

# **Data Structure and Algorithm**

**EG2202CT**

*Fourth Semester*

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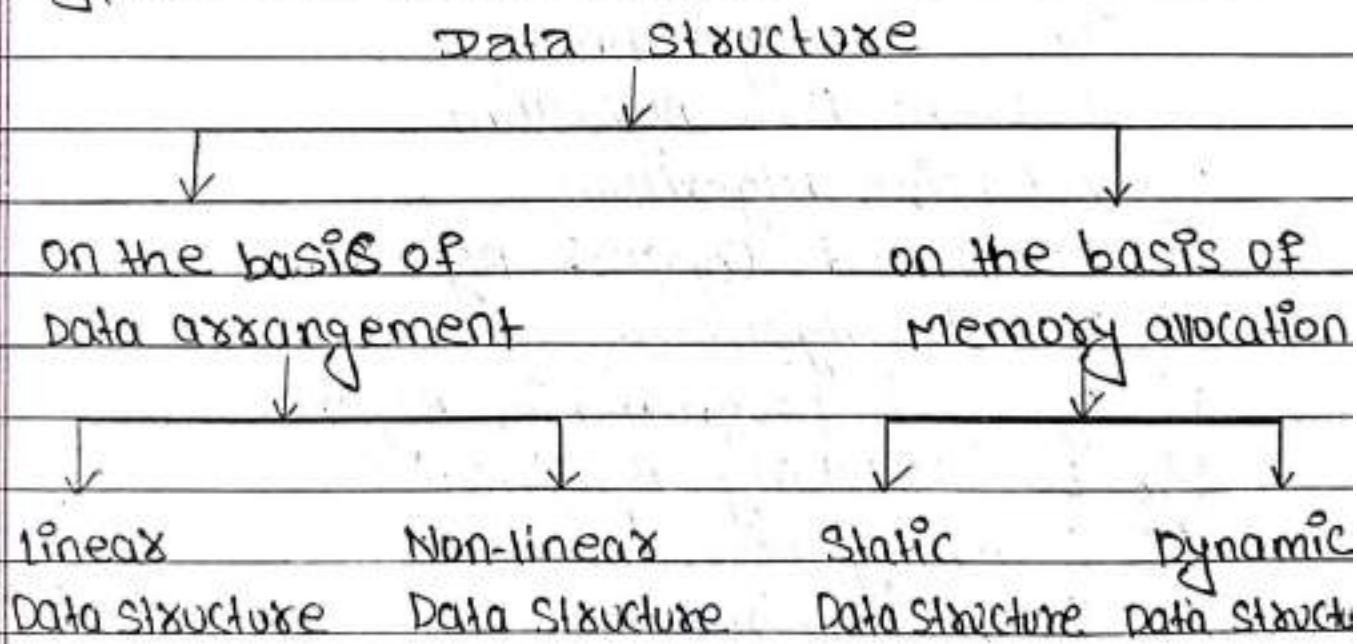
## \* Data Structure

Data structure is a way of organizing data items by considering its relationship to each other or, data structure is a way of storing data in a computer so that it can be used efficiently.

## \* Da Algorithm

An algorithm is a well defined list of steps for solving a particular problem.

## \* Types of Data structure.



## 1.1 \* Algorithm

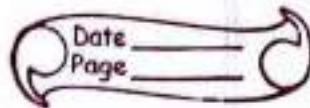
Algorithm is a set of commands that must be followed for a computer to perform calculations or other problems solving operations. It is a finite set of instructions carried out in a specific order to perform a particular task. It is not the entire program or code. It is simple logic to a problem represented as informal description in the form of statements or pseudo code.

### \* Types of Algorithm.

- 1.) Brute Force Algorithm
- 2.) Recursive Algorithm
- 3.) Divide and Conquer's Algorithm
- 4.) Greedy Algorithm
- 5.) Dynamic programming Algorithm
- 6.) Backtracking Algorithm
- 7.) Searching Algorithm
- 8.) Sorting Algorithm.

#### 1.) Brute Force Algorithm

→ Brute force algorithm are simple and straight forward solutions straight that use trial and error to solve problems. It blindly iterates all possible solutions to search one or more than one solution.



that make Solve a function.

### 2) Recursive Algorithm.

- A method that breaks a problem into smaller, smaller problems and repeatedly applies itself to solve them until reaching a base case is recursive algorithm.

### 3) Divide and conquer Algorithm

- This algorithm breaks a problem into sub-problems, solves a single sub problem and merge the solutions to get the final solutions. It consists of the following three steps: Divide, Solve, Integrate.

### 4) Greedy Algorithm.

- It makes locally optimal choices at each step in the hope of finding a global optimum useful for optimization problems but may not always lead to the best solution.

### 5) Dynamic programming Algorithm

- The algorithm works by remembering the results of the past runs and using them to find new results. In other words, it solves complex problems by breaking them into multiple simple sub problems and then it solves each of them once and then stores

them for future use. Fibonacci Series is a good example for dynamic programming Algorithm.

### 6) Backtracking Algorithm

→ Whenever a Solution is found we track back to the failure points build on the next solutions and continue this process till we find the solution.

### 7) Searching algorithm

Searching algorithm are the ones that are used for searching elements or group of elements from a particular data structure.

### 8) Sorting Algorithm

Sorting Algorithm are the ones that are used for arranging a group of data in a particular manner (e.g. ascending, descending) according to the requirement.

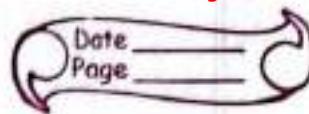
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## 1.9 \* Data Structure

Data structure is a way of organizing all data items managing and storing all data items that considers their relationship to each other. Data structure mainly specifies the following four things:

1.) Organization of data

2.) Accessing methods



- 3.) Degree of Associativity
- 4.) processing alternatives for information.

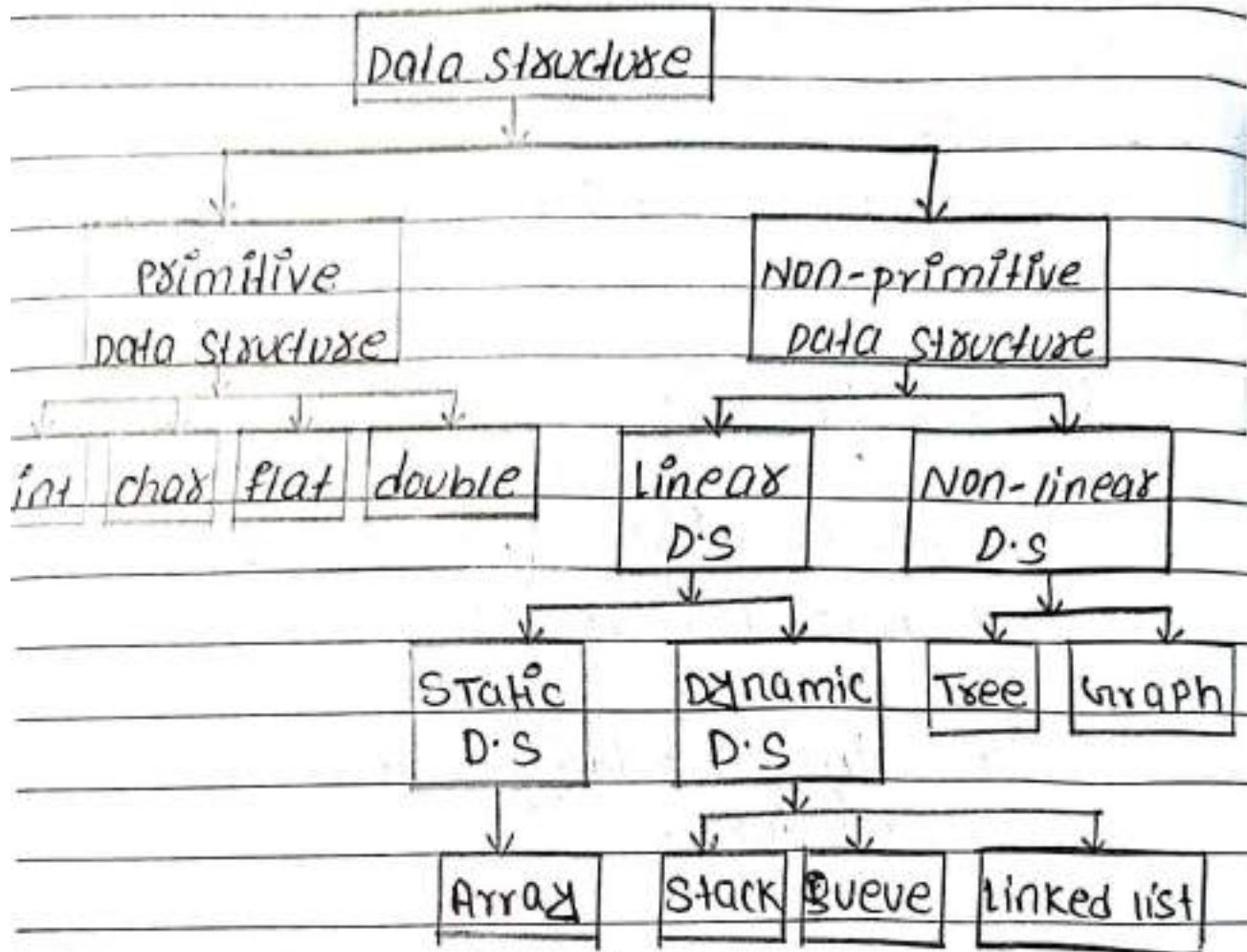
Examples of different data structures are:  
Array, stack, queue, linked list, tree, graph etc.

#### \* Operations on Datastructure.

Following are the datastructure operations

- (i) **Create** :- Create operation reserved memory space for the program elements.
- (ii) **Inserting** :- Adding new record into the data structure.
- (iii) **Deleting** :- Deleting record from the data structure.
- (iv) **Sorting** :- Arranging the records in some logical.
- (v) **Searching** :- finding data items and its location in data structure.
- (vi) **Traversing** :- Accessing each record exactly once.

## Types of data structure.

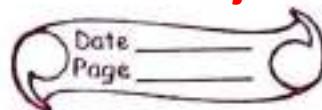


### primitive Data Structure

A primitive data structure is pre-defined by the programming language. The size and type of variable values are specified and it has no additional method. Examples are integer, character, float, double, pointer etc.

### Non-primitive Datastructure.

These data types are not actually defined by the programming language but are created by the programmer. Examples are : Array, stack, queue, linked list, Tree, graph etc.



Non-primitive data structure are further divided into two categories.

- (i) Linear Datastructure
- (ii) Non-linear Datastructure

### (i) Linear Datastructure

Linear datastructure consist of data elements arranged in a sequential manner where every element is connected to its previous and next elements. This connection helps to traverse a linear arrangement in a single level and in a single run.

Example are : Array, stack, queue, linked list etc.

### (ii) Non-Linear Datastructure

In Non-linear datastructure, data are stored without focusing on it's sequence and the data cannot be arranged and accessed in a sequence. Examples are : Tree, graph etc.

### \* Static Datastructure

Static Datastructure are datastructures where the size is allocated <sup>at</sup> the compile time. So the maximum size is fixed and cannot be changed. Examples are : Array.

## \* Dynamic Data Structure

Dynamic Datastructures are Data-structure where the size is allocated at the run time. hence the maximum size is flexible and can be changed as per requirements. Examples are: Stack, Queue, linked list etc.

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## \* Unit - 2 \*

### Stack

#### 2.1 \* Stack

Stack is a non-primitive linear Datastructure. It is an ordered list in which addition of new data item or deletion of an already existing data item is done at one end.

Called top of the stack. The insertion operation is called push operation and the deletion operation is called pop-operation.

Since all insertion and deletion take place at the same end, The last element added to the stack will be the first element removed/released from the stack. Therefore stack is (called LIFO (last in first out)) datastructure.

## \* push operation

The processing of putting a new data item onto stack is known as push operation which involves a series of steps:

- (i) checks if the stack is full.
- (ii) if the stack is full, we cannot insert data into the stack.
- (iii) if stack is not full, increments top to point next empty space.
- (iv) At data item to the stack location at where top is pointing.
- (v) Returns success.

The below figure illustrate the concept of push operation.

		Stack →	E	4
Top →	D		D	3
	C	2	C	2
	B	1	B	1
	A	0	A	0

Stack

Stack

Push operation

## \* Algorithm for push operation.

Description:-

Let Stack STACK is an array. Let MAX is the maximum of the array STACK and

JITEM is the element to be inserted.  
Top denotes the position of the Top  
element in the stack and initially we set  
Top = -1 to denote stack is empty.

Steps :-

- (i) if [TOP is equal to MAX - 1] THEN display "Stack is overflow".
- (ii) ELSE
  - (a) Set TOP = TOP + 1
  - (b) Set STACK[TOP] = JITEM
  - (c) Display "JITEM is inserted".
- (iii) END IF
- (iv) EXIT.

### \* pop-operation.

The process of deleting the existing data  
element from stack is known as pop operation.  
A pop operation may involve the following steps.

Step 1 : - check if the stack is empty.

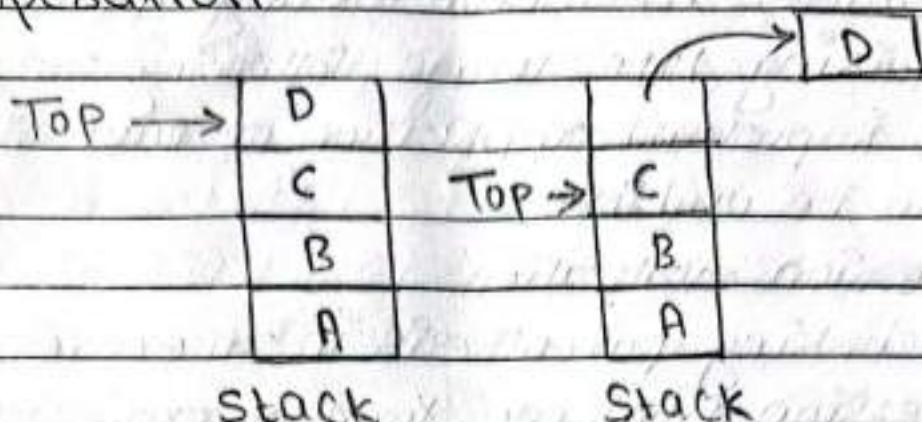
Step 2 : - if the stack is empty, display stack  
empty message and exit.

Step 3 : - if stack is not empty, delete the data  
element at which Top is pointing.

Step 4 : - decrease the value of Top by 1.

Step 5 : - Returns Success.

The below figure illustrate the concept of pop operation.



### \* Algorithm for pop operation:

Description:-

Let STK is an array having size MAX,  
 Top is used to points the top most element of  
 the array and DEL is the value to be deleted

Steps:-

- (i) if  $[\text{TOP} < 0]$  THEN Display "STK is empty / underflow"
- (ii) ELSE
  - (a) Set  $\text{DEL} = \text{STK}[\text{TOP}]$
  - (b) Set  $\text{TOP} = \text{TOP} - 1$
  - (c) Display  $\text{DEL}$  is deleted.
- (iii) END IF
- (iv) EXIT.

## 9.2 \* Application of STACK.

STACKS ARE USEFUL FOR ANY APPLICATION REQUIRING LIFO STORAGE FOLLOWING ARE SOME OF THE IMPORTANT APPLICATION OF DATA STRUCTURE.

IT CAN BE USED FOR:

- (i) Expression evaluation
- (ii) Checking parenthesis in an expression.
- (iii) Conversion from one form of expression to another.
- (iv) Memory management.
- (v) Management of undo and redo operation.
- (vi) Function call management.
- (vii) Travelling trees and graphs.
- (viii) Recursive call management.

## \* Expression.

Expression is define as the number of operands combined with several operators.

These are basically three types of Notation (way of writing expression) for an expression. These are: In-fix.

- (i) In-fix Notation  $\rightarrow A+B, x*y$
- (ii) pre-fix Notation  $\rightarrow +AB, *xy$
- (iii) post-fix Notation  $\rightarrow AB+, xy*$

## Expression Notation conversion Priority of Operators Table

Operations	Symbols	Priority
Paranthesis	( )	First
Exponentiation	\$, $\uparrow$	Second
Multiplication, division	*	Third
Addition, substraction	+, -	Fourth

### Infix to prefix conversion Rules.

Rules :-

- (i) The expression is scanned from left to right until an operator of higher priority is found.
- (ii) Once an operator of higher priority is found, the operator is shifted to its left.
- (iii) Scanning is done again from left to right and all the operators are shifted in this manner.

Examples:-

(i)  $a + b * c$  (Infix)

a + b c

+ a \* b c (prefix)

(ii)  $(a+b)*c$  (infix)

+ a b \* c

\* + a b c (prefix)

(iii)  $a + b \$ c * d$  (Infix)

a + \\$ b c \* d

+\\$bcda a + \* \\$bcd

+\\$abcd + a \* \\$bcd (prefix)

2.2.1 Rules :- (Infix to postfix conversion)

- (i) The expression is scanned from left to right until an operator of higher priority is found.
- (ii) Once an operator of higher priority is found, the operator is shifted to its right.
- (iii) Scanning is done again from left to right and all the operators are shifted in this manner.

(i)  $a+b*c$  (Infix)

$a+b*c$

$a b c * +$  (Postfix)

(ii)  $(a+b)*c$  (Infix)

$a b + * c$

$a b + c * +$  (Postfix)

(iii)  $a+b\$c*d$  (Infix)

$a+b c \$ * d$

$a+b c \$ d *$

$b c \$ d * a +$  (Postfix)

$a b c \$ d +$

(iv) Convert into prefix notation.

$(A+B)(B*C)$

$A B + * B C$

$+ A B * B C$  Prefix

$$(2) ((A \times (B * C)) / (D E))$$

$$((A \times * B C Y) / (D E))$$

$$(+ A \times B C Y) / (D E)$$

$$/ + A \times B C Y D E \text{ prefix}$$

Assignment

Infix to prefix and postfix.

$$(i) ((A+B) * (C+E)) \quad (ii) ((A+B) * (C+E))$$

$$(+ A B * + C E) \quad (A B + * C E +)$$

$$* + A B + C E \text{ (prefix)}$$

$$A B + C E + * \text{ (postfix)}$$

$$(iii) A X * (B X * ((C Y + A Y) + B Y) * (X))$$

$$(A X * (B X * ((+ C Y A Y + B Y) * X)))$$

$$(A X * (B X * (+ C Y A Y + B Y) * X))$$

$$(A X * (B X * * C Y A Y + B Y) * X)$$

$$* A X * B X * C Y A Y + B Y X \text{ (prefix)}$$

$$(iv) A X * (B X * ((C Y + A Y) + B Y) * (X))$$

$$(A X * (B X * ((+ C Y A Y + B Y) * X)))$$

$$(A X * (B X * (+ + C Y A Y B Y) * X))$$

$$(A X * (B X * * + + C Y A Y B Y X))$$

$$* A X * B X * + + C Y A Y B Y X \text{ (prefix)}$$

$$* * * + + A X A Y B Y X \text{ (prefix)}$$

$$* A X * B X * + + C Y A Y B Y X \text{ (prefix)}$$

(iii)  $A + B \uparrow C * D / (E - F)$

$A + B \uparrow C * D / - EF$

$A + \cancel{B} \uparrow BC * D / - EF$

$A + * \uparrow BCD / - EF$

$A + 1 \cancel{*} \uparrow BCD - EF$

$+ A / * \uparrow BCD - EF$  prefix

(iv)  $A + B \uparrow C * D / (E - F)$

$A + B \uparrow C * D / EF -$

$A + \cancel{BC} \uparrow * D / EF -$

$A + \cancel{BC} \uparrow D * / EF -$

$A + \cancel{BC} \uparrow D * EF - /$

$A B C \uparrow D * EF - / +$  postfix prefix

vii)  $(Ax * (Bx * ((ay + ay) + by) * (x)))$

$(Ax * (Bx * ((\cancel{ay} + + by) * (x)))$

$(Ax * (Bx * (\cancel{ay} + by + * (x)))$

$(Ax * (Bx * (\cancel{ay} + by + (x * \cancel{x})))$

$(Ax * (Bx * (\cancel{ay} + by + (x * \cancel{x} *)))$

$(Ax * Bx * \cancel{ay} + by + (x * \cancel{x} *))$

$(Ax * Bx * \cancel{ay} + by + (x * \cancel{x} * \cancel{x} *))$  postfix.

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$$A * (B + C \$ 0) - E \$ F * (G / H)$$

In postfix:-

$$A * (B + C \$) - E \$ F * (G / H)$$

$$\underline{A * B C 0 \$ +} - \underline{E \$ F * (G / H)}$$

$$\underline{A * B C 0 \$ +} - E \$ F * \underline{G H /}$$

$$A * B C 0 \$ + - \underline{E F \$ * G H /}$$

$$\underline{A B C 0 \$ + * - E F \$ G H / *}$$

$$A B C 0 \$ + * E F \$ G H / * -$$

\* WAP for stack operations.

#include &lt;stdio.h&gt;

#include &lt;conio.h&gt;

void push();

void pop();

void display();

#define max 10

int stk[max], i, choice, top = -1;

void main()

S

Home :

printf ("\n 1. push");

printf ("\n 2. pop");

printf ("\n 3. display");

printf ("\n 4. EXIT");

printf ("Enter your choice: ");

scanf ("%d", &amp;choice);

## \* switch (choice)

S

case 1 : push(); getch(); clrscr(); goto home;

case 2 : pop(); getch(); clrscr(); goto home;

case 3 : Display(); getch(); clrscr(); go to home;

case 4 : Exit();

default :

printf("In Invalid choice");

getch(); clrscr(); goto home;

2

2

⇒ void push()

S

int n;

if (top == max - 1)

{

printf("In stack is full");

2

else

{

printf("In Enter no to be inserted");

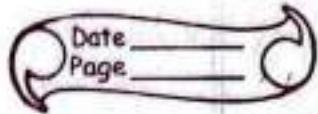
scanf("%d", &n);

top++;

stk [top] = n;

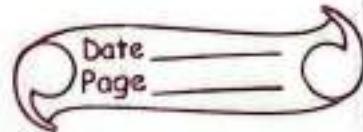
printf("In %d is inserted", n);

2



3  
 $\Rightarrow$  void pop()  
 S  
 if ( $top == -1$ )  
 S  
 printf ("The stack is empty");  
 2  
 else  
 S  
 printf ("\n%d is deleted ", STK[TOP]);  
 top = top - 1;  
 2  
 2

$\Rightarrow$  void pop()  
 S  
 int del;  
 if ( $top < 0$ )  
 S  
 printf ("The stack is empty");  
 2  
 else  
 S  
 del = STK[TOP];  
 top = -1;  
 printf ("\n%d is deleted ", del);  
 2  
 2



⇒ void display()

{

int P=top;

if (top == -1)

{

printf ("In stack is empty");

{

else

{

for (; P >= 0; P--)

{

printf ("In %d", STK[P]);

{

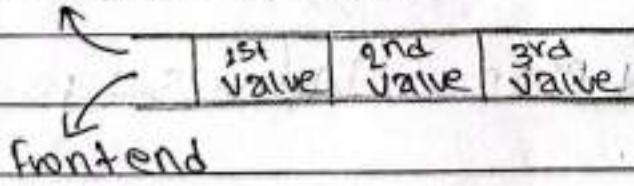
{

## 2. Queue

A queue is a non-primitive linear data structure. It is open at both of its ends. It is an ordered list in which one end is always used to insert data (called rear) and the other end is always used to remove data. In queue datastructure the insertion operation is called Enqueue operation and deletion operation is called dequeue.

Since the insertion of new data item is done only through the dedicated end (rear) and deletion of an existing data item is done through <sup>the</sup> other dedicated end (front), the first element added to the queue will be the first element to be removed from the queue. Therefore, queue is also called as FIFO (First in First Out) ordered data structure.

dequeue operation



enqueue operation

Rear end

## 2.3.1 Enqueue operation

Enqueue operations involves the following series of steps.

- 1.) checks whether the queue is full or not.
- 2.) If the queue is full then display "Queue is full".

- 3) if the queue is not empty, then increments  
rear to point makes next empty space.
- A.) Insert data element to the queue location  
where rear is pointing.
- 5.) Returns Success.

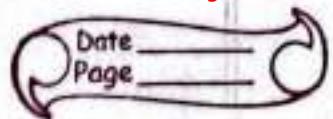
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### 2.3.2 Algorithm for Enqueue operation.

5 \* Description  
 Let queue (QUE) QUE is an array,  
 MAX is the maximum size of the array  
 QUE, ITEM is an element to be inserted,  
 FRONT and REAR denotes the front and  
 rear in of the queue. An initially its value  
 is set to -1 to denote queue is empty.

Steps:-

- 1.) IF [REAR=MAX-1] THEN display "QUE is full / overflow".
- 2.) ELSE
  - a.) if (FRONT=-1 and REAR=-1) then Set FRONT=0 - and REAR=0.
  - b.) else set REAR=REAR+1.
  - c.) end If .
  - d.) QUE (REAR)=ITEM.
  - e.) Display ITEM is inserted .
  - f.) END If .



6

void enqueue()

{

if ( $r == \text{max} - 1$ )

{

printf("Queue is full")

{

else

{

if ( $f == -1 \text{ and } r == -1$ )

{

$f = 0$ ;

$r = 0$ ;

{

else

{

$r++$ ;

{

printf("Enter a no");

scanf("%d", &item);

QUE(r) = item;

{

{

## 2.3.2 Algorithm for Dequeue Operation.

## \* Description

Let QUEUE is an array, MAX is it's maximum size and DEL is an element to be deleted. FRONT and REAR denotes the front and rear end of the queue.

Steps :-

- 1.) IF [FRONT = -1 and REAR = -1] THEN Display "Queue is underflow / Empty".
- 2.) ELSE
  - (a) Set DEL = QUEUE [FRONT]
  - (b) If [FRONT = REAR] then
    - (a.) Set FRONT = -1
    - (b.) Set REAR = -1
  - (c) else
    - Set FRONT = FRONT + 1
  - (d) end if
  - (e) Display : DEL is deleted
- (3.) END IF
- (4.) EXIT

## 2.3.6 \* Applications of QUEUE

- (1) QUEUES are widely used as waiting list for a single shared resource like printer, disk, CPU.
- (2) When data is transferred asynchronously between two processes, que is used for synchronization for example Job buffers, file input output (I/O) etc.
- (3) QUEUES are used in operating systems for handling interrupts in real time system.
- (4) Call center phone systems use queues to hold people & calling them in order.

## \* Limitation of QUEUE

As we know after ENQUEUEing and DEQUEUEing operations, the size of the queue has been reduced. If REAR reaches the last index and FRONT is not at index zero, we cannot insert element in the QUEUE even there is free spaces in the QUEUE. This is the limitation of (Linear) QUEUE.

Enqueue

Display

Exit

#include <stdio.h>

#include <conio.h>

#include <process.h>  
int que[10], item, r=-1, f=-1, choice;

void main()

{

home :

printf("In 1. Enqueue");

printf("In 2. Display");

printf("In 3. Exit");

printf("Enter your choice : ");

scanf("%d", &choice);

switch (choice)

{

Case 1 : enq(); getch(); clrscr(); goto home;

Case 2 : display(); getch(); clrscr(); goto home;

Case 3 : exit(0);

default : printf("In wrong choice"); getch(); clrscr();  
goto home;

?

?

void enq()

{

if (r == 9)

{

8  
 printf("In queue is full / overflow");

3

else

2

{ if (f == -1 && r == -1)

5

f = 0, r = 0;

3

else

2

r++;

3

printf("In enter a number");

scanf("%d", &item);

que[r] = item;

printf("In %d is inserted", item);

3

2

void display()

2

int t = f;

{ if (f == -1 && r == -1)

5

printf("In queue is empty");

3

else

2

for ( ; t < r ; t++ )

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3  
print("In 1.d", quevert);

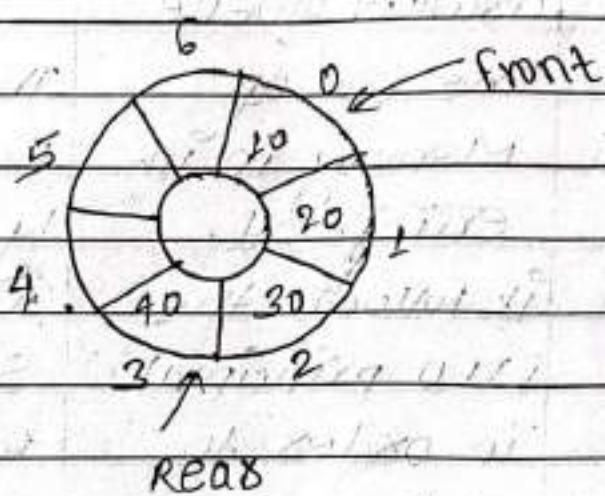
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3

3

## Circular Queue

Circular Queue is just a variation of the linear queue in which front and rear end are connected to each other to optimize up the space wastage of the linear queue and make it efficient. The below figure illustrates the concept of Circular Queue.



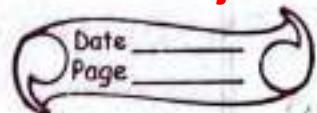
## Difference between linear queue and circular queue

Basis	Linear	Circular
1.) Meaning	The linear queue is a type of linear data structure that contains the elements in a sequential manner.	The circular queue is also linear data structure in which the last element of the queue is connected to the first element of the queue.
2.) Insertion and deletion	In linear queue insertion is done from the rear end and deletion can done	In circular queue, the insertion and deletion can done from both ends.

		and deletion is done from the front end.	In circuits from any end.
3.)	Memory Space	The memory space occupied by the linear queue is more than the circular queue.	it requires less memory as compared to linear queue.
4.)	Memory Utilization	The use of memory is inefficient.	The memory can be more efficiently utilized.
5)	Order of Execution	It follows the FIFO principle in order to perform the task.	It has no specific order for execution.

Advantage of circular queue over linear queue.

- i.) Linear queue consumes more memory as compared to circular queue.
- ii.) A circular queue uses an efficient way for memory utilization.
- iii.) In a circular queue, new data can be inserted again at a particular position after deleting previous data on that position.
- iv.) In a circular queue, an overflow message is shown when the queue is full.



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## Types of Queue

Mainly there are three types of queue. These are

- (i) Linear Queue
- (ii) Circular Queue
- (iii) Priority Queue

Enqueue

Dequeue

Display

Exit

```
#include <stdio.h>
```

```
#include <conio.h>
```

```
#include <process.h>
```

```
void enqueue();
```

```
void dequeue();
```

```
void display();
```

```
#define max 10
```

```
int enque[10], item, choice, r = -1, f = -1, del;
```

```
void main()
```

S

home:

```
printf("Enqueue()"); getch(); clrscr(); goto home;
```

```
printf("In 1. Enqueue");
```

```
printf("In 2. Dequeue");
```

```
printf("In 3. Display");
```

```
printf("In 4. Exit");
```

```

printf("Enter your choice:-");
scanf("1-d", &choice);
switch(choice)
{
    case 1 : enqueue(); getch(); clrscr(); goto home;
    case 2 : dequeue(); getch(); clrscr(); goto home;
    case 3 : display(); getch(); clrscr(); goto home;
    case 4 : Exit(0);
    default : printf("\n wrong choice"); getch(); clrscr(); goto home;
}

```

void enqueue()

```

if (r == g)

```

```

printf("In queue is full");

```

else

```

if (r == -1 && f == -1)

```

```

    f = 0; r = 0;

```

else

```

    r++;

```

```

printf("\n Enter a number");

```

Scan & l" /d ", item);

queue[r] = item;

printf("In /d is inserted", item);

}

}

void dequeue()

{

if (f == -1 && r == -1)

{

printf("In queue is empty");

}

else

del = queue[f];

if (f == r)

{

f = -1, r = -1

}

else

{

f++;

}

printf("In /d is deleted", del);

{

{

void display()

{

int t = f;

if (f == -1 && r == -1)

```
{  
    printf("\n queue is empty");  
}
```

```
else
```

```
for(l=t<=r; t++)
```

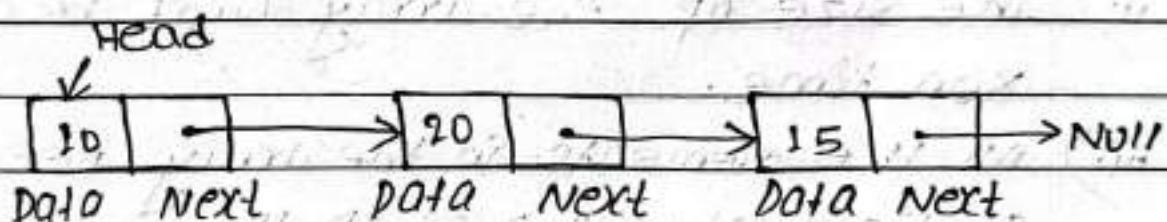
```
{  
    printf("\n %d", queue[t]);  
}
```

```
}  
}
```

## \* List / Linked List (singly linked list)

A linked list is a linear datastructure that includes a series of connected nodes. Each element in a linked list datastructure is stored in the form of a node. Here each node has two parts. One is used to store the data and the other is used to store the address of the next node.

The elements in a linked lists are linked using pointers as shown in below figure.



### \* Structure of linked list:-

each node consist :-

(i) A data item.

(ii) An address of next node

we combine both the data item and the next node address in a struct as :

struct node

{

int data;

struct node \*next;

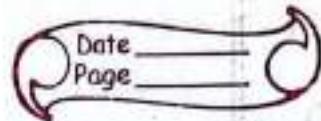
- \* Application of linked list.
- (i) Dynamic Memory Allocation (DMA)
- (ii) Implemented in stack and queue for creating dynamic environment.
- (iii) Undo functionality of softwares.
- (iv) Hash tables and graphs implementation.

- \* Advantage of linked list over array / limitation of array.

- i.) The size of array must be known in advance before using it in the program.
- ii.) The size of the array can't be changed at run time.
- iii.) All the elements in the array need to be continuously stored in the memory.
- iv.) Inserting and deleting any element in the array needs shifting of elements.

Linked list datastructure overcome all the limitation of an array some of the advantage of linked list over array are:

- i.) Sizing is no longer a problem since we don't need to define its size at the time of declaration.
- ii.) It allocates the memory dynamically so the size of linked list can be changed when ever require.



- iii) All the nodes of linked list are non-continuous stored in the memory and linked together with the help of pointers.
- iv) Insertion and deletion of nodes is easy just by adjusting the pointer parts of few nodes.

### \* Operation on Linked list:

- 1.) Node and list creation.
- 2.) Insert elements to a linked list.
- 3.) Insert at the beginning of list.
- 4.) Insert at the end of list.
- 5.) Insert at the middle of list.
- 6.) Delete elements from linked list.
- 7.) Delete the beginning element.
- 8.) Delete the end element.
- 9.) Delete the middle element.

### 2.) Node creation

struct creation of a node

S

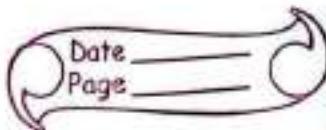
int data;

struct node \*next;

};

struct node \*head, \*ptr;

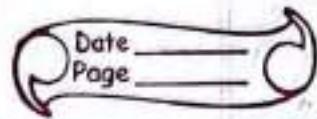
ptr = (struct node \*) malloc (sizeof(struct node));



Q.) i.) WAP to create a single node, and display its contents.

```
#include <stdio.h>
#include <conio.h>
#include <malloc.h>

struct node
{
    int data;
    struct node * next;
} * head, * ptr;
void main()
{
    int item;
    printf("Enter a Number");
    scanf("%d", &item);
    ptr = (struct node *) malloc(sizeof(struct node));
    ptr->data = item;
    ptr->next = NULL;
    clrscr();
    printf("Elements of Node is : %d", ptr->data);
    getch();
}
```



Q.2) WAP to create two nodes and link each node with each other.

```
#include <stdio.h>
#include <conio.h>
#include <malloc.h>
```

Struct node

{

int data;

Struct node \* next;

}

Struct node \*ptr1, \*ptr2;

void main()

{

int item1, item2;

printf("Enter two numbers");

scanf("%d %d", &item1, &item2);

ptr1 = (Struct node \*) malloc ( sizeof(Struct node));

ptr2 = (Struct node \*) malloc ( sizeof(Struct node));

ptr1 -> data = item1;

ptr2 -> data = item2;

ptr1 -> next = ptr2;

ptr2 -> next = NULL;

printf("\n Elements of list are :\n");

printf("%d", ptr1 -> data);

printf("%d", ptr2 -> data);

printf("%d", ptr1 -> next -> data);

getch();

2

WAP to create 5 nodes and link these nodes -

W

Q.No.4 WAP to create n nodes and link these with each other.

```
#include <stdio.h>
```

```
#include <conio.h>
```

```
#include <malloc.h>
```

```
struct mynode
```

```
{
```

```
int d;
```

```
struct mynode *link;
```

```
};
```

```
void main()
```

```
{
```

```
int n, i, item;
```

```
printf("Enter total No. of nodes");
```

```
scanf("%d", &n);
```

```
for (i=1 ; i<=n; i++)
```

```
{
```

```
ptr = (struct mynode *) malloc (sizeof (struct mynode));
```

```
printf("Enter an element");
```

```
scanf("%d", &item);
```

```
ptr->d = item;
```

```
ptr->link = NULL;
```

```
if (start == NULL)
```

```
{
```

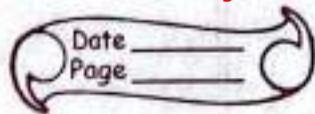
```
start = ptr;
```

```
else
```

```
{
```

```
temp = start;
```

```
@
```



15

while (*temp* → *link* != *NULL*)

{

*temp* = *temp* → *link*;

{

*temp* → *link* = *ptr*;

{

{}

*temp* = *start*;

while (*temp* != *NULL*)

{

printf("%d", *temp* → *d*);

*temp* = *temp* → *link*;

{

getch();

{

(Q-N05) WAP to create *n* nodes and links these which each other. (Addition of *n* node).

#include <stdio.h>

#include <conio.h>

#include <malloc.h>

Struct mynode

{

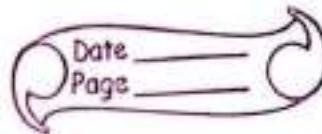
int *d*;

struct mynode \* *link*;

2 \* *start* = *NULL*, \* *ptr*, \* *temp*;

void main()

{



```

int n, i, item, sum=0;
printf("Enter total no. of nodes");
scanf("%d", &n);
for(i=1; i<=n; i++)
{
    p1=(struct MyNode *)malloc(sizeof(struct MyNode));
    printf("Enter an element");
    scanf("%d", &item);
    p1->d=item;
    p1->link=NULL;
    if (start == NULL)
        start = p1;
    else
    {
        temp = start;
        while (temp->link != NULL)
            temp = temp->link;
        temp->link = p1;
    }
    temp = start;
    while (temp != NULL)
        sum += temp->d;
}

```

(17)   
 $\text{Sum} = \text{Sum} + \text{temp} \rightarrow \text{display and assign}$   
 $\text{print} \& " \cdot \text{d}" , \text{temp} \rightarrow \text{dJj}$   
 $\text{temp} = \text{temp} \rightarrow \text{link};$

?

getch();

2

: low fri

: answer &amp; place true?

: (num / 1000, num % 1000) - place true

: ("num at later 1/1000 num")

: ("num at later 1/1000 num")

~~18-19~~ Insertion of Node in Linked List.

: Insert node → answer - 0.

(1.) Algorithm to insert node of the Beginning of list  
Description.

Suppose START is the first position in the linked list, DATA is the element to be inserted in the new node and next NEXT is the pointer to the next node in the list.

Steps :- i. Create a newNode.

ii. Input DATA to be inserted.

iii. NewNode → DATA = VAL

iv. if [START is equal to NULL] THEN

Set START = NewNode

v. ELSE

(a) Set NewNode → NEXT = START

(b) Set START = NewNode

vi. END IF

vii. Display "VAL is inserted"

viii. EXIT.

\* function for inserting a node at the Beginning  
of list.

⇒ Void InsertAtBeg ()  
S

```

int val;
struct node * newnode;
newnode=(struct node *)malloc(sizeof(struct node));
printf("\n Enter a No.");
scanf("%d", &val);
newnode->data=val;
if (start == NULL)
{
    start=newnode;
}
else
{
    newnode->next=start;
    start=newnode;
}
printf("\n%d is Inserted", val);
  
```

- Q.) 1. 1. Create  
 2. Display  
 3. InsertAtBeginning  
 4. Exit

(15)

```
#include <stdio.h>           (1) main()
#include <conio.h>           2.
#include <malloc.h>           (2) node
Struct node                (3) insertbeg()
{
    int data;               (4) insertmid()
    struct node * next;    (5) insertend()
} *start=NULL, *ptr, *newnode, *temp;
void create();               (6) display()
void display();              (7) choice()
void insertbeg();            (8) switch()
void insertmid();            (9) case 1
void insertend();            (10) case 2
void main() {                (11) case 3
    S                         (12) case 4
    int choice;               (13) default
    home:printf("\n 1.create");
    printf("\n 2.Display");
    printf("\n 3.insertbeg");
    printf("\n 4.Exit");
    printf("\n Enter your choice\n");
    scanf("%d", &choice);
}
```

Switch(choice)

S

```
Case1: create(); getch(); clrscr(); goto home;
Case2: Display(); getch(); clrscr(); goto home;
Case3: insertbeg(); getch(); clrscr(); goto home;
Case4: exit(0);
default: printf("\n Invalid"); getch(); clrscr(); goto home;
```

?

```
void create()
```

2

```
int n, i, val;
```

```
printf("\n Enter total no of node");
```

```
scanf("%d", &n);
```

```
for (i=1; i<=n; i++)
```

3

```
ptry=(start node*)malloc(sizeof(struct node));
```

```
printf("\n Enter a No");
```

```
scanf("%d", &val);
```

```
ptry->data=val;
```

```
ptry->next=NULL;
```

```
if (start==NULL)
```

4

```
Start=ptry;
```

5

```
else
```

6

```
temp=start;
```

```
while (temp->next!=NULL)
```

7

```
temp=temp->next;
```

8

```
temp->next=ptry;
```

9

- (2.) Algorithm to Insert node at the end of list.  
 \* Description

Suppose START is the first position in linked list , VAL is the element to be inserted in the new node , NEXT is the pointer to the next node in the list and TEMP is the temporary pointer to hold the node address.

Steps -> (i) Create a NewNode.

(ii) Input VAL to be inserted.

(iii) a) Set NewNode  $\rightarrow$  DATA = VAL

    b) Set NewNode  $\rightarrow$  NEXT = NULL

(iv) IF [START is equal to NULL] THEN

    (a) START = NewNode

(v) ELSE

    (a) TEMP = START

    (b) WHILE [TEMP  $\rightarrow$  NEXT != NULL]

        (c) TEMP = TEMP  $\rightarrow$  NEXT

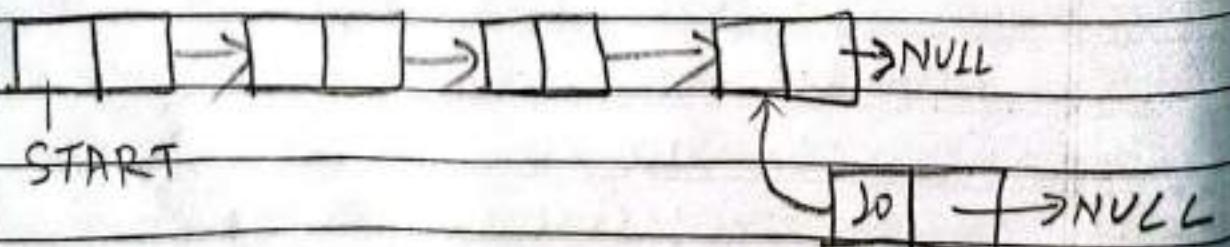
    (d) End of while

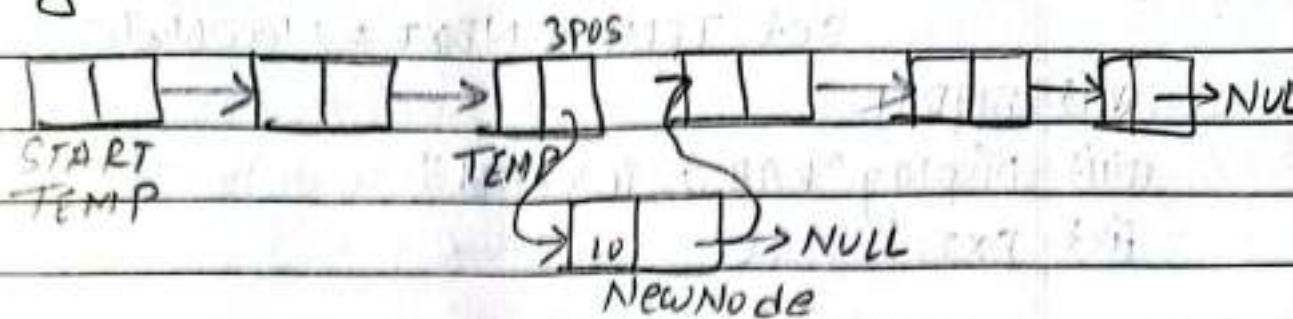
    (e) NewNode  $\rightarrow$  NEXT = TEMP

(vi) END IF

vii) Display "VAL is inserted".

viii) EXIT.



(3) Algorithm to insert node at the middle of listDescription

Suppose **START** is the first position in linked list, **VAL** is the element to be inserted in the new node, **NEXT** is the pointer to the next node in the list and **TEMP** is the temporary pointer to hold the node address. **pos** is the position where **NewNode** is inserted.

- (i) Create a **NewNode**.
- (ii) Input **VAL** to be inserted
- (iii) Input **pos** where the **NewNode** is to be inserted.
- (iv) Set **NewNode → DATA = VAL**
- b) Set **NewNode → NEXT = NULL**
- (v) if [**START** is equal to **NULL**] THEN
  - (a) **START = NewNode**
- vi. ELSE
  - (a) **TEMP = START**
  - (b) **COUNTER = 1**
  - (c) WHILE [**COUNTER <= pos**]
    - S
  - TEMP = TEMP → NEXT**
  - COUNTER** is increased by 1.

(d) Set NewNode  $\rightarrow$  NEXT = TEMP  $\rightarrow$  NEXT  
 set TEMP  $\rightarrow$  NEXT = NewNode

(vii) ENDIF

(viii) Display "VAL is inserted"

(ix) EXIT

(i) TEMP = START

(ii) START = TEMP  $\rightarrow$  NEXT

(iii) Free(TEMP)

FOR deletion at beginning

(4) Algorithm to delete first node.

Description

Let q and START are the pointers variables that hold the address of the first node.

Steps - i) if [START is equal to NULL] THEN

Display "List is not created"

(ii) ELSE

(a) Set q = START

(b) Set START = START  $\rightarrow$  NEXT

(c) Display "q  $\rightarrow$  DATA is deleted"

(d) Free(q)

(iii) ENDIF

(iv) EXIT

## (5) Algorithm to delete last node.

Description

Let  $q$  and  $START$  are the pointer variable that holds the address of first node in the list and  $p$ ,  $PREV$  is the temporary pointer variable.

steps - (i) IF [ $START$  is equal to  $NULL$ ] THEN

Display "List is not created"

(ii) ELSE

(a)  $q = START$

(b) WHILE [ $q \neq NEXT$  NOT EQUAL TO  $NULL$ ]

$PREV = q$

$q = q \rightarrow NEXT$

(c) WEND (END OF While loop)

(d)  $PREV \rightarrow NEXT = NULL$

(e) Display  $q \rightarrow DATA$  is deleted

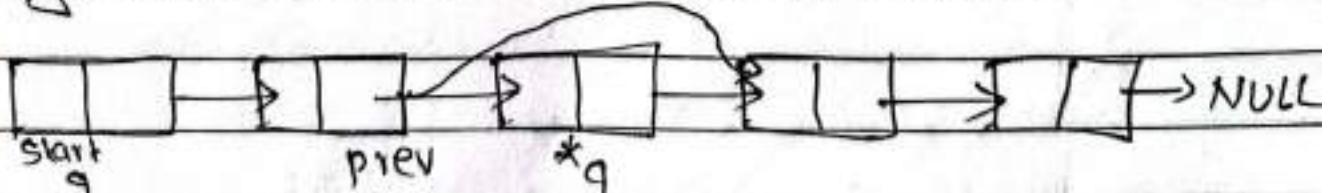
(f) free the node( $q$ )

(iii.) END IF

(iv) EXIT

08/08/24

## (6) Algorithm to delete the middle node.



Description

Let  $q$  and  $START$  are the pointer variable that holds the address of the first node in the list,  $PREV$  is the temporary pointer variable and  $pos$  is the position of the node to be deleted.

Step- (i) if [START is equal to NULL] THEN

Display "List Is Empty"

(2) ELSE

(a) Set q = START and Counter = 1

(b) WHILE [Counter < pos]

(i) Set PREV = q

(ii) q = q → NEXT

(iii) Counter = Counter + 1

(c) WEND (End of WHILE loop).

(d) PREV → NEXT = q → NEXT

(e) free the node (q)

(3) END if

\* Before the node

(Counter < pos)

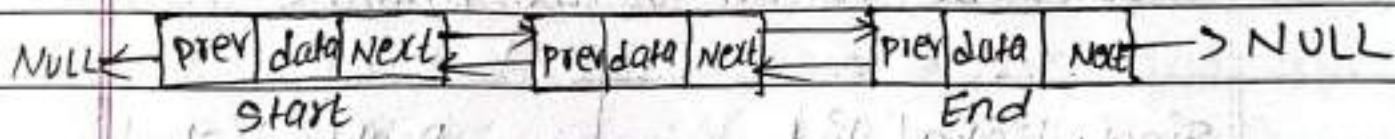
\* After the node

(Counter <= pos)

## 2. \* Doubly Linked List

Doubly linked list is a group of pointers and nodes such that each node is pointing at it's preceding as well as it's succeeding node in the list.

In Doubly linked list, each node consist of two pointers and the data. These two pointers enables the traversals of link list in both the direction, backward and forward. The below figure illustrate the concept of doubly linked list.



### \* Structure of doubly linked list.

The below structure illustrates doubly linked list structure.

struct dnode

{

  struct dnode \*prev;

  int data;

  struct dnode \*next;

}; \*start, \*end, \*ptr;

\* Advantage of Doubly linked list.

- (i) Traversal in either / both direction because convenient.
- (ii) it reduces time requirement for the program execution.
- (iii) Tree and graph datastructure can be easily represented using a doubly linked list.

\* Disadvantage of Doubly Linked list:

- (i) Each node requires extra space for storing the additional pointer.
- (ii) While manipulating the list, Extra care should be taken to manipulate both link.

### Singly Linked List

(i) Traversing is convenient in only one direction.

(ii) While deleting a node, its predecessor (previous) is required and can be found only after traversing from the beginning of the list.

(iii) Occupies less memory.

### Doubly Linked List

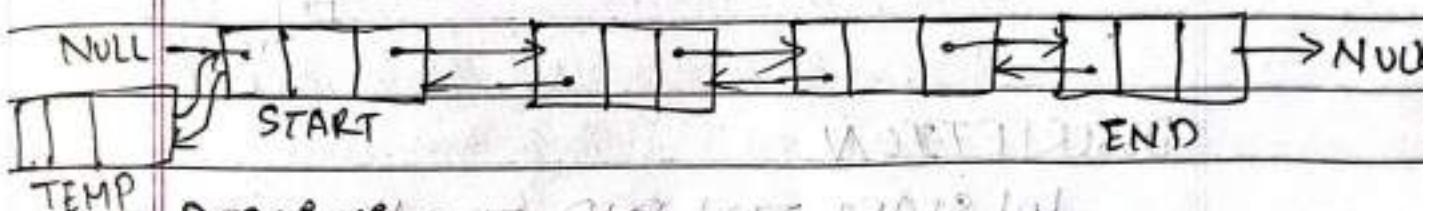
(i) Traversing can be done in both direction.

(ii) While deleting a node, its predecessor can be obtained using 'previous-link' of node. No need to traverse the list.

(iii) Occupies more memory.

- |   |  |
|---|--|
| (iv) Care is taken to modify only one link of a node. | (iv) care is taken to modify both links of a node. |
| (v) Tree & graph is not so easily simple method.      | (v)  |
- ~~is/are~~

### → Insertion at Beginning of doubly linked list



Description

Let START and END are the pointer variable that hold the address of the first and last element of the doubly linked list, VAL is the element to be inserted at the begining of the list.

Step → Create a node TEMP.

i.) a.) set TEMP → DATA = VAL

b.) set TEMP → PREV = NULL

c.) Set TEMP → NEXT = NULL

iii.) if [START is equal to NULL] THEN

a.) set START = TEMP

b.) set END = TEMP

iv.) ELSE

a.) TEMP → NEXT = START

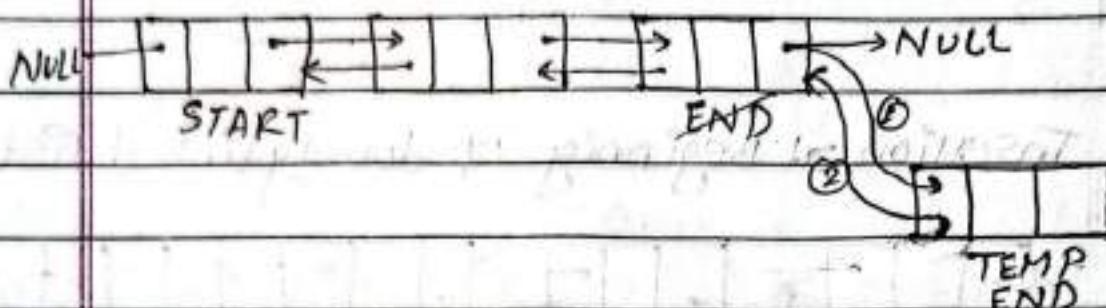
b.) START → PREV = TEMP

c)  $\text{START} = \text{TEMP}$

v) END IF

vi) EXIT.

2.) INSERTION at the END of doubly linked list.



### DESCRIPTION

let START and END are the pointers variables that hold the address of the first and last element of the doubly linked list, VAL is the element to be inserted at the beginning of the list.

Step -> 1.) Create a node TEMP

2.) a.) set  $\text{TEMP} \rightarrow \text{DATA} = \text{VAL}$

b.) set  $\text{TEMP} \rightarrow \text{PREV} = \text{NULL}$

c.) set  $\text{TEMP} \rightarrow \text{NEXT} = \text{NULL}$

3.) IF [ START is equal to NULL ] THEN

a.) Set  $\text{START} = \text{TEMP}$

b.) Set  $\text{END} = \text{TEMP}$

4.) ELSE

a.)  $\text{END} \rightarrow \text{NEXT} = \text{TEMP}$

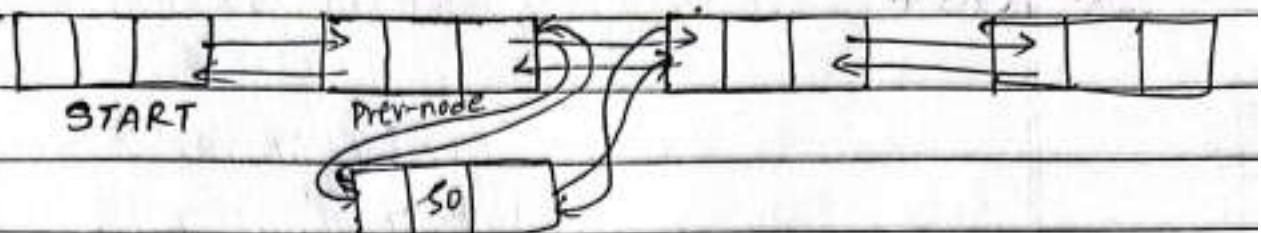
b.)  $\text{TEMP} \rightarrow \text{PREV} = \text{END}$

c.)  $\text{END} = \text{TEMP}$

5.) END IF

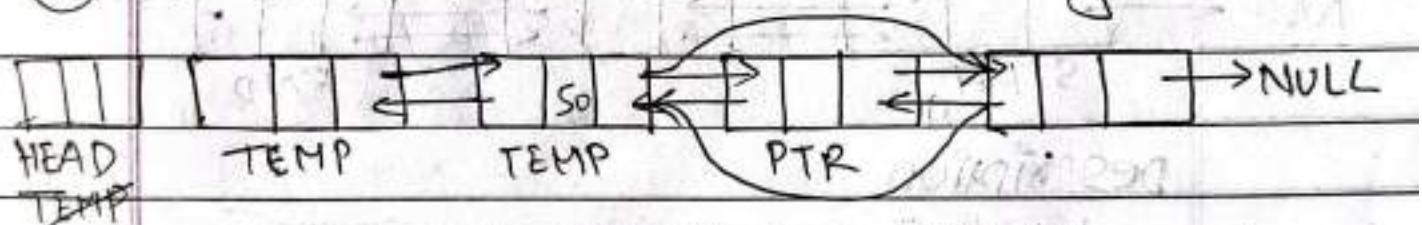
6.) EXIT.

### 3.) INSERTION at Middle of doubly linked List.



- (1) Create a new-node.
- (2) Set new-node  $\rightarrow$  data = VAL.
- (3) Set new-node  $\rightarrow$  next = prev-node  $\rightarrow$  next
- (4) Set prev-node  $\rightarrow$  next = new-node
- (5) Set new-node  $\rightarrow$  prev = prev-node
- (6) Set ~~prev~~ node  $\rightarrow$  prev (prev-node  $\rightarrow$  next)  $\rightarrow$  prev = new-node
- (7) Exit.

### 6.) Deletion of middle node in doubly linked list.

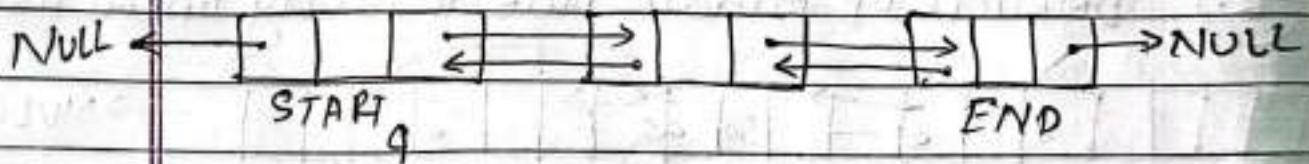


- (1) IF [HEAD = NULL] THEN  
Display "List EMPTY"
- (2) ELSE
  - (a) Set TEMP = HEAD
  - (b) Repeat While TEMP  $\rightarrow$  DATA != ITEM  
TEMP = TEMP  $\rightarrow$  NEXT
  - (c) END of While
- (3) Set PTR = TEMP  $\rightarrow$  NEXT
- (4) Set TEMP  $\rightarrow$  NEXT = PTR  $\rightarrow$  NEXT
- (5) Set PTR  $\rightarrow$  NEXT  $\rightarrow$  PREV = TEMP

(6) FREE (PTR)

(7) EXIT.

4.) Deletion of 1st / Beginning node in doubly linked list



Description

Let START and END are the pointer variable that hold the address of 1<sup>st</sup> and last node of the doubly linked list and q is the temporary node variable.

Step -> 1.) IF [START is equal to NULL] THEN

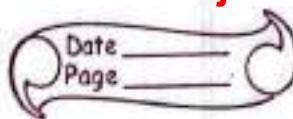
Display "List is Empty".

2.) ELSE IF [START = END] THEN

START = END = NULL

3.) ELSE

a) q = START



b)  $(q \rightarrow \text{NEXT}) \rightarrow \text{PREV} = \text{NULL}$

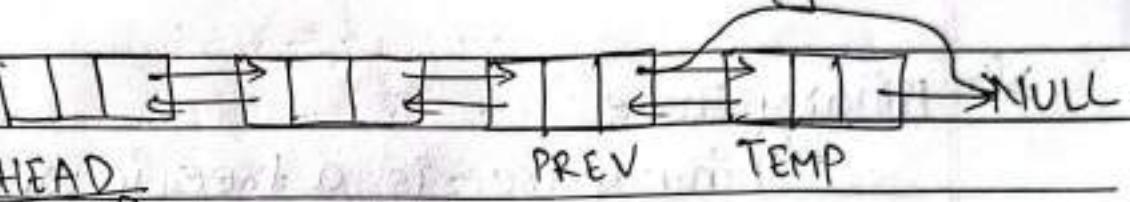
c)  $\text{START} = q \rightarrow \text{NEXT}$

4.) END IF

5.) Free the Node (q)

6.) END IF.

5.) Deletion of last node in Doubly linked list.



① If [HEAD IS NULL]

- Display "List is Empty".

② ELSE

    Set TEMP=HEAD.

    ③ Repeat while TEMP->NEXT != NULL.

        TEMP = TEMP->NEXT.

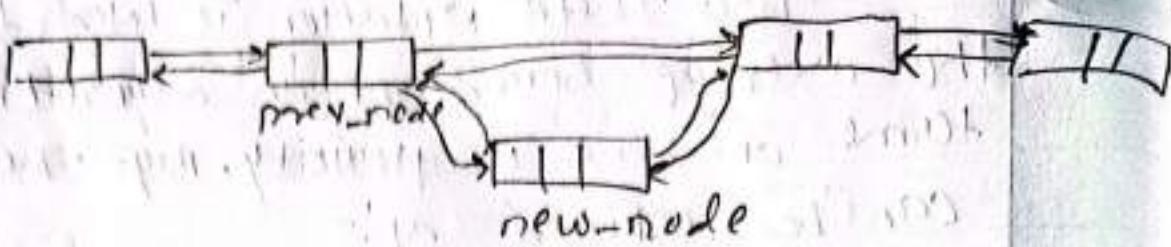
    ④ End of While.

⑤ Set TEMP->PREV->NEXT = NULL

⑥ FREE (TEMP)

⑦ EXIT.

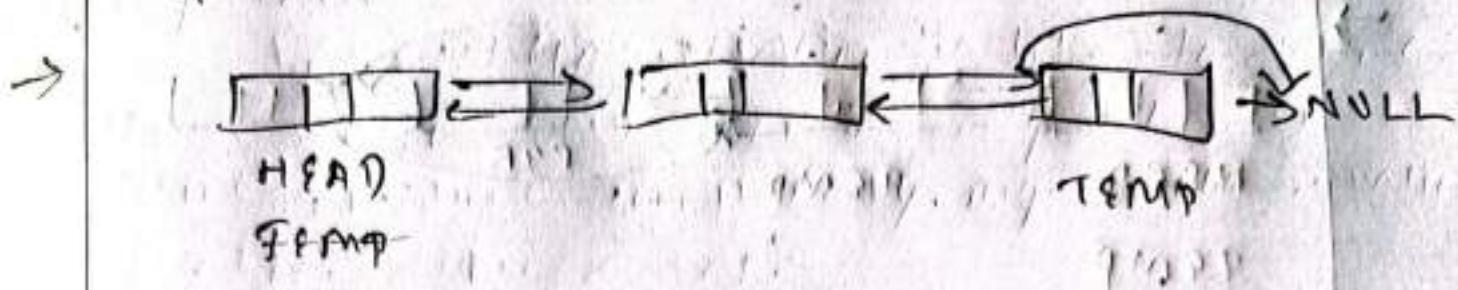
Inseartion at Middle in Doubly linked list.



Steps:-

- 1) Create a New\_Node.
  - 2) Set New\_Node → data = VAL.
  - 3) Set New\_Node → next = prev-node → next
  - 4) Set prev-node → next = new-node
  - 5) Set New\_Node → prev = prev-node
  - 6) Set (prev-node → next) → prev = new\_node
  - 7) Exit.
- $\equiv$

\* Deletion of last node in Doubly linked list.

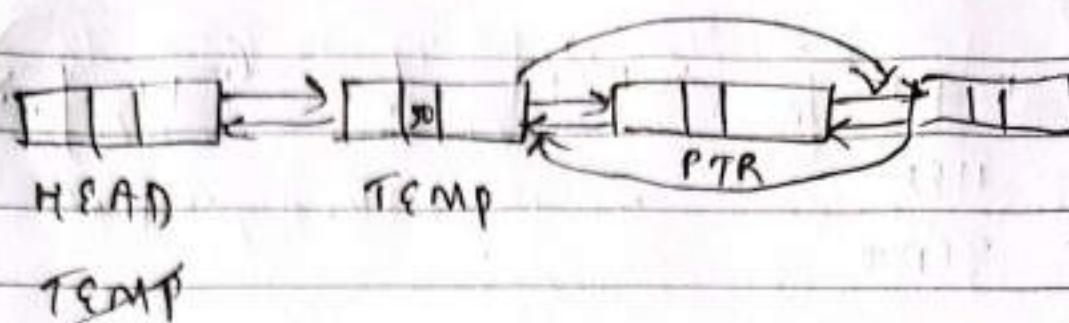


Steps :-

- 1) IF [HEAD is NULL] THEN  
Display ("List is empty").  
"End" ;
- 2) ELSE
  - (a) SET TEMP = HEAD.
  - (b) Repeat while TEMP → NEXT ≠ NULL  
 $TEMP \leftarrow TEMP \rightarrow NEXT$
  - (c) End of while loop found
  - (d) Set TEMP → PREV → NEXT = NULL
  - (e) FREE (TEMP).
  - (f) EXIT.

(AFTER THE NODE)

\* Deletion of Middle Node in Doubly linked list.



Steps:-

1) IF [HEAD = NULL] THEN  
Display "List is empty".

2) ELSE

a) Set TEMP = HEAD

b) Repeat while TEMP → DATA != ITEM

TEMP = TEMP → NEXT

c) End of While

3) Set PTR = TEMP → NEXT

4) Set TEMP → NEXT = PTR → NEXT

5) Set PTR → NEXT → PREV = TEMP

6) FREE (PTR)

7) EXIT.

## \* Tree

Trees are very flexible, versatile and powerful non-linear data structure that can be used to represent data items by processing hierarchical relationship between the grandfather and his children and grand children and so on.

## \* Binary Tree

A binary tree is a tree in which the degree of each node is less than or equal to two i.e. each node in the tree can have zero or one or two child.

or

A binary tree is a tree in which no node can have more than two sub-tree that means each node can have zero or one or two sub-trees.

## \* Tree traversal

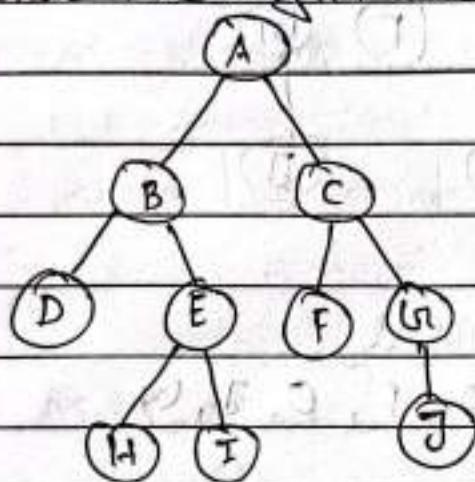
Tree traversal is a way in which each node in the tree is visited exactly once in a systematic manner. There are three ways of traversing a binary tree. These are:

- 1.) pre-order Traversal
- 2.) In-order Traversal
- 3.) post-order Traversal

## 1.) pre-order Traversal [Root-Left-Right]

The pre-order traversal of a non-empty binary tree is defined as :

- i) visit the root node.
- ii) Traverse the left sub-tree in pre-order
- iii) Traverse the right sub-tree in pre-order.



## pre-order Traversal [Root-Left-Right]

A, B, D, E, H, I, C, F, G, J

## 2.) In-order Traversal [left - Root - Right]

In the in-order traversal of a non-empty binary tree, The left sub-tree is traverse recursively before visiting the root. After visiting the root The right sub-tree is traverse recursively in the in order way.

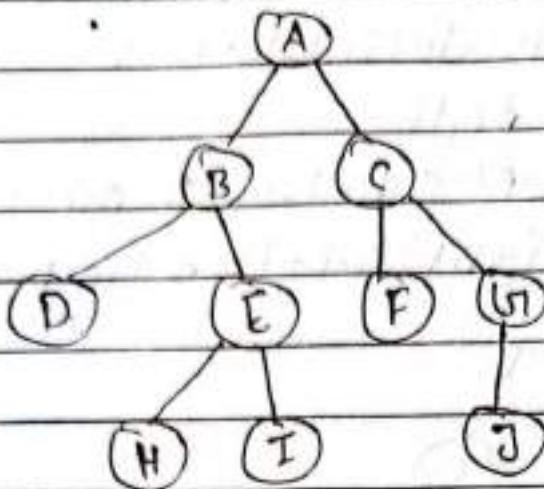
V.V.I

### Algorithm

The in-order traversal of a non-empty binary tree is defined as :

- i) In-order traverse the left sub-tree.
- ii) Visit the root of the tree.

(ii) In-order traverse the right sub-tree.



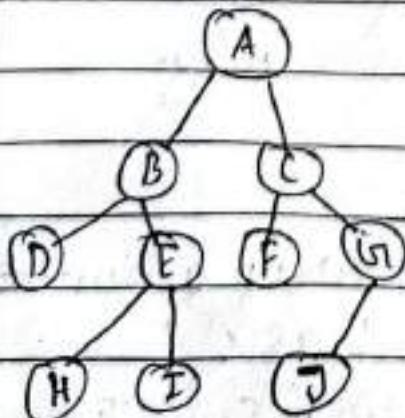
In-order traversal

D, B, H, E, I, A, F, C, J, G

(3) post-order traversal

The post-order traversal of a non-empty binary tree can be defined as :-

- (i) post order traverse the left sub-tree
- (ii) post order traverse the right sub-tree
- (iii) visit the root node.

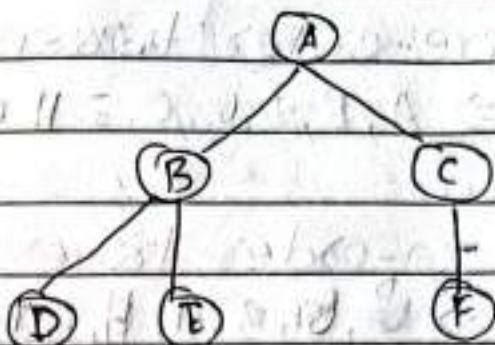


post-order traversal

D, H, I, E, B, F, J, G, C, A

Date \_\_\_\_\_  
Page \_\_\_\_\_

(2)

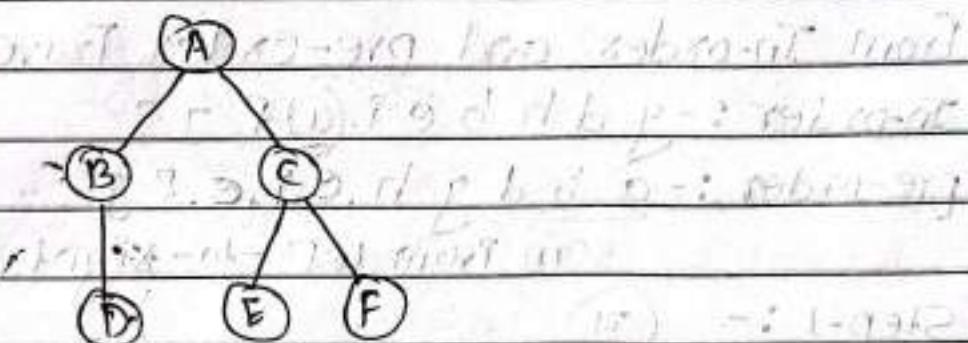


$\Rightarrow$  pre-order traversal  $\Rightarrow$  In-order traversal  
 A, B, D, E, C, F      D, B, E, A, F, C

$\Rightarrow$  post-order traversal

D, E, B, F, C, A

(2)

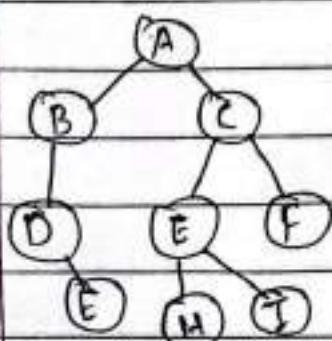


$\Rightarrow$  pre-order traversal  $\Rightarrow$  In-order traversal  
 A, B, D, C, E, F      B, D, A, E, C, F

$\Rightarrow$  post-order traversal

D, B, E, F, C, A

(3)

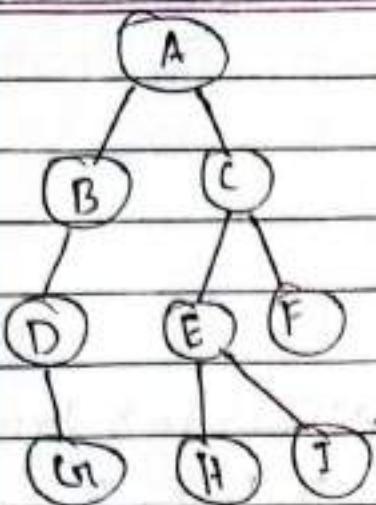


pre-order traversal  
 $\Rightarrow$  A, B, D, E, C, F, H, I, F

In-order traversal  
 $\Rightarrow$  D, E, B, A, H, E, I, F, F

post-order traversal  $\Rightarrow$  E, D, B, H, I, E, F, C, A

(4)



pre-order traversal

 $\Rightarrow A, B, D, G, C, E, H, F, I, ?$ 

In-order traversal

 ~~$\Rightarrow D, G, B, A, H, E, I, F, ?$~~ 

post-order traversal

 $\Rightarrow G, D, B, H, I, E, F, C, A$ 

2021/08/27

V.V.1

## \* Binary Tree Construction

(1) from In-order and pre-order Traversal

In-order : - g d h b e i @ f j c

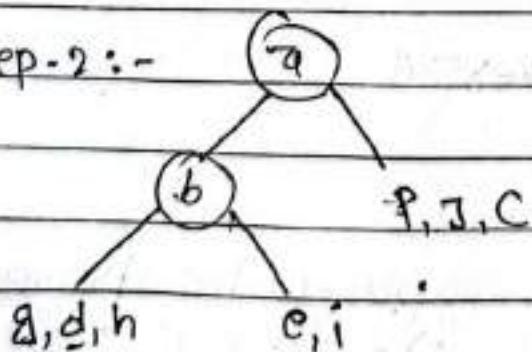
pre-order : - a b d g h , e , i , c , f , j

scan from left-to-right

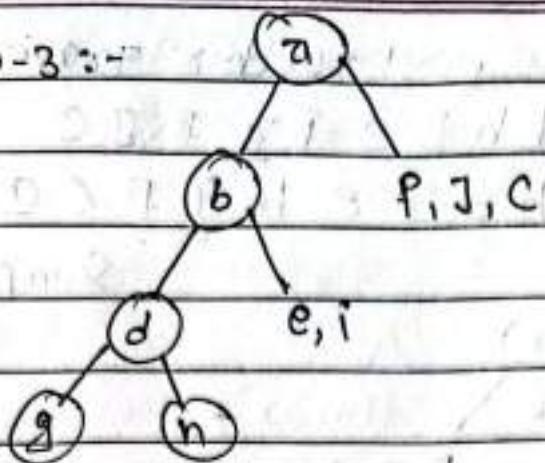
Step-1 :-



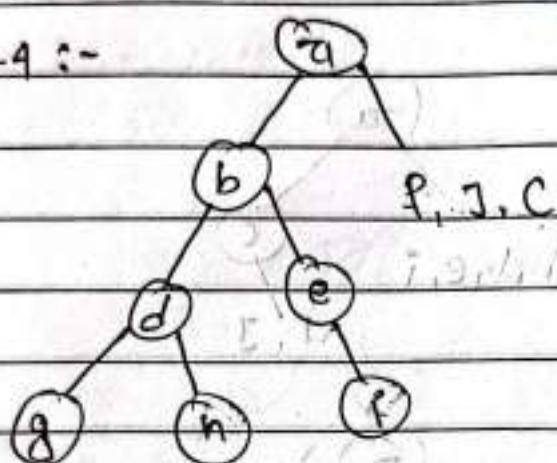
Step-2 :-



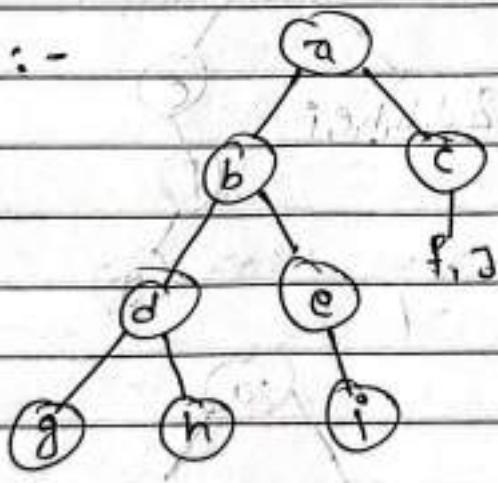
Step-3 :-



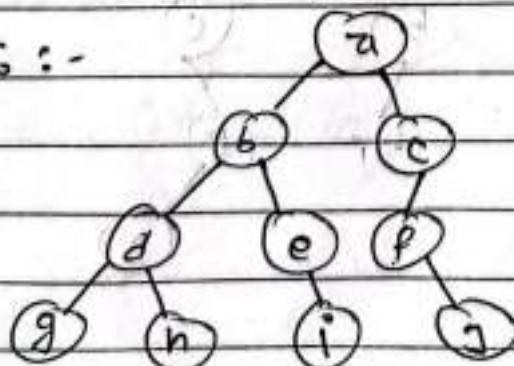
Step-4 :-



Step-5 :-



Step-6 :-



This is the required Binary Tree.

(ii)

In-order and post-order traversal

Inorder : - g d h b e i a f j c

post order : - g h d i e b j f c a

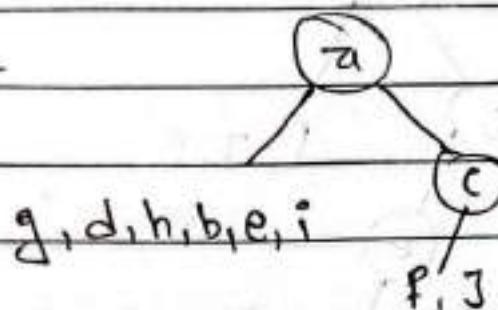
Scan from Right to Left

Step-1 :-

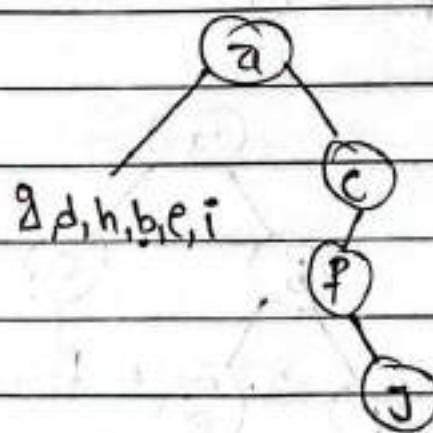


g, d, h, b, e, i      f, j, c

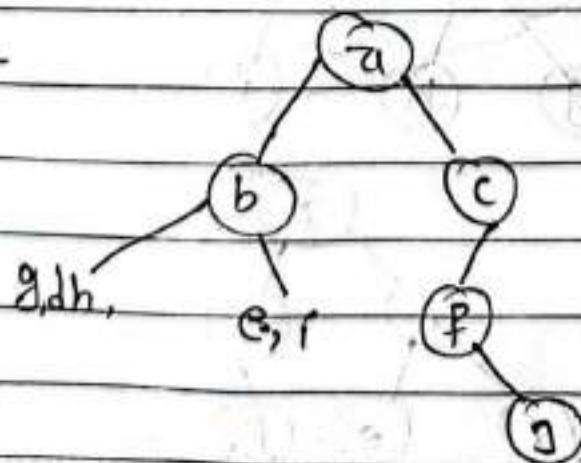
Step-2 :-



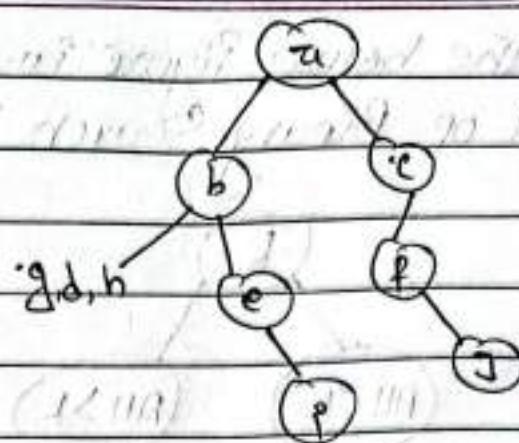
Step-3 :-



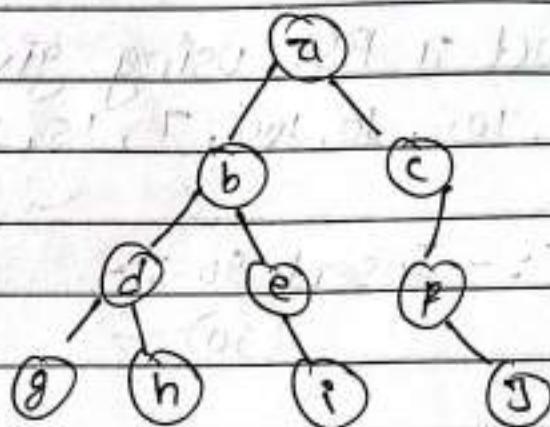
Step-4 :-



Step - 5 :-



Step - 6 :-



This is required binary Tree :-

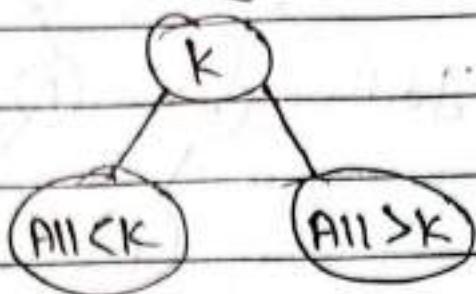
Step

### \* BST (Binary Search Tree)

A Binary Search tree is a binary tree which is either empty or satisfies the following properties

- (i) every node has a value and No two Nodes have the same values.
- (ii) If there exist a left child or left sub-tree then its value is less than the value of the root node.
- (iii) If there exist a right child or right sub-tree then its value is larger than the value of the root node.

The below figure illustrate the concept of Binary Search Tree.



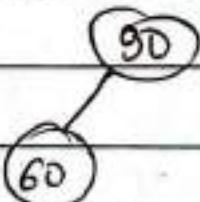
(Q.) Construct a BST using given data.

90, 60, 105, 10, 100, 7, 15, 200

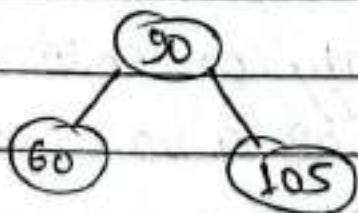
Step-1 :- Insert 90 :-



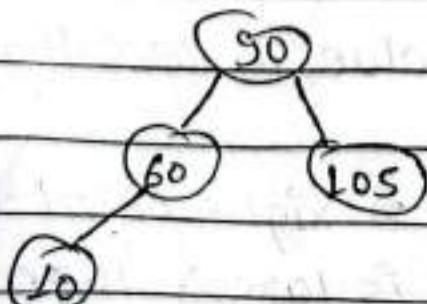
Step-2 :- Insert 60 :-



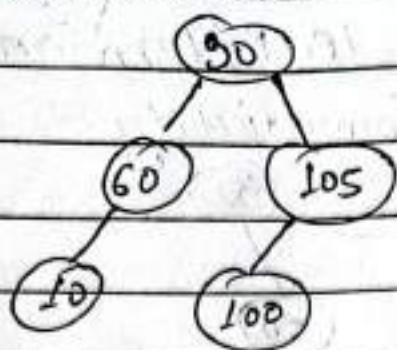
Step-3 :- Insert 105 :-



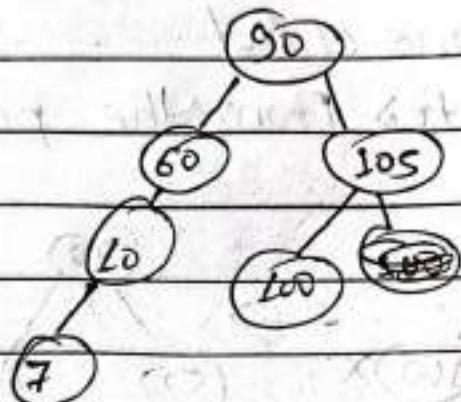
Step-4 :- Insert 10 :-



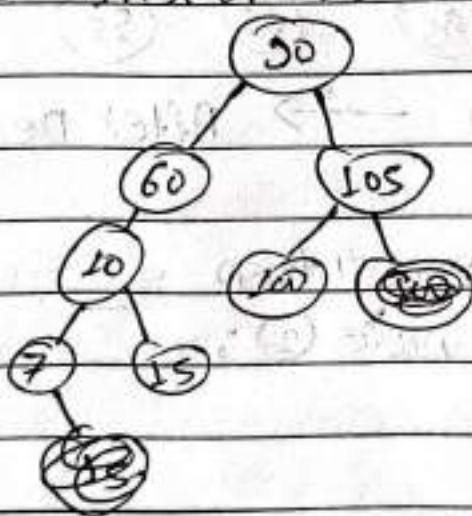
Step-5 :- Insert 100 :-



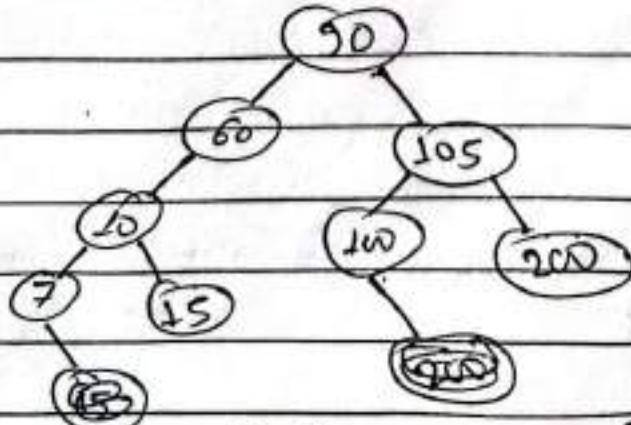
Step-6 :- Insert 7 :-



Step-7 :- Insert 15 :-



Step-8 :- Insert 200 :-

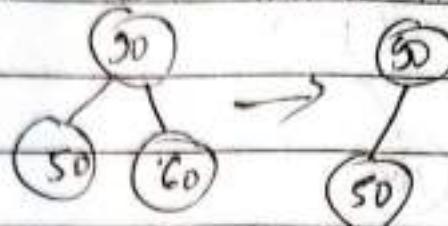


This is required Binary Search Tree.

5.5) Three cases while deleting binary tree's node.

(I.) The node is a leaf/terminal -

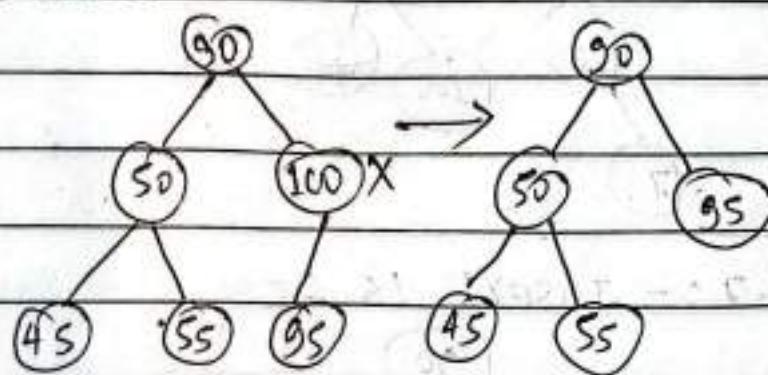
→ Delete it immediately



Before Deletion → After Deletion ..

(II) The node has one child .

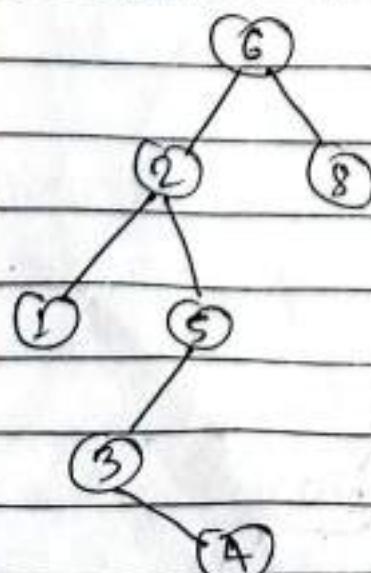
→ Adjust a pointer from the parent to bypass that node .



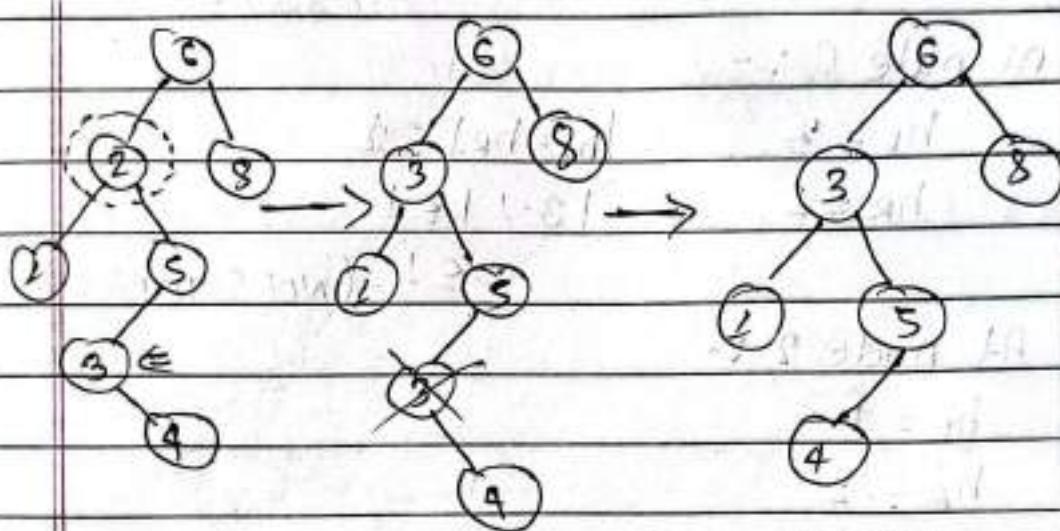
Before deletion → After deletion

(III) The node has 2 children .

for deletion of node ② :-



- ~~BSF :-~~
- Find the minimum element at a right sub-tree of the node to be deleted.
  - Copy that found minimum element to the node to be deleted.
  - Delete the found minimum element at the right sub-tree.

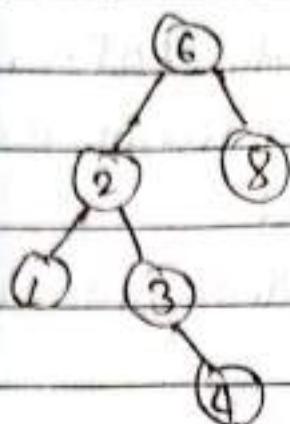


After deletion of node 2,  
we have the above BST Tree.

### ~~\* AVL Tree.~~

AVL Tree is a self balancing, height balanced Binary Search Tree whose left sub-tree and the right-sub-tree differ in height by almost 1 unit and whose left and right sub-tree are themselves AVL Tree that means for AVL Tree every node must satisfy the balance factor  $|h_L - h_R| \leq 1$  condition where  $h_L \rightarrow$  height of left sub-tree,  $h_R \rightarrow$  height of right sub-tree.

Example :-



At node 6 :-

$$h_L = 3 \quad |h_L - h_R| \leq 1$$

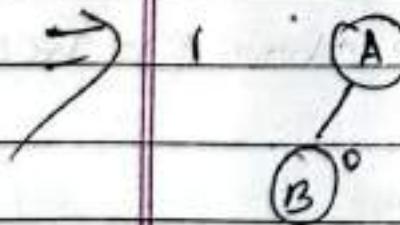
$$h_R = 1 \quad |3 - 1| \leq 1$$

$2 \leq 1$  (NOT satisfied)

At node 2 :-

$$h_L = 1$$

$$h_R = 2$$



AVL Tree

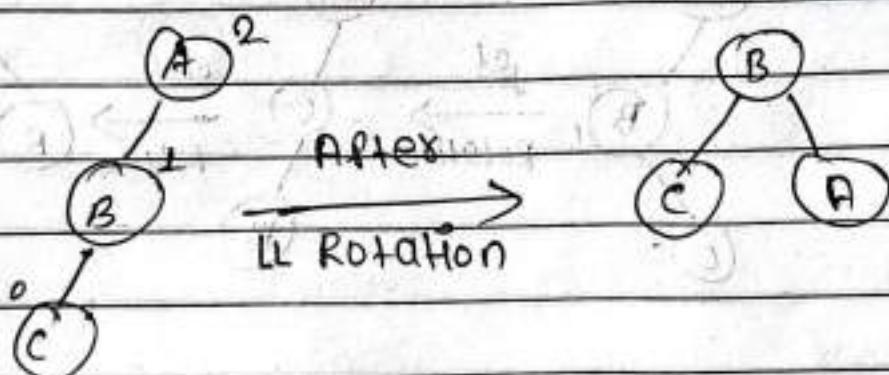
## \* Rules for Balancing unbalance tree:

- 1) The unbalance tree is balance through rotation of nodes. There are two types of rotation
- Single rotation
  - Double rotation

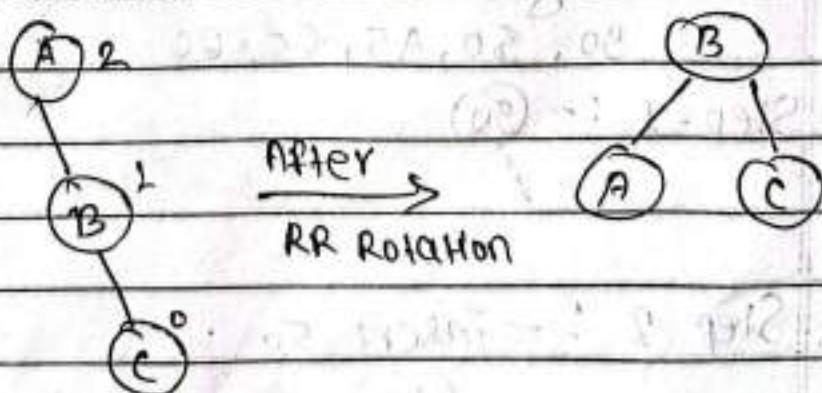
### i) Single Rotation

it is performed by:-

#### (a) LL Rotation :-



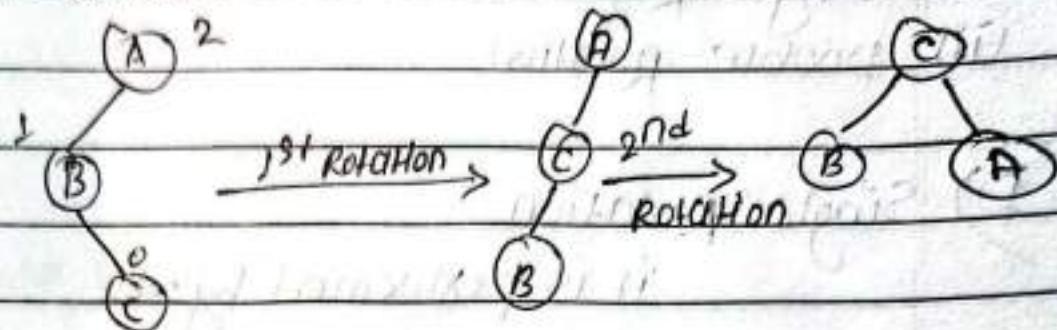
#### (b) Right RR Rotation



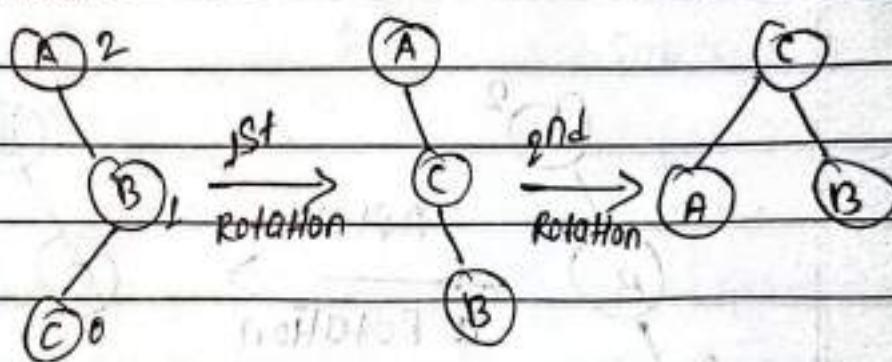
(iii) Double Rotation:-

it is performed by: LR rotation

a) LR Rotation :-



b) RL Rotation :-



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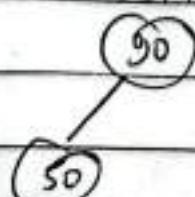
Construct BST and AVL Tree.

for the given data :-

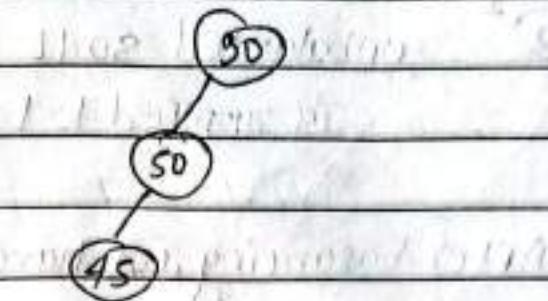
90, 50, 45, 65, 60

Step - 1 :- 90

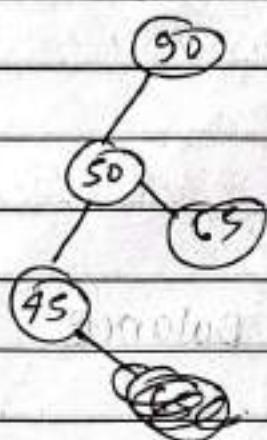
Step 2 :- insert 50



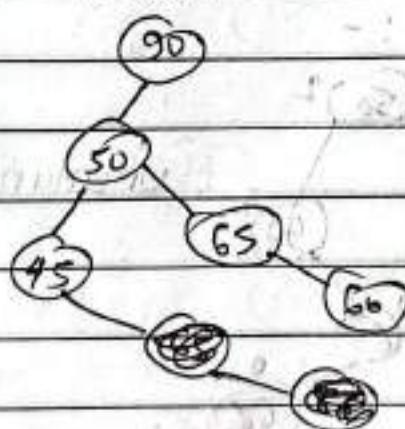
Step 3 :- insert 45



Step 4 :- insert 65



Step 5 :- insert 66

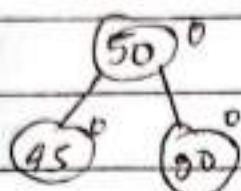
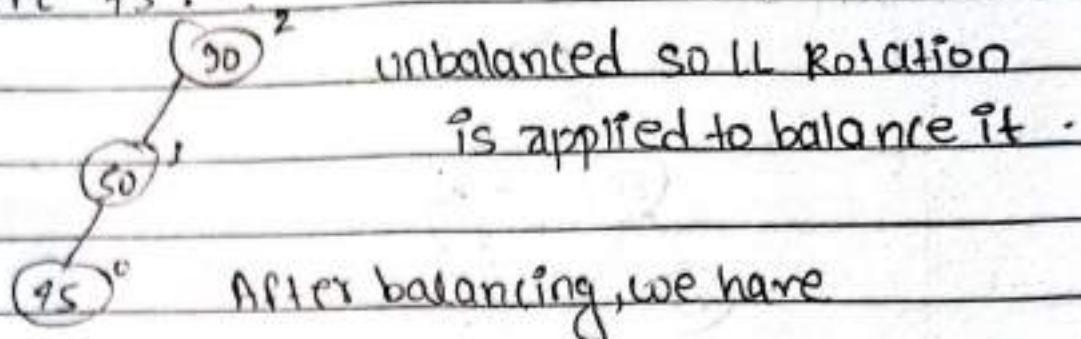


For AVL Tree

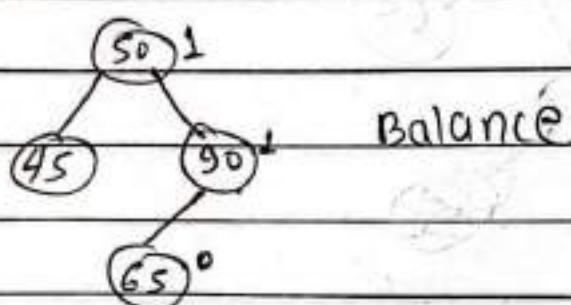
Step (i) Insert 90 :-      90      Balance

Step (ii) Insert 50 :-      90      Balance  
                              50

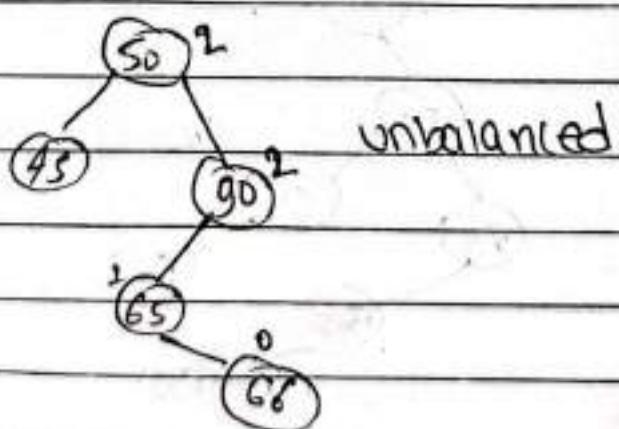
Step (III) Insert 95 :-



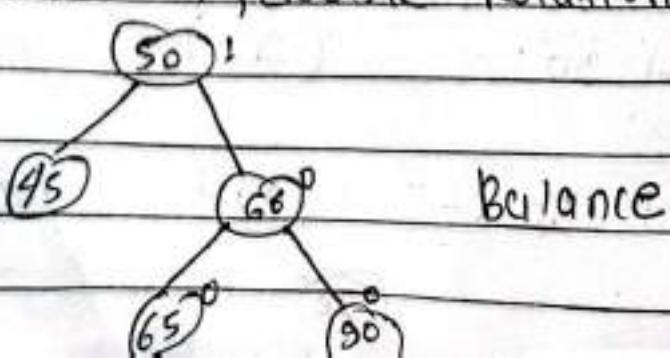
Step (IV) Insert 65 :-



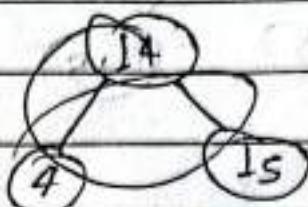
Step (V) Insert 66 :-



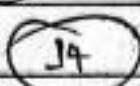
To balance it, double Rotation is applied it



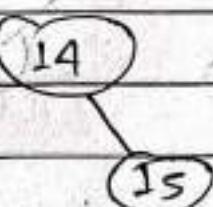
(Q1) Draw BST for 14, 15, 4, 9, 7, 18, 3, 5, 16, 9,  
20, 17, 9, 14, 5.



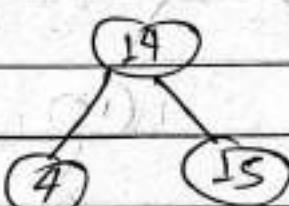
Step - (1)



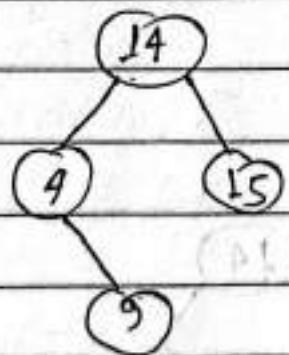
Step - (2)



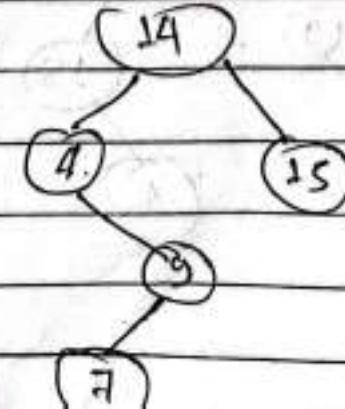
Step - (3)



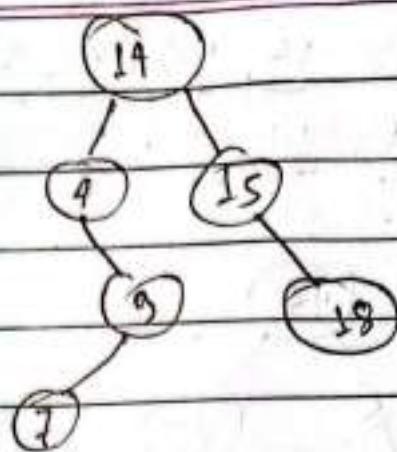
Step - (4)



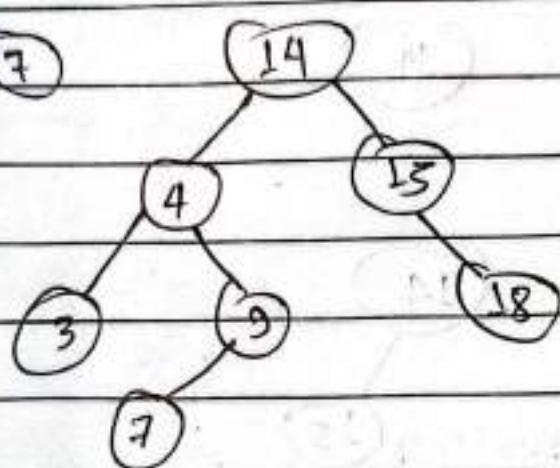
Step - (5)



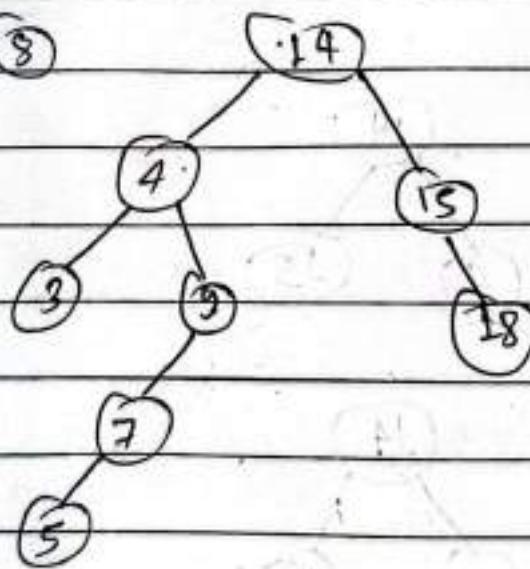
Step - 6



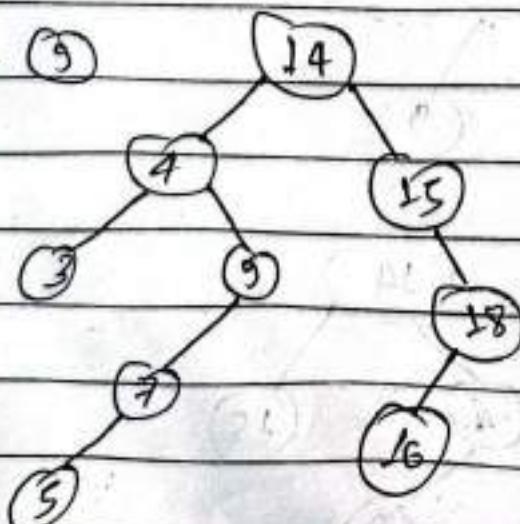
Step - 7



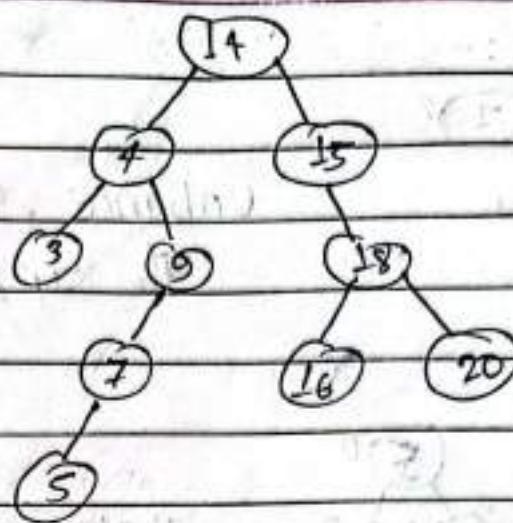
Step - 8



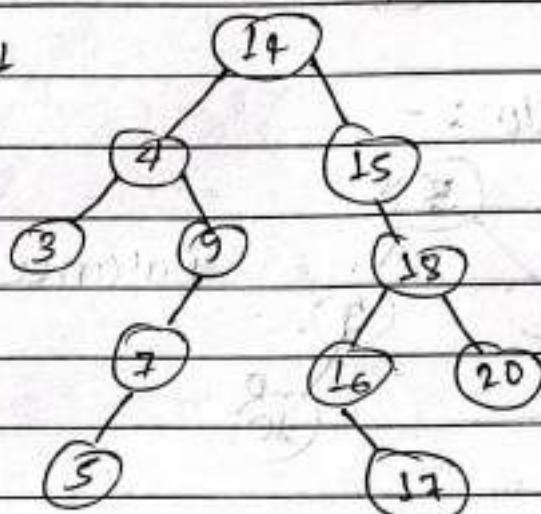
Step - 9



Step - 10



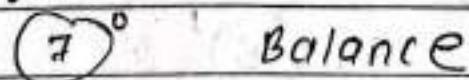
Step - 11



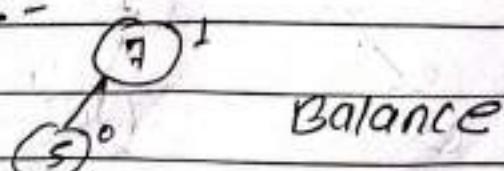
~~Step~~ · This is required BST.

- (Q) 2.) Construct an AVL tree from following .  
7, 5, 3, 10, 2, 8, 9

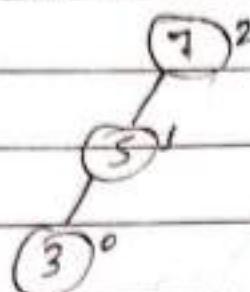
Step-1 Insert 7 :-



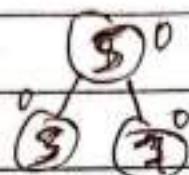
Step-① Insert 5 :-



Step-3 Insert 3 :-

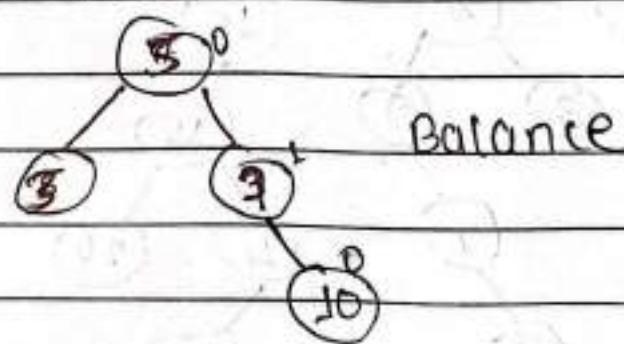


unbalance, so double rotation  
is applied it.



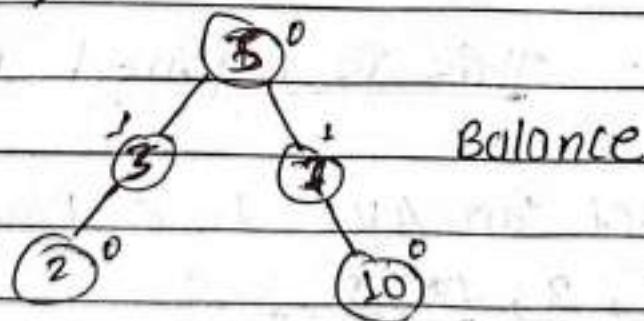
Balance

Step-4 Insert 10 :-



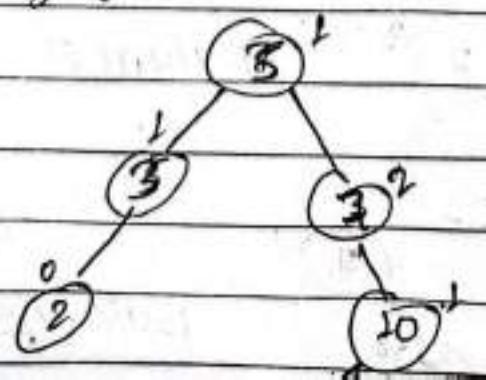
Balance

Step-5 Insert 9 :-

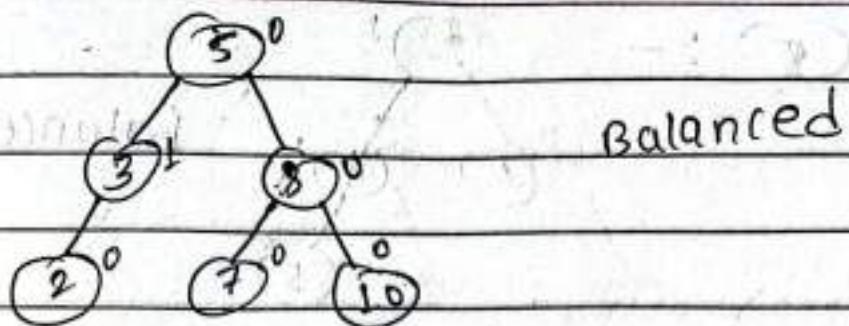


Balance

Step-6 Insert 8 :-



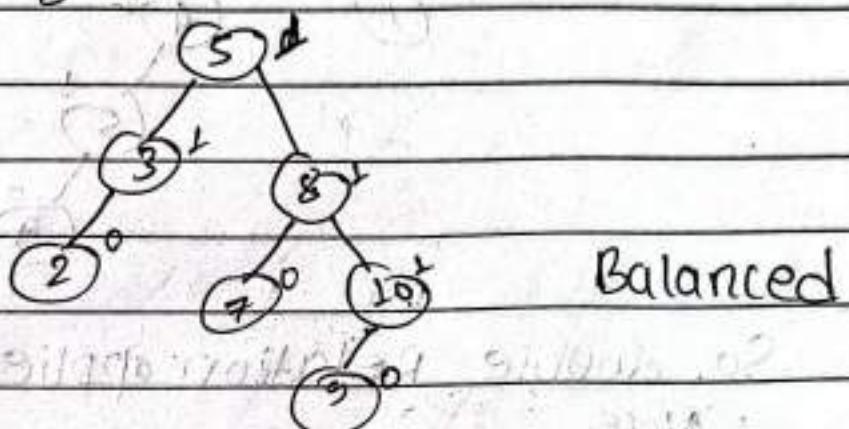
unBalanced, so  
double rotation  
applied it.

~~Step-6~~

Balanced

~~Insert.~~

Step-7 Insert 9 :-



Balanced

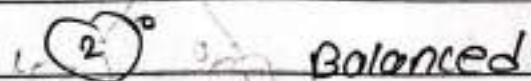
So, The required AVL tree is .

~~balance~~

Q.) 3) Construct an AVL tree from following .

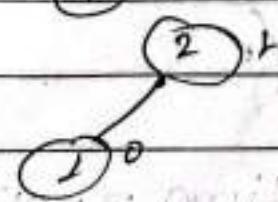
2, 1, 4, 5, 3, 6, 7

Step-1 Insert 2 :-



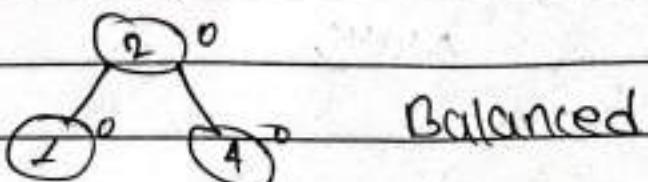
Balanced

Step-2 Insert 1 :-



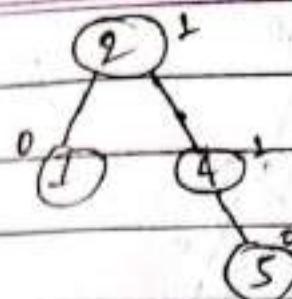
Balanced

Step-3 Insert 4 :-



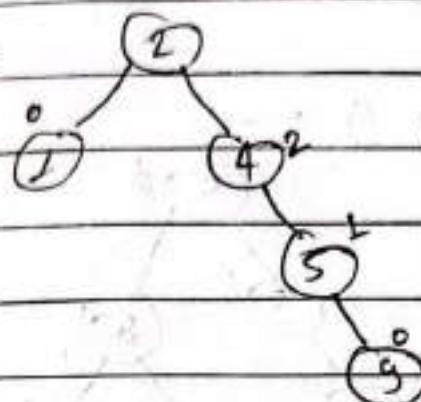
Balanced

Step-4 Insert (5) :-



Balanced

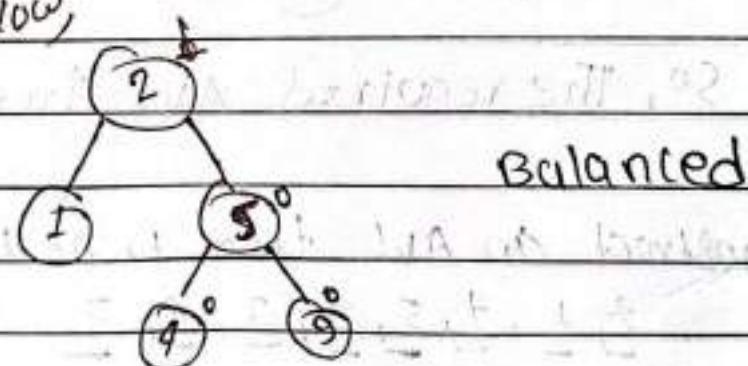
Step-5 Insert (3) :-



unbalanced

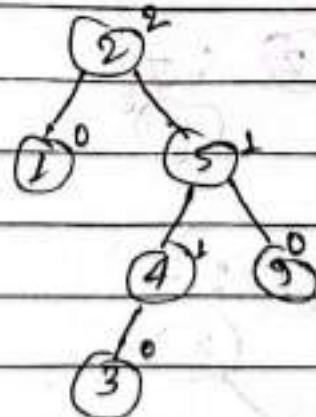
So, double relation applied it.

Now,



Balanced

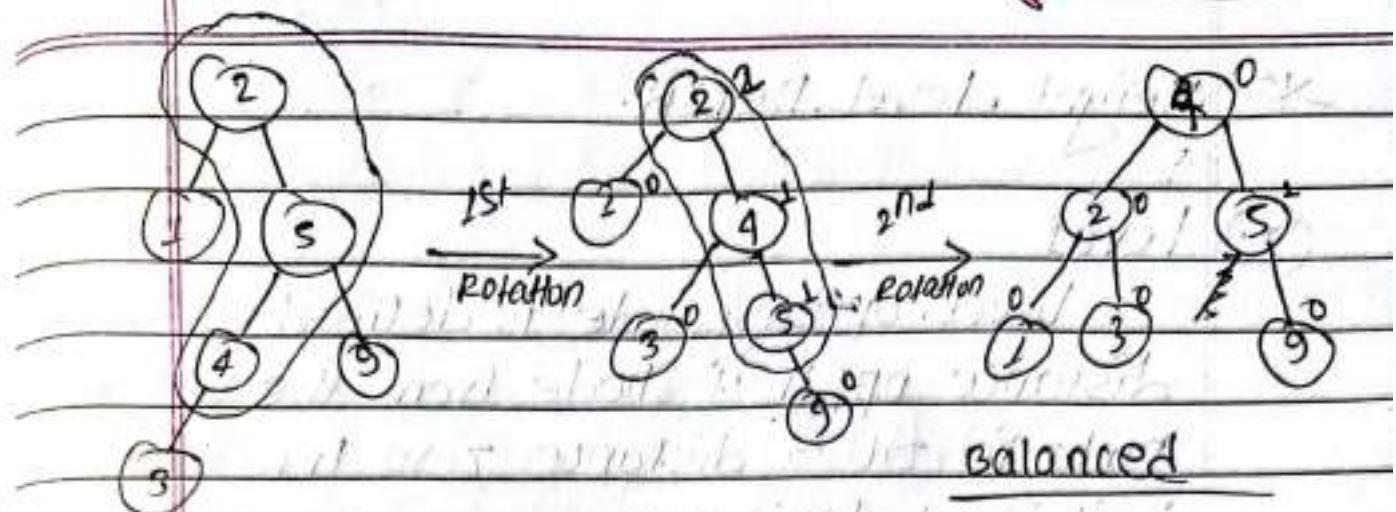
Step-6 insert (3) :-



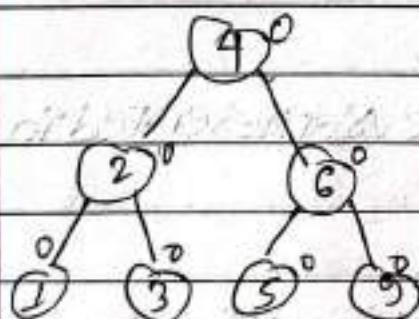
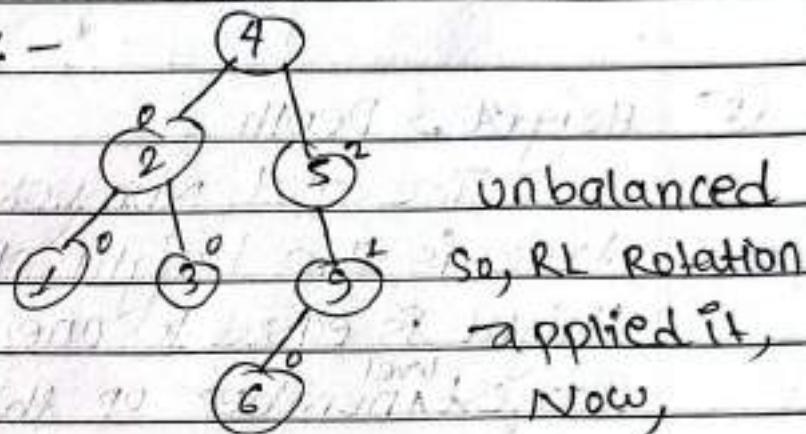
unbalanced

So, double relation applied it,

Now,

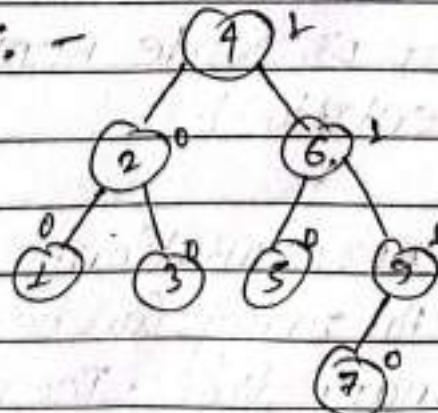


Step-7 insert 6 :-



Balanced

Step-8 Insert 7 :-



Balanced

So, The required AVL Tree.

## \* Height, level, Depth.

### (1) Level

level of any node is defined as the distance of that node from the root. Root node is at a distance zero from itself. So it is at level zero.

### (2) Height & Depth

The total number of levels in a tree is the height of the tree. So, Height is equal to one more than the largest <sup>level</sup> number of the tree.

## \* Node Representation

The tree can be represented in two ways :-

- (1) Sequential Representation using Array (static representation).
- (2) Linked list of Node Representation (dynamic representation).

### 1) Array Representation.

An array can be used to store the nodes of a binary tree. The nodes stored in an array of memory can be accessed sequentially.

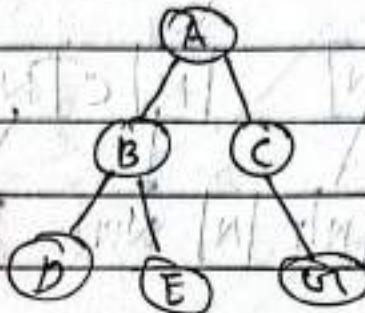


fig :- Binary Tree of Depth 3.

In array the above tree can be represented below :

0	1	2	3	4	5	6
A	B	C	D	E	F	

fig :- Array representation of the Binary tree.

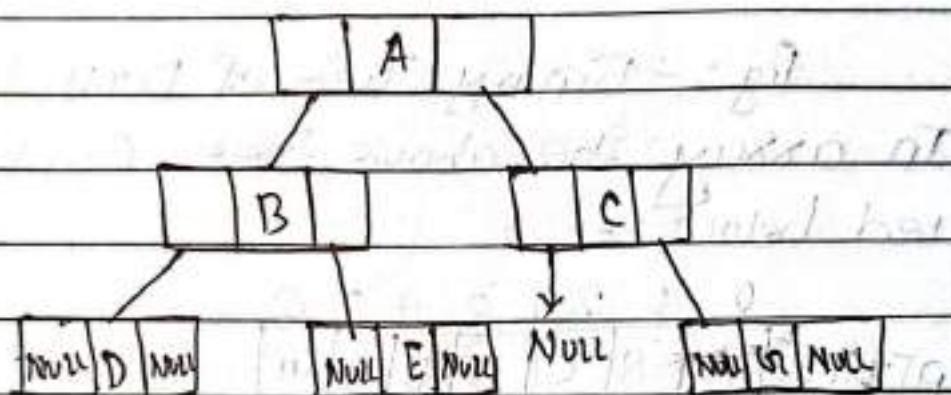
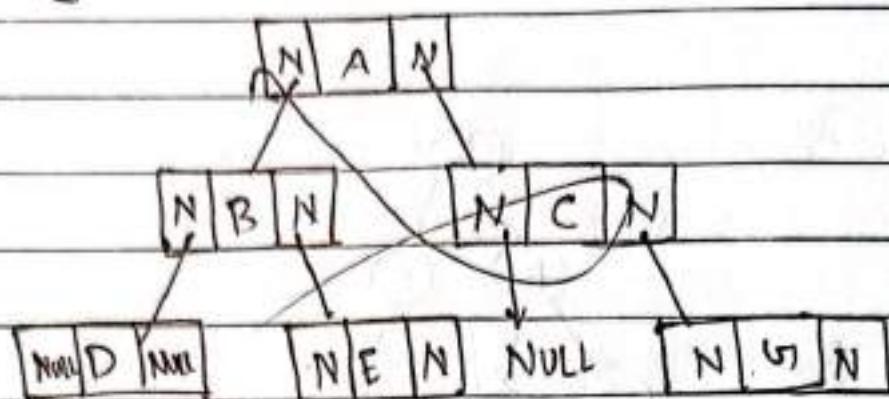
## ② Dynamic Representation.

The most popular and practical way of representing a binary tree is using linked list. In link list, every element is represented as node. A node consist of three fields such as :

left child, Information of the node and right child

A
---

The above binary tree can be represented using link list as follows:



### \* Application of Binary tree

- it is used in interpreters or compiler design for programming language.
- it is used for obtaining the pre-fix and post-fix equivalent of a given in-fix notation or an expression.
- Searching a binary tree for a particular number is easy and takes less number of comparisons in compared to linear search.
- it is used in representing arithmetic and boolean expression.
- it is used in representing priority queue.

### 3.5 priority queue

A priority queue is a queue where each element is assigned a priority.

In priority queue the elements are deleted and processed by following rules:

- (i) An element of higher priority is process before any element of lower priority.
- (ii) Two elements with the same priority are processed according to the order in which they are added to the queue.

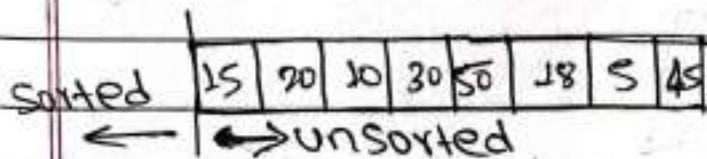
### unit-G

#### Sorting :-

##### \* Insertion Sort :-

Insertion sort algorithm arranges a list of elements in a particular order. In this algorithm, every iteration moves an element from unsorted position to sorted position until all the elements are sorted in the list.

example:- 15, 20, 10, 30, 50, 18, 5, 45



Sorted ← → unsorted

15	20	10	30	50	18	5	45
----	----	----	----	----	----	---	----

Sorted ← | → unsorted

15	20	10	30	50	18	5	45
----	----	----	----	----	----	---	----

Sorted ← | → unsorted

10	15	20	30	50	18	5	45
----	----	----	----	----	----	---	----

Sorted ← | → unsorted

10	15	20	30	50	18	5	45
----	----	----	----	----	----	---	----

Sorted ← | → unsorted

30	15	20	30	50	18	5	45
----	----	----	----	----	----	---	----

Sorted ← | → unsorted

10	15	18	20	30	50	5	45
----	----	----	----	----	----	---	----

Sorted ← | → unsorted

5	10	15	18	20	30	50	45
---	----	----	----	----	----	----	----

Sorted ← | → unsorted

5	10	15	18	20	30	45	50
---	----	----	----	----	----	----	----

## \* Types of sorting

These are two types of Sorting method.

- (i) Internal Sorting
- (ii) External Sorting

### (i) Internal Sorting

An Internal sorting is a sorting process in which all of the data are held in primary memory during the sorting process.

Internal sorting is applied when the entire collection of data to be sorted is small enough that the sorting can take place within the main memory.

### (ii) External Sorting

An external sort uses primary memory for the data currently being used sorted and secondary storage for any data that will not fit in primary memory.

External sorting method is applied to large collection of data which reside on secondary devices.

\* **Sorting :-** Sorting is a process of arranging a list of elements in a particular order (ascending and descending).

### 2. \* Selection Sort

This sorting algorithm is an inplace comparison based algorithm in which the list is divided into two parts, the sorted part at the left end and the Unsorted part at the right end.

Initially, the sorted part is empty and the unsorted part is the entire list.

The smallest element is selected from the unsorted array and the swapped with the left most element and that element becomes a part of the sorted array. This process continues moving unsorted array boundary by one element to the right. e.g:-

Selection sort : - 6, 2, 11, 7, 5

	6	2	11	7	5
Sorted ←	→ unsorted				

	2	6	11	7	5
Sorted ←	→ unsorted				

	2	5	11	7	6
Sorted ←	→ unsorted				

	2	5	6	7	11
Sorted ←	→ unsorted				

	2	5	6	7	11
Sorted ←	→ unsorted				

	2	5	6	7	11
Sorted ←	→ unsorted				

Hence, the required sorted :-

## \* Bubble Sort

Bubble Sort algorithm is comparison based algorithm in which each pair of adjacent elements is compared and the elements are swapped if they are not in-order. E.g. :- 13, 32, 26, 35, 10

PASS 1 :-

13	32	26	35	10
----	----	----	----	----

↔  
No swap is required

PASS 2 :-

13	26	32	10	35
----	----	----	----	----

↔  
No swap is required

13	32	26	35	10
----	----	----	----	----

↔  
swap is required

13	26	32	10	35
----	----	----	----	----

↔  
No swap is required

13	26	32	35	10
----	----	----	----	----

↔  
No swap is required

13	26	32	10	35
----	----	----	----	----

↔  
Swap is required

13	26	32	35	10
----	----	----	----	----

↔  
Swap is required

13	26	10	32	35
----	----	----	----	----

↔  
No swap is required

13	26	32	10	35	10
----	----	----	----	----	----

13	26	10	32	35
----	----	----	----	----

↔  
No swap is required

13	26	10	32	35
----	----	----	----	----

↔  
Swap is required

13	20	26	32	35
----	----	----	----	----

↔  
No ↔ No

PASS 4 :-

13	10	26	32	35
----	----	----	----	----

↔  
 swap is required

10	13	26	32	35
----	----	----	----	----

↔  
 No ↔ No ↔ No

10	13	26	32	35
----	----	----	----	----

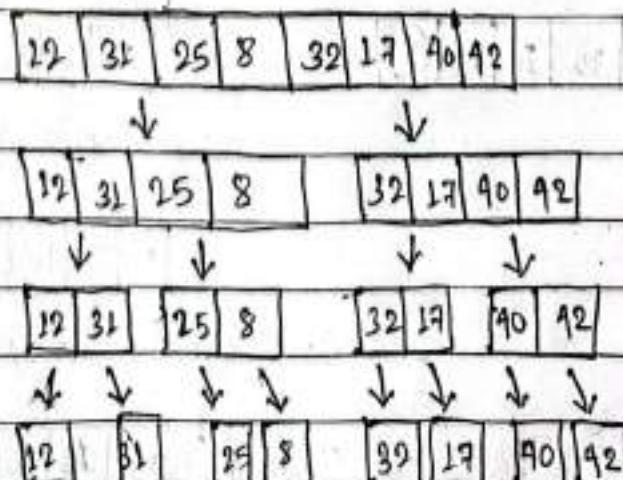
Hence, this is required sorting.

20/10/09/12

#### A. \* Merge Sort

Merge Sort is divide and conquer algorithm based on the idea of breaking down a list in to several sub-list until each sub-list consist of a single element and then after merging those sub-list in a manner that results into a sorted list. Examples :-

12, 31, 25, 8, 32, 17, 40, 42



19	31	8	25	17	32	40	42
----	----	---	----	----	----	----	----

8	12	25	31	17	32	40	42
---	----	----	----	----	----	----	----

8	12	17	25	31	32	40	42
---	----	----	----	----	----	----	----

: This is required merge sort data.

## 5 \* Quick Sort

Quick sort is a highly efficient sorting algorithm and is based on partitioning of array of data into smaller array. In quick sort a large array is divided into two arrays in which one holds values that are smaller than the specified value (pivot) and another array holds the values that are greater than the (pivot).

After that left and right sub arrays are also partitioning using the same approach. it will continue until the single elements remains in the sub array. The below figure illustrate the concept of quick sort. egs : - 24, 9, 29, 14, 19, 27

0	1	2	3	4	5
24	9	29	14	19	27

$\uparrow_p$        $\leftarrow^*$  Right to left

0	1	2	3	4	5
19	9	29	14	24	27

$\overrightarrow{*}$  Left to Right       $\uparrow_p$

0	1	2	3	4	5
19	9	24	14	29	27

$\uparrow_p$  \* ← Right to left

0	1	2	3	4	5
19	9	14	24	29	27

$\uparrow_p$

Now, Pivot value (24) is exact position.

0	1	2	3	4	5
19	9	14	(24)	29	27

$\uparrow_p$  ← Right to left  $\uparrow_p$   $\uparrow_p$  ← Right to left

0	1	2
14	9	19

← Right to left

0	1	2	3	4	5
14	9	19	(24)	27	29

← Left to right  $\uparrow_p$   $\uparrow_p$

Now, pivot value (19) is at exact position.

Now, pivot value (29) is at exact position.

0	1	2	3	4	5
14	9	(19)	(24)	27	29

$\uparrow_p$  ← Rtol

0	1	2	3	4	5
9	14	(19)	(24)	27	29

$\uparrow_p$

Now, pivot value (24) is at exact position.

0	1	2	3	4	5
9	(14)	(19)	(24)	27	29

Since, all the left side value from (9) are smaller and right side values are greater than (9), so it is also at exact position.

0	1	2	3	4	5
(9)	(14)	(15)	(24)	(27)	(29)

So, The sorted data are: - 9, 14, 15, 24, 27, 29.

## 6.\* Heap Sort

Heap Sort is the comparison based sorting algorithm that sorts an array in specific order by transforming it into a binary heap data structure. It uses the concept of Max-heap and min-<sup>heap</sup>. In Max-heap the largest element is the root node and in Min-heap, the smallest element is the root node.

### \* What is heap?

→ Heap is a complete binary tree.

Complete binary tree is a binary tree in which all the levels except the last level that is leaf node should be completely filled and all the node should be left justified.

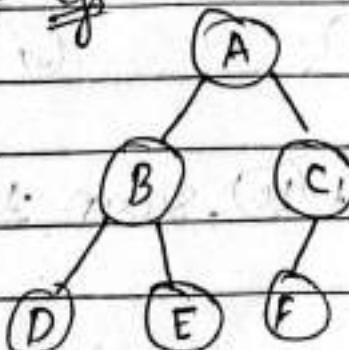
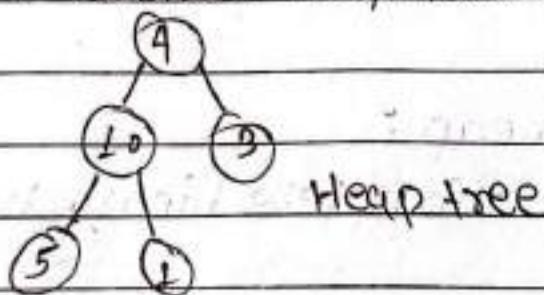


Fig:- Complete binary tree.

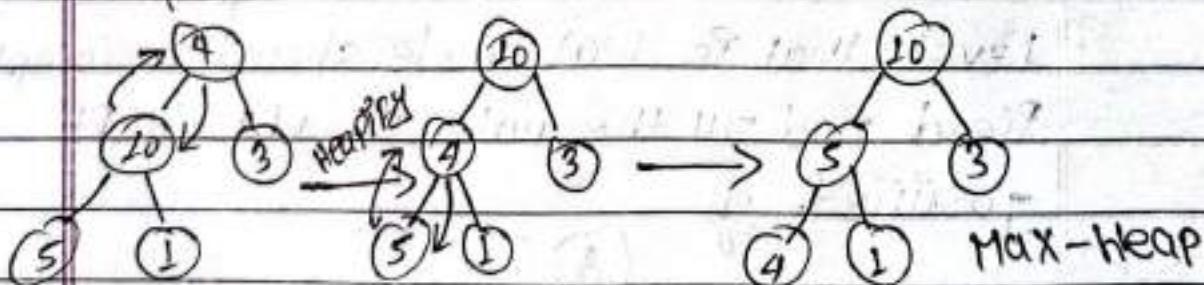
- Steps :-
- 1) Construct a complete binary tree (heap tree) with given data
  - 2) Transform the heap tree into max-heap tree.
  - 3) Delete the root element from MAX-heap tree.
  - 4) put the deleted element into the sorted stack.
  - 5) Repeats steps Two to four until max-heap until tree become empty.
  - 6) Before Display the sorted stack elements.

e.g. :- 9, 10, 3, 5, 1

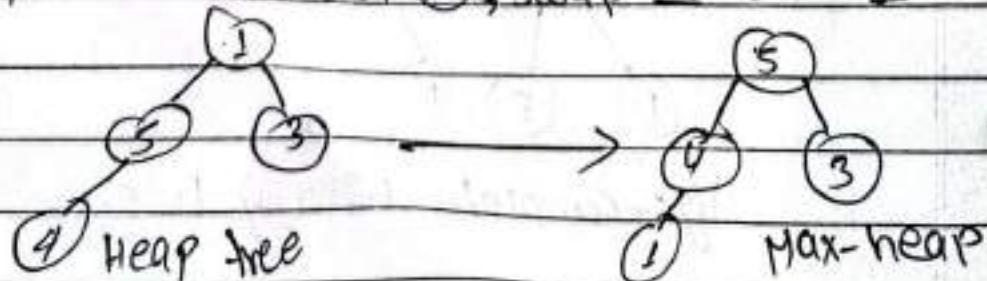
Step-1 :- Construct Heap tree

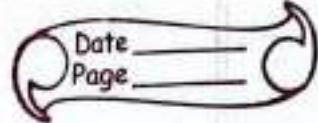


Step-2 :-

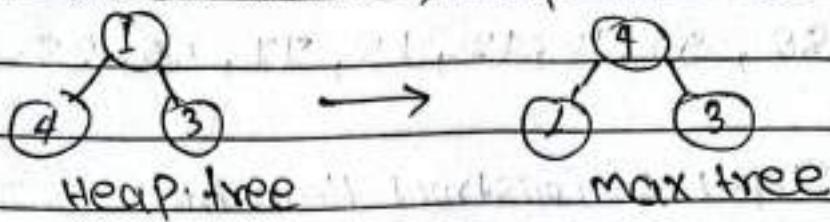


Step-3 :- Delete root 10, swap 10 with 1

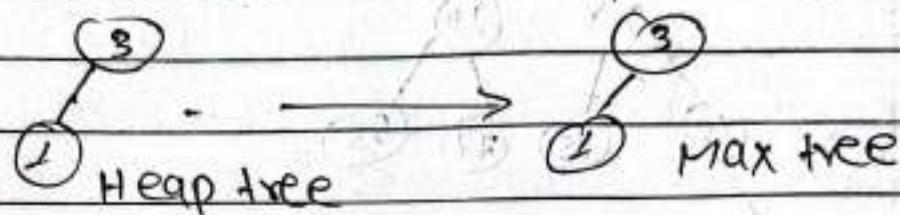




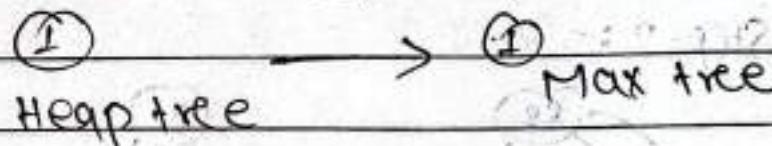
Step - 4 :- Delete Root (5), swap with (1)



Step - 5 :- Delete Root (4), swap with (3)



Step - 6 :- Delete Root (3), swap with (1)



Step - 7 :- Delete Root (1).

heap · Tree empty.

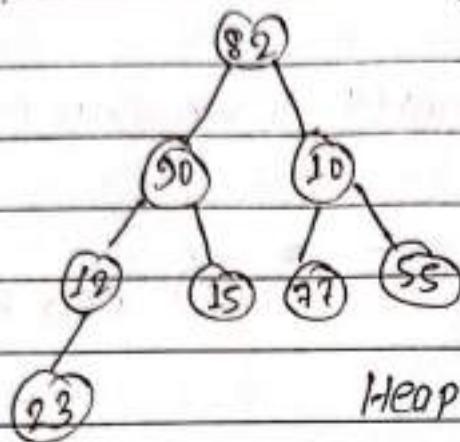
Sorted data :- 1, 3, 4, 5, 10

10	1
5	3
4	4
1	5
10	10

Q.1 Heap Tree Sort

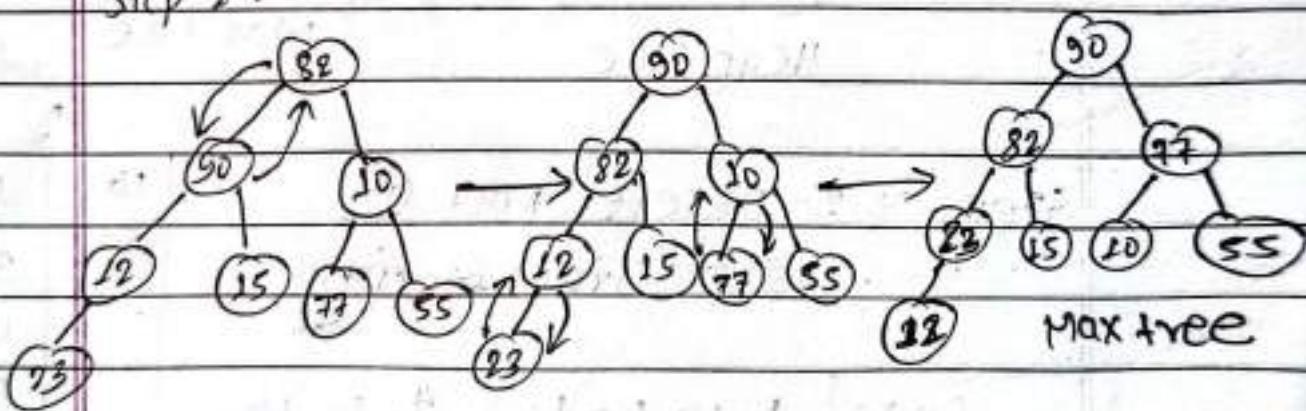
82, 90, 10, 12, 15, 77, 55, 23.

Step-1 :- Construct Heap tree

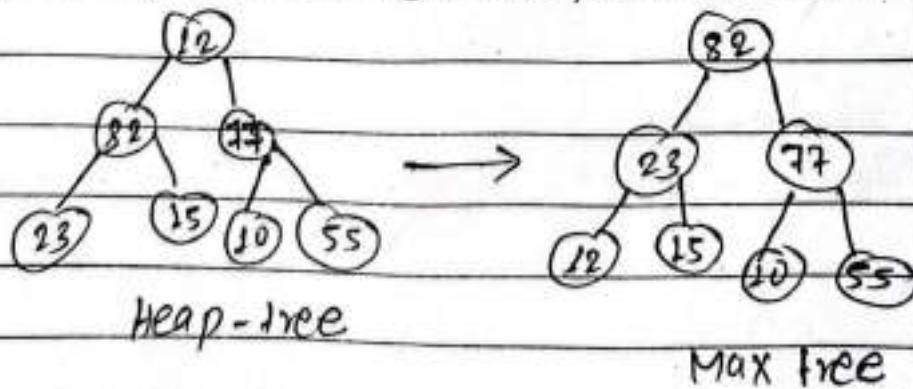


Heap tree

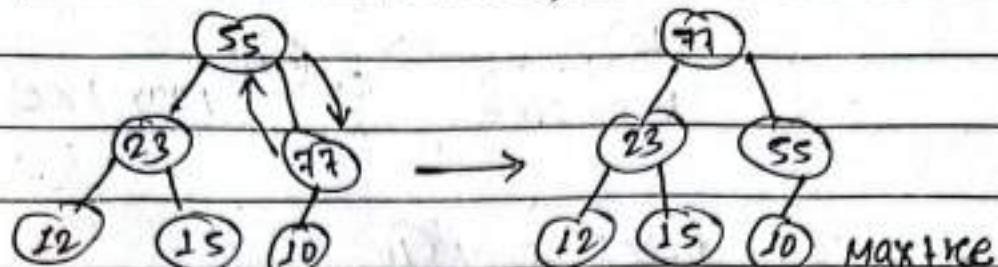
Step-2 :-



Step-3 :- Delete root 90 Swap 90 with 23

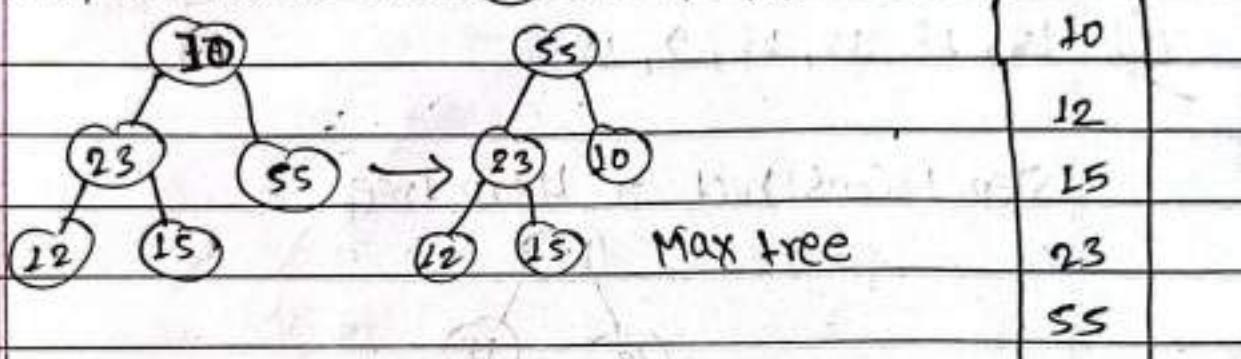


Step-4 :- Delete Root (82), swap with 55



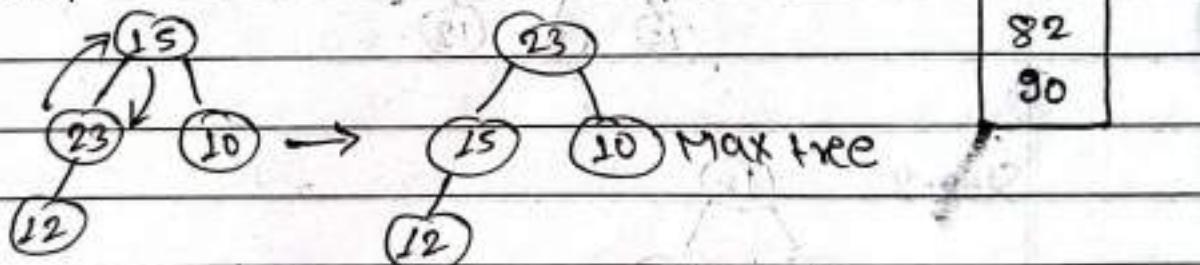
Heap tree

Step-5 :- Delete Root (77) swap ~~77~~ with 10



sorted stock

Step-6 :- Delete Root (55), swap 55 with 15

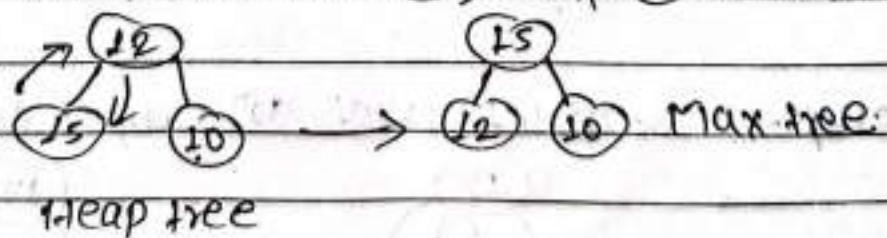


77

82

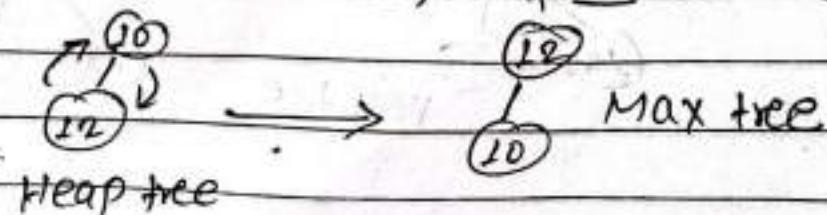
90

Step-7 :- Delete Root (23), swap 23 with 12



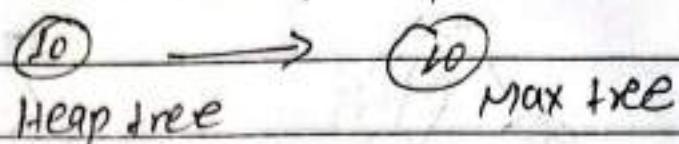
Heap tree

Step-8 :- Delete Root (15), swap 15 with 10



Max tree

Step-9 :- delete root 10, swap 12 with 10.



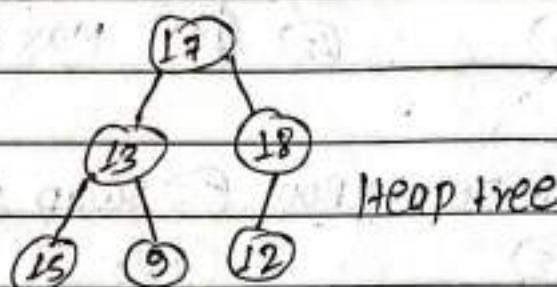
Step-10 :- Delete root 10

Heap tree empty.

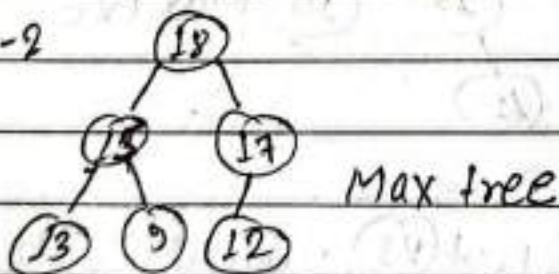
$\therefore 10, 12, 15, 23, 55, 77, 82, 90 //$

Q2. 17, 13, 18, 15, 9, 12

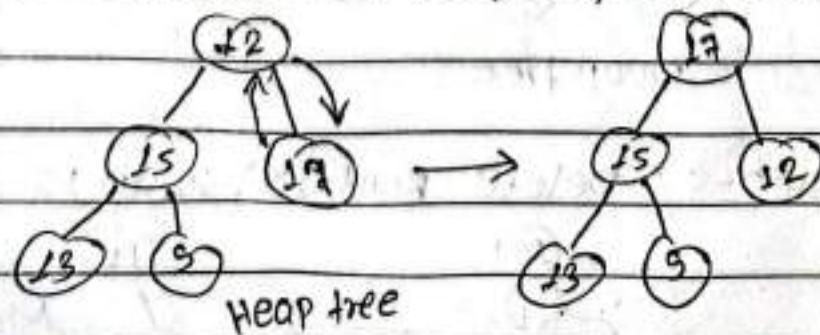
Step-1 Construct a Heap tree



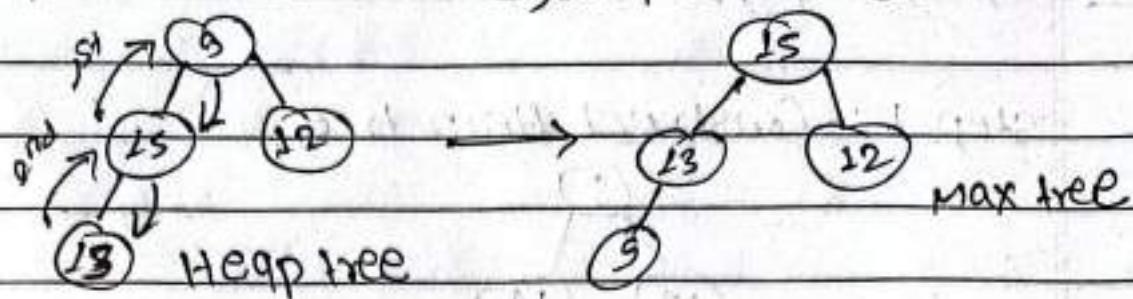
Step-2



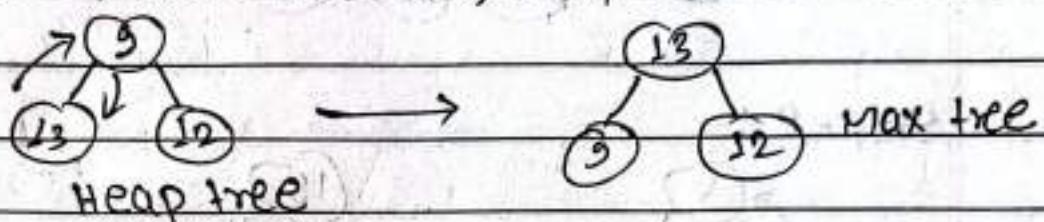
Step-3 :- Delete root 18, swap 18 with 12



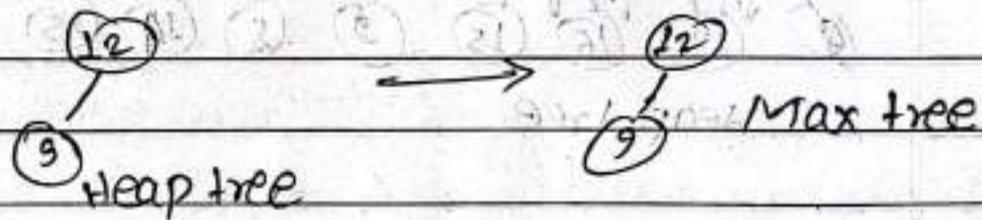
Step-4 :- Delete Root (17), swap (9) with 9.



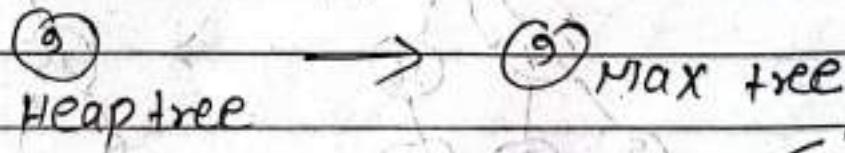
Step-5 :- Delete Root (15), swap (13) with (9).



Step-6 :- Delete Root (13), swap (12) with (12)



Step-7 :- Delete Root (12), swap (12) with (9)



Step-8 :- delete root (9)

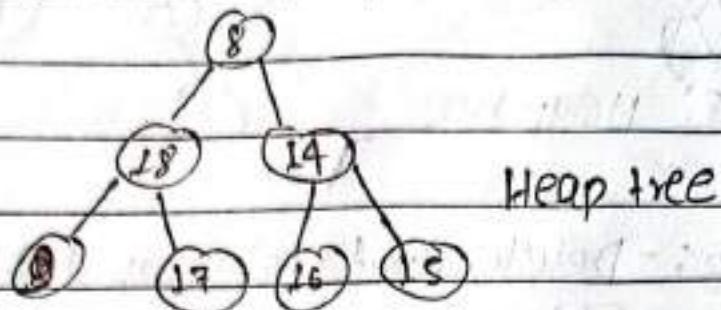
Heap tree empty.

$$\therefore 9, 12, 13, 15, 17, 18$$

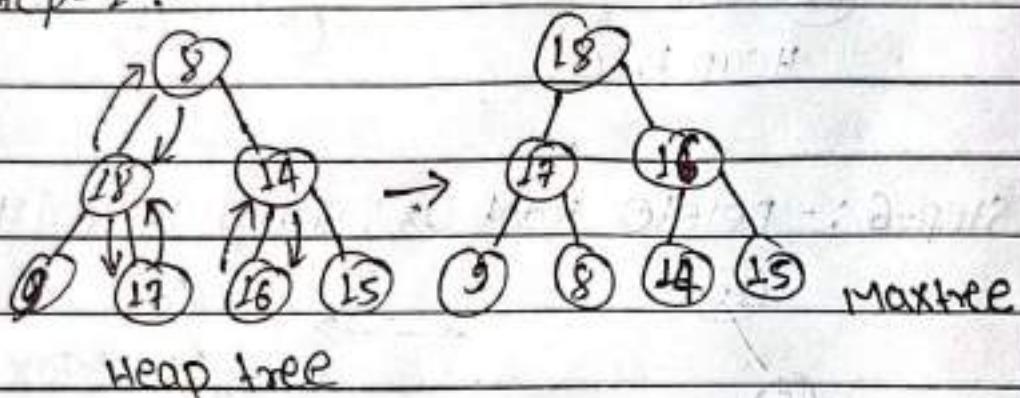
	9
	12
	13
	15
	17
	18

No.3 8, 18, 14, 9, 17, 16, 15

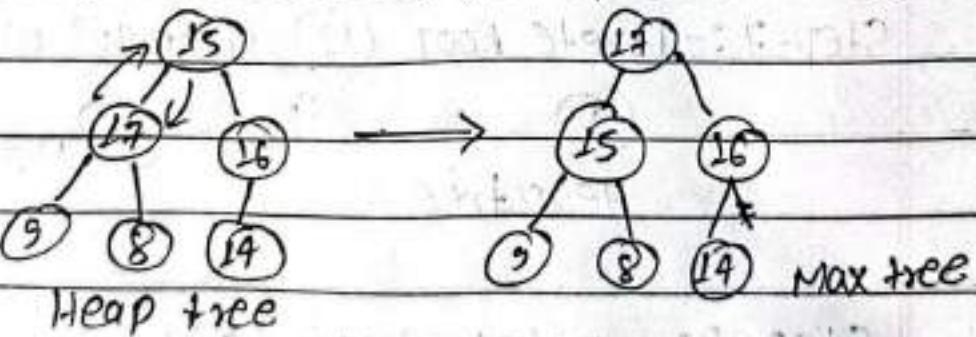
Step-1 :- Construct Heap tree.



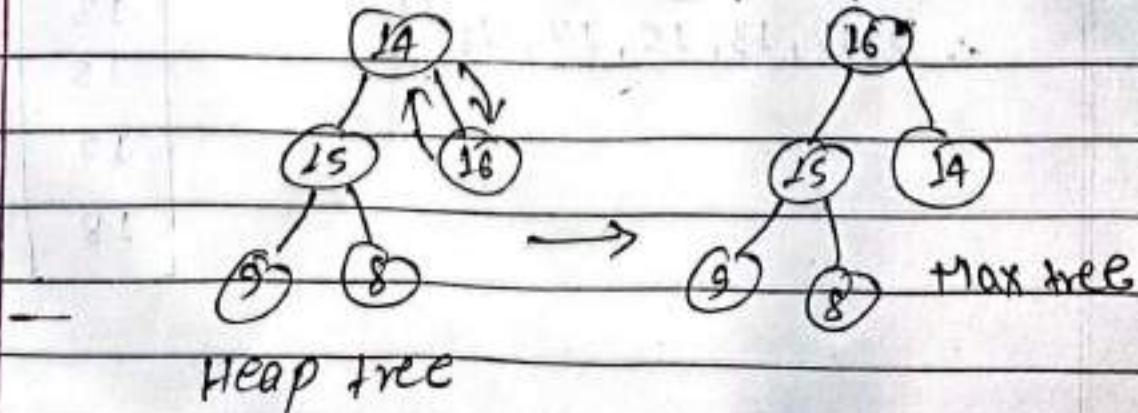
Step-2 :-



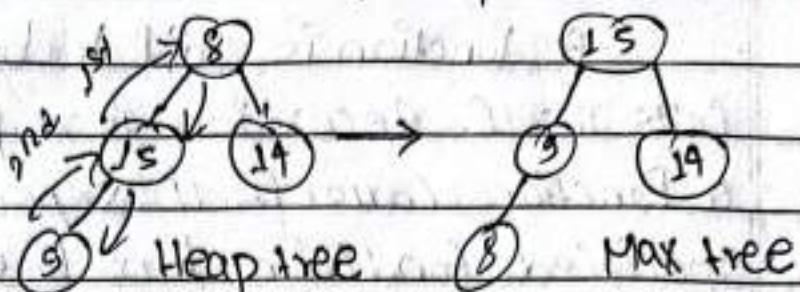
Step-3 :- Delete Root (18), Swap (18) with (15).



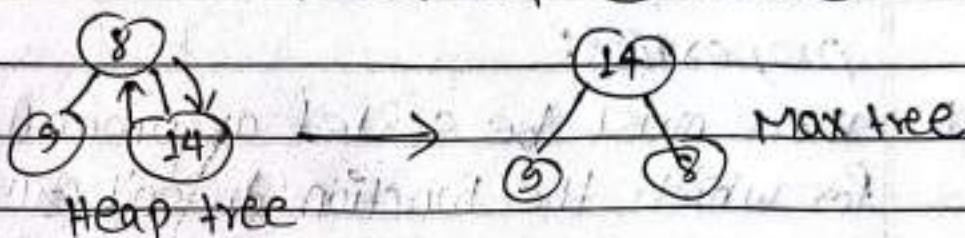
Step-4 : - Delete Root (17), swap (17) with (14).



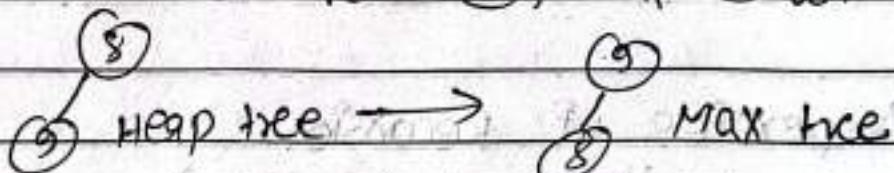
Step-5:- Delete Root (16), swap ~~(16)~~ with (8).



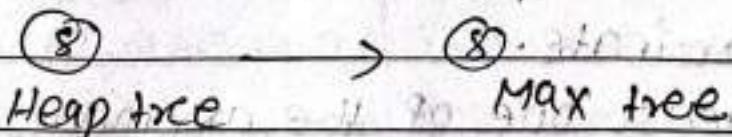
Step-6:- Delete Root (15), swap (15) with (8).



Step-7:- Delete Root (14), swap (14) with (8)



Step-8 : - Delete root (9), swap (9) with (8)



Step-9:- Delete Root (8) ~~8~~

Heap tree empty

$\therefore 8, 9, 10, 15, 16, 17, 18$  ~~18~~

8
9
14
15
16
17
18

## \* Recursion

A function is said to be recursive if it calls itself. Recursion is a process by which a function calls itself repeatedly until some specified condition has been satisfied.

A Recursive Function should have two properties:

1. There must be sorted argument (called base value) for which the function doesn't call itself.
2. Each time the function calls itself, The argument must be closer to the base value.

### 4.1 \* Properties of Recursion:

- i.) It shouldn't generate infinite numbers of calls on itself because such definitions will never terminate.
- ii.) At least one of the arguments shouldn't be expressed in terms of itself.
- iii.) There should be a terminating conditions in every recursive function which when satisfied exit the control out of the function finally.

4.2

## Recursion vs Iteration.

### Recursion

i.) Recursion is a technique of defining anything in terms of itself.

ii.) There must be an "if" statement inside the recursive function to specify stopping condition.

iii.) Not all problems have recursive solution.

iv.) Recursion is generally a worse version function to solve simple problem.

v.) The speed of <sup>recursive</sup> iteration program is <sup>slower</sup> faster than recursive because of stack overhead.

### Iteration

i) It is a process of executing statements repeatedly until some specified condition is satisfied.

Iteration involves four clear cut steps like initialization, condition, execution & update.

Any recursive problem can be solved iteratively.

Iterative counter part of a problem is more efficient in terms of memory utilization & execution speed.

The speed of iteration program is faster than recursive program.

4.3

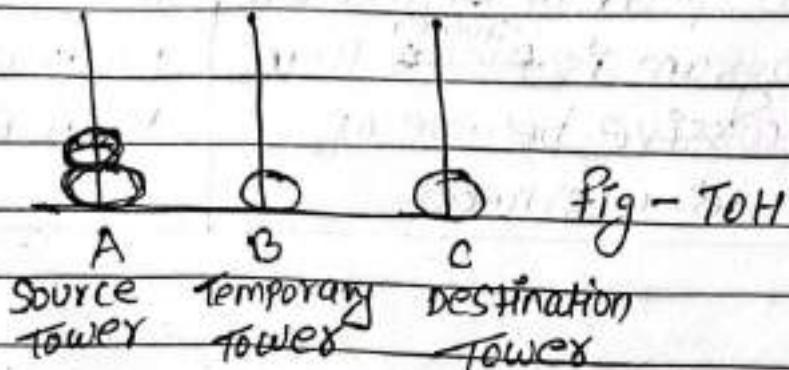
### Towers of Hanoi (TOH)

V.V.S.

TOH has Three towers, one is used as Source tower, second one is used temporary tower and the third one is used as destination tower. TOH problem is moved to disk(plate) from one tower to another using a temporary tower.

Suppose we have a Source tower A, which has finite number of disk and these disk are placed on it in a decreasing order that is largest disk is at the bottom and the smallest disk is at the top. Now we have to place all these disks on destination tower C in the same order. We can use a temporary tower B to placed the disk temporarily on tower B when ever required. Following are the rules to be followed during transferred :

- i) At any point of transformation of disk, larger disk cannot be placed on smaller disc.
- ii) Only one disk may be moved at a time.



The recursive function for solving TOH problem can be defined as:

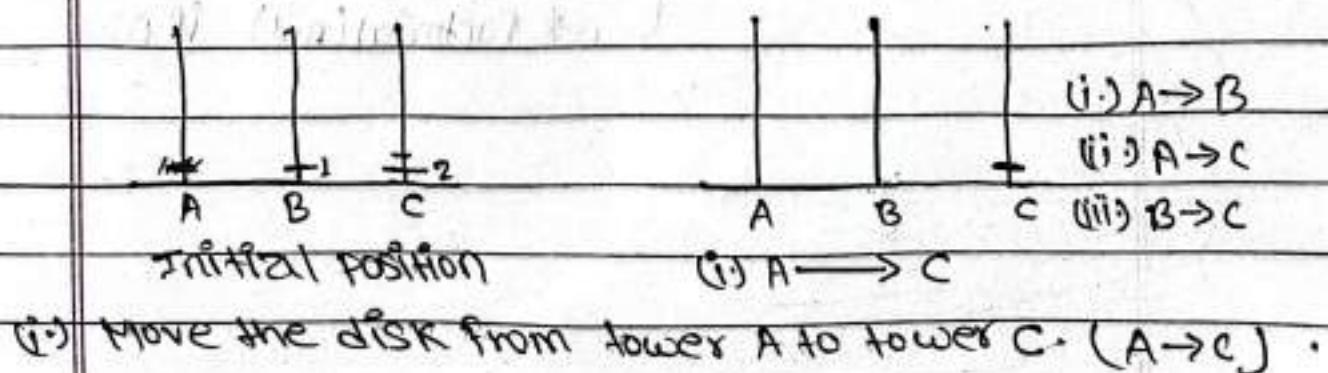
$$t(n, \text{Source}, \text{Temp}, \text{dest}) = \begin{cases} \text{Move disk } 1 \text{ from (if } n=1) \\ \text{Source to dest} \\ t(n-1, \text{source}, (\text{if } n>1) \\ \text{dest, temp}) \\ \text{move } n^{\text{th}} \text{ disk from} \\ \text{Source to dest to } t(n-1, \text{temp, source, dest}) \end{cases}$$

\* Algorithm for TOH.

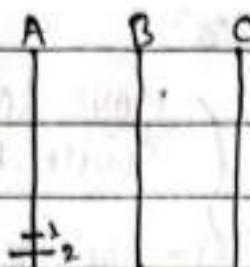
To move  $n$  disk from tower A to C using B as temporary tower, the general solution can be written as :

- (i) If  $n=1$  Move the single disk from A to C and stop.
- (ii) Move upper  $n-1$  disks from A to B using C as the temporary tower.
- (iii) Move  $n^{\text{th}}$  disk from A to C.
- (iv) Move  $n-1$  disk from B to C using A as the temporary tower.

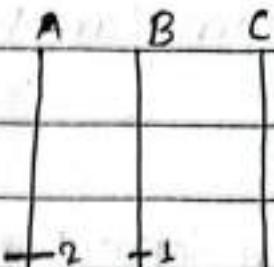
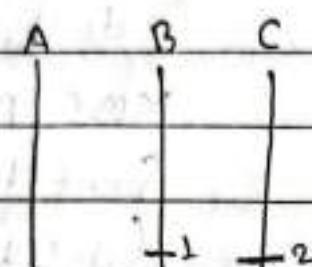
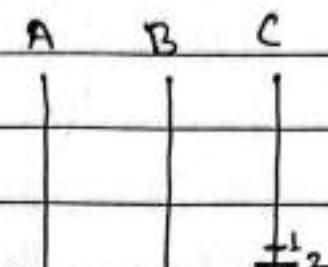
Eg :- for  $n=1$  [i.e. no of disk on source tower A is one].



for  $n=2$  [i.e. no of disks on source tower A is two].



(a) Initial position

(i)  $A \rightarrow B$ (ii)  $A \rightarrow C$ (iii)  $B \rightarrow C$ 

(i) Move disk 1 from A to B

(ii) Move disk 2 from A to C

(iii) Move disk 1 from B to C

08/03/18

#### 4.4 Factorial

The factorial of a number is the product of the integer values from 1 to the number, recursively the factorial algorithm can be define as below:

$$\text{factorial}(n) = \begin{cases} 1 & \text{if } n=0 \\ n * \text{factorial}(n-1) & \text{if } n>0 \end{cases}$$

program

```
#include <stdio.h>
```

```
#include <conio.h>
```

```
long int fact(int n);
```

S

```
if(n==0)
```

```
    return 1;
```

else

```
    return(n * fact(n-1));
```

2

```
void main();
```

S

```
i (int x,y);
```

```
printf("In Enter an integer value");
```

```
scanf("%d", &x);
```

```
y=fact(x);
```

```
printf("n factorial=%d",y);
```

```
getch();
```

2

# The below figure illustrate the recursive concept of factorial up to 3.

fact(3)=3×fact(2)

fact(3)=3×2=6

fact(2)=2×fact(1)

fact(2)=2×1=2

fact(1)=1×fact(0)

fact(1)=1×1=1

fig:-fact(3) recursive concept.

fact(0)=1

## \* Fibonacci Numbers [Leonardo Fibonacci]

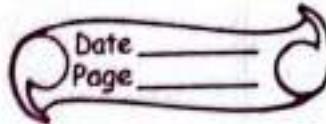
Fibonacci Numbers are a series in which each number is the sum of the previous two numbers. The first few numbers in Fibonacci series are 0, 1, 1, 2, 3, 5, 8, 13... where, 0 & 1 is the base value of Fibonacci Series.

Recursively the Fibonacci algorithm can be defined as :

$$\text{fib}(n) = \begin{cases} 1 & \text{if } n=1 \\ \text{fib}(n-1) + \text{fib}(n-2) & \text{if } n>1 \end{cases}$$

### Program

```
#include <stdio.h>
#include <conio.h>
int fib (int n)
{
    if (n<=1)
        return n;
    else
        return (fib(n-1)+fib(n-2));
}
void main()
{
    int n, x;
    printf("Enter the number of terms");
}
```



scanf("%d", &n);

for (i=0; i<n; i++)

S

x = fib(i);

printf("%d", x);

2

getch();

2

\* Graph

Graph is a non-hierarchical data structure that consists of nodes and set of edges that establish relationships (connections) between the nodes. OR,

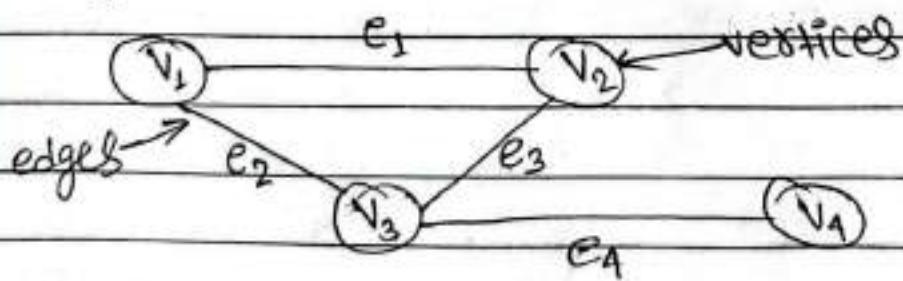
A graph is a collection of nodes called vertices and line segments [edge or arc] that connect pairs of nodes.

OR,

A graph 'G' is defined as set of  $(V, E)$  where,  $V$  is a finite and non-empty set of vertices and  $E$  is the set of edges.

## \* Components of Graph.

The below figure illustrates graph with its edges and vertices component.



The two components are :-

\* vertex:- Vertex is the point or node used to store the data elements. In the above graph the vertex 'V' can be represented as

$$V = \{v_1, v_2, v_3, v_4\}$$

\* Edge :- Edge is a line or arc used to connect vertices in the above graph the edge 'F' can be represented as

$$E = \{e_1, e_2, e_3, e_4\}$$

$$= \{(v_1+v_2), (v_1+v_3), (v_2+v_3), (v_3+v_4)\}$$

\* Types of Graph

A graph can be of Two Types:-

(i) Directed Graph [Digraph]

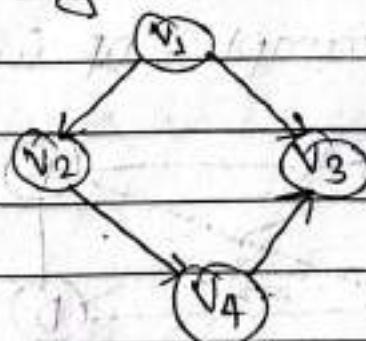
(ii) Undirected Graph

(i) Directed Graph

A Directed Graph is a graph which has ordered pair of vertices  $(u,v)$ . In directed graph, A direction is associated with each edge that means  $(u,v)$  is not equal to  $(v,u)$ .

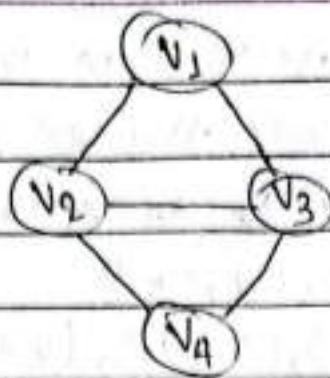
The below figure illustrate the directed

(ii) Undirected Graph



(iii) Undirected Graph

Undirected Graph is a graph which has unordered pair of vertices. If there is an edge between vertices  $(u \& v)$  then it can be represented as either  $(u,v)$  or  $(v,u)$  in undirected graph. The below figure illustrate the concept of undirected graph.



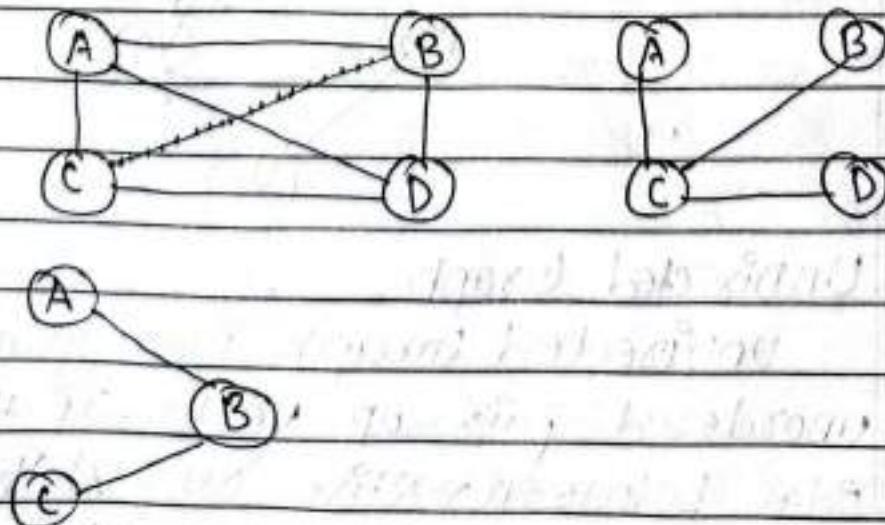
### \* Undirected Graph Connectivity.

There are two types of connectivity of undirected graph.

- (i) Connected graph.
- (ii) Disconnected graph

#### (i) Connected graph

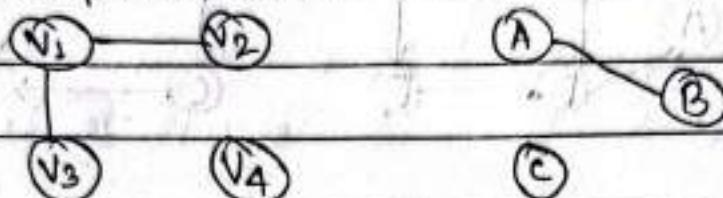
An undirected graph is said to be connected if there exist a path from any vertex to any other vertex. A connected graph of "N" vertices has at least " $N-1$ " edges. Examples of connected graphs are



## (ii) DisConnected graph (unconnected graph)

An undirected graph is disconnected graph if there does not exist at least one path from any vertex to any other vertex.

Examples are :-



## \* Directed Bigraph Connectivity

There are two types of connectivity of directed graph.

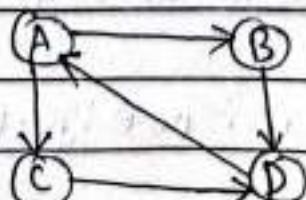
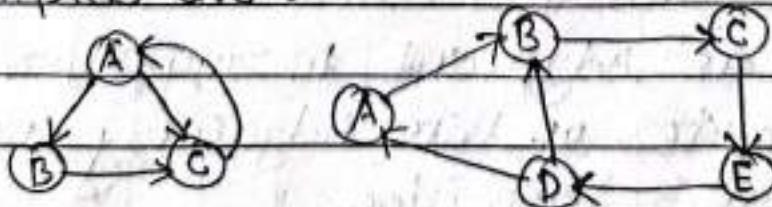
i) Strongly Connected graph.

ii) Weakly Connected graph.

## i) Strongly Connected graph

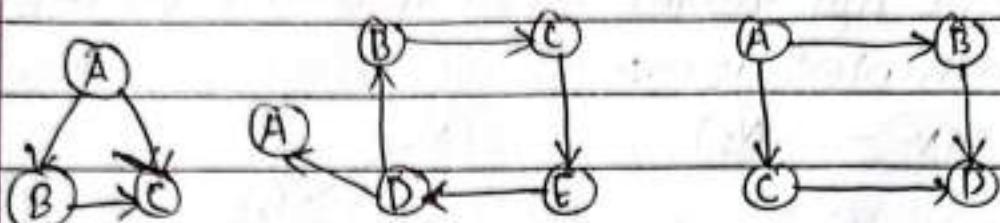
A directed graph is strongly connected if there is a directed path from any vertex of graph to any other vertex.

Examples are :-



### (ii) Weakly Connected Digraph

A directed graph is weakly connected if at least two vertices are not connected.



### \* Representation of Digraph

There are two main standard ways of representing a graph in the memory of a computer.

- (i) Sequential representation using Adjacency Matrix.
- (ii) Link representation using linked list of Adjacency list.

### // (i) Adjacency Matrix

Adjacency Matrix is a matrix that maintains the information of adjacent vertices. That means this matrix tells us whether a vertex is adjacent to any other vertex or not. The entries of this adjacency matrix are fined using this definition.

$$A(i,j) = \begin{cases} 1 & \text{if there is an edge from vertex } i \text{ to vertex } j \\ 0 & \text{if there is no edge from vertex } i \text{ to vertex } j. \end{cases}$$

In this method the size of the graph must be known before the program starts.

For example :-

(a) For Directed graph :-

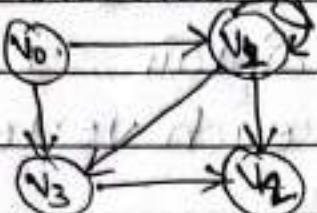
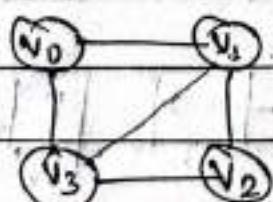


Fig :- Directed graph

The above directed graph can be represented in Adjacency Matrix form as below :-

	$v_0$	$v_1$	$v_2$	$v_3$	
$v_0$	0	1	0	1	
$v_1$	0	0	1	1	
$v_2$	0	0	0	0	
$v_3$	0	0	1	0	

(b) For Undirected graph :-



	$v_0$	$v_1$	$v_2$	$v_3$	
$v_0$	0	1	0	1	
$v_1$	1	0	1	1	
$v_2$	0	1	0	1	
$v_3$	1	1	1	0	

Fig :- Undirected graph

The above undirected graph can be represented in Adjacency Matrix form as above :-

## ii.) Adjacency List

In Adjacency List method, we maintain two linked list. The first linked list is the vertex list that keeps track off all the vertices in the graph and second linked list is the edge list that maintains a list of adjacent vertices for each vertex.

The following figure shown a directed graph and its adjacency list structure.

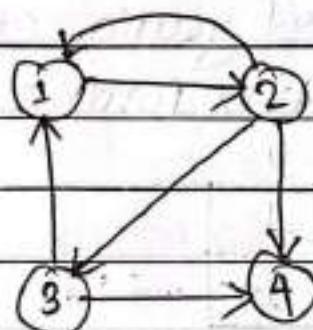
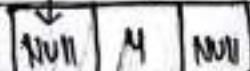
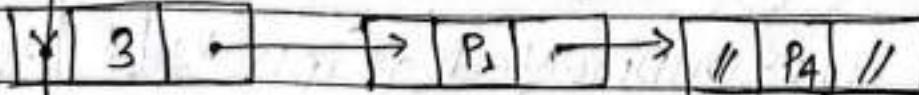
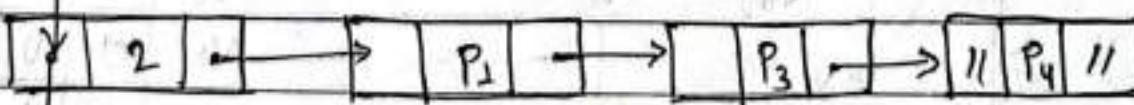
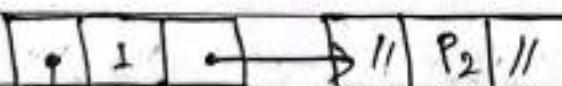


Fig :- Directed graph



## \* Graph Traversal

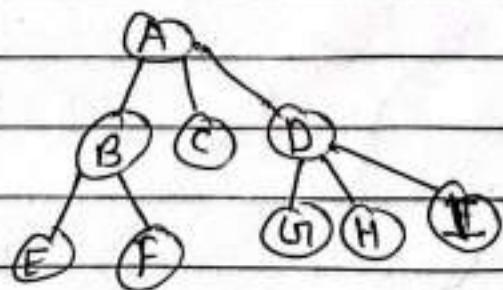
- 1.) Depth First Search / Traversal [DFS/T]
- 2.) Breadth First Search / Traversal [BFS/T]

### 1.) Depth First Search / Traversal [DFS/T]

The idea of this algorithm is to make a path as long as possible and then go back to add branches also as long as possible.

In DFS, we process all of a vertex descendants before we move to an adjacent vertex. In this method, we first visit the starting vertex and then pick up any path that starts from the starting vertex and all the vertices in the path till we reach a data ending. It is implemented using Stack.

For example, let us take a graph and perform a depth first search traversal taking 'A' as starting vertex.



DFS :- A, B, E, F, D, G, H, I, C

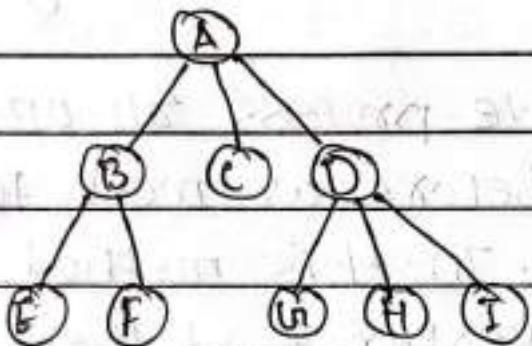
A, C, B, F, E, D, H, G, I.

## ② Breadth first-Search (BFS) Traversal:

In this technique, the starting vertex is visiting first and then visit all the vertices adjacent to the starting vertex. After this way pick this <sup>adjacent</sup> vertices one by one and visit these adjacent vertices and this process goes on.

BFS is implemented by using queue.

For example, the BFS Traversal of the below graph is written as



BFS :- A, B, C, D, E, F, G, H, I

7.4.)

### Hashing:-

Hash is a datastructure that allow the storage and retrieval of data in an average time which does not depend at on the collection size. So Hashing is a method to store data in a way that storing, searching, inserting and deleting data is fast.

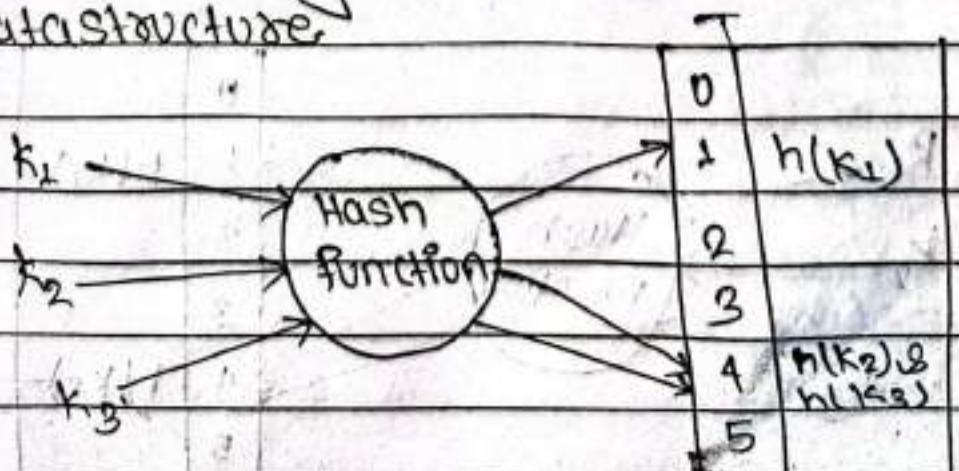
7.4.1)

### Hash Table,

it is a datastructure in which the location of a data item is determined directly as a function of the data item itself rather than by a sequence of comparisons.

A Hash Table is made up of two parts:  
an array (where actual data is stored) and  
a mapping function (hash functions).

The below figure illustrate the hash table datastructure.



7.4.3)

Hash Collision or Hash clash

A hash collision occurs when the hash function generates the same address for different keys.

7.4.4)

Techniques to solve Hash Collision.

A technique used to resolve the collision generated by a hash function is called Collision Resolution Techniques. The possible collision resolutions techniques are as follows:

(1)

chaining

(2)

open addressing

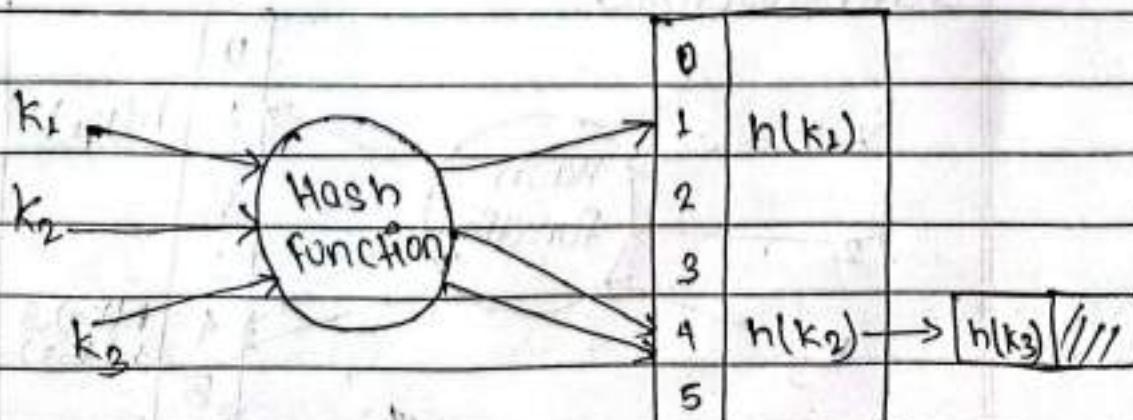
(3)

Re-hashing

(4)

chaining collision

It creates a linked list of all items whose keys hash to the same values. In this method, linked lists are maintained for keys that have same hash address.



Example:-

Let us consider the insertion of elements 5, 28, 19, 15, 20, 33, 12, 17, 10 into a chained Hash table. Let us suppose that Hash table has ~~9 slots~~ <sup>9 slots</sup> and the Hash function be " $h(k) = k \bmod 9$ "

Soln

We create a chained table with 9 slots.

0	NULL	Insert(5) :-						
1	NULL	→	28			$h(5) = 5 \bmod 9 \Rightarrow 5$		
2						Create a linked list at slot 5		
3						and insert it at the linked list		
4								
5				5			Insert(28) :-	
6						$h(28) = 28 \bmod 9 \Rightarrow 1$		
7								
8							Insert(19) :-	

Fig :- Initial Step of  
chained Hash Table.

Insert(15) :-

$$h(15) = 15 \bmod 9 \Rightarrow 6$$

Insert(20) :-

$$h(20) = 20 \bmod 9 \Rightarrow 2$$

Insert(33) :-

$$h(33) = 33 \bmod 9 \Rightarrow 6$$

Insert (12) :-

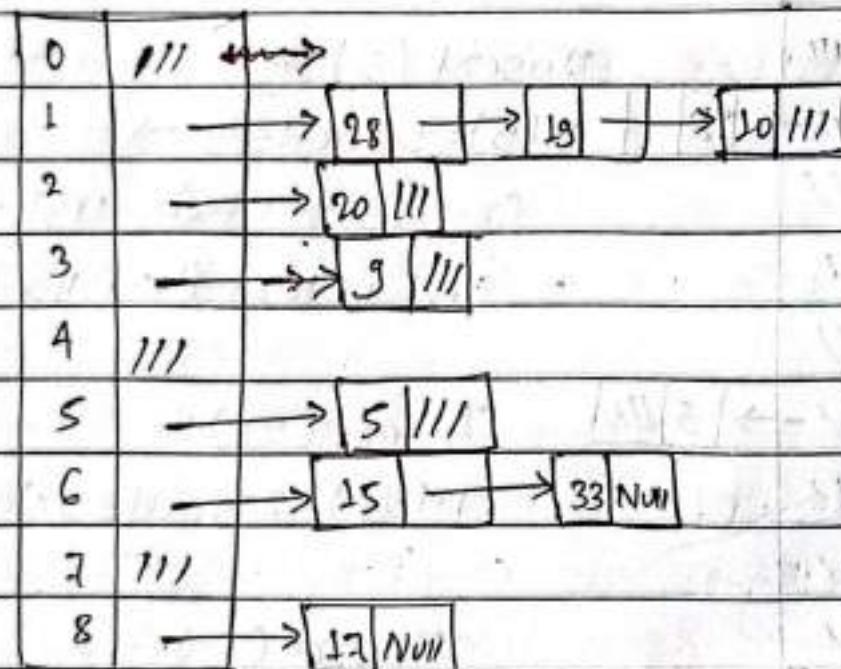
$$h(12) = 12 \text{ mode } 9 \Rightarrow 3$$

Insert (17) :-

$$h(17) = 17 \text{ mode } 9 \Rightarrow 8$$

Insert (10) :-

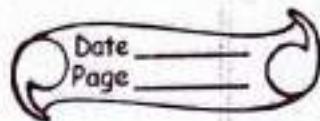
$$h(10) = 10 \text{ mode } 9 \Rightarrow 1$$



## ② open addressing

Open Addressing is a method of collision resolution in which a hash collision is resolved by "probing" or searching until either the target record is found or an unused array slot is found. Some of the well known probe sequences are:

- (1) Linear probing
- (2) Quadratic probing
- (3) Double hashing



### (a) Linear probing

Linear probing uses the following Hash functions.

$$h(k, i) = [h'(k) + i] \bmod m \text{ for } i = 0, 1, 2, \dots, m-1$$

where,

$m \rightarrow$  is the size of Hash table.

$$h'(k) = k \bmod m$$

$i$  is the probe number.

Example:-

Consider inserting the keys 76, 26, 37, 59, 91, 65, 88 into a Hash table of size  $m=11$ , using linear probing. Further consider that the primary Hash function is  $h'(k) = k \bmod m$ .

	0	1	2	3	4	5	6	7	8	9	10
T	21	65			26	37	59	.	88		76

Fig:- Initial stage of the Hash Table.

As we know,

for linear probing :-  $h(k, i) = [h'(k) + i] \bmod m$

$$\text{We have } h'(k) = k \bmod m$$

$$\text{and } m=11$$

Insert (76)

$$h(76+0) = [76 \bmod 11 + 0] \bmod 11$$

$$k \quad i \\ \underline{= 10}$$

11) 76 G

66

10

Insert (26) :-

$$h(26, 0) = [26 \bmod 11 + 0] \bmod 11 \\ = 4$$

Insert (37) :-

$$h(37, 0) = [37 \bmod 11 + 0] \bmod 11 \\ = 4$$

Since, slot T[4] is occupied, the next prob sequence is.

Insert (39) :- Calculated as

$$h(39, 1) = [39 \bmod 11 + 1] \bmod 11 = 5$$

Insert (59) :-

$$h(59, 0) = [59 \bmod 11 + 0] \bmod 11 = 4$$

Since, slot T[4] is occupied, the next prob Sequence is calculated as

$$h(59, 1) = [59 \bmod 11 + 1] \bmod 11 = 5$$

Again, slot T[5] is occupied, the next prob Sequence is calculated as

$$h(59, 2) = [59 \bmod 11 + 2] \bmod 11 = 6$$

Insert (21) :-

$$h(21, 0) = [21 \bmod 11 + 0] \bmod 11 = 10$$

Since, slot T[10] is occupied, the next prob sequence is calculated as

$$h(21, 1) = [21 \bmod 11 + 1] \bmod 11 = 0$$

Insert (65) :-

$$h(65, 0) = [65 \bmod 11 + 0] \bmod 11 = 10$$

Since, slot  $T[10]$  is occupied, the next prob sequence is calculated as

$$h(65, 1) = [65 \bmod 11 + 1] \bmod 11 = 0$$

Since, slot  $T[0]$  is occupied, the next prob sequence is calculated as

$$h(65, 2) = [65 \bmod 11 + 2] \bmod 11 = 1$$

Insert (88) :-

$$h(88, 0) = [88 \bmod 11 + 0] \bmod 11 = 8$$

05/09/23

This is required linear probing

7.4.9)

### Collision Resolution Technique

Open Hashing

chaining

Closed Hashing

Linear probing  
Quadratic probing  
Double Hashing

(b) Quadratic probing

In Quadratic probing, Instead of probing the next slot linearly, the algorithm uses a quadratic function to determine the next prob position. The probing sequence follows a quadratic pattern until an empty slot is found.

The quadratic probing uses the following Hash function.

$$h(k, i) = [h'(k) + c_1 i + c_2 i^2] \bmod m$$

for  $i = 0, 1, 2, \dots, m-1$

where,  $c_1$  &  $c_2 \neq 0$ , are auxiliary constants.

Example :-

Consider Inserting the keys 76, 26, 37, 59, 21, 65, 88 into a Hash table of size  $M=11$  using quadratic probing with  $c_1=1$  &  $c_2=3$ . Further consider that the primary Hash function is " $h'(k)=k \bmod m$ ".

Soln

$$\begin{aligned} \text{we have } h(k, i) &= [h'(k) + c_1 i + c_2 i^2] \bmod m \\ &= [h'(k) \bmod 11 + i + 3i^2] \bmod 11 \end{aligned}$$

0	1	2	3	4	5	6	7	8	9	10
T	88	11	65	21	26	11	11	59	37	11

Fig:- Initial stage of Hash Table.

Insert 76 :-

$$h(76, 0) = [76 \bmod 11 + 0 + 0] \bmod 11 \Rightarrow 10$$

Since, slot  $T[10]$  is free, insert 76 at slot 10.

Insert 26 :-

$$h(26, 0) = [26 \bmod 11 + 0 + 0] \bmod 11 \Rightarrow 4$$

Since, slot  $T[4]$  is free, Insert 26 at slot 4.

Insert 37 :-

$$h(37, 0) = [37 \bmod 11 + 0 + 0] \bmod 11 \Rightarrow 4$$

Since,  $T[4]$  is occupied, the next prob sequence

is computed as

$$h(37,1) = [37 \bmod 11 + 1 + 3] \bmod 11 \Rightarrow 8$$

Insert 59 :-

$$h(59,0) = [59 \bmod 11 + 0 + 0] \bmod 11 \Rightarrow 4$$

Since,  $T[4]$  is occupied, the next prob sequence is computed as

$$h(59,1) = [59 \bmod 11 + 1 + 3] \bmod 11 \Rightarrow 8$$

Now,  $T[8]$  is occupied, the next prob sequence is computed as

$$h(59,2) = [59 \bmod 11 + 2 + 12] \bmod 11 \Rightarrow 7$$

Insert 21 :-

$$h(21,0) = [21 \bmod 11 + 0 + 0] \bmod 11 \Rightarrow 10$$

Since,  $T[10]$  is occupied, the next prob sequence is computed as

$$h(21,1) = [21 \bmod 11 + 1 + 3] \bmod 11 \Rightarrow 3$$

Insert 65 :-

$$h(65,0) = [65 \bmod 11 + 0 + 0] \bmod 11 \Rightarrow 10$$

Since,  $T[10]$  is occupied, the next prob sequence is computed as

$$h(65,1) = [65 \bmod 11 + 1 + 3] \bmod 11 \Rightarrow 3$$

Since,  $T[3]$  is occupied, the next prob sequence is computed as

$$h(65,2) = [65 \bmod 11 + 2 + 12] \bmod 11 \Rightarrow 2$$

Insert 88 :-

$$h(88,0) = [88 \bmod 11 + 0 + 0] \bmod 11 \Rightarrow 0$$

### C) Double Hashing

it uses two hash function to determine the probe sequence. it occupies the second hash function to calculate and offset which is then used to find the next slot to probe. double Hashing uses a hash function of the form

$$h(k, i) = [h_1(k) + ih_2(k)] \bmod m$$

for  $i = 0, 1, 2, \dots, m-1$

where,  $h_1(k) = k \bmod m$

$$h_2(k) = k \bmod m'$$

and  $m'$  is lightly less than  $m$ .

Example:-

Considered inserting the keys 76, 96, 37, 59, 21, 65, 88 in a Hash Table of size  $m=11$  using double Hashing. Further consider that the auxiliary Hash function are :-

" $h_1(k) = k \bmod 11$ " and " $h_2(k) = k \bmod 9$ "

Soln

	0	1	2	3	4	5	6	7	8	9	10
T	88	65	21		26	37			59	76	

We have,  $- h(k, i) = [(k \bmod 11) + i(k \bmod 9)] \bmod 11$

Insert 76 :-

$$h(76, 0) = [(76 \bmod 11) + 0] \bmod 11 \\ = 10$$

Insert 26 :-

$$h(26, 0) = [(26 \bmod 11) + 0] \bmod 11 = 4$$

Insert 37 :-

$$h(37, 0) = [(37 \bmod 11) + 0] \bmod 11 \Rightarrow 4$$

Since,  $T[4]$  is occupied, the next prob sequence is computed as

$$h(37, 1) = [(37 \bmod 11) + (37 \bmod 9)] \bmod 11 \\ = 5$$

Insert 59 :-

$$h(59, 0) = [(59 \bmod 11) + 0] \bmod 11 \Rightarrow 4$$

Since,  $T[4]$  is occupied, the next prob sequence is computed as

$$h(59, 1) = [(59 \bmod 11) + (59 \bmod 9)] \bmod 11 \Rightarrow 9$$

Insert 21 :-

$$h(21, 0) = [(21 \bmod 11) + 0] \bmod 11 \Rightarrow 10$$

Since,  $T[10]$  is occupied, the next prob sequence is computed as

$$h(21, 1) = [(21 \bmod 11) + (21 \bmod 9)] \bmod 11 \Rightarrow 2$$



Inserted 65 :-

$$h(65, 0) = [(65 \bmod 11) + 0] \bmod 11 \Rightarrow 10$$

Since,  $T[10]$  is occupied, the next probe sequence is computed as

$$h(65, 1) = [(65 \bmod 11) + (65 \bmod 9)] \bmod 11 \Rightarrow 1$$

Insert 88 :-

$$h(88, 0) = [(88 \bmod 11) + 0] \bmod 11 \Rightarrow 0$$

## \* Searching

Searching is the process of finding the presence of desired data items in the list of data items.

Basically there are two types of searching

- (i) Sequential Search
- (ii) Binary Search

### (i) Sequential Search (Linear Search)

Sequential Search is a very simple search algorithm. In this type of search, we start searching at the beginning of a list of items and looks at each turn until we either found the required data or search at end of list.

Algorithm for Sequential Searching.

- (1) Set  $I = 1$
- (2) If  $I > N$  THEN  
    goto Step 7
- (3) If  $A[I] = k$  Then  
    goto Step 6
- (4) Set  $I = I + 1$
- (5) Go to Step 2
- (6) print value  $k$  is found at index  $I$  and goto Step 8.
- (7) print value  $k$  is not found.
- (8) EXIT.

## (ii) Binary Search

This search algorithm works on the principle of divide and conquer. For this algorithm, the data collection should be in the sorted form.

Binary Search looks for a particular item by comparing the middle most item of the collection. If a match occurs, then the index of item is returned. If the middle item is greater than the item, then the item is searched in the sub-array to the left of the middle item. Otherwise, the item is searched in the sub-array to the right of the middle item. This process continues on the sub-array as well until the size of the sub-array reduces to zero.

	0	1	2	3	4	5	6	7	8	9
Search 23	2	5	8	12	16	23	38	56	72	91

	0	1	2	3	4	5	6	7	8	9
23 > 16 So,	2	5	8	12	16	23	38	56	72	91

take 2<sup>nd</sup> half    ↑mid

	0	1	2	3	4	5	6	7	8	9
23 < 56, So	2	5	8	12	16	23	38	56	72	91

take 1<sup>st</sup> half    ↓low      ↑mid      ↓high

	0	1	2	3	4	5	6	7	8	9
found 23 at	2	5	8	12	16	23	38	56	72	91

Index 5

↑low      ↓high  
mid

## \* Spanning Tree

A spanning tree is the sub-set of graph in which has all the vertices covered with minimum possible number of edges. Hence a spanning tree does not have cycles and cannot be disconnected.

Example :-

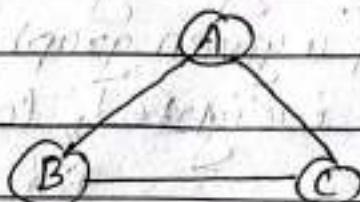


fig :- Graph

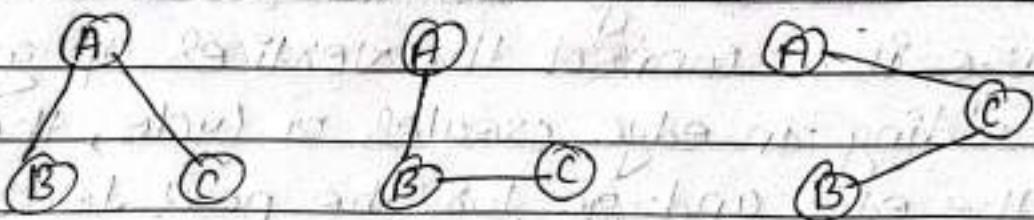


fig :- Spanning Trees of above graph.

## \* Minimum Spanning Tree (MST)

In a weighted graph, a minimum (MST) is a spanning tree that has minimum weight than all other spanning tree of the same graph.

## MST Algorithm

### (1) KXUSKAL'S ALGORITHM

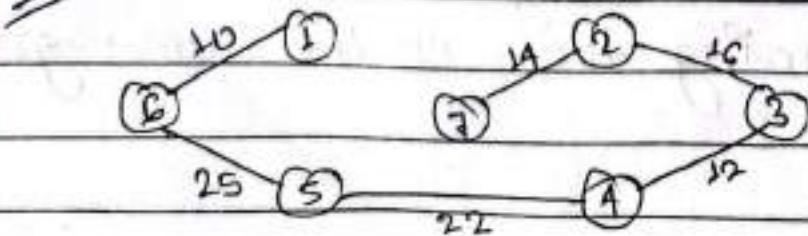
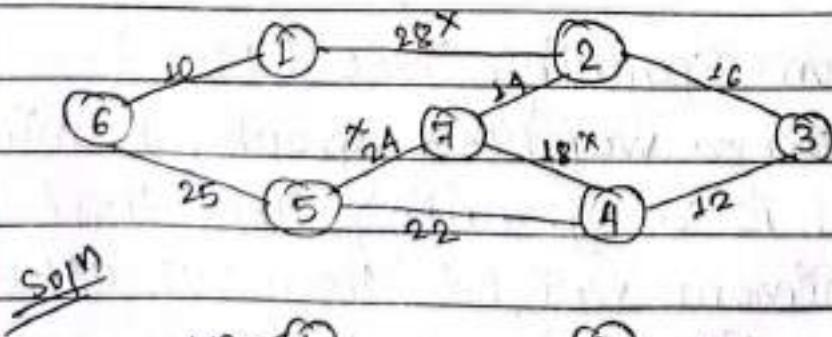
### (2) PRIM'S ALGORITHM

### (1) KXUSKAL'S ALGORITHM

This is a famous greedy algorithm it is used for finding the MST of a given graph. It is applied to the graph which must be weighted, connected and undirected.

Steps :-

- (1) Sort all the arrays from low weight to high weight.
- (2) Take the arrays with the lowest weight and use it to connect the vertices of graph. If adding an edge creates a cycle, then <sup>then</sup> reject the edge and go for the next least weight edge.
- (3) Keep adding edges until all the vertices are connected and a MST is obtained.



$$\text{Weight of MST} = 10 + 10 + 16 + 18 + 22 + 25 = 99 \text{ units}$$

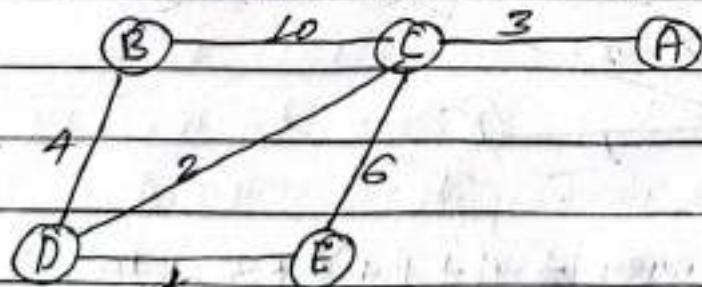
## ② Prim's Algorithm

prim's algorithm is use to find the MST. it takes a graph as input and finds the sub-set of the edges of that graph which:

- (i) form a tree that includes every vertex.
- (ii) Has the minimum sum of weights among all the trees that can be formed from the graph

Steps :-

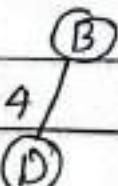
- (a) Initialize the minimum spanning tree with a vertex chosen at random.
- (b) Find all the edges that connect the tree to new vertices, find the minimum and it to the tree.
- (c) Keep repeating 'step-2' until we get a minimum spanning tree.



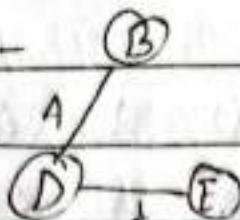
Step-1 :- B is chosen as random vertex for mst.

(B)

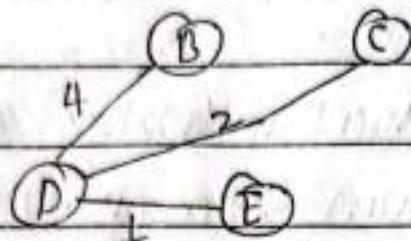
Step-2 :-



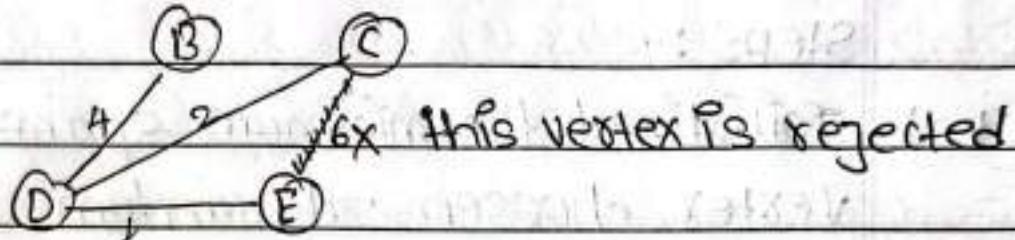
Step-3 :-



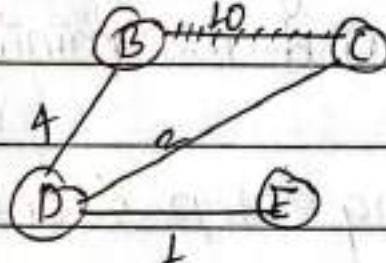
Step-4 :-



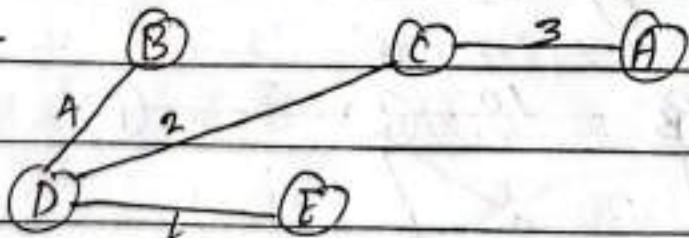
Step-5 :-



Step-6 :-



Step-7 :-



Weight of MST =  $1 + 4 + 1 + 2 + 3 = 10$

1.3.)

## Algorithm Analysis

Algorithm Analysis is an important part of Computational Complexity theory. It is used to determine the amount of time and space required to execute it. The analysis of algorithm is based on Time Complexity and Space Complexity.

- (i) Time Complexity
- (ii) Space Complexity

### (i) Time Complexity

It is the required time of the algorithm to solve the given problem. To estimate the Time Complexity we need to consider the cost of each fundamental instruction and the number of times the instruction is executed.

### (ii) Space Complexity

problem solving using computer requires memory to hold temporary data or final result while the program is in execution. The amount of memory required by the algorithm to solve given problem is called space complexity of the algorithm.

## \* Asymptotic Notations :-

Asymptotic Notation of an algorithm is an mathematical representation of its time complexity. In this notation, we use only the most significant times in the complexity of that algorithm.

The time complexity is represented by three ways:-

- (i) Big-Oh ( $O$ )
- (ii) Big-Omega ( $\Omega$ )
- (iii) Big-Theta ( $\Theta$ )

### (i) Big-oh ( $O$ ): -

Big-oh Notation is used to define the upper bound of an algorithm in terms of time complexity. Big-oh can be defined as:-

The function  $f(n) = O(g(n))$  iff  $\exists$  positive constants such that

$$f(n) \leq c * g(n) \quad \forall n > n_0.$$

Def-2 :- Consider Function  $f(n)$  as time complexity of an algorithm and  $g(n)$  is the most significant term.

The function  $f(n) = O(g(n))$  iff  
 $f(n) \leq c * g(n)$  for all  $n > n_0$ ,  
 $c > 0$  and  $n_0 \geq 1$ .

Example

$$f(n) = 2n + 3$$

or  $2n + 3 \leq 2n + 3n$

$$2n + 3 \leq 5n$$

$$\therefore f(n) = O(n)$$

## 2) Big-O

Big-O is used to define the lower bound of an algorithm in terms of time complexity. Big-O notation can be defined as:

The function  $f(n) = O(g(n))$  iff  $\exists$  positive constants such that  $f(n) \geq c * g(n) \forall n > n_0$ .

Consider Function  $f(n)$  as time complexity of an algorithm function  $g(n)$  is the most significant term. The function  $f(n) = \mathcal{O}(g(n))$  if and only if  $f(n) \geq c * g(n)$  for all  $n \geq n_0$ , ( $c > 0$  and  $n_0 \geq 1$ )

for example:-  $f(n) = 2n + 3$

$$2n + 3 \geq 2n$$

$$f(n) \geq g(n)$$

$$\therefore f(n) = \mathcal{O}(g(n))$$

### Big- $\Theta$

Big- $\Theta$  notation is used to define the average bound of an algorithm in terms of time complexity. Big- $\Theta$  notation can be defined as:

The function  $f(n) = \Theta(g(n))$  iff  $\exists$  positive constants  $c_1, c_2$  and  $n_0$  such that

$$c_1 \times g(n) \leq f(n) \leq c_2 \times g(n) \quad \forall n \geq n_0$$