1. a) If the equation $x^2 + 2(k+2)x + 9k = 0$ has equal roots. Find k.

➢ **Solution**

By, question roots are equal so,

Discriminant is Zero $i.e,\ b^2 - 4ac = 0$

$$\{2(k+2)\}^2 - 4.1.9k = 0$$

$$or,\quad 4(k+2)^2 - 4.9k = 0$$

$$or,\quad (k+2)^2 - 9k = 0$$

$$or,\quad k^2 + 2k + 4 - 9k = 0$$

$$or,\quad k^2 - 7k + 4 = 0$$

Solving we get, $k = \frac{7 \pm \sqrt{49-16}}{2} = \frac{7 \pm \sqrt{33}}{2}$

Hence, the value of $k = \frac{7+\sqrt{33}}{2}, \frac{7-\sqrt{33}}{2}$

b) Find the conjugate of the complex number $\frac{3+4i}{3-4i}$

➢ **Solution**

Given,

Let $Z = \frac{3+4i}{3-4i}$ **to find** $\bar{Z} = ?$

$$Z = \frac{3+4i}{3-4i} \times \frac{3+4i}{3+4i}$$

$$Z = \frac{(3+4i)^2}{9-16i^2} = \frac{9+24i+16i^2}{9-16i^2}$$

$$= \frac{9+24i-16}{9+16} \qquad [\because\ i^2 = -1\,]$$

$$= \frac{24i-7}{25}$$

$$Z = \frac{24i}{25} - \frac{7}{25}$$

$$= -\frac{7}{25} + \frac{24}{25}i$$

$$\bar{Z} = -\frac{7}{25} + \frac{24}{25}i$$

$$= -\frac{7}{25} - \frac{24}{25}$$

if $Z = a + bi$

$\because \bar{Z} = \overline{a+bi} = a - bi$

2. a) If $A = \begin{pmatrix} 2 & -1 \\ 0 & 3 \end{pmatrix}$ and $B = \begin{pmatrix} 3 & 1 \\ -2 & 4 \end{pmatrix}$ find the matrix of $(AB)^T$

➢ **Solution**

Given,

$$A = \begin{pmatrix} 2 & -1 \\ 0 & 3 \end{pmatrix} \text{ and } B = \begin{pmatrix} 3 & 1 \\ -2 & 4 \end{pmatrix}$$

Now,

$$AB = \begin{pmatrix} 1 & -2 & 3 \\ -1 & 2 & 1 \end{pmatrix} \begin{pmatrix} 2 & 3 \\ 3 & 1 \\ 1 & 2 \end{pmatrix}$$

$$AB = \begin{pmatrix} 2 & -1 \\ 0 & 3 \end{pmatrix} \begin{pmatrix} 3 & 1 \\ -2 & 4 \end{pmatrix}$$

$$= \begin{pmatrix} 2 \times 3 - 1 \times -2 & 2 \times 1 - 1 \times 4 \\ 0 \times 3 + 3 \times -2 & 0 \times 1 + 3 \times 4 \end{pmatrix}$$

$$AB = \begin{pmatrix} 8 & -2 \\ -6 & 12 \end{pmatrix}$$

To find,

$$(AB)^T$$

$$= \begin{pmatrix} 8 & -2 \\ -6 & 12 \end{pmatrix}^T$$

$$= \begin{pmatrix} 8 & -6 \\ -2 & 12 \end{pmatrix}$$

$$\therefore (AB)^T = \begin{pmatrix} 8 & -6 \\ -2 & 12 \end{pmatrix}$$

b) If w be the cube root of unity prove that:

$$(1 - \omega+\omega^2)(1 + \omega-\omega^2) = 4$$

➢ **Solution**

Given,

LHS $= (1 - \omega+\omega^2)(1 + \omega-\omega^2)$

$= 1 + \omega-\omega^2 - \omega-\omega^2 + \omega^3 + \omega^2 + \omega^3 - \omega^4$

$= 1 + \omega-\omega^2 - \omega-\omega^2 + \omega^3 + \omega^2 + \omega^3 - \omega^3.\omega$

$[\because \omega^2 \times \omega = \omega^3 = 1]$

$= 1 + \omega-\omega^2 - \omega-\omega^2 + 1 + \omega^2 + 1 - \omega$

$= 3-\omega^2 - \omega$

$= 3 + 1 - 1-\omega^2 - \omega$

$= 4 - (1+\omega + \omega^2)$ $[\because 1 + \omega^2 + \omega = 0]$

$= 4 - 0$

$= 4$ Proved.

3. a) Find the equation of parabola vertex (3,2) and focus (3, 4).

➢ **Solution**

Since, X-coordinates and focus are equal so, axis of parabola is parallel to y-axis.

Here, Vertex (h, k) = (3, 2) and focus (h , k + b) = (3, 4)

$\therefore$ **b = 1 , h = 3 and k = 2**

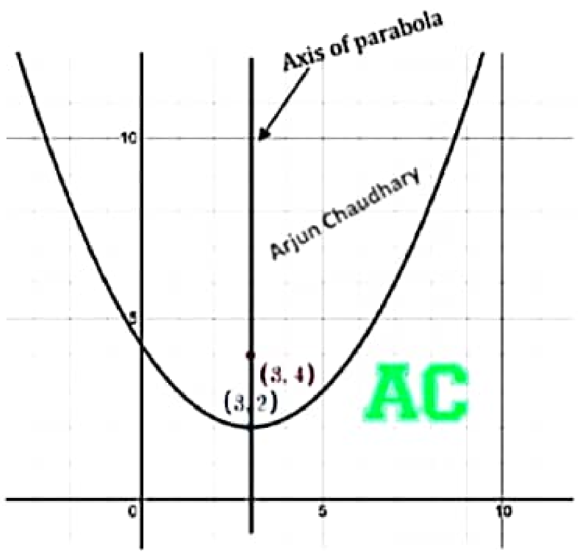
Required equation of parabola is

$$(x-h)^2 = 4b(y-k)$$

$$or, (x-3)^2 = 4 \times 1(y-2)$$

$$or, x^2 - 6x + 9 = 4(y-2)$$

$$\mathbf{or,\ x^2 - 6x - 4y + 17 = 0}$$



b) Find the foci of the hyperbola $3x^2 - 4y^2 = 36$.

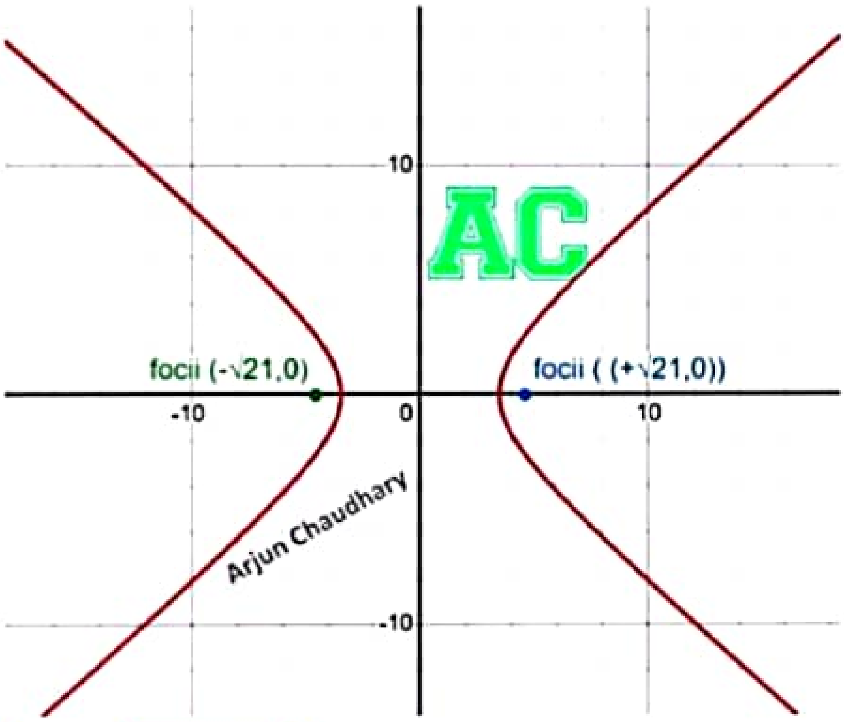
➢ **Solution**

Given,

$$3x^2 - 4y^2 = 36$$

$$\frac{3x^2 - 4y^2}{36} = 1$$

$$\frac{x^2}{12} - \frac{y^2}{9} = 1$$



Comparing with $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$

$$a^2 = 12, \quad b^2 = 9$$

$$\Longrightarrow a = 2\sqrt{3}, \qquad b = 3$$

Center of ellipse (0,0)

$$\text{Vertices} = (\pm a, 0) = (\pm 2\sqrt{3}, 0)$$

$$\text{Eccentricity}, e = \sqrt{1 + \frac{b^2}{a^2}} = \sqrt{1 + \frac{9}{12}} = \sqrt{\frac{21}{12}}$$

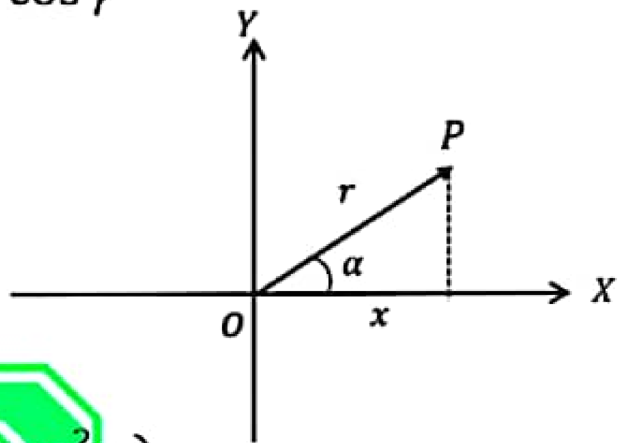
$$\text{For focii} \quad S(\pm ae, 0) = \left(\pm 2\sqrt{3} \times \sqrt{\frac{21}{12}}, 0\right) = (\pm\sqrt{21}, 0)$$

4. a) Find the direction cosines of a line which are equally Inclined to the axes.

➢ **Solution**

Let l, m, n are direction cosines in the x, y, z axes and α, β, γ are the angles made with d.c along respective axes $i.e,$

$$l = \cos\alpha\,, \mathrm{m} = \cos\beta \text{ and } n = \cos\gamma$$



$\cos\alpha = \frac{x}{r}$ or, $x = r.\cos\alpha$

Similarly, $y = r.\cos\beta$ *and* $z = r.\cos\gamma$

Squaring both sides, and adding

$$x^2 + y^2 + z^2 = r^2\ (\cos^2\alpha + \cos^2\beta + \cos^2\gamma)$$

As, $OP = r = \sqrt{x^2 + y^2 + z^2} \Rightarrow r^2 = r^2(\cos^2\alpha + \cos^2\beta + \cos^2\gamma)$

$$\mathbf{\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1 \quad or, [\,l^2 + m^2 + n^2 = 1\,]}$$

By question, equally inclined to the axes $i.e.,\ \alpha = \beta = \gamma$

$$or, \quad \cos^2\alpha + \cos^2\alpha + \cos^2\alpha = 1$$

$$or, \quad 3\cos^2\alpha = 1$$

$$or, \quad \cos^2\alpha = \frac{1}{3}$$

$$\Rightarrow \cos\alpha = \pm\frac{1}{\sqrt{3}}$$

Hence, $\cos\alpha = \cos\beta = \cos\gamma = \pm\frac{1}{\sqrt{3}}$, $\quad l = m = n = \pm\frac{1}{\sqrt{3}}$

Alternative solution:

Let l, m, n be d.c of lines making α, β, γ with postives axes respectively.

We have relation , $\boldsymbol{l^2+m^2+n^2=1} \cdots\cdots\cdots\cdots\cdots (\boldsymbol{i})$

Also we have, $l=\cos\alpha$, $\mathrm{m}=\cos\beta$ *and* $n=\cos\gamma$

Equation (i) becomes,

$$\cos^2\alpha+\cos^2\beta+\cos^2\gamma=1$$

By question, equally inclined to the axes $i.e.$, $\alpha=\beta=\gamma$

or, $\cos^2\alpha+\cos^2\alpha+\cos^2\alpha=1$

or, $3\cos^2\alpha=1$

or, $\cos^2\alpha=\frac{1}{3}$

$\Rightarrow \cos\alpha=\pm\frac{\boldsymbol{1}}{\boldsymbol{\sqrt{3}}}$

Hence, $\cos\alpha=\cos\beta=\cos\gamma=\pm\frac{1}{\sqrt{3}}$

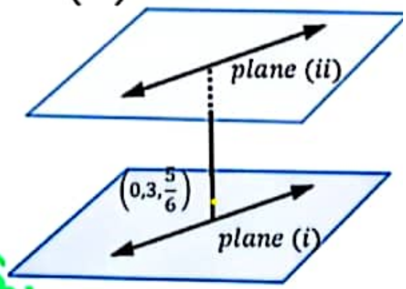
$$\boldsymbol{l=m=n=\pm\frac{1}{\sqrt{3}}}$$

b) Find the distance between the parallel planes $3x + 2y - 6z = 1$ and $6x + 4y - 12z + 9 = 0$.

➢ **Solution**

Given lines,

$$3x + 2y - 6z - 1 = 0 \cdots\cdots\cdots\cdots (i)$$
$$6x + 4y - 12z + 9 = 0 \cdots\cdots\cdots\cdots (ii)$$



When $x = 0, y = 3$ in (i) Then,

$$3 \times 0 + 2 \times 3 - 6z - 1 = 0$$
$$z = \frac{5}{6}$$

$\therefore$ Point $\left(0,3,\frac{5}{6}\right)$ lies on the plane (i)

Let 'P' be length of perpendicular drawn from point $\left(0,3,\frac{5}{6}\right)$ lies on the plane $6x + 4y - 12z + 9 = 0$ is

$$P = \left|\frac{6 \times 0 + 4 \times 3 - 12 \times \frac{5}{6} + 9}{\sqrt{6^2 + 4^2 + (-12)^2}}\right|$$

$$P = \left|\frac{11}{\sqrt{196}}\right|$$

$$P = \left|\frac{11}{14}\right|$$

$$\therefore \mathbf{P = \frac{11}{14} = 0.785\ unit.}$$

5. a) If $\vec{a} = 2\vec{i} + 3\vec{j}$ and $\vec{b} = -\vec{i} + \vec{j}$ find the unit vector along $2\vec{a} - 3\vec{b}$.

➢ **Solution**

Given,

$$2\vec{a} - 3\vec{b} = 2(2\vec{i} + 3\vec{j}) - 3(-\vec{i} + \vec{j})$$
$$= 4\vec{i} + 6\vec{j} + 3\vec{i} - 3\vec{j}$$
$$or,\ 2\vec{a} - 3\vec{b} = 7\vec{i} + 3\vec{j}$$

$$|2\vec{a} - 3\vec{b}| = \sqrt{7^2 + 3^2} = \sqrt{58}$$

Unit vector along $\widehat{2\vec{a} - 3\vec{b}} = \frac{7\vec{i} + 3\vec{j}}{\sqrt{58}} = \frac{7}{\sqrt{58}}\vec{i} + \frac{3}{\sqrt{58}}\vec{j}$

b) If $|\vec{a}+\vec{b}| = |\vec{a}-\vec{b}|$.Prove that a is perpendicular to $\vec{b}$

➢ **Solution**

Given, $|\vec{a}+\vec{b}| = |\vec{a}-\vec{b}|$

Squaring both side,

$$|\vec{a}+\vec{b}|^2 = |\vec{a}-\vec{b}|^2$$

$$(\vec{a}+\vec{b})^2 = (\vec{a}-\vec{b})^2$$

$$(\vec{a})^2 + 2\vec{a}\vec{b} + (\vec{b})^2 = (\vec{a})^2 - 2\vec{a}\vec{b} + (\vec{b})^2$$

$$a^2 + 2\vec{a}.\vec{b} + b^2 = a^2 - 2\vec{a}.\vec{b} + b^2$$

$$\Rightarrow 2\vec{a}.\vec{b} = -2\vec{a}\vec{b}$$

$$\Rightarrow 4\vec{a}.\vec{b} = 0$$

$$\vec{a}.\vec{b} = 0$$

Hence , $\vec{a}$ *is perpendicular to* $\vec{b}$

6. a) Find the area of the triangle whose two sides are determined by the vectors $\overrightarrow{2i} - \vec{j} + \vec{k}$ and $3\vec{i} + 4\vec{j} - \vec{k}$.

➢ **Solution**

Given,

$Let, \vec{a} = \overrightarrow{2i} - \vec{j} + \vec{k}$ *and* $\vec{b} = 3\vec{i} + 4\vec{j} - \vec{k}$

Then, $\vec{a} \times \vec{b} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 2 & -1 & 1 \\ 3 & 4 & -1 \end{vmatrix}$

$$= (1-4)\vec{i} - (-2-3)\vec{j} + (8+3)\vec{k}$$

$$= -3\vec{i} + 5\vec{j} + 11\vec{k}$$

$$|\vec{a} \times \vec{b}| = -3\vec{i} + 5\vec{j} + 11\vec{k}$$

$$= \sqrt{(-3)^2 + (5)^2 + 11^2} = \sqrt{155}$$

$$Area\ of\ triangle = \frac{1}{2} \times |\vec{a} \times \vec{b}| = \frac{\sqrt{155}}{2}$$

$$\therefore The\ required\ area\ of\ triangle = \frac{\sqrt{155}}{2} sq.units$$

b) Two coin are tossed simultaneously. Find the probability of getting at least one head

➢ **Solution**

$$P = \text{Prob. of getting head in one tossed} = \frac{1}{2}$$

$$q = \text{Prob. of getting tail} = 1 - p = 1 - \frac{1}{2}$$

$$n = \text{no. of trails} = 2$$

we have, $P(r) = \text{Prob. of getting } \boldsymbol{r} \text{ head in } \boldsymbol{n} \text{ trails}$

$$P(r) = C(n, r)\, p^r q^{n-r} \ldots\ldots\ldots\ldots (i)$$

To find : at least 1 head $i.e,\ r \geq 1$

$$P(r \geq 1) = 1 - P(r < 1)$$

$$P(r \geq 1) = 1 - P(r = 0)$$

From (i)

$$P(r \geq 1) = 1 - C(2,0)\, p^0 q^{2-0}$$

$$P(r \geq 1) = 1 - 1 \times 1.\left(\frac{1}{2}\right)^2$$

$$P(r \geq 1) = 1 - \frac{1}{4} = \frac{3}{4}$$

$\therefore$ Prob. of getting at least one heads $= \frac{3}{4}$

7. a) If mean and median of the given data are 3.5 and 4.5. Find Mode.

➢ **Solution**

Mean = 3.5
Median = 4.5

We have relation,

$$\text{Mean} - \text{mode} = 3(\text{mean} - \text{median})$$

or, $3.5 - \text{mode} = 3(3.5 - 4.5)$

or, $3.5 - \text{mode} = -3$

or, $\text{mode} = 3 + 3.5 = \mathbf{6.5}$

Hence, Mode = 6.5

b) If $n = 15, \sigma_x = 3.2, \sigma_y = 3.4$ and $\sum(x - \bar{x})(y - \bar{y}) = 122$. Find the correlation coefficient between the two variables.

➢ **Solution**

$n = 15$

$\sigma_x = 3.2$, $\sigma_y = 3.4$

$\sum(x - \bar{x})(y - \bar{y}) = 122$

$$\text{Coeff. of correlation, (r)} = \frac{\frac{1}{n}\sum(x - \bar{x})(y - \bar{y})}{\sigma_x \times \sigma_y}$$

$$\text{(r)} = \frac{\frac{1}{15} \times 122}{3.2 \times 3.4} = \mathbf{0.7475}$$

8) Prove the quadratic equation $ax^2 + bx + c = 0$ have not more than two roots.

➢ **Solution**

The given quadratic equation is $\mathbf{ax^2 + bx + c = 0}$.

For, if possible, let α, β, γ be three different roots of quadratics equation;

$$ax^2 + bx + c = 0 \qquad \text{----------}(a \neq 0)$$

Then, since each of these values must satisfy the equation;
We have,

$$a\alpha^2 + b\alpha + c = 0 \qquad \cdots\cdots\cdots\cdots (i)$$
$$a\beta^2 + b\beta + c = 0 \qquad \cdots\cdots\cdots\cdots (ii)$$
$$a\gamma^2 + b\gamma + c = 0 \qquad \cdots\cdots\cdots\cdots (iii)$$

From equation (i) and (ii) by subtraction; we get,

$$a(\alpha^2 - \beta^2) + b(\alpha - \beta) = 0$$

Since $\alpha \neq \beta$, divide out by $\alpha - \beta$; Then,

$$a(\alpha + \beta) + b = 0$$

Similarly, from equation (ii) and (iii); we get

$$a(\beta + \gamma) + b = 0$$

Hence by subtraction; we get

$a(\alpha - \gamma) = 0$ which is impossible , since, by hypothesis $a \neq 0$, and α is not equal to γ.

Hence, there cannot be more than two different roots.

9) Prove that :

$$\begin{vmatrix} x & y & z \\ x^2 & y^2 & z^2 \\ yz & zx & xy \end{vmatrix} = (y-z)(z-x)(x-y)(xy+yz+zx)$$

➢ **Solution**

$$\text{LHS} = \begin{vmatrix} x & y & z \\ x^2 & y^2 & z^2 \\ yz & zx & xy \end{vmatrix}$$

$$= \begin{vmatrix} x & y-x & z-x \\ x^2 & y^2-x^2 & z^2-x^2 \\ yz & zx-yz & xy-yz \end{vmatrix}$$

[Applying $C_2 \to C_2 - C_1,\ C_3 \to C_3 - C_1$]

$$= \begin{vmatrix} x & y-x & z-x \\ x^2 & y^2-x^2 & z^2-x^2 \\ yz & -z(y-x) & -y(z-x) \end{vmatrix}$$

$$= (y-x)(z-x)\begin{vmatrix} x & y-x & z-x \\ x^2 & y^2-x^2 & z^2-x^2 \\ yz & -z & -y \end{vmatrix}$$

[Taking $y-x$ as a common from C_2 and z − x form C_3

$$= (y-x)(z-x)\begin{vmatrix} x & 1 & 1 \\ x^2 & y+x & z+x \\ yz & -z & z-y \end{vmatrix}$$

[Applying $C_3 \to C_3 - C_2$]

$$= (y-x)(z-x)(z-y)\begin{vmatrix} x & 1 & 0 \\ x^2 & y+x & 1 \\ yz & -z & 1 \end{vmatrix}$$

$$= (y-x)(z-x)(z-y)\begin{vmatrix} x & 1 & 0 \\ x^2-yz & x+y+z & 0 \\ yz & -z & 1 \end{vmatrix}$$

[Applying $R_2 \to R_1 - R_3$]

$$= (y-x)(z-x)(z-y).1\begin{vmatrix} x & 1 \\ x^2-yz & x+y+z \end{vmatrix}$$

$$= (y-x)(z-x)(z-y)(x^2+xy+zx-x^2+yz)$$

$$= (y-z)(z-x)(x-y)(xy+yz+zx)$$

$= \text{RHS}$ **Proved**

Or

Solve by row equivalent matrix method or Cramer's rule:

$$x + y + z - 1$$
$$x + 2y + 3z = 4$$
$$x + 3y + 7z = 13$$

➢**Solution,**

Solving by Row equivalent matrix method,

The augmented matrix is

$$\begin{bmatrix} 1 & 1 & 1 & : & 1 \\ 1 & 2 & 3 & : & 4 \\ 1 & 3 & 7 & : & 13 \end{bmatrix}$$

$$\underset{R_2 \to R_2 - 2R_1, R_3 \to R_3 - R_1}{\sim} \begin{bmatrix} 1 & 1 & 1 & : & 1 \\ 0 & 1 & 2 & : & 3 \\ 0 & 2 & 6 & : & 12 \end{bmatrix}$$

$$\underset{R_1 \to R_1 - R_2, R_3 \to R_3 - 2R_1}{\sim} \begin{bmatrix} 1 & 0 & -1 & : & -2 \\ 0 & 1 & 2 & : & 3 \\ 0 & 0 & 2 & : & 6 \end{bmatrix}$$

$$\underset{R_3 \to \frac{1}{2}R_3}{\sim} \begin{bmatrix} 1 & 0 & -1 & : & -2 \\ 0 & 1 & 2 & : & 3 \\ 0 & 0 & 1 & : & 3 \end{bmatrix}$$

$$\underset{R_1 \to R_1 + R_3, R_2 \to R_2 - 2R_3}{\sim} \begin{bmatrix} 1 & 0 & 0 & : & 1 \\ 0 & 1 & 0 & : & -3 \\ 0 & 0 & 1 & : & 3 \end{bmatrix}$$

Hence,

$$x = 1,\ y = -3,\ z = 3 \ \ i.e., (1, -3, 3)$$

Solving by Cramer' rule,

$$x + y + z - 1$$
$$x + 2y + 3z = 4$$
$$x + 3y + 7z = 13$$

We have,

$$D = \begin{vmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \\ 1 & 3 & 7 \end{vmatrix} = 1(14-9) - 1(7-3) + 1(3-2)$$
$$= 5 - 4 + 1$$
$$= \mathbf{2}$$

$$D_1 = \begin{vmatrix} 1 & 1 & 1 \\ 4 & 2 & 3 \\ 13 & 3 & 7 \end{vmatrix} = 1(14-9) - 1(28-39) + 1(12-26)$$
$$= 5 + 11 - 14$$
$$= \mathbf{2}$$

$$D_2 = \begin{vmatrix} 1 & 1 & 1 \\ 1 & 4 & 3 \\ 1 & 13 & 7 \end{vmatrix} = 1(28-39) - 1(7-3) + 1(13-4)$$
$$= -11 - 4 + 9$$
$$= \mathbf{-6}$$

$$D_3 = \begin{vmatrix} 1 & 1 & 1 \\ 1 & 2 & 4 \\ 1 & 3 & 13 \end{vmatrix} = 1(26-12) - 1(13-4) + 1(3-2)$$
$$= 14 - 9 + 1$$
$$= \mathbf{6}$$

Now,

$$x = \frac{D_1}{D} = \frac{2}{2} = 1$$

$$y = \frac{D_2}{D} = \frac{-6}{2} = -3$$

$$z = \frac{D_3}{D} = \frac{6}{2} = 3$$

Hence,

$$\boldsymbol{x = 1,\ y = -3,\ z = 3\ i.e., (1, -3, 3)}$$

10) Maximize $x = 3x + 5y$ subject to constraints $3x + 2y \leq 180, \leq 4, y \leq 6$ and $x, y \geq 0$.

➢**Solution,**

When $x = 0, y = 0$
$3.0 + 2.0 \leq 18 \Rightarrow 0 \leq 18$ (true) so, it is directed towards origin.

The boundary lines of the given inequalities are

$$3x + 2y = 18 \quad \cdots\cdots\cdots\cdots (i)$$

From equation (i)

$3x + 2y = 18$	
x	y
0	9
6	0

So,C(0,9) & D(6,0) are passes through equation (i)

$$x \leq 4 \quad \cdots\cdots\cdots\cdots (ii)$$

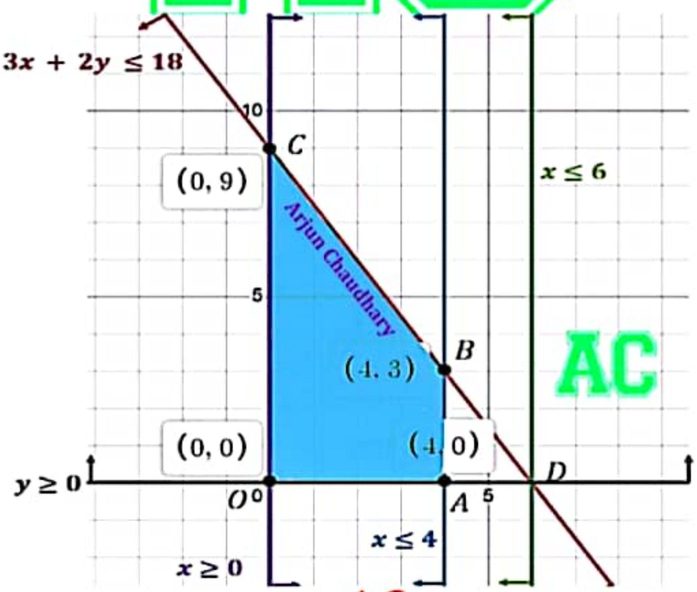
When, $x = 0, y = 0$
$0 \leq 4$ (true) so, it is directed towards origin.

$$x \leq 6 \quad \cdots\cdots\cdots\cdots (iii)$$

When, $x = 0, y = 0$
$0 \leq 6$ (true) so, it is directed towards origin.

Also,

From graph the vertices B & C are calculated by solving Equation.



11) Using De-Moivre's theorem. Find the fourth roots of unity.

➢ **Solution,**

Let $z^4 = 1$

$or,\ z^4 = \cos 0° + i\ \sin 0° = \cos\ n\,360° + i\ \sin\ n\,360°$

$\therefore z = (\cos\ n\,360° + i\ \sin\ n\,360°)^{\frac{1}{4}}$, where $n = 0, 1, 2, 3.$

$$= \cos\left(\frac{n\,360°}{4}\right) + i\ \sin\left(\frac{n\,360°}{4}\right)$$

$$= \cos\ (n\,90°) + i\ \sin\ (n\,90°)$$

For $\mathbf{n = 0}, z = \cos 0° + i \sin 0° = 1$

For $\mathbf{n = 1}, z = \cos 90° + i \sin 90° = i$

For $\mathbf{n = 2}, z = \cos 180° + i \sin 180° = -1$

For $\mathbf{n = 3}, z = \cos 270° + i \sin 270° = -i$

$\therefore$ **Required roots are $1, -1,\ i, -i$, $i.e,\ \pm 1,\ \pm i$**

or) Define absolute value of a complex number. If z and w are two complex numbers, prove that: $|z + w| \le |z| + |w|$

➢ Let $Z = a + bi$ be a complex number Then, Absolute value of complex number is denoted by $|Z|$ and given by

$$|Z| = \sqrt{a^2 + b^2}$$

$$|z + w| \le |z| + |w|$$

$$\Rightarrow |z| + |w| \ \ge\ |z + w|$$

➢ **Proof**

Let $z = a + ib$ and $w = c + id$, so that $z + w = a + c + i(b + d)$, then

$|z| = \sqrt{a^2 + b^2}, |w| = \sqrt{c^2 + d^2}$

$Also,\ |z + w| = \sqrt{(a + c)^2 + (b + d)^2}$

$Now,\ |z + w| \le |z| + |w|$ will be true

If $\sqrt{(a + c)^2 + (b + d)^2} \le \sqrt{a^2 + b^2} + \sqrt{c^2 + d^2}$

$i.e, (a + c)^2 + (b + d)^2 \le a^2 + b^2 + c^2 + d^2 + 2\sqrt{(a^2 + c^2) + (b^2 + d^2)}$

i.e. $ac + bd \leq \sqrt{(a^2 + c^2) + (b^2 + d^2)}$

i.e. $a^2c^2 + b^2d^2 + 2abcd \leq (a^2 + c^2) + (b^2 + d^2)$

i.e. $2abcd \leq a^2d^2 + b^2c^2$

i.e. $0 \leq 2abcd - a^2d^2 + b^2c^2$

i.e. $0 \leq (ad - bc)^2$ which is true all real number a, b, c, d

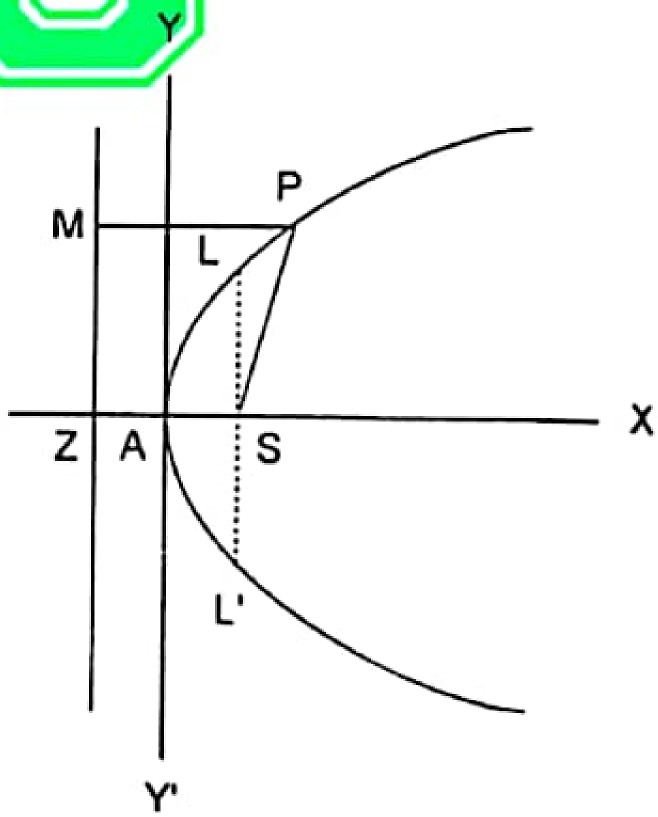
$\therefore \; |z + w| \leq |z| + |w|$

12) Define Parabola. Find the equation of the parabola in the standard form $y^2 = 4ax$.

- **Parabola :-** A plane curve generated by a point moving so that its distance from a fixed point is equal to its distance from a fixed line.

- 2nd Part :

Solution,



Let S be focus and ZM, the directrix of the parabola. SZ is drawn perpendicular to ZM. Let A be the middle point of SZ, so that SA = AZ. Then A is the vertex and ZAS is the axis of the parabola.

To determine the equation of a parabola in the standard form, take the vertex A at the origin, the focus S on the $x-axis$ so that the axis of the parabola is the $x-axis$ and the directrix is parallel to the $y-axis$.

Let AS $= a$ .Thus the coordinates of Z, A and S are Respectively $(-a, 0)$, $(0, 0)$ and $(a, 0)$, and the equation of the directrix is $x + a = 0$.

Let P(x, y) be any point on the parabola. Join PS and draw PM perpendicular to ZM.

Then, $\quad PS = PM.$

$\Rightarrow \quad PS^2 = PM^2$

$or, \quad (x-a)^2 + (y-0)^2 = \left(\frac{x+a}{\sqrt{1}}\right)^2$

$or, \quad y^2 = (x+a)^2 - (x-a)^2$

$or, \quad y^2 = (x+a+x-a)(x+a-x+a)$

$\Rightarrow \boldsymbol{y^2 = 4ax.}$

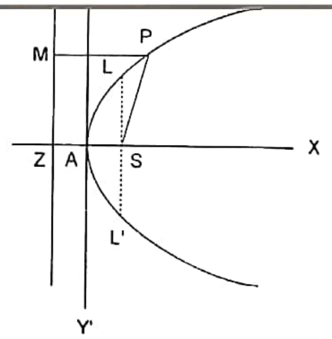
13) Show that $x^2 + 4y^2 - 4x + 24y + 24 = 0$ represents the equation of an ellipse. Find center vertices focus and length of axes.

➢ **Solution**

Given,

$$x^2 + 4y^2 - 4x + 24y + 24 = 0$$
$$x^2 - 4x + 4 + 4y^2 + 24y + 20 = 0$$

Let S be focus and ZM, the directrix of the parabola. SZ is drawn perpendicular to ZM. Let A be the middle point of SZ, so that SA $=$ AZ. Then A is the vertex and ZAS is the axis of the parabola.



To determine the equation of a parabola in the standard form, take the vertex A at the origin, the focus S on the $x-axis$ so that the axis of the parabola is the $x-axis$ and the directrix is parallel to the $y-axis$.

Let AS $= a$ .Thus the coordinates of Z, A and S are Respectively $(-a, 0)$, $(0,0)$ and $(a, 0)$, and the equation of the directrix is $x + a = 0$.

Let P(x, y) be any point on the parabola. Join PS and draw PM perpendicular to ZM.

Then, PS $=$ PM.

$\Rightarrow$ $PS^2 = PM^2$

or, $(x-a)^2 + (y-0)^2 = \left(\dfrac{x+a}{\sqrt{1}}\right)^2$

or, $y^2 = (x+a)^2 - (x-a)^2$

or, $y^2 = (x+a+x-a)(x+a-x+a)$

$\Rightarrow$ $\boldsymbol{y^2 = 4ax}$.

13) Show that $x^2 + 4y^2 - 4x + 24y + 24 = 0$ represents the equation of an ellipse. Find center vertices focus and length of axes.

➢ **Solution**

Given,

$x^2 + 4y^2 - 4x + 24y + 24 = 0$

$x^2 - 4x + 4 + 4y^2 + 24y + 20 = 0$

$$(x-2)^2 + 4(y^2+6y) + 20 = 0$$

$$(x-2)^2 + 4(y^2+2.3y+9) - 36 + 20 = 0$$

$$(x-2)^2 + 4(y+3)^2 - 16 = 0$$

$$(x-2)^2 + 4(y+3)^2 = 16$$

$$\frac{(x-2)^2 + 4(y+3)^2}{16} = 1$$

$$\frac{(x-2)^2}{16} + \frac{(y+3)^2}{4} = 1$$

Which is equation of ellipse.

Comparing with $\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$

$$a^2 = 16, \quad b^2 = 4, h = 2, k = -3$$

$$\Rightarrow a = 4, \quad b = 2$$

Since, a > b So, the major *axis* is along $x - axis$

Center of ellipse $(h, k) = (2, -3)$

Vertices = $(h \pm a, \mathrm{k}) = (2 \pm 4, 0)$ i.e $(6,0)$ & $(-2,0)$

$$\text{Eccentricity}, e = \sqrt{1 - \frac{b^2}{a^2}} = \sqrt{1 - \frac{4}{16}} = \sqrt{\frac{12}{16}} = \frac{\sqrt{3}}{2}$$

For focii $S(\mathrm{h} \pm \mathrm{ae}, \mathrm{k}) = \left(2 \pm 4 \times \frac{\sqrt{3}}{2}, -3\right) = \mathbf{(2 \pm 2\sqrt{3}, -3)}$

OR) Determine the equation of the hyperbola in the standard position with focus at (-7, 0) and eccentricity $\frac{7}{4}$.

➢ **Solution**

Given, focus = $(-7,0)$ and Eccentricity , $e = \frac{7}{4}$

Standard Equation of hyperbola,

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$

Focus $(\pm ae, 0) = (-7,0)$

$\Rightarrow -ae = -7 \Rightarrow a.\frac{7}{4} = 7 \Rightarrow \boldsymbol{a = 4}$

Eccentricity , $e = \sqrt{1 + \frac{b^2}{a^2}} \Rightarrow \frac{49}{16} = 1 + \frac{b^2}{16}$

$or,\ \frac{49}{16} - 1 = \frac{b^2}{16}\ or,\ \frac{33}{16} = \frac{b^2}{16} \Rightarrow b^2 = 33$

Standard Equation of hyperbola,

$$\frac{x^2}{16} - \frac{y^2}{33} = 1$$

14) Find the equation of the plane through the points $(1,2,1),(2,2,2)$ and $(0,1,0)$.

➢**Solution,**

Equation of the plane through $(1,2,1)$

$$a(x-1)+b(y-2)+c(z-1)=0 \quad \cdots\cdots\cdots (i)$$

Since,

Equation (i) Passes through (2,2,2) & (0,1,0)

So,

$$1.a+0.b+1.c=0 \quad \cdots\cdots\cdots (ii)$$

$$and, \quad -a-b-c=0$$

$$a+b+c=0 \quad \cdots\cdots\cdots (iii)$$

Solving, (ii) & (iii) by cross multiplication method,

$$\frac{a}{(0+1)}=\frac{b}{(1-1)}=\frac{c}{(1-0)}$$

$$or, \quad \frac{a}{1}=\frac{b}{0}=\frac{c}{1}=K \ (say)$$

$$a=K, \qquad b=0, \qquad c=K$$

Equation (i) becomes,

$$K(x-1)-0(y-2)-K(Z-1)=0$$

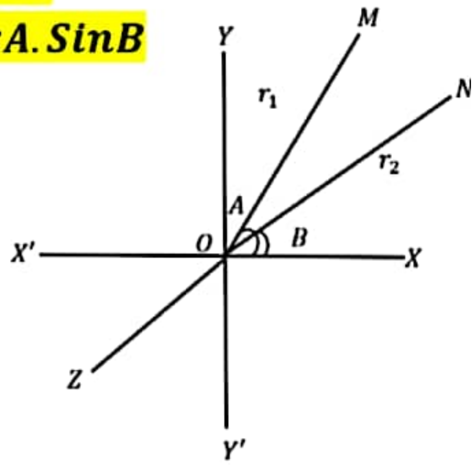
$$or, \ K(x-1-z+1)=0$$

$$\boxed{\therefore \ x-z=0}$$

Which is Eq^n of plane.

16) Using vector method, prove that :

$$Sin(A - B) = SinA.CosB - CosA.SinB$$



➢ **Solution,**

Let XOX' and YOY', the two mutually perpendicular straight lines represent $x - axis$ and $y - axis$ respectively.

Let $= \angle XOM = A$

and $\angle NOX' = B$

so that $\angle MON = (A - B)$

Let OM $= r_1$ and ON $= r_2$. Then the co-ordinates of M and N are $(r_1 \cos A, r_1 \sin A)$ and $(r_2 \cos B, r_2 \sin B)$

So, position vector are:

So, $\overrightarrow{OM} = (r_1 \cos A, r_1 \sin A) = r_1 \cos A\ \vec{i} + r_1 \sin A\, \vec{j}$

and $\overrightarrow{ON} = (r_2 \cos B, r_2 \sin B)$
$= r_2 \cos B\ \vec{i} + r_2 \sin B\, \vec{j}$

$$\overrightarrow{OM} \times \overrightarrow{ON} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ r_1 \cos A & r_1 \sin A & 0 \\ r_2 \cos B & r_2 \sin B & 0 \end{vmatrix}$$

Taking common r_1 and r_2 from R_2 and R_3,

$$= r_1 r_2 \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ \cos A & \sin A & 0 \\ \cos B & \sin B & 0 \end{vmatrix}$$

$$= r_1 r_2 [\vec{i}(0 - 0) - \vec{j}(0 - 0) + \vec{k}(\cos A \sin B - \sin A \cos B)]$$

$$= r_1 r_2 (\cos A \sin B - \sin A \cos B)\vec{k}$$

Magnitude of $\overrightarrow{OM} \times \overrightarrow{ON}$

$\therefore |\overrightarrow{OM} \times \overrightarrow{ON}| = r_1 r_2 (\cos A \sin B - \sin A \cos B)$

Since $(A - B)$ is the angle between OM and ON, So

Direction cosines of line (1)=$(l_1, m_1, n_1) = (\frac{1}{\sqrt{14}}, \frac{2}{\sqrt{14}}, \frac{-3}{\sqrt{14}})$

Again, Substituting $l = -2m$ in $eqn(i)$

$-2m + m + n = 0 \quad \Rightarrow n = -m \quad \Rightarrow \frac{n}{-1} = \frac{m}{1}$

Equating $\frac{l}{-2} = \frac{m}{1}$ and $\frac{n}{-1} = \frac{m}{1}$

$$\Rightarrow \frac{l}{-2} = \frac{m}{1} = \frac{n}{-1}$$

$$\Rightarrow \frac{l}{-2} = \frac{m}{1} = \frac{n}{-1} = \frac{l^2 + m^2 + n^2}{\sqrt{(-2)^2 + 1^2 + (-1)^2}}$$

$$\Rightarrow \frac{l}{-2} = \frac{m}{1} = \frac{n}{-1} = \frac{1}{\sqrt{6}}$$

$$\Rightarrow l = \frac{-2}{\sqrt{6}}$$

$$\Rightarrow m = \frac{1}{\sqrt{6}}$$

$$\Rightarrow n = \frac{-1}{\sqrt{6}}$$

Direction cosines of line (2)=$(l_2, m_2, n_2) = (\frac{-2}{\sqrt{6}}, \frac{1}{\sqrt{6}}, \frac{-1}{\sqrt{6}})$

Let l_1, m_1, n_1 and l_2, m_2, n_2 be direction cosines of two lines and θ be the angle between them so,

$$\cos\theta = l_1.l_2 + m_1.m_2 + n_1.n_2$$

$$\cos\theta = \frac{1}{\sqrt{14}}.\frac{-2}{\sqrt{6}} + \frac{2}{\sqrt{14}}.\frac{1}{\sqrt{6}} + \frac{-3}{\sqrt{14}}.\frac{-1}{\sqrt{6}}$$

$$\cos\theta = \frac{-2 + 2 + 3}{\sqrt{84}} = \frac{3}{\sqrt{84}}$$

$$\theta = \cos^{-1}(\frac{3}{\sqrt{84}})$$

$$2lm - 2l \times -(l+m) + m \times -(l+m) = 0$$

or, $2lm + 2l^2 + 2lm - ml - m^2 = 0$

or, $2l^2 + 3ml - m^2 = 0$

or, $\dfrac{2l^2 + 3ml - m^2}{m^2} = \dfrac{0}{m^2}$ Dividing by both side m^2

or, $2\left(\dfrac{l}{m}\right)^2 + 3\dfrac{l}{m} - 1 = 0$, which is qudratic in $\dfrac{l}{m}$

$$\frac{l}{m} = \frac{-3 \pm \sqrt{9 - 4.2.(-2)}}{2 \times 2}$$

$$\frac{l}{m} = \frac{-3 \pm \sqrt{25}}{4} = \frac{-3 \pm 5}{4}$$

Taking + ve sign, $\dfrac{l}{m} = \dfrac{-3+5}{4} = \dfrac{1}{2} \Rightarrow l = \dfrac{m}{2}$

Taking − ve sign, $\dfrac{l}{m} = \dfrac{-3-5}{4} = -2 \Rightarrow l = -2m$

Substituting $l = \dfrac{m}{2}$ in eqn(i)

$$\frac{m}{2} + m + n = 0 \quad \Rightarrow n = -\frac{3}{2}m \quad \Rightarrow \frac{n}{-3} = \frac{m}{2}$$

Equating $l = \dfrac{m}{2}$ and $\dfrac{n}{-3} = \dfrac{m}{2}$

$$\Rightarrow \frac{n}{-3} = \frac{m}{2} = \frac{l}{1}$$

$$\Rightarrow \frac{l}{1} = \frac{m}{2} = \frac{n}{-3} = \frac{l^2 + m^2 + n^2}{\sqrt{1^2 + 2^2 + (-3)^2}}$$

15) Find the ratio in which the *yz* plane divides the line joining (4, 6, 7) and (-1,2,5). Also find the coordinates of the point in the yz.

➢ **Solution**

Let *yz-plane* divides the line joining points A(4, 6, 7) and B (− 1, 2, 5) in the ratio k:1 at P(x, y, z).

In *yz-plane* $x = 0$.

$$\therefore 0 = \frac{k(-1) + 4}{k + 1}$$

$or,\ \ k = 4$

The ratio is $k:1,\ i.e, 4:1$

Hence, the point

$$P(x, y, z) = P\left(0, \frac{4.2 + 6}{5}, \frac{4.5 + 7}{5}\right)$$

$$= P\left(0, \frac{14}{5}, \frac{27}{5}\right)$$

OR) Find the angle between two lines whose direction cosines are given by $l + m + n = 0$ *and* $2lm - mn + 1nl = 0$.

➢ **Solution,**

Given Equation,

$$l + m + n = 0 \quad \cdots\cdots\cdots (i)$$

$$or,\ \ m = -(l + n) = 0$$

$$or,\ \ l = -(n + m)$$

$$2lm - 2nl + mn = 0 \quad \cdots\cdots\cdots (ii)$$

Eliminating **n** so, Subsitituting Value of n in eqn(ii),

19) Calculate Karl Pearson's coefficient of correlation from the following data:

Age of Husband	23	22	24	23	26	27
Age of Wives	20	18	20	21	21	22

➢ **Solution,**

Age of husband (x)	Age of Wives (y)	$x = X - \bar{x}$	$y = Y - \bar{y}$	xy	x^2	y^2
23	20	−1.17	−0.33	0.3861	1.3689	0.1089
22	18	−2.17	−2.33	5.0561	4.7089	5.4289
24	20	−0.17	−0.33	0.0561	0.0289	0.1089
23	21	−1.17	0.67	−0.7839	1.3689	0.4489
26	21	1.83	0.67	1.2261	3.3489	0.4489
27	22	2.83	1.67	4.7261	8.0089	2.7889
$\sum x = 145$	$\sum y = 122$			$\sum xy = 10.67$	$\sum x^2 = 18.83$	$\sum y^2 = 9.33$

$n = 6$

Now,

$$\bar{X} = \frac{\sum x}{n} = \frac{145}{6} = 24.17\,, \quad \bar{Y} = \frac{\sum y}{n} = \frac{122}{6} = 20.33$$

Using Product moment formula ,

$$\text{Coeff. of correlation, } (r) = \frac{\sum xy}{\sqrt{\sum x^2}\ \sqrt{\sum y^2}}$$

$$= \frac{10.67}{\sqrt{18.83}\sqrt{9.33}}$$

$$\therefore r = 0.89$$

So,

$$\bar{x} = a + \frac{\sum fd'}{N} \times h$$

$$= 25 + \frac{(5)}{22} \times 10 = \mathbf{27.27}$$

Mean $(\bar{x}) = \mathbf{27.27}$

Standard Deviation $(\sigma) = \sqrt{\frac{\sum fd'^2}{N} - \left(\frac{\sum fd'}{N}\right)^2} \times h$

$$= \sqrt{\frac{31}{22} - \left(\frac{5}{22}\right)^2} \times 10$$

$$= \sqrt{1.41 - 0.052} \times 10$$

$$= 1.165 \times 10$$

$$= \mathbf{11.65}$$

Coefficient of Variation $= \frac{\sigma}{\bar{x}} \times 100\%$

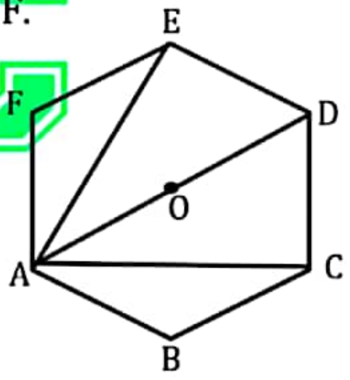
$$= \frac{11.65}{27.27} \times 100$$

$$= \mathbf{42.72\%}$$

Or) If $ABCDEF$ is a regular hexagon and O is its center, then show that: $\overrightarrow{AB} + \overrightarrow{AC} + \overrightarrow{AD} + \overrightarrow{AE} + \overrightarrow{AF} = 6\overrightarrow{AO}$

➢**Solution,**

If $ABCDEF$ is a regular hexagon, then we can suppose O is its center of the regular hexagon ABCDEF.



$Now,\ \overrightarrow{AB} + \overrightarrow{AC} + \overrightarrow{AD} + \overrightarrow{AE} + \overrightarrow{AF}$

$= \overrightarrow{AD} + (\overrightarrow{AB} + \overrightarrow{AE})\ \overrightarrow{AC} + \overrightarrow{AF}$

$= \overrightarrow{AD} + (\overrightarrow{ED} + \overrightarrow{AE})\ \overrightarrow{AC} + \overrightarrow{AF}$

$= \overrightarrow{AD} + \overrightarrow{AD} + (\overrightarrow{AC} + \overrightarrow{CD})$

$= \overrightarrow{AD} + \overrightarrow{AD} + \overrightarrow{AD}$

$= 3\overrightarrow{AD}$

$= 3 \times 2\,\overrightarrow{AO}$ $\quad [\because \overrightarrow{AD} = 2\,\overrightarrow{AO}]$

$\therefore 6\overrightarrow{AO}$

18) Calculate coefficient of variation from the following data:

Class	0-10	10-20	20-30	30-40	40-50
Frequency	2	4	6	7	3

➢**Solution,**

Class x	f	Mid value (M)	$d' = \frac{m-a}{10}$ $(a = 25)$, $h = 10$	d'^2	Fd'	Fd'^2
0-10	2	5	-2	4	-4	8
10-20	4	15	-1	1	-4	4
20-30	6	25	0	0	0	0
30-40	7	35	1	1	7	7
40-50	3	45	2	4	6	12
	N=22				$\sum fd' = 5$	$\sum fd'^2 = 31$

$$\sin(A-B)=\frac{|\overrightarrow{OM}\times\overrightarrow{ON}|}{|\overrightarrow{OM}|\times|\overrightarrow{ON}|}$$

or, $$\sin(A-B)=\frac{r_1r_2\,(\cos A\sin B-\sin A\cos B)}{r_1r_2}$$

Hence, $\sin(A-B)=\sin A\cos B-\cos A\sin B.$

17) Prove that the three vectors $\vec{a}-2\vec{b}+3\vec{c}$, $-2\vec{a}+3\vec{b}-4\vec{c}$ and $-\vec{b}+2\vec{c}$ *are coplanar*.

➢ **Solution,**

$\vec{r_1}=\vec{a}-2\vec{b}+3\vec{c}$

$\vec{r_2}=-2\vec{a}+3\vec{b}-4\vec{c}$

$\vec{r_3}=-\vec{b}+2\vec{c}$

To show: $\vec{r_1},\vec{r_2}$ *and* $\vec{r_3}$ **are coplanar**

➢ **If three vector are coplanar, one can expressed as the sum of the scalar multiples of the other two.**

$\vec{r_3}=x\vec{r_1}+y\vec{r_2}$ where, x *and* y are scalars

$or, -\vec{b}+2\vec{c}=x(\vec{a}-2\vec{b}+3\vec{c})+y(-2\vec{a}+3\vec{b}-4\vec{c})$

$or\ \ -\vec{b}+2\vec{c}=(x-2y)\vec{a}+(-2x+3y)\vec{b}+(3x-4y)\vec{c}$

Equating Coefficient of likes vectors,

$$(x-2y)=0\cdots\cdots\cdots(i)$$
$$-2x+3y=-1\cdots\cdots\cdots(ii)$$
$$3x-4y=2\cdots\cdots\cdots(iii)$$

Solving eqn (i) *and* (ii) *weget* , $x=2$ ***and*** $y=1$

Put , $x=2$ *and* $y=1$ in eqn (iii)

$3\times2-4\times1=2$

$2=2,$ **satisfied**

Hence, Three vector are coplanar and its can expresssed as

$$\vec{r_3}=2\vec{r_1}+\vec{r_2}$$

$$P(5) = C(6,5).\left(\frac{1}{5}\right)^5.\left(\frac{4}{5}\right)^{6-5}$$

$$= \frac{6!}{(6-5)!5!}\ \frac{1}{5^5} \times \frac{4}{5}$$

$$= 15 \times \frac{1}{3125} \times \frac{4}{5}$$

$$= \frac{24}{15625}$$

$$P(6) = C(6,6).\left(\frac{1}{5}\right)^6.\left(\frac{4}{5}\right)^{6-6}$$

$$= \frac{6!}{(6-6)!\,6!}\ \frac{1}{5^6} \times \left(\frac{4}{5}\right)^0$$

$$= 1 \times \frac{1}{15625} \times 1$$

$$= \frac{1}{15625}$$

$$P(r \geq 4) = P(4) + P(5) + P(6)$$

$$= \frac{48}{3125} + \frac{24}{15625} + \frac{1}{15625}$$

$$= \frac{53}{3125}$$

$$= 0.01696$$

$$= 1.696\%$$

Hence, the probability that out of six men, four or more will contact the disease is 1.696%

OR) The incidence of occupations diseases in an industry is such that the workman have a 20% chance of suffering from it. What is the probability that out of six men, four or more will contact the disease?

➢**Solution,**

P = probability of Suffering from diseases = 20%

$$P = \frac{1}{5}$$

$$q = 1 - \frac{1}{5} = \frac{4}{5}$$

$$\mathbf{n = 6}$$

we have,

To find : $P(r \geq 4) = ?$

$$P(r) = C(n,r)\, p^r q^{n-r}$$

$$P(r \geq 4) = P(4) + P(5) + P(6) \quad \cdots\cdots\cdots (i)$$

$$\mathbf{P(4) = C\,(6,4).\left(\frac{1}{5}\right)^4.\left(\frac{4}{5}\right)^{6-4}}$$

$$= \frac{6!}{(6-4)!4!}\ \frac{1}{5^4} \times \left(\frac{4}{5}\right)^2$$

(b) P (Both are girls) = ?

$$
\begin{aligned}
M &= \text{no. of favorable case.} \\
&= \text{selection of 2 girls out of 60} \\
&= c\,(60,2) \\
&= \frac{60!}{(60-2)!\ \ 2!} \\
&= 1770
\end{aligned}
$$

Hence, $\text{P(both are girls)} = \frac{M}{n} = \frac{c(60,2)}{c(100,2)}$

$$= \frac{1770}{4950}$$

$$= \frac{59}{165}$$

(C) P (one boy and one girls) = ?

$$
\begin{aligned}
M &= \text{no. of favorable case.} \\
&= \text{selection of 1 girls out of 60 and selection of 1 boys out of 40} \\
&= c\,(60,1) \times c\,(40,1) \\
&= \frac{60!}{(60-1)!\ \ 1!} \times \frac{40!}{(40-1)!\ \ 1!} \\
&= 60 \times 40 \\
&= 2400
\end{aligned}
$$

$\therefore$ P (one boy and one girls) $= \frac{M}{n} = \frac{2400}{4950}$

$$= \frac{16}{33}$$

20) A class consists of 40 boys and 60 girls. It two students are chosen at random, what will be the probability that. (a) Both are Boys (b) Both are Girls (C) One Boy and one Girl.

➢Solution

$n(B) = 40, \; n(G) = 60,$

$n(S) = 60 + 40 = 100$

Now,

When two students are chosen randomly then, Possible outcomes , $n =$ selection of 2 std out of 100

$= c(100, 2)$

$= \dfrac{100!}{(100-2)! \; 2!}$

$= 4950$

(a) P (Both are Boys) = ?

$M =$ no. of favorable case.

$=$ selection of 2 boys out of 40

$= c\,(40,2)$

$= \dfrac{40!}{(40-2)! \; 2!}$

$= 780$

Hence, $P(\text{both are boys}) = \dfrac{M}{n} = \dfrac{c(40,2)}{c(100,2)}$

$= \dfrac{780}{4950}$

$= \dfrac{5}{33}$