq^n of this tangent line is

$$(y - y_1) = m(x - x_1)$$

$$\Rightarrow y - 1 = \frac{1}{3}(x-1) \Rightarrow 3y - 3 = x - 1$$

$$\Rightarrow \boxed{y = \frac{x+2}{3}}$$

3.3 Derivative as a f^n.

(Imp) See, Example 1 & 2 and try to understand it

Exercise 3.3 (4) $g(t) = \frac{1}{\sqrt{t}}$

We know that $g'(t) = \lim_{h\to0} \frac{g(t+h) - g(t)}{h}$ by definition

$$= \lim_{h\to0} \left(\frac{\frac{1}{\sqrt{t+h}} - \frac{1}{\sqrt{t}}}{h}\right) \Rightarrow \lim_{h\to0} \left(\frac{\sqrt{t} - \sqrt{t+h}}{h\sqrt{t}\,(\sqrt{t+h})}\right) \times \frac{(\sqrt{t} + \sqrt{t+h})}{(\sqrt{t} + \sqrt{t+h})}$$

$$\Rightarrow \lim_{h\to0} \frac{\cancel{t} - \cancel{t} - h}{h\sqrt{t}\,\sqrt{t+h}\,(\sqrt{t} + \sqrt{t+h})}$$

$$\lim_{h\to0} \frac{-1}{\sqrt{t}\sqrt{t+h}\,(\sqrt{t} + \sqrt{t+h})} = \frac{-1}{\sqrt{t}\sqrt{t}\,(\sqrt{t}+\sqrt{t})} = \frac{-1}{2t\sqrt{t}}$$

$$\boxed{g'(t) = -\frac{1}{2t\sqrt{t}}}$$

Imp

$1 + \frac{1}{2} = 3/2$

$1t^{-\frac{1}{2}-1}$

$= -\frac{1}{2}t^{-3/2}$

Since the root f^n is defined only for non-negative value and the rational f^n is defined only for non-zero denominator value, both $g(t)$ and $g(t)$ are defined for only $t \in (0, \infty)$. (Important point to note)

Try to solve other problems in this exercise yourself!

3.4 Review of derivative

Remember all the rules and derivatives of trigonometric & transcendental (log & e^x) functions. Apply these rules and solve for the derivatives of any order for the given problem.

Exercise 3.4 (8) differentiate

(a) $f(x) = 3x^2 - 2\cos x$

$$\Rightarrow \frac{dy}{dx} = \frac{d}{dx}(3x^2 - 2\cos x) = \frac{d(3x^2)}{dx} - \frac{d(2\cos x)}{dx}$$

$$= 6x + \underline{\underline{2\sin x}}$$

(b) $g(\theta) = e^{2\theta}(\tan\theta - \theta)$.

$$\frac{dg(\theta)}{d\theta} = e^{2\theta}\frac{d(\tan\theta - \theta)}{d\theta} + (\tan\theta - \theta)\frac{d(e^{2\theta})}{d\theta}$$

$$= e^{2\theta}(\sec^2\theta - 1) + (\tan\theta - \theta)\frac{de^{2\theta}}{d(2\theta)}\cdot\frac{d(2\theta)}{d\theta}$$

$$= e^{2\theta}(\sec^2\theta - 1) + (\tan\theta - \theta)\, e^{2\theta}.2$$

$$= e^{2\theta}[\sec^2\theta - 1) + 2(\tan\theta - \theta)]$$

and so on ------

Try solving all problems of the exerci

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