

1. a) By using De-Moivres theorem find the cube root of i.

➤ Solution :

$$\begin{aligned}\text{Let } z^3 &= i = 0 + 1.i = \cos 90^\circ + i \sin 90^\circ \\&= \cos(90^\circ + k.360^\circ) + i \sin(90^\circ + k.360^\circ) \\ \therefore z &= \{\cos(90^\circ + k.360^\circ) + i \sin(90^\circ + k.360^\circ)\}^{\frac{1}{3}} \\&= \cos \frac{90^\circ + k.360^\circ}{3} + i \sin \frac{90^\circ + k.360^\circ}{3} \\&= \cos(30^\circ + k.120^\circ) + i \sin(30^\circ + k.120^\circ)\end{aligned}$$

Where, $k = 0, 1, 2$. For $k = 0, 1, 2$, the roots are,

$$\cos 30^\circ + i \sin 30^\circ, \cos 150^\circ + i \sin 150^\circ, \cos 270^\circ + i \sin 270^\circ$$

$$\text{i.e., } \frac{\sqrt{3}}{2} + \frac{1}{2}i, -\frac{\sqrt{3}}{2} + \frac{1}{2}i, -i.$$

b) If ω be a cube root of unity, prove that:

$$(2 + \omega + \omega^2)^3 + (1 + \omega - \omega^2)^8 - (1 - 3\omega + \omega^2)^4 = 1$$

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➤ Solution,

LHS,

$$\begin{aligned}&= (2 + \omega + \omega^2)^3 + (1 + \omega - \omega^2)^8 - (1 - 3\omega + \omega^2)^4 \\&= (1 + 1 + \omega + \omega^2)^3 + (1 + \omega - \omega^2)^8 - (1 - 3\omega + \omega^2)^4 \\&\quad [\because 1 + \omega + \omega^2 = 0] \\&= (1 + 0)^3 + (-\omega^2 - \omega^2)^8 - (\omega - 3\omega)^4 \\&= 1 + (-2\omega^2)^8 - (-4\omega)^4 \\&= 1 + 256\omega^{16} - 256\omega^{14} \\&= 1 + 256(\omega^3)^5 \omega - 256\omega^3 \cdot \omega \\&\quad [\because \omega^3 = 1] \\&= 1 + 256\omega - 256\omega \\&= 1\end{aligned}$$

$\therefore \text{RHS} = \text{LHS}$, Proved.

2. a) Find the area of the triangles determined by the Vectors $2\vec{i} - \vec{j} + \vec{k}$ and $3\vec{i} + 4\vec{j} - \vec{k}$

➤ **Solution**

$$\text{Let, } \vec{a} = 2\vec{i} - \vec{j} + \vec{k} \text{ and } \vec{b} = 3\vec{i} + 4\vec{j} - \vec{k}$$

$$\begin{aligned}\text{Then, } \vec{a} \times \vec{b} &= \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 2 & -1 & 1 \\ 3 & 4 & -1 \end{vmatrix} \\ &= (1 - 4)\vec{i} - (-2 - 3)\vec{j} + (8 + 3)\vec{k} \\ &= -3\vec{i} + 5\vec{j} + 11\vec{k}\end{aligned}$$

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$$|\vec{a} \times \vec{b}| = -3\vec{i} + 5\vec{j} + 11\vec{k}$$

$$= \sqrt{(-3)^2 + (5)^2 + 11^2} = \sqrt{155}$$

$$\text{Area of triangle} = \frac{1}{2} \times |\vec{a} \times \vec{b}| = \frac{\sqrt{155}}{2}$$

$$\therefore \text{The required area of triangle} = \frac{\sqrt{155}}{2} \text{ sq. units}$$

b) Find the equation of the plane through the points (4, 5, 1), (3, 9, 4) and (-4, 4, 4).

➤ **Solution,**

Equation of the plane through (4, 5, 1) is

$$a(x - 4) + b(y - 5) + c(z - 1) = 0 \dots\dots\dots (i)$$

Since,

Equation (i) Passes through (3, 9, 4) & (-4, 4, 4)

So,

$$a(3 - 4) + b(9 - 5) + c(4 - 1) = 0$$

$$\text{or, } -a + 4b + 3c = 0 \dots\dots\dots (ii)$$

$$a(-4 - 4) + b(4 - 5) + c(4 - 1) = 0$$

$$\text{or, } -8a + b + 3c = 0 \dots\dots\dots (iii)$$

Solving, (ii) & (iii) by cross multiplication method,

$$\frac{a}{12 + 3} = \frac{b}{-24 + 1} = \frac{c}{1 + 32}$$

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$$\text{or, } \frac{a}{15} = \frac{b}{-21} = \frac{c}{33} = K \text{ (say)}$$

$$a = 15K, \quad b = -21K, \quad c = 33K$$

Equation (i) becomes,

$$15K(x - 4) - 21K(y - 5) + 33K(Z - 1) = 0$$

$$\text{or, } 5(x - 4) - 7(y - 5) + 11(z - 1) = 0$$

$$\therefore 5x - 7y + 11z - 4 = 0$$

Which is the required Eqⁿ of plane.

3. a) Solve by using Cramer's rule or Row equivalent matrix:

$$x + 2y - 3z = 9$$

$$2x - y + 2z = -8$$

$$3x - y - 4z = 3$$

➤ **Solution,** Row equivalent matrix method,

The augmented matrix is

$$\left[\begin{array}{ccc|c} 1 & 2 & -3 & 9 \\ 2 & -1 & 2 & -8 \\ 3 & -1 & -4 & 3 \end{array} \right]$$

$$R_2 \rightarrow R_2 - 2R_1, R_3 \rightarrow R_3 - 2R_1 \quad \left[\begin{array}{ccc|c} 1 & 2 & -3 & 9 \\ 0 & -5 & 8 & -26 \\ 0 & -7 & 5 & -24 \end{array} \right]$$

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$$R_2 \rightarrow -\frac{1}{5}R_2$$

$$\left[\begin{array}{ccc|c} 0 & 2 & -3 & 9 \\ 0 & 1 & \frac{-8}{5} & \frac{26}{5} \\ 0 & -7 & 5 & -24 \end{array} \right]$$

$$R_1 \rightarrow R_1 - 2R_2, R_3 \rightarrow R_3 - 7R_2$$

$$\left[\begin{array}{ccc|c} 1 & 0 & \frac{1}{5} & \frac{-7}{5} \\ 0 & 1 & \frac{-8}{5} & \frac{26}{5} \\ 0 & 0 & \frac{-31}{5} & \frac{62}{5} \end{array} \right]$$

$$R_3 \rightarrow -\frac{5}{31}R_3$$

$$\left[\begin{array}{ccc|c} 1 & 0 & \frac{1}{5} & \frac{-7}{5} \\ 0 & 1 & \frac{-8}{5} & \frac{26}{5} \\ 0 & 0 & -1 & -2 \end{array} \right]$$

$$R_1 \rightarrow R_1 - \frac{1}{5}R_3, R_2 \rightarrow R_2 + \frac{8}{5}R_3$$

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & -1 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & -2 \end{array} \right]$$

Hence,

$$x = -1, y = 2, z = -2.$$

Or

$$x + 2y - 3z = 9$$

$$2x - y + 2z = -8$$

$$3x - y - 4z = 3$$

Solving by Cramer's rule :

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$$\begin{aligned}\text{Let } D &= \begin{vmatrix} 1 & 2 & -3 \\ 2 & -1 & 2 \\ 3 & -1 & -4 \end{vmatrix} = 1(4 + 2) - 2(-8 - 6) + (-3)(-2 + 3) \\ &= 6 + 28 - 3 \\ &= 31\end{aligned}$$

$$\begin{aligned}D_1 &= \begin{vmatrix} 1 & 2 & -3 \\ -8 & -1 & 2 \\ 3 & -1 & -4 \end{vmatrix} = 9(4 + 2) - 2(32 - 6) + (-3)(8 + 3) \\ &= 54 - 52 - 33 \\ &= -31\end{aligned}$$

$$\begin{aligned}D_2 &= \begin{vmatrix} 1 & 9 & -3 \\ 2 & -8 & 2 \\ 3 & 3 & -4 \end{vmatrix} = 1(32 - 6) - 9(-8 - 6) + (-3)(-6 + 24) \\ &= 26 + 126 - 90 \\ &= 62\end{aligned}$$

$$\begin{aligned}D_3 &= \begin{vmatrix} 1 & 2 & 9 \\ 2 & -1 & -8 \\ 3 & -1 & 3 \end{vmatrix} = 1(-3 - 8) - 2(6 + 24) + 9(-2 + 3) \\ &= -11 - 60 + 9 \\ &= -62\end{aligned}$$

Now,

$$x = \frac{D_1}{D} = \frac{-31}{31} = -1, \quad y = \frac{D_2}{D} = \frac{62}{31} = 2, \quad z = \frac{D_3}{D} = \frac{-62}{31} = -2$$

Hence,

$$x = -1, \quad y = 2, \quad z = -2.$$

b) Prove that:

$$\begin{vmatrix} b+c & a & b \\ c+a & c & d \\ a+b & b & c \end{vmatrix} = (a+b+c)(a-c)^2$$

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$$\triangleright \text{LHS} = \begin{vmatrix} b+c & a & b \\ c+a & c & a \\ a+b & b & c \end{vmatrix}$$

$$= \begin{vmatrix} 2(a+b+c) & a+b+c & a+b+c \\ c+a & c & a \\ a+b & b & c \end{vmatrix} \quad (R_1 \rightarrow R_1 + R_2 + R_3)$$

$$= (a+b+c) \begin{vmatrix} 2 & 1 & 1 \\ c+a & c & a \\ a+b & b & c \end{vmatrix} \quad (\text{Taking } a+b+c \text{ common from } R_1)$$

$$= (a+b+c) \begin{vmatrix} 1 & 1 & 1 \\ a & c & a \\ a & b & c \end{vmatrix} \quad (C_1 \rightarrow C_1 - C_2)$$

$$= (a+b+c) \begin{vmatrix} 1 & 0 & 0 \\ a & c-a & 0 \\ a & b-a & c-a \end{vmatrix} \quad (C_2 \rightarrow C_2 - C_1, C_3 \rightarrow C_3 - C_1)$$

$$= (a+b+c) \begin{vmatrix} c-a & 0 \\ b-a & c-a \end{vmatrix}$$

$$= (a+b+c) [(c-a)^2 - 0]$$

$$= (a+b+c) (c-a)^2$$

$\therefore \text{RHS} = \text{LHS}$ Proved.

4) Sand is pouring from a pipe at the rate of $18 \text{ cm}^3/\text{sec}$. The falling sand forms a cone on the ground in such a way that the height of the cone is one-sixth of the radius of the base. How fast is height of the cone increasing when its height is 3 cm ?

➤ **Solution,**

Let r be the radius of the base, h be the height and v be the volume of the sand cone at time t .

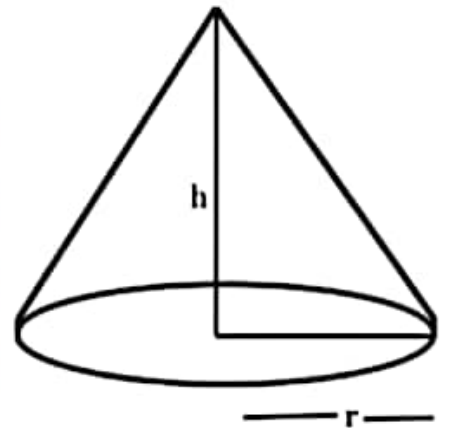
By question,

$$h = \frac{1}{6} r$$

or, $r = 6h$

$$\frac{dv}{dt} = \frac{18 \text{ cm}^3}{\text{sec}}$$

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$$h = 3 \text{ cm}, \quad \frac{dh}{dt} = ?$$

We have, $v = \frac{1}{2} \pi r^2 h = \frac{1}{3} \pi (6h)^2 h = 12\pi h^3$

$$\text{or, } \frac{dv}{dt} = \frac{d}{dt} 12\pi h^3 = 12\pi \frac{dh^3}{dt} = 36\pi h^2 \frac{dh}{dt}$$

$$\text{or, } 18 = 36\pi \times (3)^2 \times \frac{dh}{dt}$$

$$\therefore \frac{dh}{dt} = \frac{1}{18\pi} \text{ cm/sec.}$$

5) A man who has got 144 metres of fencing material Wishes to enclose a rectangular garden. Find the maximum area he can enclose.

➤Solution,

Let x be the length and y be the breath of the garden. If P be perimeter and A be the area of the garden, then

$$P = 2(x + y)$$

$$\text{or, } 144 = 2(x + y)$$

$$\text{or, } x + y = 72 \quad \dots\dots\dots (i)$$

$$\text{and } A = xy \quad \dots\dots\dots (ii)$$

Now, eliminating y between (i) and (ii), we have

$$A = x(72 - x)$$

$$\frac{dA}{dx} = 72 - 2x$$

$$\text{and, } \frac{d^2A}{dx^2} = -2$$

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For maximum area A ,

$$\frac{dA}{dx} = 0$$

$$\text{or, } 72 - 2x = 0$$

$$\therefore x = 36$$

$$\text{When } x = 36 \quad y = 72 - 36 = 36$$

$$\text{At } x = 36 \quad \frac{d^2A}{dx^2} = -2 \text{ which is negative.}$$

Hence, the area is maximum when $x = 36$ and $y = 36$ and the maximum area $A = 36 \times 36 = 1296 \text{ sq.mtrs}$

6) Find the Area bounded by the curve $y^2 = 16x$ and the line $y = 2x$.

➤ **Solution,**

Given curve and line are $y^2 = 16x$ and $y = 2x$ respectively.

Eliminating y between them, we have,

$$(2x)^2 = 16x$$

$$\text{or, } x^2 = 4x$$

$$\text{or, } x(x - 4) = 0$$

Either $x = 0$ or $x = 4$

∴ Ordinates are $x = 0, x = 4$.

Hence the required area, $A = \int_0^4 (y_1 - y_2) dx$, where $y_1 = 4\sqrt{x}$ and $y_2 = 2x$.

$$= \int_0^4 (y_1 - y_2) dx$$

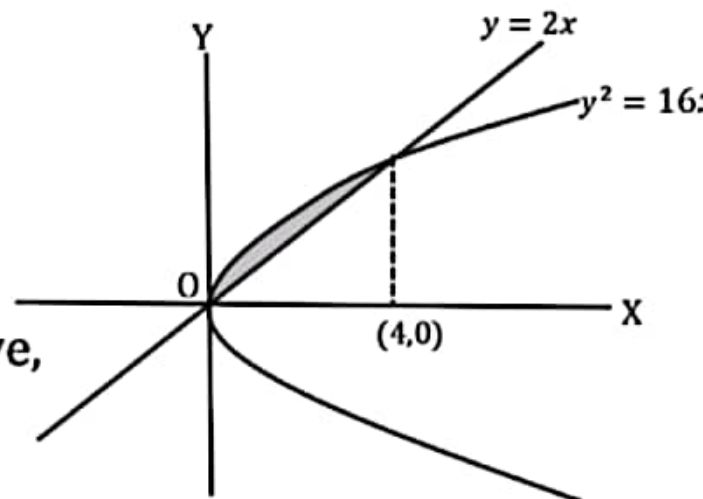
$$= \int_0^4 (4\sqrt{x} - 2x) dx$$

$$= \left[\frac{8}{3} x^{\frac{3}{2}} - x^2 \right]_0^4$$

$$= \frac{8}{3} 4^{\frac{3}{2}} - 4^2$$

$$= \frac{64}{3} - 16$$

$$\therefore A = \frac{16}{3} \text{ sq. units}$$



OR) Find the area bounded by the curve $\frac{x^2}{16} + \frac{y^2}{9} = 1$

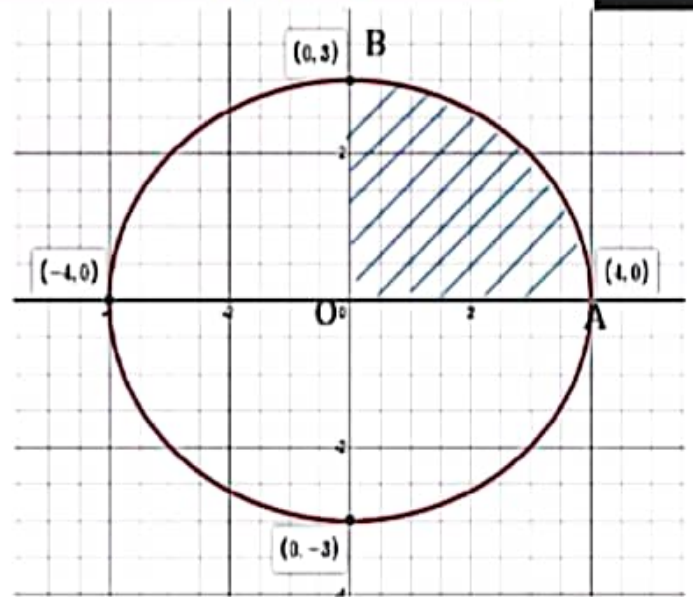
➤ **Solution,**

Given Equation,

$$\frac{x^2}{16} + \frac{y^2}{9} = 1$$

$$y^2 = 9\left(1 - \frac{x^2}{16}\right)$$

$$\Rightarrow y = \frac{3}{4}\sqrt{16 - x^2}$$



Its center is (0,0) and is symmetrical about both x - axis and y - axis .

Total area Bounded by Curve = $4 \times$ Area of OAB which lies between the ordinates $x = 0, x = 4$.

Hence, Total area of the Curve, $A = 4 \times \text{area of OAB}$

$$A = 4 \int_0^4 y \, dx$$

$$= 4 \int_0^4 \frac{3}{4} \sqrt{16 - x^2} \, dx$$

$$\text{let } x = 4 \sin \theta, \quad dx = 4 \cos \theta \, d\theta$$

$$\text{Limits, } x = 0 \rightarrow \theta = 0 \quad \& \quad x = 4 \rightarrow \theta = \frac{\pi}{2}$$

$$A = 4 \times \frac{3}{4} \int_0^{\frac{\pi}{2}} \sqrt{16 - 16 \sin^2 \theta} \cdot 4 \cos \theta \, d\theta$$

$$= 3 \int_0^{\frac{\pi}{2}} 4 \cos \theta \cdot 4 \cos \theta \, d\theta$$

$$= 3 \times 16 \int_0^{\frac{\pi}{2}} \cos^2 \theta \, d\theta$$

$$= 3 \times 16 \times \int_0^{\frac{\pi}{2}} \frac{(1 + \cos 2\theta) d\theta}{2}$$

$$= \frac{3 \times 16}{2} \left[\theta + \frac{\sin 2\theta}{2} \right]_0^{\frac{\pi}{2}}$$

$$= 3 \times 8 \left(\frac{\pi}{2} + 0 \right)$$

$$= 24 \times \frac{\pi}{2}$$

$$= 12\pi \text{ sq. units}$$

\therefore Area Bounded By the Curve is 12π sq. unit

7) Prove that the line joining the points $(1, 2, 3)$ and $(-1, -2, -3)$ is perpendicular to the line joining the points $(-2, 1, 5)$ and $(3, 3, 2)$.

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The direction ratios of the line joining the points $(1, 2, 3)$ and $(-1, -2, -3)$ are $-2, -4, -6$.

The direction ratios of the line joining the points $(-2, 1, 5)$ and $(3, 3, 2)$ are $3 - (-2), 1 - 5, 2 - 5$,
i.e. $5, -4, -3$

Now,

$$-2(5) - 4(-4) - 6(-3) = 0$$

$$\text{or, } -10 + 16 + 18 = 0$$

So, the lines joining the given points are perpendicular to each other.

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8) Maximize and minimize $F = 9x + 7y$ subject to $x + 2y \leq 7$,
 $x - y \leq 4$, $x \geq 0$, $y \geq 0$.

➤ Solution,

When $x = 0, y = 0$

$0 + 2 \cdot 0 \leq 7 \Rightarrow 0 \leq 7$ (true) so, it is directed towards origin.

The boundary lines of the given inequalities are

$$x + 2y = 7 \quad \dots\dots\dots (i)$$

$$x - y = 4 \quad \dots\dots\dots (ii)$$

From equation (i)

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$x + 2y = 7$	
x	y
0	$\frac{7}{2}$
7	0

So, $C\left(0, \frac{7}{2}\right)$ & $(7, 0)$ are passes through equation (i)

From equation (ii),

When, $x = 0, y = 0$

$0 \leq 4$ (true) so, it is directed towards origin.

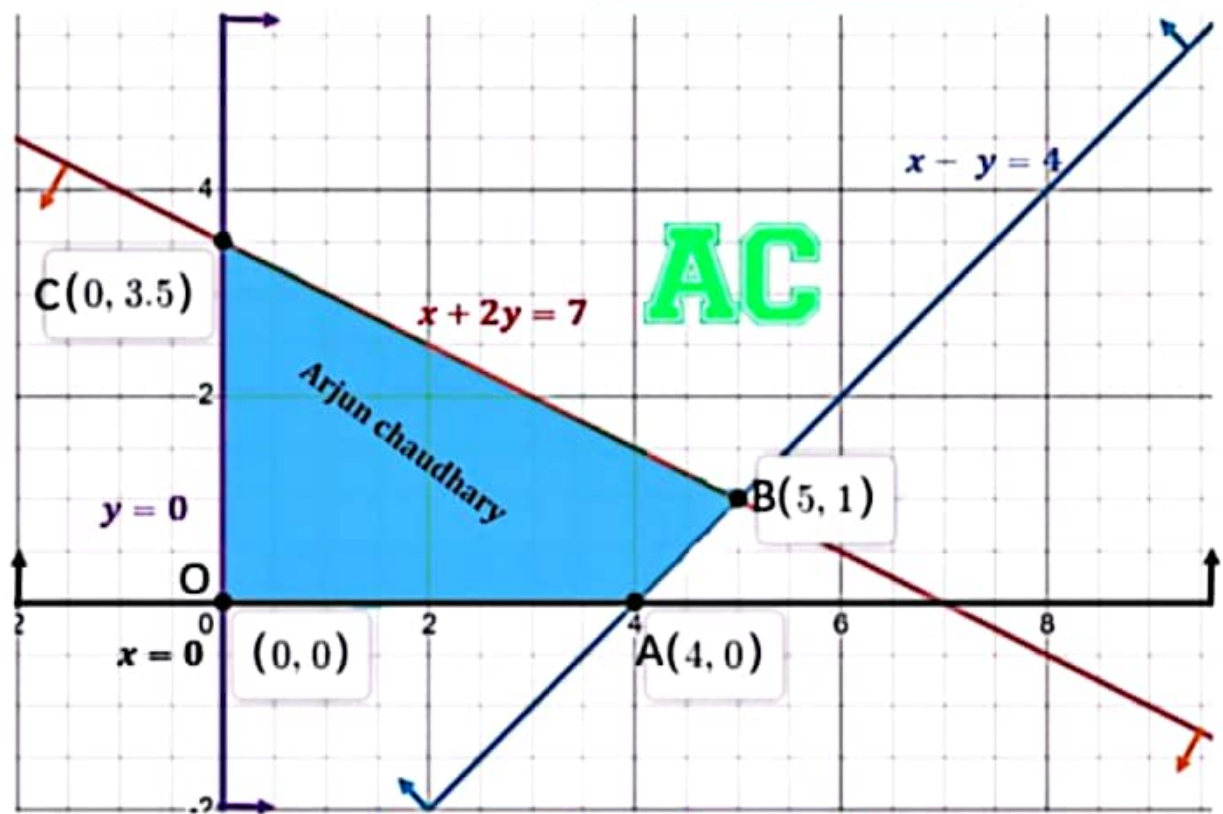
$x - y = 4$	
x	y
0	4
4	0

So, $(0, 4)$ & $A(4, 0)$ are passes through equation (ii)

When, $x = 0, y = 0$

$0 \leq 6$ (true) so, it is directed towards origin.

Also, From graph the vertices B are calculated by solving equation



For Vertices B,

Solving equation (i) and (ii)

We get, $B(5, 1)$

Hence, The Vertices of ABCD are $O(0, 0)$, $A(4, 0)$, $B(5, 1)$ and $C(0, 3.5)$.

Vertices	$F = 9x + 7y$
$O(0, 0)$	$9 \times 0 + 7 \times 0 = 0$ (MIN)
$A(4, 0)$	$9 \times 4 + 7 \times 0 = 36$
$B(5, 1)$	$9 \times 5 + 7 \times 1 = 52$ (MAX)
$C(0, 3.5)$	$9 \times 0 + 7 \times 3.5 = 24.5$

Hence, Maximum Value = 52 at $C(5, 1)$

Minimum Value = 0 at $O(0, 0)$

9) A committee of 3 is to be formed from 6 men and 4 women.
What is the probability that all three are women?

➤ **Solution,**

The selection of member in the committee can made as follows.

men(6)	women (4)	Selection of 3
0	3	$C(6,0) \times C(4,3)$

The Probability that all three are women.

$$= C(6,0) \times C(4,3)$$

$$= \frac{6!}{6!0!} \times \frac{4!}{1!3!}$$

$$= 1 \times 4$$

$$= 4$$

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10) A company that produces 10% of its products are defective among such 6 products find the probability that one is defective.

➤ **Solution**

$$P = \text{Prob. of getting defective product} = \frac{10}{100} = \frac{1}{10}$$

$$q = \text{Prob. of getting tail} = 1 - p = 1 - \frac{1}{10} = \frac{9}{10}$$

$$n = 6$$

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we have,

$P(r) = \text{Prob. of getting } r \text{ defective product in } n \text{ products}$

$$P(r) = C(n, r) p^r q^{n-r} \dots \dots \dots (i)$$

To find : probability that one is defective, $r = 1$

From (i)

$$P(r = 1) = C(6,1) p^1 q^{6-1}$$

$$P(r = 1) = 6 \times \left(\frac{1}{10}\right)^1 \cdot \left(\frac{9}{10}\right)^5$$

$$P(r = 1) = 0.3543 = 35.43\%$$

∴ Probability that one is defective = 35.43%

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11) Calculate the quartile deviation (Q.D.) and its coefficient from the following data :

Class	10-15	15-20	20-25	25-30	30-35	35-40	40-45
Frequency	2	4	6	7	3	1	5

➤Solution

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Class	Frequency (F)	C.F
10-15	2	2
15-20	4	6
20-25	6	12
25-30	7	19
30-35	3	22
35-40	1	23
45-45	5	28

$$N = 28$$

For Q_1 :

$$Q_1 = \left(\frac{N}{4}\right)^{th} \text{ item} = \frac{28}{4} = 7^{th} \text{ item}$$

C.f is just greater than 7 and is 12. So, it lies between 20 – 25.

$$L = 20, F = 6, C.f = 6, i = 5$$

$$Q_1 = L + \frac{\frac{N}{4} - C.f}{f} \times i$$

$$= 20 + \frac{7 - 6}{6} \times 5$$

$$= 20 + 0.83$$

$$Q_1 = 20.83$$

For Q_3 :

$$Q_3 = 3\left(\frac{N}{4}\right)^{th} \text{ item} = 3 \times 7 = 21^{th} \text{ item}$$

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C.f is just greater than 21 and is 22. So, it lies between 30-35.

$$L = 30, f = 3, C.f = 19, i = 5$$

$$Q_3 = L + \frac{\frac{3N}{4} - C.f}{f} \times i$$

$$= 30 + \frac{21 - 19}{3} 5$$

$$= 30 + 3.33$$

$$Q_3 = 33.33$$

$$\begin{aligned} \text{Coefficient of Q. D} &= \frac{Q_3 - Q_1}{Q_3 + Q_1} \\ &= \frac{33.33 - 20.83}{33.33 + 20.836} \\ &= \frac{12.47}{54.16} \end{aligned}$$

$$\therefore \text{Coefficient of Q. D} = 0.230$$

12) Find the regression equation of y on x from the following data Also, estimate the value of y when $x = 5$.

X	2	4	6	8	10	12
Y	5	6	13	16	23	24

➤ **Solution,**

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x	y	x^2	$x y$
2	5	4	10
4	6	16	24
6	13	36	78
8	16	64	128
10	13	100	130
12	24	144	288
$\sum x = 42$	$\sum y = 77$	$\sum x^2 = 364$	$\sum xy = 658$

Here, $n = 6$

$$\bar{x} = \frac{\sum x}{n} = \frac{42}{6} = 7, \bar{y} = \frac{\sum y}{n} = \frac{77}{6} = 12.83$$

$$b_{yx} = \frac{n \sum xy - \sum x \sum y}{n \sum x^2 - (\sum x)^2} = \frac{6 \times 658 - 42 \times 77}{6 \times 364 - (42)^2} = \frac{714}{420} = 1.7$$

Hence, the regression y on x is ,

$$y - \bar{y} = b_{yx} (x - \bar{x})$$

$$\text{or, } y - 12.83 = 1.7 (x - 7)$$

$$\text{or, } y - 12.83 = 1.7 x - 11.9$$

Therefore, $y = 1.7 x + 0.93$

∴ The regression Equation y on x is , $y = 1.7 x + 0.93$

$$\text{When } x = 5 \text{ then, } y = 1.7 \times 5 + 0.93 = 9.43$$

13) Show that the vectors: $\vec{i} + \vec{j} + \vec{k}$, $2\vec{i} + 3\vec{j} - \vec{k}$, $-\vec{i} - 2\vec{j} + 2\vec{k}$ are co-planar.

➤ **Solution**

$$\vec{r}_1 = \vec{i} + \vec{j} + \vec{k}$$

$$\vec{r}_2 = 2\vec{i} + 3\vec{j} - \vec{k}$$

$$\vec{r}_3 = -\vec{i} - 2\vec{j} + 2\vec{k}$$

To show: \vec{r}_1, \vec{r}_2 and \vec{r}_3 are coplanar

➤ If three vector are coplanar, one can expressed as the sum of the scalar multiples of the other two.

$$\vec{r}_3 = x\vec{r}_1 + y\vec{r}_2 \quad \text{where, } x \text{ and } y \text{ are scalars}$$

or, $-\vec{i} - 2\vec{j} + 2\vec{k} = x(\vec{i} + \vec{j} + \vec{k}) + y(2\vec{i} + 3\vec{j} - \vec{k})$

$$-\vec{i} - 2\vec{j} + 2\vec{k} = (x + 2y)\vec{i} + (x + 3y)\vec{j} + (x - y)\vec{k}$$

Equating Coefficient of likes vectors,

$$(x + 2y) = -1 \quad \dots\dots\dots (i)$$

$$x + 3y = -2 \quad \dots\dots\dots (ii)$$

$$x - y = 2 \quad \dots\dots\dots (iii)$$

Solving eqn (i) and (ii) we get, $x = 1$ and $y = -1$

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Put, $x = 1$ and $y = -1$ in eqn (iii)

$$1 \times 1 - 1 \times -1 = 2$$

$$2 = 2, \text{ satisfied.}$$

Hence, Three vector are coplanar and its can expressed as

$$\vec{r}_3 = \vec{r}_1 - \vec{r}_2$$