Unit 4

Solving System of Linear Eqns

A system of linear equations is a collection of linear equations involving the same set of variables. The values of variables that satisfy each of the linear equations is the solution of the given system of linear equations.

The general form of a system of linear equations is,

$$a_{11}x_1 + a_{12}x_2 + \cdots + a_{1n}x_n = b_1$$
$$a_{21}x_1 + a_{22}x_2 + \cdots + a_{2n}x_n = b_2$$
$$\cdots\cdots\cdots\cdots$$
$$a_{n1}x_1 + a_{n2}x_2 + \cdots + a_{nn}x_n = b_n$$

The system of linear equations can be expressed in matrix form as $AX = B$.

$$\underbrace{\begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \cdots & \cdots & \cdots & \cdots \\ a_{n1} & a_{n2} & \cdots & a_{nn} \end{bmatrix}}_{A} \underbrace{\begin{bmatrix} x_1 \\ x_2 \\ \cdots \\ x_n \end{bmatrix}}_{X} = \underbrace{\begin{bmatrix} b_1 \\ b_2 \\ \cdots \\ b_n \end{bmatrix}}_{B}$$

Where,

A is the coefficient matrix.
X ~~B~~ is the column vector.
B is the column matrix of constants.

The coefficient matrix combined with constant matrix is called augmented matrix.

$$\begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} & : & b_1 \\ a_{21} & a_{22} & \cdots & a_{2n} & : & b_2 \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ a_{n1} & a_{n2} & \cdots & a_{nn} & : & b_n \end{bmatrix}$$ is an augmented matrix.

A system of linear equations is said to be homogeneous if the constant matrix $B = 0$.

Note:-

A system of linear equations is said to be well conditioned if small change in the value of ~~variables~~ coefficients leads to minute change in other values. Otherwise it is called ill conditioned.

Eg: $\begin{bmatrix} 1 & 2 \\ 2 & 3 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 4 \\ 7 \end{bmatrix}$

Here, $x = 2$, $y = 1$

If we make small change in coefficients,

$$\begin{bmatrix} 1.001 & 2.001 \\ 2.001 & 3.001 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 4 \\ 7 \end{bmatrix}$$

Then, $x = 2.003$, $y = 0.997$

Since the effect of change is minute, we can say given system of linear eqn is well conditioned.

Types of Solution

The method of solving system of linear equations has two fundamental approaches. They are:

a) Elimination/Direct Method.
b) Iterative/Indirect Method.

In elimination method, the given system of linear equations is reduced to a form from which solution can be easily obtained. Methods like Gauss-Elimination method, Gauss-Jordan method, Cholesky's method etc fall under this category. This method give exact solution.

In iterative method, the initial assumptions are made for the unknowns and the values are refined in successive iterations, untill they reach some level of accuracy. The Jacobi's method, Gauss-Siedal method etc fall under this category. This method give approximate solution.

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Well conditioned & Ill conditioned system of linear equations :-

A system of linear eqⁿˢ are said to be well conditioned if small change in the value of any variable leads to minute change in other values, if small change in value of a variable lead to large change in other, then the system of linear equations is said to be ill conditioned system.

consider the system of linear equations,

Eg: $\begin{bmatrix} 1 & 2 \\ 2 & 3 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 4 \\ 7 \end{bmatrix}$

Here,

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$$

If we make small change in coefficients,

$$\begin{bmatrix} 1.001 & 2.001 \\ 2.001 & 3.001 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 4 \\ 7 \end{bmatrix}$$

then

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 2.003 \\ 0.997 \end{bmatrix}$$

Here, the system is well conditioned.

Gauss Elimination Method

In this method, the unknowns are eliminated successively and the system of equations is reduced to an upper triangular or echelon form, from which the solution can be easily obtained by backward substitution method.

Step-1 : Represent the system of linear eqns in matrix form.

Step-2 : Obtain the augmented matrix.

Step-3 : Transform the augmented matrix into upper triangular form or echelon form.

Note:

$$\left[\begin{array}{ccc:c} a_1' & b_1' & c_1' & d_1 \\ 0 & b_2' & c_2' & d_2 \\ 0 & 0 & c_3' & d_3 \end{array}\right] \Rightarrow \text{upper triangular form}$$

$$\left[\begin{array}{ccc:c} 1 & b_1' & c_1' & d_1' \\ 0 & 1 & c_2' & d_2' \\ 0 & 0 & 1 & d_3' \end{array}\right] \Rightarrow \text{Echelon form}$$

Step-4: Find the equations Corresponding to the upper triangular form (or echelon form)

Step-5: Use backward substitution to find the desired unknowns.

Q. Solve the given system of linear equations using Gauss-Elimination method.

$$2x_1 + x_2 + x_3 = 5$$
$$4x_1 - 6x_2 = -2$$
$$-2x_1 + 7x_2 + 2x_3 = 9$$

→ Writing in matrix form,

$$\begin{bmatrix} 2 & 1 & 1 \\ 4 & -6 & 0 \\ -2 & 7 & 2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 5 \\ -2 \\ 9 \end{bmatrix}$$

The augmented matrix is,

$$\left[\begin{array}{ccc:c} 2 & 1 & 1 & 5 \\ 4 & -6 & 0 & -2 \\ -2 & 7 & 2 & 9 \end{array}\right]$$

$R_1 \leftarrow R_1/2$, $R_2 \leftarrow R_2 + 2R_3$

$$\left[\begin{array}{ccc:c} 1 & 1/2 & 1/2 & 5/2 \\ 0 & 8 & 4 & 16 \\ -2 & 7 & 2 & 9 \end{array}\right]$$

$R_3 \leftarrow R_3 + 2R_1$

$$\left[\begin{array}{ccc:c} 1 & 1/2 & 1/2 & 5/2 \\ 0 & 8 & 4 & 16 \\ 0 & 8 & 3 & 14 \end{array}\right]$$

$R_2 \leftarrow R_2 - R_3$

$$\left[\begin{array}{ccc:c} 1 & 1/2 & 1/2 & 5/2 \\ 0 & 0 & 1 & 2 \\ 0 & 8 & 3 & 14 \end{array}\right]$$

$R_2 \leftrightarrow R_3$

$$\left[\begin{array}{ccc:c} 1 & 1/2 & 1/2 & 5/2 \\ 0 & 8 & 3 & 14 \\ 0 & 0 & 1 & 2 \end{array}\right]$$

$R_2 \leftarrow R_2/8$

$$\left[\begin{array}{ccc:c} 1 & 1/2 & 1/2 & 5/2 \\ 0 & 1 & 3/8 & 14/8 \\ 0 & 0 & 1 & 2 \end{array}\right]$$

The eqns corresponding to above echelon form are,

$$x_1 + x_2/2 + x_3/2 = 5/2$$
$$x_2 + 3x_3/8 = 14/8$$
$$x_3 = 2$$

By backward substitution, we get

$$x_3 = 2$$
$$x_2 = 1$$
$$x_1 = 1.$$

Gauss-Elimination Method with Partial Pivoting :-

In native Gauss elimination method, we select a random row to perform any row operation. This may take multiple arithmetic calculations and hence multiple round-off error may occur, which leads to incorrect result. To avoide this problem, we can use partial pivote to solve the problem.

A row selected to eliminate a variable from another row is called pivot row. In the Gauss-elimination with pivot, we use the pivot to eliminate a variable from another row. This method differs from the native method of Gauss-elimination only in choosing the pivot. It is easier for programming as well.

Steps

1. Find an element from the leftmost column of coefficient matrix having largest absolute value. This gives the first pivot.

2. Make sure that pivot is in first row. If not, perform row interchange to make pivot in the first row.

3. Perform necessary row operations to convert the augmented matrix to upper triangular or row echelon form.
4. After the first pass, ignore the first row & first column and repeate all the steps for the remaining submatrix.

Q. Solve the given system of linear equations using Gauss-elimination method with partial pivoting.

$$2x_1 + 2x_2 + x_3 = 6$$
$$4x_1 + 2x_2 + 3x_3 = 4$$
$$x_1 + x_2 + x_3 = 0$$

→ Writing in matrix form,

$$\begin{bmatrix} 2 & 2 & 1 \\ 4 & 2 & 3 \\ 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 6 \\ 4 \\ 0 \end{bmatrix}$$

The augmented matrix is

$$\begin{bmatrix} 2 & 2 & 1 & : & 6 \\ 4 & 2 & 3 & : & 4 \\ 1 & 1 & 1 & : & 0 \end{bmatrix}$$

In first column, largest element with highest absolute value is 4. This is first pivot.

Interchanging R_1 & R_2,

$R_1 \leftrightarrow R_2$

$$\left[\begin{array}{ccccc} 4 & 2 & 3 & : & 4 \\ 2 & 2 & 1 & : & 6 \\ 1 & 1 & 1 & : & 0 \end{array}\right]$$

$R_1 \leftarrow R_1/4$

$$\left[\begin{array}{ccccc} 1 & 1/2 & 3/4 & : & 1 \\ 2 & 2 & 1 & : & 6 \\ 1 & 1 & 1 & : & 0 \end{array}\right]$$

$R_2 \leftarrow R_2 - 2R_1$

$R_3 \leftarrow R_3 - R_1$

$$\left[\begin{array}{ccccc} 1 & 1/2 & 3/4 & : & 1 \\ 0 & 1 & -1/2 & : & 4 \\ 0 & 1/2 & 1/4 & : & -1 \end{array}\right]$$ $\because$ 1 is pivot.

$R_3 \leftarrow R_3 - 1/2 R_2$

$$\left[\begin{array}{ccccc} 1 & 1/2 & 3/4 & : & 1 \\ 0 & 1 & -1/2 & : & 4 \\ 0 & 0 & 1/2 & : & -3 \end{array}\right]$$

$R_3 \leftarrow 2R_3$

$$\left[\begin{array}{ccccc} 1 & 1/2 & 3/4 & : & 1 \\ 0 & 1 & -1/2 & : & 4 \\ 0 & 0 & 1 & : & -6 \end{array}\right]$$

Here, by backward substitution, we get

$x_3 = -6$

$x_2 = 1$

$x_1 = 1$

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Q. Using Gauss-elimination with partial pivoting, solve the given system of linear equation.

$$x_1 - x_2 + 3x_3 = 13$$
$$4x_1 - 2x_2 + x_3 = 15$$
$$-3x_1 - x_2 + 4x_3 = 8$$

Q. Solve the following system of linear equations using Gauss-elimination method. Use partial pivoting if necessary.

$$2x_2 + x_4 = 0$$
$$2x_1 + 2x_2 + 3x_3 + 2x_4 = -2$$
$$4x_1 - 3x_2 + x_4 = -7$$
$$6x_1 + x_2 - 6x_3 - 5x_4 = 6$$

Q. What is pivoting? Why it is necessary? Solve the given system of linear equations using Gauss-elimination or Gauss-Seidal method.

$$x_1 + 10x_2 + x_3 = 24$$
$$10x_1 + x_2 + x_3 = 15$$
$$x_1 + x_2 + 10x_3 = 33$$

Gauss-Jordan Method:

It is another varient of Gauss-elimination method. It is different from the Gauss-elimination method in the following points.

- # In this method, we convert the coefficient matrix into diagonal matrix.
- # Backward substitution is not needed. Final solution is obtained directly from the augmented matrix.
- # This method needs nearly double row operation than the Gauss-elimination method.

Steps:

1. Obtaine the augmented matrix from the given system of linear equations.

2. Perform the suitable row operations such that, the coefficient in 1st row & 1st column become 1 & remaining elements in the column become 0.

3. Perform the suitable row operations such that the element in 2nd row and 2nd column become 1 & remaining elements in the column become 0.

4. Repeat the process for all the columns. For n^{th} column, element in n^{th} row & n^{th} column becomes 1 and remaining elements in the column become 0.

5. Values of unknowns can be directly obtained from the final normalized diagonal matrix.

Q. Solve the following system of linear equations using Gauss-Jordan method.

$$6x - y + z = 13$$
$$x + y + z = 9$$
$$10x + y - z = 19$$

Ans: $x = 2$
$y = 3$
$z = 4$

→ Writing in matrix form,

$$\begin{bmatrix} 6 & -1 & 1 \\ 1 & 1 & 1 \\ 10 & 1 & -1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 13 \\ 9 \\ 19 \end{bmatrix}$$

The augmented matrix is

$$\left[\begin{array}{ccc:c} 6 & -1 & 1 & 13 \\ 1 & 1 & 1 & 9 \\ 10 & 1 & -1 & 19 \end{array}\right]$$

$R_1 \leftrightarrow R_2$

$$\left[\begin{array}{ccc:c} 1 & 1 & 1 & 9 \\ 6 & -1 & 1 & 13 \\ 10 & 1 & -1 & 19 \end{array}\right]$$

$R_2 \leftarrow R_2 - 6R_1$, $R_3 \leftarrow R_3 - 10R_1$

$$\left[\begin{array}{ccc:c} 1 & 1 & 1 & 9 \\ 0 & -7 & -5 & -41 \\ 0 & -9 & -11 & -71 \end{array}\right]$$

$R_2 \leftarrow R_2 - R_3$

$$\left[\begin{array}{ccc:c} 1 & 1 & 1 & 9 \\ 0 & 2 & 6 & 30 \\ 0 & -9 & -11 & -71 \end{array}\right]$$

$R_2 \leftarrow R_2/2$

$$\left[\begin{array}{ccc:c} 1 & 1 & 1 & 9 \\ 0 & 1 & 3 & 15 \\ 0 & -9 & -11 & -71 \end{array}\right]$$

$R_3 \leftarrow R_3 + 9R_2$

$$\left[\begin{array}{ccc:c} 1 & 1 & 1 & 9 \\ 0 & 1 & 3 & 15 \\ 0 & 0 & 16 & 64 \end{array}\right]$$

$R_1 \leftarrow R_1 - R_2, \quad R_3 \leftarrow R_3/16$

$$\left[\begin{array}{ccc:c} 1 & 0 & -2 & -6 \\ 0 & 1 & 3 & 15 \\ 0 & 0 & 1 & 4 \end{array}\right]$$

$R_1 \leftarrow R_1 + 2R_3, \quad R_2 \leftarrow R_2 - 3R_3$

$$\left[\begin{array}{ccc:c} 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & 3 \\ 0 & 0 & 1 & 4 \end{array}\right]$$

$\therefore \quad x = 2$

$y = 3$

$z = 4$

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Inverse of matrix using Gauss-Jordan Method

The inverse of a matrix A is denoted by A^{-1} and the product AA^{-1} gives identity matrix. Only the square matrix can have inverse.

$$AA^{-1} = I \quad (I \text{ is identity matrix of order same as } A)$$

Steps

1. Augment the coefficient matrix with the identity matrix of same order.

$$\left[\begin{array}{ccc:ccc} a_{11} & a_{12} & a_{13} & 1 & 0 & 1 \\ a_{21} & a_{22} & a_{23} & 0 & 1 & 0 \\ a_{31} & a_{32} & a_{33} & 0 & 0 & 1 \end{array}\right]$$

2. Apply Gauss-Jordan method & transform the given matrix into identity matrix.

$$\left[\begin{array}{ccc:ccc} 1 & 0 & 0 & a_{11}' & a_{12}' & a_{13}' \\ 0 & 1 & 0 & a_{21}' & a_{22}' & a_{23}' \\ 0 & 0 & 1 & a_{31}' & a_{32}' & a_{33}' \end{array}\right]$$

3. The augmented matrix reduces to inverse matrix.

Q. Find the inverse of given matrix using Gauss-Jordan method.

$$\begin{bmatrix} 1 & 3 & 0 \\ -2 & 3 & 1 \\ 0 & 1 & 1 \end{bmatrix}$$

→ Augmenting the given matrix with identity matrix,

$$\left[\begin{array}{ccc:ccc} 1 & 3 & 0 & 1 & 0 & 0 \\ -2 & 3 & 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 & 0 & 1 \end{array}\right]$$

$R_2 \leftarrow R_2 + 2R_1$

$$\left[\begin{array}{ccc:ccc} 1 & 3 & 0 & 1 & 0 & 0 \\ 0 & 9 & 1 & 2 & 1 & 0 \\ 0 & 1 & 1 & 0 & 0 & 1 \end{array}\right]$$

$R_2 \leftarrow R_2 - R_3$

$$\left[\begin{array}{ccc:ccc} 1 & 3 & 0 & 1 & 0 & 0 \\ 0 & 8 & 0 & 2 & 1 & -1 \\ 0 & 1 & 1 & 0 & 0 & 1 \end{array}\right]$$

$R_2 \leftarrow R_2/8$

$$\left[\begin{array}{ccc:ccc} 1 & 3 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1/4 & 1/8 & -1/8 \\ 0 & 1 & 1 & 0 & 0 & 1 \end{array}\right]$$

$R_1 \leftarrow R_1 - 3R_2$, $R_3 \leftarrow R_3 - R_2$

$$\left[\begin{array}{ccc:ccc} 1 & 0 & 0 & 1/4 & -3/8 & 3/8 \\ 0 & 1 & 0 & 1/4 & 1/8 & -1/8 \\ 0 & 0 & 1 & -1/4 & -1/8 & 9/8 \end{array}\right]$$

$\therefore$ The inverse of given matrix is

$$\begin{bmatrix} 1/4 & -3/8 & 3/8 \\ 1/4 & 1/8 & -1/8 \\ -1/4 & -1/8 & 9/8 \end{bmatrix}$$ Ans

Matrix Factorization To solve the system of lin. eq's

This approach is based on the fact that any square matrix(A) can be expressed as a product of a lower triangular matrix (L) and an upper triangular matrix (U) provided that all the minors of A are non-zero.

ie. $a_{11} \neq 0$, $\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix} \neq 0$, $\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} \neq 0$

We can use the concept of matrix factorization to solve the system of linear equations. There are different methods like Doolittle method, Cholesky's method etc. which are based on matrix factorization. Such methods which are based on matrix factorization are also known as LU-factorization method or Triangular method or LU-Decomposition method.

Assume a system of linear equations with 3 unknowns
We can represent the system of linear equations in matrix form as $AX = B$ ——— (i)

where $A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$ is a coefficient matrix.

$X = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$ is a vector of unknowns &

$B = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}$ is a vector of constants.

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We can represent A as a product lower triangular matrix L and upper triangular matrix U as;
$A = LU$ where

$$L = \begin{bmatrix} l_{11} & 0 & 0 \\ l_{21} & l_{22} & 0 \\ l_{31} & l_{32} & l_{33} \end{bmatrix} \quad \& \quad U = \begin{bmatrix} u_{11} & u_{12} & u_{13} \\ 0 & u_{22} & u_{23} \\ 0 & 0 & u_{33} \end{bmatrix}$$

Here
$LU = A$ can be used to determine L & U matrices. ~~and A LUX = B and~~

Eqn (i) can be written as

$$LUX = B$$

or, $LC = B$ where $C = UX$

Now, we can determine C using $LC = B$ and finally X can be obtained using $UX = C$.

Doolittle Method

It is a LU-Decomposition method for solving the system of linear equations, where the coefficient matrix can be expressed in the form of product of lower triangular matrix L & upper triangular matrix U and the diagonal elements of L are unit element (1).

Working Procedure:-

1. Represent the system of linear equations in matrix form as $AX = B$.

2. Assume $A = LU$ where L is lower triangular matrix with unit diagonal and U is the upper triangular matrix.

3. Solve $LU = A$ to determine L and U.

4. Consider $AX = B$

or, $LUX = B$

or, $LC = B$ where $UX = C$ assumed.

& $C = \begin{bmatrix} c_1 \\ c_2 \\ c_3 \end{bmatrix}$

5. Solve $LC = B$ to find C.

6. Solve $UX = C$ to find X (the final solution). by using backward substitution.

Q. Solve
$$3x_1 + 2x_2 + x_3 = 24$$
$$2x_1 + 3x_2 + 2x_3 = 14$$
$$x_1 + 2x_2 + 3x_3 = 14$$
using Doolittle LU decomposition method.

→ Representing the system of eqns using matrix form $AX = B$,

$$A = \begin{bmatrix} 3 & 2 & 1 \\ 2 & 3 & 2 \\ 1 & 2 & 3 \end{bmatrix}, \quad X = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}, \quad B = \begin{bmatrix} 24 \\ 14 \\ 14 \end{bmatrix}$$

Let A ~~can the~~ be expressed as the product of L & U where L is a lower triangular matrix having unit diagonal & U be an upper triangular matrix.

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ie. $L = \begin{bmatrix} 1 & 0 & 0 \\ l_{21} & 1 & 0 \\ l_{31} & l_{32} & 1 \end{bmatrix}$ & $U = \begin{bmatrix} u_{11} & u_{12} & u_{13} \\ 0 & u_{22} & u_{23} \\ 0 & 0 & u_{33} \end{bmatrix}$

Here,

$$LU = A$$

or, $\begin{bmatrix} 1 & 0 & 0 \\ l_{21} & 1 & 0 \\ l_{31} & l_{32} & 1 \end{bmatrix} \begin{bmatrix} u_{11} & u_{12} & u_{13} \\ 0 & u_{22} & u_{23} \\ 0 & 0 & u_{33} \end{bmatrix} = \begin{bmatrix} 3 & 2 & 1 \\ 2 & 3 & 2 \\ 1 & 2 & 3 \end{bmatrix}$

Now

or $u_{11} = 3$

$u_{12} = 2$

$u_{13} = 1$

$l_{21} \cdot u_{11} = 2 \Rightarrow l_{21} = 2/3$

$l_{21} \cdot u_{12} + 1 . u_{22} = 3 \Rightarrow u_{22} = 4/3$

$l_{21} \cdot u_{13} + u_{23} + 0 = 2 \Rightarrow u_{23} = 4/3$

$l_{31} \cdot u_{11} = 1 \Rightarrow l_{31} = 1/3$

$l_{31} \cdot u_{12} + l_{32} \cdot u_{22} = 2 \Rightarrow l_{32} = 1$

$l_{31} \cdot u_{13} + l_{32} \cdot u_{23} + u_{33} = 3 \Rightarrow u_{33} = 4/3$

$\therefore L = \begin{bmatrix} 1 & 0 & 0 \\ 2/3 & 1 & 0 \\ 1/3 & 1 & 1 \end{bmatrix}$ $U = \begin{bmatrix} 3 & 2 & 1 \\ 0 & 4/3 & 4/3 \\ 0 & 0 & 4/3 \end{bmatrix}$

Here $AX = B$

or, $LUX = B$

or, $LC = B$ where $UX = C$. & $C = \begin{bmatrix} C_1 \\ C_2 \\ C_3 \end{bmatrix}$ assume.

Taking $LC = B$

or, $$\begin{bmatrix} 1 & 0 & 0 \\ 2/3 & 1 & 0 \\ 1/3 & 1 & 1 \end{bmatrix} \begin{bmatrix} c_1 \\ c_2 \\ c_3 \end{bmatrix} = \begin{bmatrix} 24 \\ 14 \\ 14 \end{bmatrix}$$

Now,

$$c_1 = 24$$
$$\tfrac{2}{3}c_1 + c_2 = 14 \Rightarrow c_2 = -2$$
$$\tfrac{1}{3}c_1 + c_2 + c_3 = 14 \Rightarrow c_3 = 8$$

$$\therefore C = \begin{bmatrix} 24 \\ -2 \\ 8 \end{bmatrix}$$

Now, using $UX = C$

$$\begin{bmatrix} 3 & 2 & 1 \\ 0 & 4/3 & 4/3 \\ 0 & 0 & 4/3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 24 \\ -2 \\ 8 \end{bmatrix}$$

or, $$3x_1 + 2x_2 + x_3 = 24$$
$$\tfrac{4}{3}x_2 + \tfrac{4}{3}x_3 = -2$$
$$\tfrac{4}{3}x_3 = 8$$

By backward substitution method,

$$x_3 = 6$$
$$x_2 = -15/2 \quad -6$$
$$x_1 = 11 \quad 10$$

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Cholesky's Algorithm

It is a matrix ~~LU~~ Decomposition method for solving the system of linear equations, where the coefficient matrix A can be expressed in the form of product of lower triangular matrix L and it's transpose L^T. It is different from Doolittle method in the following points.

- It uses decomposition of coefficient matrix A in the form of product of lower triangular matrix & its transpose.
- The diagonal elements of lower triangular matrix are not necessarily be unit element (1).

Note: This method is applicable only if the coefficient matrix A is symmetric i.e. $A = A^T$ (transpose of A).

Working Procedure:

1. Represent the system of linear equations in matrix form as $AX = B$.

2. Check if A is symmetric or not. If A is not symmetric, we cannot use the Cholesky method.

3. Assume $A = LL^T$ where L is lower triangular matrix and L^T is transpose of L.

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4. Solve $LL^T = A$ to determine L & L^T.

5. Consider $AX = B$

or, $LL^T X = B$

or, $LC = B$ where $C = L^T X = \begin{bmatrix} c_1 \\ c_2 \\ \vdots \end{bmatrix}$

6. Solve $LC = B$ to determine C.

7. Solve $L^T X = C$ to determine X by using backward substitution.

Q. Solve the following system of linear equations by using cholesky's method.

$$x + 2y + 3z = 5$$
$$2x + 8y + 22z = 6$$
$$3x + 22y + 82z = -10$$

Ans: $x = 2$ $y = 3$ $z = -1$

Jacobi Method

This is an iterative method to solve the system of linear equations. Unlike direct methods such as Gaussian elimination method & others, in this method, we get the approximate solution but not the exact solution. In this method, an initial guess is made and follow the iterative procedure to converge the guess into approximately exact solution.

This method is applicable to solve the system of linear equations for which the coefficient matrix is strictly diagonal. ~~A matrix is said to be strictly diagonal if for each row, the diagonal element is largest in magnitude & follow the~~
It is found that the Jacobi method has no guarantee of convergence if the coefficient matrix is not strictly diagonal.

Working procedure:

1. Represent the system of linear equations in matrix form $(AX = B)$.

$$\underbrace{\begin{bmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{bmatrix}}_{A} \underbrace{\begin{bmatrix} x \\ y \\ z \end{bmatrix}}_{X} = \underbrace{\begin{bmatrix} d_1 \\ d_2 \\ d_3 \end{bmatrix}}_{B}$$

$$a_1x + b_1y + c_1z = d_1$$
$$a_2x + b_2y + c_2z = d_2$$
$$a_3x + b_3y + c_3z = d_3$$

where, A = Coefficient matrix
X = Column vector of unknowns
B = Column vector of constants

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2. Check whether the coefficient matrix A is strictly diagonal or not. A is said to be strictly diagonal if,

$$|a_1| > |b_1| + |c_1|$$
$$|b_2| > |a_2| + |c_2|$$
$$|c_3| > |a_3| + |b_3|$$

If these conditions are not satisfied, directly, we can rearrange/reorder the equations and check again. If the coefficient matrix A is not strictly diagonal, then there is no guarantee of solution.

3. Derive the expressions to solve for x, y & z (the unknowns).

$$x = \frac{1}{a_1}(d_1 - b_1 y - c_1 z) \quad \text{from first row of A.}$$

$$y = \frac{1}{b_2}(d_2 - a_2 x - c_2 z) \quad \text{from second row of A.}$$

$$z = \frac{1}{c_3}(d_3 - a_3 x - b_3 y) \quad \text{from 3rd row of A.}$$

4. Make an initial guess ~~for~~ x_0, y_0 & z_0 (usually 0, 0, 0).

5. Use the initial guess to find the first approximations x_1, y_1 & z_1.

6. Use the previous approximations to find the next approximations.

7. Repeat step 6 untill the ~~desired~~ new approximation is obtained upto the desired precision. (or there is neglisible difference between two consecutive approximations.

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Q. Solve the following system of linear equations using Jacobi-Iteration method.

$$6x_1 - 2x_2 + x_3 = 11 \quad \text{——} \quad (1)$$

$$-2x_1 + 7x_2 + 2x_3 = 5 \quad \text{——} \quad (2)$$

$$x_1 + 2x_2 - 5x_3 = -1 \quad \text{——} \quad (3)$$

→ Representing the system of eqns in matrix form

$$\begin{bmatrix} 6 & -2 & 1 \\ -2 & 7 & 2 \\ 1 & 2 & -5 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 11 \\ 5 \\ -1 \end{bmatrix}$$

Here the coefficient matrix is strictly diagonal, Hence we can apply Jacobi method.

from first equation

$$x_1 = \frac{1}{6}(11 + 2x_2 - x_3) \quad \text{——} \quad (4)$$

from second equation

$$x_2 = \frac{1}{7}(5 + 2x_1 - 2x_3) \quad \text{——} \quad (5)$$

from third equation

$$x_3 = \frac{1}{5}(1 + x_1 + 2x_2) \quad \text{——} \quad (6)$$

let the Initial guess be $x_1 = x_2 = x_3 = 0$.

first approximations are,

$$(x_1)_1 = \frac{1}{6}(11 + 0 + 0) = 1.833$$

$$(x_2)_1 = \frac{1}{7}(5 + 0 - 0) = 0.714$$

$$(x_3)_1 = \frac{1}{5}(1 + 0 + 0) = 0.200$$

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Second approximations (iteration-2)

$(x_1)_2 = \frac{1}{6}(11 + 2\times 0.714 - 0.2) = 2.038$

$(x_2)_2 = \frac{1}{7}(5 + 2\times 1.833 - 2\times 0.2) = 1.181$

$(x_3)_2 = \frac{1}{5}(1 + 1.833 + 2\times 0.714) = 0.852$

Tabulating the approximations of successive iterations

Iteration	x_1	x_2	x_3
1.	1.833	0.714	0.200
2.	2.038	1.181	0.852
3.	2.085	1.053	1.080
4.	2.004	1.001	1.038
~~5.~~	~~1.994~~		

$\therefore x_1 = 2, \; x_2 = 1 \;\&\; x_3 = 1$

(Program & Algorithm in Lab)

CSIT-70

Q. Solve the following system of eqns using Gaussian elimination or Gauss-Seidel method.

$x_1 + 10x_2 + x_3 = 24$

$10x_1 + x_2 + x_3 = 15$

$x_1 + x_2 + 10x_3 = 33$

Q. $10x - 5y - 2z = 3$

$4x - 10y + 3z = -3$

$x + 6y + 10z = -3$

strictly diagonally dominant

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Gauss-Seidel Method

It is an iterative method to solve a system of linear equations. It is very similar to Gauss-Jacobi method. In fact it is a modification of Gauss-Jacobi method in which most recent values (or approximations) are used in the successive iterations. This method is also called as the method of successive displacement.

Like Jacobi method, this method is applicable to solve the system of linear equations for which the coefficient matrix is strictly diagonal, otherwise the method has no guarantee of convergence.

Working principle:

1. Represent the system of linear equations in matrix form. $(AX = B)$

$$\underbrace{\begin{bmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{bmatrix}}_{A} \underbrace{\begin{bmatrix} x \\ y \\ z \end{bmatrix}}_{X} = \underbrace{\begin{bmatrix} d_1 \\ d_2 \\ d_3 \end{bmatrix}}_{B}$$

$$a_1x + b_1y + c_1z = d_1$$
$$a_2x + b_2y + c_2z = d_2$$
$$a_3x + b_3y + c_3z = d_3$$

where $A =$

$B =$

$X =$

2. Check whether the coefficient matrix (A) is strictly diagonal or not. A is said to be strictly diagonal if

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If these conditions aren't directly satisfied we may reorder the eqns & check again.
If A is not strictly diagonal, there is no guarantee of solution.

3. Derive the expressions to solve for unknowns as

$$x_n = \frac{1}{a_1}(d_1 - b_1 y_{n-1} - c_1 z_{n-1}) \text{ from first row of A.}$$

$$y_n = \frac{1}{b_2}(d_2 - a_2 x_{n-1} - c_2 z_{n-1}) \text{ from } 2^{nd} \text{ eqn}$$

$$z_n = \frac{1}{c_3}(d_3 - a_3 x_{n-1} - b_3 y_{n-1}) \text{ from } 3^{rd} \text{ eqn.}$$

4. Make an initial guess ~~for~~ y_0 & z_0 (usually $y_0 = z_0 = 0$)

5. Use initial guesses to find first approximations as

$$x_1 = \frac{1}{a_1}(d_1)$$

$$y_1 = \frac{1}{b_2}(d_2 - a_2 x_1 - 0)$$

$$z_1 = \frac{1}{c_3}(d_3 - a_3 x_1 - b_3 y_1)$$

6. Again find the next approximations as

$$x_2 = \frac{1}{a_1}(d_1 - b_1 y_1 - c_1 z_1)$$

$$y_2 = \frac{1}{b_2}(d_2 - a_2 x_2 - c_2 z_1)$$

$$z_2 = \frac{1}{c_3}(d_3 - a_3 x_2 - b_3 y_2)$$

7. Repeat step-6 untill the new approximations are obtained upto the desired precision. (Or there is negligible difference between two successive approximations)

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Q. Solve the following system of equations using Gauss-Seidel method.

$$x_1 + 10x_2 + x_3 = 24$$
$$10x_1 + x_2 + x_3 = 15$$
$$x_1 + x_2 + 10x_3 = 33$$

→ Reordering the equations:

$$10x_1 + x_2 + x_3 = 15$$
$$x_1 + 10x_2 + x_3 = 24$$
$$x_1 + x_2 + 10x_3 = 33$$

Representing in matrix form

$$\begin{bmatrix} 10 & 1 & 1 \\ 1 & 10 & 1 \\ 1 & 1 & 10 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 15 \\ 24 \\ 33 \end{bmatrix}$$

Here the Coeff. matrix is strictly diagonal. Hence we can apply Gauss-Seidel method.

From first equation

$$x_1 = \frac{1}{10}(15 - x_2 - x_3) \quad \text{——— (i)}$$

from 2nd

$$x_2 = \frac{1}{10}(24 - x_1 - x_3) \quad \text{——— (ii)}$$

from 3rd

$$x_3 = \frac{1}{10}(33 - x_1 - x_2) \quad \text{——— (iii)}$$

let the initial guess be $x_2 = 0$, $x_3 = 0$.

Then the first approximations are

$$(x_1)_1 = 15/10 = 1.500$$

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$$(x_2)_1 = \frac{1}{10}(24 - 1.5 - 0) = 2.250$$

$$(x_3)_1 = \frac{1}{10}(33 - 1.5 - 2.25) = 2.925$$

second approximations are:

$$(x_1)_2 = \frac{1}{10}(15 - 2.25 - 2.925) = 0.9825$$

$$(x_2)_2 = \frac{1}{10}(24 - 0.9825 - 2.925) = 2.0093$$

$$(x_3)_2 = \frac{1}{10}(33 - 0.9825 - 2.0093) = 3.0008$$

~~Third~~ Tabulating the successive approximations

Iteration	x_1	x_2	x_3
~~Guess~~ (0)	~~15~~ -	Guess 0	Guess 0
1.	1.5	2.25	2.925
2.	0.9825	2.0093	3.0008
3.	0.999	2.0000	3.0001
4.	0.9999	2	3.00001

$\therefore x_1 = 1 \quad x_2 = 2 \quad x_3 = 3$

Q. Solve the following system of linear equations using Gauss-Seidel iterative method.

$$6x_1 - 2x_2 + x_3 = 11$$
$$-2x_1 + 7x_2 + 2x_3 = 5$$
$$x_1 + 2x_2 - 5x_3 = -1$$

Q. $2x + 15y + 2 = 33$
$x - y + 10z = 25$
$13x + 2y + 3z = 46$

Eigen Value & Eigen ~~Matrix~~ (Vector)

Let A be a $n \times n$ square matrix. If there exists non trivial column vector X as a solution of the matrix equation $AX = \lambda X$; where λ is a constant, then X is called an Eigen vector of matrix A and the corresponding constant λ is called Eigen value of A.

Here, $AX = \lambda X$

or, $AX - \lambda X = 0$

or, $(A - \lambda I)X = 0$

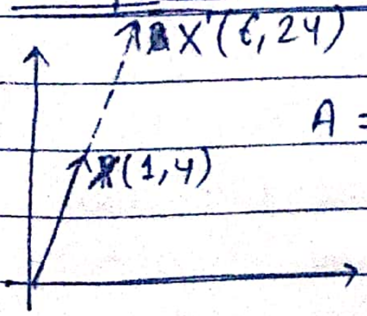
The above matrix equation has non trivial solutions if and only if ~~the~~

$$|A - \lambda I| = 0 \quad \text{——} \quad (1)$$

Here eqn (1) is also known as characteristic eqn of A

Note: Eigen vector is often called as latent vector.

Concept:- (Geometric meaning)



$$A = \begin{bmatrix} -6 & 3 \\ 4 & 5 \end{bmatrix} \quad X = \begin{bmatrix} 1 \\ 4 \end{bmatrix}$$

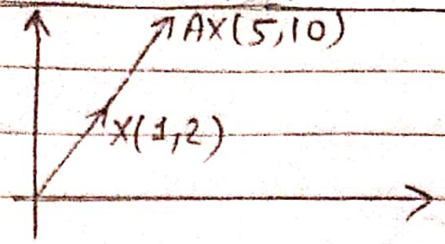
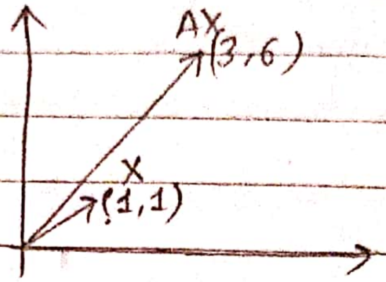
$$AX = \begin{bmatrix} -6 & 3 \\ 4 & 5 \end{bmatrix} \begin{bmatrix} 1 \\ 4 \end{bmatrix}$$

$$= 6 \begin{bmatrix} 1 \\ 4 \end{bmatrix}$$

Eigen value $(\lambda) = 6$

Eigen vector $(X) = \begin{bmatrix} 1 \\ 4 \end{bmatrix}$

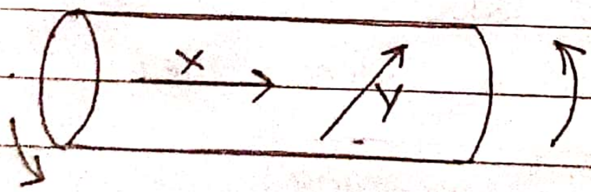
Geometrically, Eigen vector X of matrix A is that vector which when transformed by A, doesn't change its direction, rather it is a scalar multiple of itself. The scalar factor is called as Eigen value.

A	X	AX	Result
$\begin{bmatrix} 1 & 2 \\ 2 & 4 \end{bmatrix}$	$\begin{bmatrix} 1 \\ 2 \end{bmatrix}$		X is Eigen vector of A with eigen value = 5.
	$\begin{bmatrix} 1 \\ 1 \end{bmatrix}$		X is not an Eigen Vector of A.

Eigen value problem is a problem of scaling without rotation.

Another

Take a rubber cylinder & draw two lines X & Y as shown:



If we squize the cylinder about its access, the line X undergoes scaling and rotation whereas Y undergo scaling but not rotation. Hence we can call X not an Eigen vector & Y as Eigen vector of the transformation (squizing) matrix. Eigen value is the factor by which vector Y has been scaled.

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Q. Find the Eigen vector corresponding to the largest Eigen value of the matrix $\begin{bmatrix} 1 & -2 \\ -5 & 4 \end{bmatrix}$

→ Let $A = \begin{bmatrix} 1 & -2 \\ -5 & 4 \end{bmatrix}$

The characteristic equation for A is
$|A - \lambda I| = 0$ where λ is Eigen value, I is identity matrix

or, $\begin{vmatrix} 1-\lambda & -2 \\ -5 & 4-\lambda \end{vmatrix} = 0$

or, $\lambda^2 - 5\lambda - 6 = 0$

or, $(\lambda - 6)(\lambda + 1) = 0$

$\therefore$ $\lambda = +6, -1$

largest Eigen value $= 6$.

Now for $\lambda = 6$,

Let the Eigen vector $X = \begin{bmatrix} x \\ y \end{bmatrix}$

We have $AX = \lambda X$

or, $(A - \lambda)X = 0$

or, $\begin{bmatrix} 1-\lambda & -2 \\ -5 & 4-\lambda \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = 0$

or, $\begin{bmatrix} -5 & -2 \\ -5 & -2 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = 0$ $\quad \because \lambda = 6$

or, $\begin{bmatrix} -5 & -2 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = 0$ $\quad \because R_2 \leftarrow R_2 - R_1$

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or, $-5x-2y=0$

or, $5x=-2y$

or, $\frac{x}{-2}=\frac{y}{5}$

let $\frac{x}{-2}=\frac{y}{5}=k$

$\therefore$ $x=-2k$

$y=5k$

let $k=1$ $\Rightarrow$ $\begin{bmatrix} x \\ y \end{bmatrix}=\begin{bmatrix} -2 \\ 5 \end{bmatrix}$ is desired Eigen vector.

Q. Find Eigen vector corresponding to largest Eigen value of matrix $A=\begin{bmatrix} 2 & 2 & 1 \\ 1 & 3 & 1 \\ 1 & 2 & 2 \end{bmatrix}$

→ The characteristic equation of matrix A is

$|A-\lambda I|=0$

or, $\begin{vmatrix} 2-\lambda & 2 & 1 \\ 1 & 3-\lambda & 1 \\ 1 & 2 & 2-\lambda \end{vmatrix}=0$

or, $(\lambda-1)(\lambda^2-6\lambda+5)=0$

or, $\lambda=1,1,5$

The larges Eigen value is 5.

Now for $\lambda=5$, let eigen vector be $X=\begin{bmatrix} x \\ y \\ z \end{bmatrix}$

we have $AX = \lambda X$

or $(A - \lambda I)X = 0$

or,
$$\begin{bmatrix} 2-5 & 2 & 1 \\ 1 & 3-5 & 1 \\ 1 & 2 & 2-5 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = 0$$

or,
$$\begin{bmatrix} -3 & 2 & 1 \\ 1 & -2 & 1 \\ 1 & 2 & -3 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = 0$$

Solving we get
$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

Power Method :-

Power method is an iterative approach to find the largest eigen value and the corresponding eigen vector for a given matrix.

Eigen value problem is a mathematical tool that has applications in different areas of physics & engineering. The vibration & oscillation problems are commonly eigen value problems.

A square matrix A of size $n \times n$ can have maximum of n different eigen values, $\lambda_1, \lambda_2, \dots, \lambda_n$. Any eigen value λ_i is the dominant eigen value if $|\lambda_i| > |\lambda_j| \ \forall \ j = 1, \dots, n$, $i \neq j$, the corresponding eigen vector is called dominant eigen vector.

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Working Principle/Procedure:

1. Make an initial guess for eigen vector $X_0 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$

(Note: For given square matrix of size $n \times n$, we choose initial guess X_0 as a column vector of size $n \times 1$.)

2. Determine the first approximations λ_1 & X_1 using the matrix equation $\lambda_1 X_1 = A X_0$

3. Determine the ~~next~~ approximations λ_i & X_i using the matrix equation $\lambda_i X_i = A X_{i-1}$

4. Repeat step 3 untill the eigen value & eigen vectors are obtained upto the desired precision or the there is neglisible differences between the successive iterations.

Q. Find the largest eigen value & the corresponding eigen vector corresponding to the following matrix

$$\begin{bmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{bmatrix}.$$

→ Here, $A = \begin{bmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{bmatrix}$

We can determine the different approximations of eigen value & eigen vector using the following matrix equation.

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$$\lambda_i X_i = A X_{i-1} \quad \text{———} \quad (i)$$

Let initial guess for eigen vector be $x_0 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$.

For first approximation, $(i=1)$

$$\lambda_1 X_1 = A X_0$$

$$= \begin{bmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$

$$= \begin{bmatrix} 2 \\ -1 \\ 0 \end{bmatrix}$$

$$= 2 \begin{bmatrix} 1 \\ -0.5 \\ 0 \end{bmatrix}$$

$$\therefore \lambda_1 = 2, \quad X_1 = \begin{bmatrix} 1 \\ -0.5 \\ 0 \end{bmatrix}$$

For second approximation $(i=2)$

$$\lambda_2 X_2 = A X_1$$

$$= \begin{bmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{bmatrix} \begin{bmatrix} 1 \\ -0.5 \\ 0 \end{bmatrix}$$

$$= \begin{bmatrix} 2.5 \\ -2 \\ 0.5 \end{bmatrix}$$

$$= 2.5 \begin{bmatrix} 1 \\ -0.8 \\ 0.2 \end{bmatrix}$$

$$\therefore \lambda_2 = 2.5, \quad X_2 = \begin{bmatrix} 1 \\ -0.8 \\ 0.2 \end{bmatrix}$$

For 3rd approximation $(i=3)$

$$\lambda_3 X_3 = A X_2$$

$$= \begin{bmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{bmatrix} \begin{bmatrix} 1 \\ -0.8 \\ 0.2 \end{bmatrix}$$

$$= \begin{bmatrix} 2.8 \\ -2.8 \\ 1.2 \end{bmatrix}$$

$$= 2.8 \begin{bmatrix} 1 \\ -1 \\ 0.43 \end{bmatrix}$$

$$\therefore \lambda_3 = 2.8, \quad X_3 = \begin{bmatrix} 1 \\ -1 \\ 0.43 \end{bmatrix}$$

For 4th approximation $(i=4)$

$$\lambda_4 X_4 = A X_3$$

$$= \begin{bmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{bmatrix} \begin{bmatrix} 1 \\ -1 \\ 0.43 \end{bmatrix}$$

$$= \begin{bmatrix} 3 \\ -3.43 \\ 1.86 \end{bmatrix}$$

$$= 3.43 \begin{bmatrix} 0.88 \\ -1 \\ 0.54 \end{bmatrix}$$

$$\therefore \lambda_4 = 3.43 \;\&\; X_4 = \begin{bmatrix} 0.88 \\ -1 \\ 0.54 \end{bmatrix}$$

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For 5^{th} approximation $(i=5)$

$$\lambda_5 X_5 = A X_4$$

$$= \begin{bmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{bmatrix} \begin{bmatrix} 0.88 \\ -1 \\ 0.54 \end{bmatrix}$$

$$= \begin{bmatrix} 2.76 \\ -3.42 \\ 2.08 \end{bmatrix}$$

$$= 3.42 \begin{bmatrix} 0.80 \\ -1 \\ 0.61 \end{bmatrix}$$

$$\therefore \lambda_5 = 3.42, \quad X_5 = \begin{bmatrix} 0.80 \\ -1 \\ 0.61 \end{bmatrix}$$

For 6^{th} approximation $(i=6)$

$$\lambda_6 X_6 = A X_5$$

$$= \begin{bmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{bmatrix} \begin{bmatrix} 0.8 \\ -1 \\ 0.61 \end{bmatrix}$$

$$= \begin{bmatrix} 2.6 \\ -3.41 \\ 2.22 \end{bmatrix}$$

$$= 3.41 \begin{bmatrix} 0.76 \\ -1 \\ 0.65 \end{bmatrix}$$

$$\therefore \lambda_6 = 3.41, \quad X_6 = \begin{bmatrix} 0.76 \\ -1 \\ 0.65 \end{bmatrix}$$

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for 7^{th} approximation ($i=7$)

$$\lambda_7 X_7 = A X_6$$

$$= \begin{bmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{bmatrix} \begin{bmatrix} 0.76 \\ -1 \\ 0.65 \end{bmatrix}$$

$$= \begin{bmatrix} 2.52 \\ -3.41 \\ 2.3 \end{bmatrix}$$

$$= 3.41 \begin{bmatrix} 0.74 \\ -1 \\ 0.67 \end{bmatrix}$$

$\therefore \lambda_7 = 3.41$ & $X_7 = \begin{bmatrix} 0.74 \\ -1 \\ 0.67 \end{bmatrix}$

Since $\lambda_6 = \lambda_7$ & $X_6 \simeq X_7$, we can conclude that:

Largest eigen value $= 3.41$

Corresponding eigen vector $= \begin{bmatrix} 0.74 \\ -1 \\ 0.67 \end{bmatrix}$.

2018

Q. What do you mean by eigen-value eigen-vector problem? Find the largest eigen-value correct to two significant digits & corresponding eigen vector for the following matrix using power method.

$$A = \begin{bmatrix} 2 & 4 & 1 \\ 0 & 1 & 3 \\ 1 & 0 & 3 \end{bmatrix}.$$

[2+6]