

Applied Mechanics

(For Diploma I Yrs. II Part)

2nd Semester

(Engineering All)

By

Arjun Chaudhary

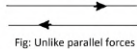
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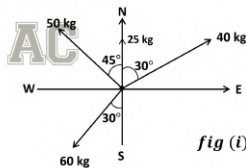
Unlike parallel forces

- The force, whose line of action is parallel to each other but they act in the opposite direction.



b) Find the magnitude and direction of the resultant force for the following force: 40kg acting at N30°E, 25kg acting at North, 50kg acting at N45°W and 60kg acting at S30°W.

- The system is shown below in the figure (i):-



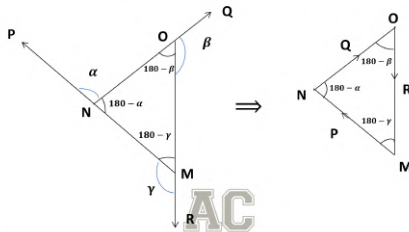
Now,

Resolving all the forces horizontally i.e., East-West line.(x-axis)

$$\begin{aligned}\sum H &= 40 \sin 30^\circ - 50 \sin 45^\circ - 60 \sin 30^\circ \\ &= 20 - 35.35 - 30 \\ &= -45.35 \text{ kg}\end{aligned}$$

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Let angle between Q & R force are 'β' i.e., ∠QOM = 'β'
, angle between P & Q forces are 'α' i.e., ∠PNO = 'α'
, angle between P & R forces are 'γ' i.e., ∠RMN = 'γ'



Now,

$$\begin{aligned}\angle MON &= 180^\circ - \angle PNO, \quad \angle NMO = 180^\circ - \angle RMN, \quad \angle MON = 180^\circ - \angle QOM \\ &= 180^\circ - \alpha \quad \quad \quad = 180^\circ - \gamma \quad \quad \quad = 180^\circ - \beta\end{aligned}$$

Using Sine law in Δ MON

$$\begin{aligned}\frac{P}{\sin(\angle MON)} &= \frac{Q}{\sin(\angle NMO)} = \frac{R}{\sin(\angle MON)} \\ \frac{P}{\sin(180^\circ - \beta)} &= \frac{Q}{\sin(180^\circ - \gamma)} = \frac{R}{\sin(180^\circ - \alpha)} \\ \frac{P}{\sin \beta} &= \frac{Q}{\sin \gamma} = \frac{R}{\sin \alpha}\end{aligned}$$

Proved.

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S.No Exam Year, Month

1. 2078 Magh Regular/Back
2. 2079 Falgun Regular/Back
3. New Model Question

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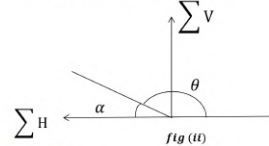
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Resolving all the forces Vertically i.e., North-south line.(y-axis)

$$\begin{aligned}\sum V &= 25 + 40 \cos 30^\circ + 50 \cos 45^\circ - 60 \cos 30^\circ \\ &= 43.034 \text{ kg}\end{aligned}$$

As $\sum H = -ve$, and $\sum V = +ve$ i.e. It lies on 2nd Quadrant



Let "R" be Resultant Force That make θ angles with horizontal

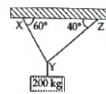
$$\begin{aligned}R &= \sqrt{(\sum H)^2 + (\sum V)^2} \\ &= \sqrt{(-45.35)^2 + (43.034)^2} \\ &= 62.518 \text{ kg}\end{aligned}$$

$$\alpha = \tan^{-1} \left(\frac{\sum V}{\sum H} \right) = \tan^{-1} \left(\frac{43.034}{-45.35} \right) = -43.49^\circ$$

$$\text{actual } \theta = 180 - 43.49 = 136.50^\circ \text{ [from fig (ii)]}$$

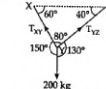
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b) Find the force in the cable XY and YZ as shown in figure.



➤ Solution:

The free body diagram of the given figure is as;



The remaining angles at point are determined by geometrically. Now,

Let, T_{xy} = Tension in XY cable T_{yz} = Tension in YZ cable

Using Lami's theorem; we get,

$$\frac{T_{xy}}{\sin 130^\circ} = \frac{T_{yz}}{\sin 150^\circ} = \frac{200}{\sin 80^\circ}$$

Taking first and last expression; we get,

$$T_{xy} = \frac{200}{\sin 80^\circ} \times \sin 130^\circ$$

$$\therefore T_{xy} = 155.572 \text{ kg}$$

Similarly;

$$T_{yz} = \frac{200}{\sin 80^\circ} \times \sin 150^\circ$$

$$\therefore T_{yz} = 101.543 \text{ kg}$$

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Applied Mechanics (Engg. All) 3rd Sem
(2078) Question Paper Solution.

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1. a) What do you mean by rigid and Deformed body? Explain

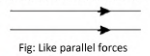
Coplanar, Non-coplanar, like parallel and Unlike parallel Force.

- **Rigid body** is the combination of two or more particles which are so interconnected that they does not change their relative position when force is applied on it. Otherwise it is called **deformable body**.
- When all forces are acting in the same plane, they are called **coplanar** force.
- If the lines of action of forces do not lie in the same plane then the forces are called **non-coplanar** force.

- Parallel forces are those which do not meet at any point though they have same effect on the body on which they act. The parallel forces are classified as:-

Like parallel forces

- The force, whose line of action is parallel to each other and they act in the same direction.

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2. a) State and prove Lami's Theorem.

➤ Statement

"If three coplanar forces acting at a point be in equilibrium, then each force is proportional to the sine of the angle between the other two."

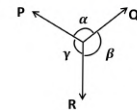
Mathematically;

$$\frac{P}{\sin \beta} = \frac{Q}{\sin \gamma} = \frac{R}{\sin \alpha}$$

where, P, Q and R are three forces and β, γ, α are angles as shown in the figure.

Proof

Let us consider three coplanar forces P, Q and R. Let the opposite angles of the three forces be β, γ and α respectively as in the figure.



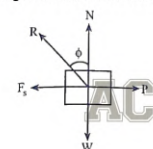
Forming triangle with these forces,

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3. a) Define the terms friction and angle of friction. And also show the sliding and Tripling condition of the body with sketch.

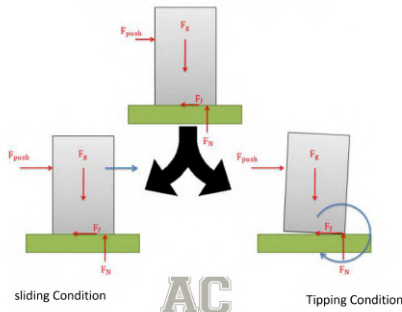
- When a body slides over another body, a force is exerted a surface of contact by the stationary body on a moving body. The resisting force is called the force of friction and acts in the direction opposite to the direction of motion.

- Angle of friction (φ) is defined as the angle which the resultant of normal reaction and limiting frictional force makes with the normal reaction.



- The sliding and Tripling condition of the body with sketch

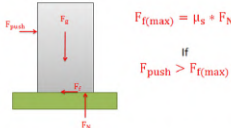
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Tipping Condition

➤ The sliding condition of the body with sketch



If Applied (push) force is greater than Friction force then, body will slide i.e no tipping

$$\begin{aligned} \sum f_y = 0; R &= W \cos \theta + F \sin \theta \dots (i) \\ \sum f_x = 0; W \sin \theta + f_k &= F \cos \theta \dots (ii) \end{aligned}$$

Using equations (i) on eqn (ii), we get

$$W \sin \theta + \mu_k (W \cos \theta + F \sin \theta) = F \cos \theta \quad [\because f_k = \mu_k R]$$

$$W(\sin \theta + \mu_k \cos \theta) = F(\cos \theta - \mu_k \sin \theta)$$

$$\therefore F = \frac{W(\sin \theta + \mu_k \cos \theta)}{(\cos \theta - \mu_k \sin \theta)}$$

$$= \frac{50(\sin 30^\circ + 0.4 \cos 30^\circ)}{(\cos 30^\circ - 0.4 \sin 30^\circ)}$$

$$\therefore F_{\max} = 63.54N$$

AC

➤ For minimum Horizontal Force i.e., Body Sliding Down Motion

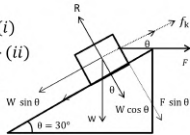
At Equilibrium,

$$\sum f_y = 0; R = W \cos \theta + F \sin \theta \dots (i)$$

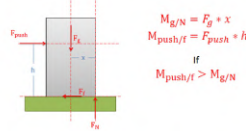
$$\sum f_x = 0; W \sin \theta = f_k + F \cos \theta \dots (ii)$$

Using equations (i) on eqn (ii), we get

$$[\because f_k = \mu_k R]$$



➤ Tripling condition of the body with sketch



If Moment due to push force is greater than Moment due to Weight of block such that Moment taken at Edge B then, body will begin to Tip

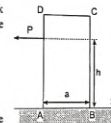
OR,

➤ Derivation of Condition of Tipping and sliding of a block

Here tipping refers to the turning of block ABCD due to the formation of couple. The condition for no tipping is $h = \frac{a}{2\mu_k}$

$$\text{or, } h < \frac{a}{2\mu_k} \quad \text{i.e. } h \leq \frac{a}{2\mu_k}$$

h is point of application of force P above the base of block AB.



$$W \sin \theta = \mu_k (F \sin \theta + W \cos \theta) + F \cos \theta$$

$$W [\sin \theta - \mu_k (\cos \theta)] = F(\mu_k \sin \theta + \cos \theta)$$

$$\begin{aligned} \therefore F &= \frac{W(\sin \theta - \mu_k \cos \theta)}{(\mu_k \sin \theta + \cos \theta)} \\ &= \frac{50(\sin 30^\circ - 0.4 \cos 30^\circ)}{(0.4 \sin 30^\circ + \cos 30^\circ)} \end{aligned}$$

$$\therefore F_{\min} = 7.2N$$

4. a) Define centroid and find the relation for centroid for rectangle.

➤ The centroid or centre of area is defined as the point where the whole area of the figure is assumed to be concentrated. It is analogous to centre of gravity when a body has area but not weight.

➤ Consider a rectangle ABCD of breadth b and height h. Consider strip of thickness dy located at a distance y as shown.

Elemental area $dA = b \, dy$

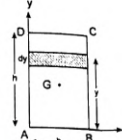
Moment about x-axis $dM_x = y \, dA$

= by dy

Area of rectangle $A = b \times h$

Let \bar{Y} be the distance of centroid from x-axis.

Then from moment principle:



5. a) What do you mean by statically determinate structure? What are the different types of beam?

➤ A statically determinate beam is one for which the reactions and internal stresses Developed in the plane member can be completely be determined by using the equation of static equilibrium. i.e. $\sum H=0$, $\sum V=0$ and $\sum M=0$. For example; simply supported beams, cantilever beams, three hinged arches, etc.

➤ The different types of beam are:-

1) Simply supported Beam

When both ends are supported on simple supports.

2) Continuous Beam

If beam supported more than two supports then the beam is continuous beam.

3) Cantilever Beam

One end of the beam is fixed and other end is free.

4) Propped Cantilever Beam

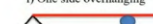
One end of the beam is fixed and other end supported on simple support.

5) Fixed Beam:

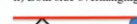
A beam whose both ends are rigidly fixed or built into its supporting walls and columns.

6) Overhanging Beam

i) One side overhanging



ii) Both side overhanging



Proof:

At extreme point of tipping i.e. at just tipping condition, the block will be in equilibrium and reaction R will be at point A.

So, at equilibrium

$$i) \sum M_A = 0 \rightarrow +ve$$

$$mg \frac{a}{2} - P \cdot h = 0$$

$$P \cdot h = mg \frac{a}{2}$$

$$ii) \sum F_x = 0 \rightarrow +ve$$

$$H \cdot R - P = 0$$

$$\therefore P = \mu_k R$$

$$iii) \sum F_y = 0 \uparrow +ve$$

$$R - mg = 0$$

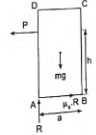
$$R = mg$$

Combining all three equations above

$$\mu_k R \cdot h = R \frac{a}{2}$$

$$h = \frac{a}{2\mu_k}$$

So, for no tipping $h \leq \frac{a}{2\mu_k}$. Proved.



AC

b) A load of 50N is lying on inclined plane whose inclination with the horizontal is 30°. If the coefficient of friction between load plane is 0.4. Find the maximum and minimum horizontal force which will keep the load in equilibrium.

➤ For maximum Horizontal Force i.e., Body Sliding Up Motion

$$A \times \bar{y} = \int dM_x$$

$$bh \times \bar{y} = \int_0^h y \, dy = b \frac{y^2}{2} \Big|_0^h = \frac{b}{2} h^2 \cdot \frac{1}{bh}$$

$$\therefore \bar{y} = \frac{h}{2}$$

$$\text{Similarly, } \bar{x} = \frac{b}{2}$$

$$\therefore (\bar{x}, \bar{y}) = \left(\frac{b}{2}, \frac{h}{2} \right)$$

b) Find the MOI of the given section about centroidal axis parallel to the x-x axis.

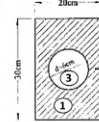


Fig	\bar{X}	\bar{Y}	Area
1(+ve)	$\frac{20}{2} = 10 \text{ cm}$	$\frac{30}{2} = 15 \text{ cm}$	$30 \times 20 = 600 \text{ cm}^2$
2(+ve)	$20 + \frac{5}{3} = 21.67 \text{ cm}$	$20 + \frac{10}{3} = 23.33 \text{ cm}$	$\frac{1}{2} \times 5 \times 10 = 25 \text{ cm}^2$
3(-ve)	10cm	15cm	$\pi(3)^2 = 28.27 \text{ cm}^2$

$$\bar{X} = \frac{\sum AX}{\sum A} = \frac{A_1 \bar{X}_1 + A_2 \bar{X}_2 - A_3 \bar{X}_3}{A_1 + A_2 - A_3}$$

b) Discuss about the method of analysis of truss.

➤ They are two method of analysis of truss are:-

a) Method of joints

➤ The solution proceeds from one joint to the other. At every joint the forces are concurrent as the centre line of each member at a joint meet at a point. Equation of equilibrium at a joint is

$$\sum F_x = 0; \quad \sum F_y = 0;$$

Thus only two unknown forces can be determined. So we need to start with a joint with two unknown force at most.

b) Method of sections

➤ This method is more economical than the method of joints, if forces in only few members of the truss are desired. Divide the truss into two imaginary section by passing a cut (straight or crooked) through the truss such that the member in which force is to be determined is cut. To solve problem more than one cut may be required. Take one cut section of truss and use

$$\sum M = 0; \quad \sum F_y = 0; \quad \sum F_x = 0;$$

to solve the unknown forces.

The section has to be such that it does not cut more than three members in which the forces are to be determined.

MOI about Centroidal X-axis,

$$I_{CX-CX} = \left[\frac{bh^3}{12} + A_1(\bar{Y} - Y_1)^2 \right] + \left[\frac{bh^3}{36} + A_2(\bar{Y} - Y_2)^2 \right] - \left[\frac{\pi r^4}{4} + A_3(\bar{Y} - Y_3)^2 \right]$$

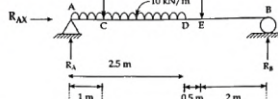
$$= \left[\frac{20 \times 30^3}{12} + 600 \times (15.348 - 15)^2 \right] + \left[\frac{5 \times 10^3}{36} + 25(15.348 - 23.33)^2 \right] - \left[\frac{\pi \times 3^4}{4} + 28.27(15.348 - 15)^2 \right]$$

$$= 45072.662 + 1719.746 - 67.040$$

$$I_{CX-CX} = 46725.368 \text{ cm}^4$$

c) A simply supported beam of 5m span carries of the UDL load of 10KN/m from left side of the beam to the center and 5KN each at 1m and 2m from the left and right hand support respectively. Find the support reaction.

➤ By Question,



➤ Let R_A and R_B be the reactions at support A and B respectively. Let, R_{AX} be the horizontal reaction at support A.

$$\sum R_X = 0$$

$$\therefore R_{AX} = 0$$

Taking moment at B and equating the same; we get,
(+ve) $\sum M_B = 0$

$$\text{or, } R_A \times 5 - \left\{ 5 \times 2 + 5 \times 4 + 10 \times 2.5 \times \left(2.5 + \frac{2.5}{2} \right) \right\} = 0$$

$$\text{or, } R_A = \frac{123.75}{5}$$

$$\therefore R_A = 24.75 \text{ kN } (\uparrow)$$

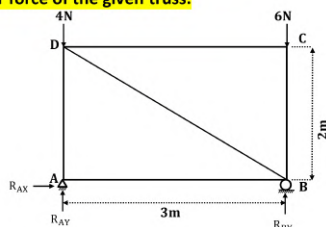
and $(\uparrow +ve) \sum F_Y = 0$

$$\text{or, } R_A + R_B - (5 + 5 + 10 \times 2.5) = 0$$

$$\text{or, } R_B = 35 - 24.75$$

$$\therefore R_B = 10.25 \text{ kN } (\uparrow)$$

6. a) Find the member force of the given truss.



➤ Calculation of Support Reaction:

Let R_{AX} & R_{AY} be the horizontal reactions & Vertical reactions at support A. Let, R_{BY} be the reaction at support B.

$$(\rightarrow) +ve \sum F_X = 0$$

$$\therefore R_{AX} = 0$$

Taking moment at A and equating the same; we get,

$$(+ve) \sum M_A = 0$$

$$R_{BY} \times 3 - 6 \times 3 = 0$$

$$R_{BY} = \frac{6 \times 3}{3}$$

$$\therefore R_{BY} = 6 \text{ N } (\uparrow)$$

$$(\uparrow) +ve \sum F_Y = 0$$

$$R_{AY} + R_{BY} - 4 - 6 = 0$$

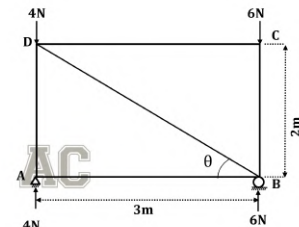
$$R_{AY} + 6 - 4 - 6 = 0$$

$$\therefore R_{BY} = 4 \text{ N } (\uparrow)$$

$$\tan \theta = \frac{2}{3}$$

$$\theta = \tan^{-1} \left(\frac{2}{3} \right)$$

$$\theta = 33.69^\circ$$



➤ Analysis Using Method of Joints:

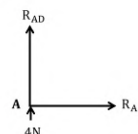
At Joint A

$$(\rightarrow) +ve \sum F_X = 0$$

$$\therefore R_{AB} = 0$$

$$(\uparrow) +ve \sum F_Y = 0$$

$$R_{AD} + 4 = 0 \Rightarrow R_{AD} = -4 \text{ N}$$



At Joint B

$$(\rightarrow) +ve \sum F_X = 0$$

$$R_{BA} + R_{BD} \cos \theta = 0$$

$$0 + R_{BD} \cos \theta = 0$$

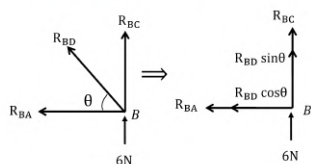
$$\Rightarrow R_{BD} = 0 \text{ N}$$

$$(\uparrow) +ve \sum F_Y = 0$$

$$R_{BC} + 6 + R_{BD} \sin \theta = 0$$

$$R_{BC} + 6 + 0 \cdot \sin \theta = 0$$

$$\Rightarrow R_{BC} = -6 \text{ N}$$



At Joint C

$$(\rightarrow) +ve \sum F_X = 0$$

$$-R_{CB} = 0 \Rightarrow R_{CB} = 0 \text{ N}$$

For Checking:

$$(\uparrow) +ve \sum F_Y = 0$$

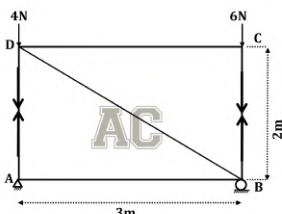
$$-R_{BC} - 6 = 0$$

$$\Rightarrow R_{BC} = -6 \text{ N (Match)}$$

Hence,

Member	AB	BC	CD	BD	AD
Member Force(N)	0	6 (C)	0	0	4 (C)

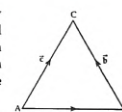
Where, (C) indicate the nature of force is compressive



b) Write Short notes on: (Any Two)

i) Triangle Law of Forces

➤ It states that, "If two forces acting simultaneously on a particle be represented in magnitude and direction by the two sides of a triangle, taken in order then their resultant may be represented in magnitude and direction by the third side of the triangle, taken in opposite order."



In the adjoining figure, \vec{a} and \vec{b} be the two forces acting simultaneously at a particle then their resultant becomes \vec{c} represented by third side of a triangle in the opposite direction.

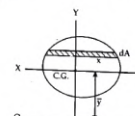
Mathematically;
 $\vec{a} + \vec{b} = \vec{c}$

ii) Parallel axis Theorem for moment of Inertia

➤ It states that "The moment of inertia of a plane area about any axis parallel to the centroidal axis is equal to the sum of moment of inertia about a parallel centroidal axis and the product of area and square of the distance between them (i.e., to areas)." Mathematically;

$$I_{XX} = I_{XX'} + Ay^2$$

Let us consider a plane area 'A' having XX and YY centroidal axis as shown in figure. For this let us consider an axis parallel to XX axis at a distance \bar{y} from this axis.



Take an elementary area dA at distance y from XX'' -axis.

We know,

$$I_{XX} = \sum y^2 dA = \int y^2 dA$$

Also,

$$I_{XX} = \sum (\bar{y} + y)^2 dA = \sum (\bar{y}^2 + 2\bar{y}y + y^2) dA$$

$$= \bar{y}^2 \sum dA + 2\bar{y} \sum y dA + \sum y^2 dA = \bar{y}^2 \sum dA + 2\bar{y} \times 0 + I_{XX'}$$

$$= I_{XX'} + A\bar{y}^2 \quad \left[\because \sum y dA = 0, \text{ because first moment of area about centroidal axis} = 0 \right]$$

iii) Different types of load and support in structure.

➤ The different types of load are:-

1) Point Load

It is assumed to act at a point. It may be vertical, horizontal or inclined. Unit: KN, Kg etc.

2) Uniformly distributed Load (UDL)

Load is distributed along the structure uniformly.

It is in rectangular form. Unit: KN/m, t/m, Kg/m etc.

Point Load = Length * Height

= Area of Rectangle (WL)

3) Uniformly Varying Load (UVL)

The load is vary from point to point in triangular form.

Unit: KN/m, t/m, Kg/m etc.

Point Load = $\frac{1}{2}$ * Length * Height

= Area of Triangle (0.5WL)

This point load assumed to be acted from centroid of the triangle.

****Extra,**

4) Wind load: Load due to wind

5) Seismic load: Load due to earthquake

6) Hydrostatic load: Load due to water

7) Snow load: Load due to snow and considered only in snowfall places.

8) Dead load and Live load

• Dead load is the self load or load of permanent structure.

• Live loads are movable loads.

9) Static Load and Dynamic load

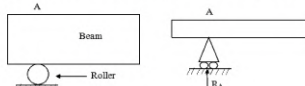
• Static load are applied gradually but they don't move.

• Dynamic load gives rise to load and variation as well.

➤ The different types of Support are:-

a) Roller support

➤ It gives rise to one force which is perpendicular to the plane supporting the plane.



No. of reaction = 1

b) Hinge support

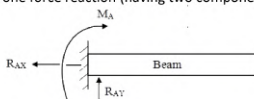
➤ It gives rise to one force reaction whose direction is unknown. It can be resolved into two forces along x and y axes.



No. of reaction = 2

c) Fixed support

➤ It gives rise to one force reaction (having two components) and one reaction moment.



No. of reaction = 3

-The End-

Applied Mechanics (Engg. All) 2nd Sem

(2019) Question Paper Solution.

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1. Define applied mechanics with its scope in engineering.

➤ Applied mechanics also known as engineering mechanics is the branch of engineering which deals with the laws of mechanics as applied to the solution of engineering problems.

➤ Scope of applied mechanics :-

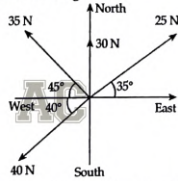
- It is coupled with more specialized knowledge subject such as strength of materials, theory of structure, hydrostatics, etc.
- It is foundation and an indispensable pre-requisite to the study of most engineering sciences.
- It is used in the analysis of forces acting on roof and bridge, truss, rocket propulsion, etc.
- Basics for study and understand structural engineering design of buildings.

b) The following forces act at a point, find the magnitude and direction of resultant force.

- 25N inclined at 35° towards north of east.
- 30N towards north.
- 35N inclined at 45° towards north of west.
- 40N inclined at 40° towards south of west.

> Solution

The system is shown below in the figure.



Now, resolving all the forces horizontally i.e., along East-West line.

$$(\pm ve) \sum H = 25 \cos 35^\circ + 30 \cos 90^\circ - 35 \cos 45^\circ - 40 \cos 40^\circ$$

$$\sum H = 20.47 + 0 - 24.74 - 30.64 = -34.91 \text{ N}$$

and, resolving vertically; we get,

$$(\uparrow +ve) \sum V = 25 \sin 35^\circ + 30 \sin 90^\circ + 35 \sin 45^\circ - 40 \sin 40^\circ$$

$$= 14.33 + 30 + 24.74 - 25.71 = 43.359 \text{ N}$$

Hence, magnitude of resultant (R);

$$R = \sqrt{(\sum H)^2 + (\sum V)^2} = \sqrt{(-34.91)^2 + (43.359)^2} = 55.66 \text{ N}$$

Now, for the direction of the resultant force;

Let θ = angle which the resultant force makes with East.

$$\tan \theta = \frac{\sum V}{\sum H} = \frac{43.359}{-34.91} = -1.242$$

$$\theta = \tan^{-1}(-1.242) = -51.16^\circ$$

Since, $\sum H$ is negative and $\sum V$ is positive.

Hence, θ lies between 90° and 180° .

$$\text{so, actual } \theta = 180^\circ - 51.16^\circ = 128.84^\circ$$

2. a) Define Free Body Diagram. Write down the steps to draw FBD.

> The force analysis of a structure is made in a simplified way by considering the equilibrium of a portion of the structure. For that, the portion is drawn separately showing applied forces, self weight and reactions at the point of contact with other bodies. The resulting diagram is known as **Free Body Diagram (FBD)**. So, FBD is a sketch of body (space diagram) drawn in such a way that it shows all the reaction forces, applied forces and moments on the body.

> The steps to draw FBD are :-

- FBD should have no external supports or connections.
- The self weight ($W = mg$) should be indicated with vertical downward arrow.
- The reactions from the supports and connections should be indicated.
- The uncut member force should not be shown in FBD.
- Tension in rope or string is directed towards support.
- The adopted coordinate system and sense of unknown force should be shown in FBD.

b) State and prove Lami's Theorem.

> **Statement**

"If three coplanar forces acting at a point be in equilibrium, then each force is proportional to the sine of the angle between the other two."

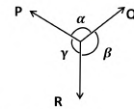
Mathematically;

$$\frac{P}{\sin \beta} = \frac{Q}{\sin \gamma} = \frac{R}{\sin \alpha}$$

where, P, Q and R are three forces and β, γ, α are angles as shown in the figure.

Proof

Let us consider three coplanar forces P, Q and R. Let the opposite angles of the three forces be β, γ and α respectively as in the figure.

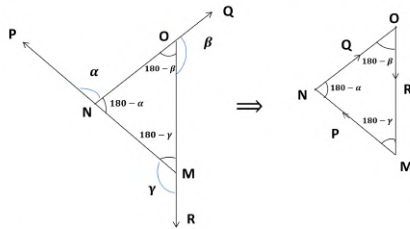


Forming triangle with these forces,

Let angle between Q & R force are ' β ' i.e., $\angle QOM = \beta$

, angle between P & Q forces are ' α ' i.e., $\angle PNO = \alpha$

, angle between P & R forces are ' γ ' i.e., $\angle RMN = \gamma$

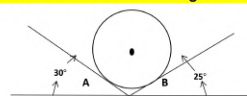


$$\begin{aligned} \angle MON &= 180^\circ - \angle PNO, \quad \angle NMO = 180^\circ - \angle RMN, \quad \angle MON = 180^\circ - \angle QOM \\ &= 180^\circ - \alpha, \quad = 180^\circ - \gamma, \quad = 180^\circ - \beta \end{aligned}$$

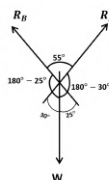
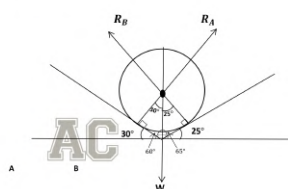
Using Sine law in ΔMON

$$\begin{aligned} \frac{P}{\sin(\angle MON)} &= \frac{Q}{\sin(\angle NMO)} = \frac{R}{\sin(\angle MNO)} \\ \frac{P}{\sin(180^\circ - \beta)} &= \frac{Q}{\sin(180^\circ - \gamma)} = \frac{R}{\sin(180^\circ - \alpha)} \\ \frac{P}{\sin \beta} &= \frac{Q}{\sin \gamma} = \frac{R}{\sin \alpha} \quad \text{Proved.} \end{aligned}$$

c) A spherical ball of weight 200N rests in a V-shaped trough, whose sides are inclined shown in figure. Find the reaction at each side.



> **Solution,**



Applying Lami's Theorem ;

$$\frac{R_A}{\sin(180^\circ - 25^\circ)} = \frac{R_A}{\sin(180^\circ - 30^\circ)} = \frac{W}{\sin(55^\circ)}$$

$$\text{Or, } \frac{R_A}{\sin(155^\circ)} = \frac{R_A}{\sin(150^\circ)} = \frac{200}{\sin(55^\circ)}$$

Taking first and last ,

$$\frac{R_A}{\sin(155^\circ)} = \frac{200}{\sin(55^\circ)}$$

$$R_A = \frac{200}{\sin(55^\circ)} \times \sin(155^\circ) = 103.184 \text{ N}$$

Similarly, Taking Second and last

$$\frac{R_A}{\sin(150^\circ)} = \frac{200}{\sin(55^\circ)}$$

$$R_B = \frac{200}{\sin(55^\circ)} \times \sin(150^\circ) = 122.077 \text{ N}$$

3. a) Define friction. Write down the laws of friction.

> When a body slides over another body, a force is exerted a surface of contact by the stationary body on a moving body. The resisting force is called the force of friction and acts in the direction opposite to the direction of motion.

> **The laws of friction are as follows;**

- The force of static friction always acts in a direction, opposite to that in which the body tends to move.
 - The magnitude of the force of friction is exactly equal to the force, which tends to move the body.
 - The magnitude of the following friction bears a constant ratio, to the normal reaction between the two surfaces.
- i. e., $\frac{F}{R} = \text{Constant}$
- $\therefore \frac{F}{R} = \mu;$
- Where, μ is the coefficient of friction.
- F is the limiting friction.
- R is the normal reaction.
- The force of friction is depends upon the roughness of the surfaces.
 - The force of friction is independent of the area of the contact between two surfaces.

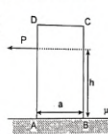
b) Explain the condition of tipping and sliding of a block.

> **The Condition of Tipping and sliding of a block**

Here tipping refers to the turning of block ABCD due to the formation of couple. The condition for no tipping is $h = \frac{a}{2\mu_s}$

$$\text{or, } h < \frac{a}{2\mu_s} \quad \text{i.e. } h \leq \frac{a}{2\mu_s}$$

h is point of application of force P above the base of block AB.



Proof:

At extreme point of tipping i.e. at just tipping condition, the block will be in equilibrium and reaction R will be at point A.

So, at equilibrium

$$\text{i) } \sum M_A = 0 \rightarrow +ve$$

$$mg \frac{a}{2} - P \cdot h = 0$$

$$P \cdot h = mg \frac{a}{2}$$

$$\text{ii) } \sum F_x = 0 \rightarrow +ve$$

$$\mu_s R - P = 0$$

$$\therefore P = \mu_s R$$

$$\text{iii) } \sum F_y = 0 \uparrow +ve$$

$$R - mg = 0$$

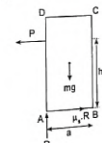
$$R = mg$$

Combining all three equations above

$$\mu_s R \cdot h = R \frac{a}{2}$$

$$h = \frac{a}{2\mu_s}$$

So, for no tipping $h \leq \frac{a}{2\mu_s}$. Proved.



or) A force of 100N pulls a body of weights 150N up in an inclined plane. The force being applied parallel to inclined plane. The inclined plane makes an angle of 30° with the horizontal. Calculate the coefficient of friction.

> **Solution,**

Given that;

Weight of body (W) = 150 N

Angle of inclination (θ) = 30°

Let, ' R ' be the normal reaction, μ be the coefficient of the friction and θ be an angle of inclination of plane.

We know that;

$$\begin{aligned} \text{Frictional force (F)} &= \mu R \\ &= \mu W \cos \theta \\ &= \mu \times 150 \cos 30^\circ \end{aligned} \quad (1)$$

Again, we have from figure;

$$F + W \sin \theta = 100$$

$$F + 150 \sin 30^\circ = 100$$

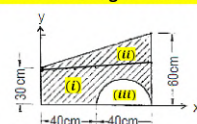
$$\text{or, } \mu \times 150 \cos 30^\circ + 150 \sin 30^\circ = 100 \quad [\text{From equation (1)}]$$

$$\text{or, } \mu \times 150 \cos 30^\circ = 100 - 150 \sin 30^\circ$$

$$\text{or, } \mu = \frac{100 - 150 \sin 30^\circ}{150 \cos 30^\circ}$$

\therefore Coefficient of friction (μ) = 0.192

4. a) Locate the centroid of give shaded area.



> **Solution**

➤ Calculation of Support Reaction:

Let R_{AX} & R_{AY} be the horizontal reactions & Vertical reactions at support A. Let, R_{BY} be the reaction at support B.

$$(\rightarrow) + \sum F_X = 0$$

$$\therefore R_{AX} = 0$$

Taking moment at A and equating the same; we get,

$$(\uparrow) \sum M_A = 0$$

$$R_{BY} \times 6 - 20 \times 6 = 0$$

$$R_{BY} = \frac{20 \times 6}{6}$$

$$\therefore R_{BY} = 20 \text{ N } (\uparrow)$$

$$(\uparrow) + \sum F_Y = 0$$

$$R_{AY} + R_{BY} - 40 - 20 = 0$$

$$R_{AY} + 20 - 40 - 20 = 0$$

$$\therefore R_{AY} = 40 \text{ N } (\uparrow)$$

$$\tan \theta = \frac{4}{6}$$

$$\theta = \tan^{-1} \left(\frac{2}{3} \right)$$

$$\theta = 33.69^\circ$$

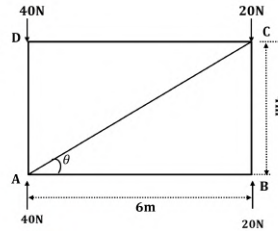
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➤ Analysis Using Method of Joints:

At Joint B

$$(\rightarrow) + \sum F_X = 0$$

$$-R_{BA} = 0$$

$$\therefore R_{BA} = 0$$

$$(\uparrow) + \sum F_Y = 0$$

$$R_{BC} + 20 = 0 \Rightarrow R_{AD} = -20 \text{ N}$$

At Joint A

$$(\rightarrow) + \sum F_X = 0$$

$$R_{AB} + R_{AC} \cos \theta = 0$$

$$0 + R_{AC} \cos \theta = 0$$

$$\Rightarrow R_{AC} = 0 \text{ N}$$

$$(\uparrow) + \sum F_Y = 0$$

$$R_{AD} + 40 + R_{AC} \sin \theta = 0$$

$$R_{AD} + 40 + 0 \cdot \sin \theta = 0$$

$$\Rightarrow R_{AD} = -40 \text{ N}$$

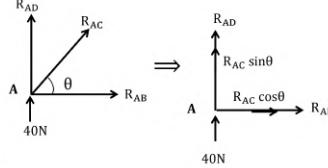
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At Joint D

$$(\rightarrow) + \sum F_X = 0$$

$$-R_{DC} = 0 \Rightarrow R_{DC} = 0 \text{ N}$$

For Checking:

$$(\uparrow) + \sum F_Y = 0$$

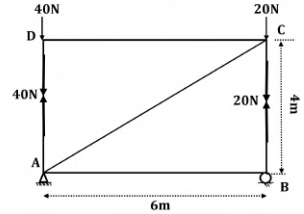
$$-R_{DA} - 40 = 0$$

$$\Rightarrow R_{DA} = -40 \text{ N (Match)}$$

Hence,

Member	AB	BC	CD	BD	AD
Member Force(N)	0	20 (C)	0	0	40 (C)

Where, (C) indicate the nature of force is compressive



-The End-

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Similarly, taking second and third; we get,

$$F_{BC} = \frac{500}{\sin 60^\circ} \times \sin 225^\circ = 557.677 \text{ N}$$

Hence,

$$\text{Tension in the string } (F_{AC}) = 408.248 \text{ N}$$

and, Reaction at the point of contact (F_{BC}) = 557.677 N

3. a) Write down the laws of friction.

➤ The laws of friction are as follows;

- The force of static friction always acts in a direction, opposite to that in which the body tends to move.
- The magnitude of the force of friction is exactly equal to the force, which tends to move the body.
- The magnitude of the following friction bears a constant ratio, to the normal reaction between the two surfaces.

$$\text{i. e., } \frac{F}{R} = \text{Constant}$$

$$\therefore \frac{F}{R} = \mu;$$

Where, μ is the coefficient of friction.

F is the limiting friction.

R is the normal reaction.

- The force of friction is depends upon the roughness of the surfaces.
- The force of friction is independent of the area of the contact between two surfaces.

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b) A body (Car) of weights 150N is pulled up on a rough inclined plane whose inclination to the horizontal plane is 30° by force of 100N being applied parallel to inclined plane. Calculate the angle of friction.

➤ Solution,

Given that;

$$\text{Weight of body } (W) = 150 \text{ N}$$

$$\text{Angle of inclination } (\theta) = 30^\circ$$

Let, 'R' be the normal reaction, μ be the coefficient of the friction and ϕ be an angle of inclination of plane.

We know that;

$$\begin{aligned} \text{Frictional force } (F) &= \mu R \\ &= \mu W \cos \theta \\ &= \mu \times 150 \cos 30^\circ \end{aligned}$$

Again, we have from figure;

$$F + W \sin \theta = 100$$

$$F + 150 \sin 30^\circ = 100$$

$$\text{or, } \mu \times 150 \cos 30^\circ + 150 \sin 30^\circ = 100 \quad [\text{From equation (1)}]$$

$$\text{or, } \mu \times 150 \cos 30^\circ = 100 - 150 \sin 30^\circ$$

$$\text{or, } \mu = \frac{100 - 150 \sin 30^\circ}{150 \cos 30^\circ}$$

$$\text{Coefficient of friction } (\mu) = 0.192$$

$$\therefore \text{angle of friction } (\phi) = \tan^{-1} (0.192) = 10.86^\circ$$

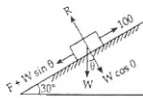
4. Locate the centroid of the area shown with respect to the given reference axis.

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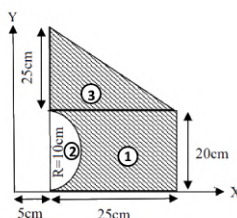
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➤ Solution:

To determine the centroid of the area shown we divide the composite figure into three parts as shown for simplicity.



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$$\begin{aligned} \text{We know, } \bar{x} &= \frac{a_1 x_1 - a_2 x_2 + a_3 x_3}{a_1 - a_2 + a_3} \\ &= \frac{(25 \times 20) \left(5 + \frac{25}{2} \right) - \left(\frac{\pi (10)^2}{2} \right) \left(5 + \frac{4(10)}{3\pi} \right) + \left(\frac{1}{2} \times 25 \times 25 \right) \left(5 + \frac{25}{3} \right)}{(25 \times 20) - \left(\frac{\pi (10)^2}{2} \right) + \left(\frac{1}{2} \times 25 \times 25 \right)} \\ &= \frac{8750 - (1452.06) + 4166.66}{500 - 157.08 + 312.5} = \frac{11464.66}{655.42} = 17.49 \text{ cm} \\ \bar{y} &= \frac{a_1 y_1 - a_2 y_2 + a_3 y_3}{a_1 - a_2 + a_3} = \frac{500(10) - 157.08(10) + 312.5 \left(20 + \frac{25}{3} \right)}{655.42} \\ &= \frac{5000 - 157.08 + 8854.16}{655.42} = 18.74 \text{ cm} \\ \text{Hence, Centroid } (\bar{x}, \bar{y}) &= (17.49, 18.74) \text{ cm} \end{aligned}$$

5. a) State parallel axis theorem.

➤ It states that "the moment of inertia of a plane area about any axis parallel to the centroidal axis is equal to the sum of moment of inertia about a parallel centroidal axis and the product of area and square of the distance between them (i.e., to areas)." Mathematically;

$$I_{xx} = I_{xx'} + A \bar{y}^2$$

b) Prove that MOI inertia of triangle about centroidal X-X axis is $\frac{bh^3}{36}$, where b is the breadth of triangle and h is the height of triangle.

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> Solution:-

Let us consider a triangle as shown in the figure with the assumed reference axes. Let 'b' be its base and 'h' be its height from base to the vertex. Now, consider a strip of width dy at a distance of 'y' from base.

From similar triangles OAB and ACD; we have,

$$\frac{x}{b} = \frac{(h-y)}{h}$$

or, $x = \frac{b(h-y)}{h}$

M.I. of the strip about the base (x) = $dA y_s^2$

$$\text{and, } dI_x = dA y_s^2 = \frac{b(h-y)}{h} dy \times y^2$$

M.I. of whole area about base;

$$I_{\text{base}} = \int dI_x = \int_0^h \frac{b}{h} (h-y) y^2 dy$$

$$= \frac{b}{h} \left[h y^3 - \frac{y^4}{4} \right]_0^h = \frac{b}{h} \left[\frac{h^4}{3} - \frac{h^4}{4} \right] = \frac{bh^3}{12}$$

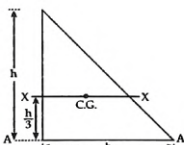
$$\text{M.I. about base} = \frac{bh^3}{12}$$

According to parallel axis theorem; we have,

$$I_{\text{base}} = I_{XX} + A X^2$$

$$\text{or, } I_{XX} = I_{\text{base}} - \frac{Ah^2}{9}$$

$$= \frac{bh^3}{12} - \frac{1}{2} \times b \times h \times \frac{h^2}{9}$$



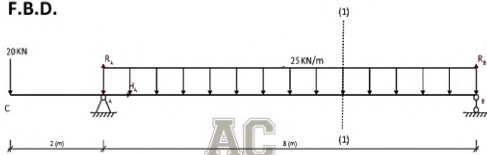
$$\text{and } \uparrow (+ve) \sum F_y = 0$$

$$\text{or, } R_A + R_B - 25 \times 8 - 20 = 0$$

$$\text{or, } R_B = 220 - 125$$

$$\therefore R_B = 95 \text{ kN } (\uparrow)$$

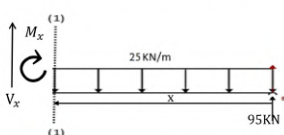
F.B.D.



> Calculation Of Shear Force and Bending Moment

For member AB:

Taking the right part of section (1)-(1) from x distance from B,



7. Determine the forces in each member of truss as shown in the figure and state whether it is tension or compression.



> Solution:

Let us first calculate the reactions at supports.

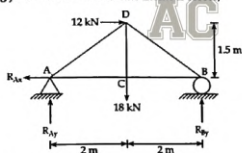
$$R_{Ay} \times 4 = 18 \times 2 + 12 \times 1.5$$

$$\therefore R_{Ay} = 4.5 \text{ kN } (\uparrow)$$

$$R_{By} = 18 - 4.5 = 13.5 \text{ kN } (\uparrow)$$

$$\text{and, } R_{Ax} = 12 \text{ kN } (\leftarrow)$$

Now, using joint method to calculate member forces:



Joint A

$$\sum F_y = 0$$

$$\text{or, } R_{AD} \sin 36.87^\circ + 4.5 = 0$$

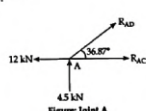
$$\text{or, } R_{AD} = -7.5 \text{ kN}$$

$$\therefore R_{AD} = 7.5 \text{ kN } (C)$$

$$\sum F_x = 0$$

$$\text{or, } R_{AC} + R_{AD} \cos 36.87^\circ - 12 = 0$$

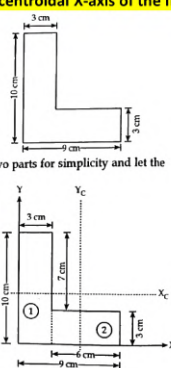
$$\therefore R_{AC} = 18 \text{ kN } (T)$$



$$= \frac{bh^3}{12} - \frac{bh^3}{18}$$

$$= \frac{bh^3}{36}$$

or) Find the moment of inertia about the centroidal X-axis of the figure shown below.



> Solution:

Let us divide the composite figure into two parts for simplicity and let the axis be as shown in the figure.

Now, finding centroid; we get,

$$y_1 = \frac{10}{2} = 5$$

$$y_2 = \frac{3}{2} = 1.5$$

$$\bar{y} = \frac{a_1 y_1 + a_2 y_2}{a_1 + a_2}$$

$$= \frac{(10 \times 3) (5) + (6 \times 3) (1.5)}{30 + 18}$$

$$= \frac{150 + 27}{48}$$

$$= 3.68 \text{ cm}$$

Now, for moment of inertia;

$$I_{XX} = (I_{CG})_X + A_1 h_1^2 + (I_{CG})_X + A_2 h_2^2$$

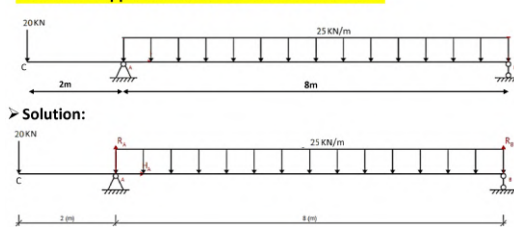
$$= \frac{b_1 d_1^3}{12} + b_1 d_1 (\bar{y} - y_1)^2 + \frac{b_2 d_2^3}{12} + b_2 d_2 (\bar{y} - y_2)^2$$

$$= \frac{3(10)^3}{12} + (3 \times 10)(3.68 - 5)^2 + \frac{6(3)^3}{12} + (6 \times 3)(3.68 - 1.5)^2$$

$$= 250 + 52.272 + 13.5 + 85.5432$$

$$= 401.31 \text{ cm}^4$$

6. Find the support reaction and draw SFD and BMD.



> Solution:

Let H_A & R_A be the horizontal reactions & Vertical reactions at support A. Let, R_B be the reaction at support B.

$$(\rightarrow) +ve \sum F_X = 0$$

$$\therefore H_A = 0$$

Taking moment at B; we get,

$$(+\odot) \sum M_B = 0$$

$$\text{or, } 20 \times 10 - R_{AY} \times 8 + 25 \times 8 \times \left(\frac{8}{2}\right) = 0$$

$$\text{or, } R_A = \frac{1000}{8}$$

$$\therefore R_A = 125 \text{ kN } (\uparrow)$$

For member CA: Shear Force

$$C_L = 0 \text{ kN}$$

$$C_R = -20 \text{ kN}$$

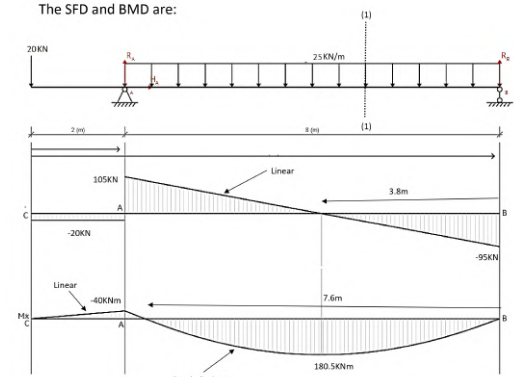
$$A_L = -20 = -20 \text{ kN}$$

$$C_R = -20 + 125 = 105 \text{ kN}$$

Bending Moment; From right to Left

$$M_C = 125 \times 2 - 25 \times 8 \times (2 + 4) + 95 \times 10 = 0 \text{ kNm}$$

The SFD and BMD are:



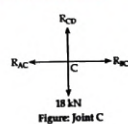
Joint C

$$\sum F_y = 0$$

$$\text{or, } R_{CD} = 18 \text{ kN } (T)$$

$$\sum F_x = 0$$

$$\text{or, } R_{AC} = R_{BC} = 18 \text{ kN } (T)$$



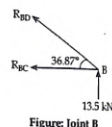
Joint B

$$\sum F_y = 0$$

$$\text{or, } R_{BD} \sin 36.87^\circ + 13.5 = 0$$

$$\therefore R_{BD} = -22.5 \text{ kN} = 22.5 \text{ kN } (C)$$

Hence,



Member	AD	AC	CD	BC	BD
Force (kN)	7.5	18	18	18	22.5
Nature	C	T	T	T	C